Technical Report

Telemetry Antenna for Lincoln Experimental Satellites LES-1 and LES-2

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TELEMETRY ANTENNA FOR LINCOLN EXPERIMENTAL SATELLITES LES-1 AND LES-2

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ABSTRACT

The telemetry antenna used on the first two Lincoln Experimental Satellites consists of four short stubs equally spaced around, and parallel to, the spin axis of the satellite. A detailed description of the antenna and its transmission-line system is presented. Theoretical and model studies leading to the design of this antenna are discussed. Calculated and measured performance data are presented and compared.

Accepted for the Air Force
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Lt Colonel, USAF
Chief, Lincoln Laboratory Office
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Telemetry Antenna</td>
<td>1</td>
</tr>
<tr>
<td>III. Theoretical Study</td>
<td>2</td>
</tr>
<tr>
<td>IV. Model Study</td>
<td>6</td>
</tr>
<tr>
<td>V. Conclusions</td>
<td>8</td>
</tr>
<tr>
<td>Acknowledgment</td>
<td>9</td>
</tr>
<tr>
<td>References</td>
<td>9</td>
</tr>
<tr>
<td>Appendix – Computer Programs</td>
<td>25</td>
</tr>
</tbody>
</table>
TELEMETRY ANTENNA FOR LINCOLN EXPERIMENTAL SATELLITES
LES-1 AND LES-2

I. INTRODUCTION

Major goals in the design and fabrication of the first Lincoln Experimental Satellite (LES) telemetry antenna were:

(a) Omnidirectional radiation patterns.
(b) Gain and efficiency high enough to provide more than the minimum effective radiated power required by the telemetry link.
(c) Minimum shadowing of the satellite's solar cells by the antenna elements.
(d) A mechanical configuration compatible with the launch package.
(e) Small-size and light-weight radiating elements, transmission lines, and impedance matching networks.
(f) Ability to withstand environmental extremes of temperature, radiation, acceleration, shock and vibration during launch and in orbit.

Several possible antenna configurations were considered, and some were tried experimentally before the present design was adopted. The size and shape of the satellite and its launch shroud imposed severe limitations on the types of antennas that could be used. Both slot and stub radiating elements were considered, slot elements being particularly attractive because, being flush, they would not shadow the solar panels. However, they occupy area which could be covered with solar cells; for this reason, small-diameter stub elements were chosen. Tested on the satellite were 1-, 2-, 4- and 6-element configurations at the VHF telemetry frequency. Following is a description of the geometry finally selected.

II. TELEMETRY ANTENNA

The antenna consists of four $\frac{1}{2}$-wavelength ($\lambda$) unipoles mounted on the lower corner of the triangular panels in the upper hemisphere of the satellite (Fig. 1). These elements are fed in phase rotation; that is, the phase of the excitation voltage to each element is delayed an amount equal to its angular displacement from a reference element. Delay is provided by a coaxial transmission-line feed harness inside the satellite. The coaxial line is the semi-rigid type with a helical-cut dielectric. Each line was measured electrically to accomplish the phasing; impedance transformers were inserted at the junction of the lines to each pair of antennas to maintain a constant impedance throughout the phasing network. Each 7-inch-long stub antenna element was matched to 50 ohms by placing a shorted shunt stub across the coaxial feed line. The polarization of the present 4-element system is invariant with rotation of the satellite because the radiators are placed symmetrically about, and parallel with, the spin axis of the satellite.
III. THEORETICAL STUDY

An investigation of radiating slotted spheres, for another application, was in progress during the development of the LES telemetry antenna. Since a small annular slot on a sphere was thought to be a good approximation to a short stub on a regular polyhedron, the mathematical model of the slotted sphere was used to calculate the telemetry antenna radiation characteristics. First, the radiation pattern of a single, small annular slot on an equivalent sphere was calculated. The geometry and coordinate system are shown in Fig. 2.

Radius (a) of the equivalent sphere is the mean of the maximum and minimum distance from the center of the 26-side polyhedron to its surface. A zonal slot is formed by the intersection of the sphere and the two cones defined by $\theta_1 = \alpha$. Therefore, the slot width is $2\alpha$.

With uniform slot excitation, the radiation field is a function of $\theta$ only. Relative field strength is given by

$$E = \sum_{n=1}^{\infty} \frac{j^n(2n+1) ka}{4\alpha n(n+1) [k ah_n^2(ka)]'} \frac{dP_n^{(1)}(\cos \theta)}{d\theta} \int_{\theta_1-\alpha}^{\theta_1+\alpha} \frac{dP_n(\cos \theta)}{d\theta} \sin \theta \ d\theta$$

(1)

where $h_n^2$ is the spherical Hankel function of the second kind; $[k ah_n^2(ka)]'$ is the first derivative of the product of this function and its argument, with respect to the argument; $k$ is the phase constant $2\pi/\lambda$; $P_n^{(1)}(\cos \theta)$ is the zero-degree associated Legendre function of the first kind; and $j = \sqrt{-1}$.

Equation (1) represents a solution to Maxwell's equations as a series of spherical harmonics. The method of derivation is due to Stratton and Chu of M.I.T. Convergence of the series is rapid, and less than ten terms are needed for practical values of the relative field at any value of $\theta$. This field is, of course, constant with $\varphi$. In general, $E$ is a complex quantity (having a real and an imaginary part) at each point in the space around the sphere, because the Hankel function is complex. Equation (1) was programmed for solution by the IBM 7094 computer.

Figure 3 is a plot of the relative magnitude of $E$ vs $\theta$ for a single small slot ($\theta_1 = 5^\circ$, $\alpha = 1^\circ$) on a sphere ($ka = 1.62$) equivalent to a single stub on the polyhedron satellite body. For comparison, it is superimposed on a plot of the measured relative field of a single short stub on a test model of the satellite at 240 MHz. Agreement is excellent.

Figure 4 shows the calculated and measured radiation patterns of two elements which are on opposite sides of the sphere and the polyhedron, respectively. As before, the elements on the sphere are small annular slots, while those on the polyhedron are short stubs fed in phase opposition at 240 MHz. Agreement between the two patterns is good, with differences being attributed to imperfect excitation of the test model as evidenced by the asymmetrical measured pattern.

Results similar to those shown in Figs. 3 and 4 confirmed the validity of the assumptions on which the theoretical study was based. The investigation continued with the result that four equally spaced elements on a great circle of the sphere were found to be inadequate to meet the requirements, while six elements would be satisfactory. A computer program was formulated that would calculate the radiation pattern of any number of elements, equally spaced on a great circle of a sphere. This was combined with a random number program to introduce random errors in the phase and amplitude of the assumed excitation. These programs are presented in the Appendix.
Six elements, equally spaced on a great circle of a sphere for which $ka = 1.62$, were calculated to produce a radiation pattern omnidirectional to within $\pm 0.3 \text{ db}$ in the plane of the elements. The corresponding measured pattern was omnidirectional to within $\pm 1.3 \text{ db}$. Again the difference was attributed to errors in excitation of the test model, but now this theory could be tested by introducing assumed limits to phase and amplitude errors into the computer program. This would then rapidly calculate patterns resulting from a random distribution of these errors. From several hundred patterns calculated this way, the average was found to be omnidirectional within the limits shown in Table I. Listed are the assumed limits in the standard deviation in phase and magnitude of the excitation voltages; the phase was considered to be off by no more than $10^\circ$, and the magnitude by no more than 20 percent. Opposite each set of error limits is the resulting range of departure from an omnidirectional pattern and the mean of the pattern.

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>± Departure from Omnidirectional</th>
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<tbody>
<tr>
<td>Phase (deg)</td>
<td>Magnitude (percent)</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Range (db)</td>
<td>Mean (db)</td>
</tr>
<tr>
<td>1.0 to 4.5</td>
<td>3.3</td>
</tr>
<tr>
<td>0.7 to 3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1.6 to 3.3</td>
<td>2.5</td>
</tr>
<tr>
<td>0.8 to 1.8</td>
<td>1.3</td>
</tr>
</tbody>
</table>

A magnitude error of 10 percent is quite likely, since the voltage-standing-wave ratio (VSWR) measured at a typical element is about 1.1 after careful adjustment of the matching network. Similarly, phase errors are likely to be quite small, for transmission line lengths were measured and equalized electrically. Therefore, the last case listed in Table I is most likely, and the mean departure from omnidirectional does correspond with the measured value.

The foregoing was cited mainly to show the usefulness of this type computer program. It can be used either to determine the effect of random errors or to establish tolerances for particular parameters. To do this experimentally would be impractical because of the time and labor required to make all the incremental changes and to measure the results.

Each element of the proposed 6-element antenna would be limited in length to $\frac{1}{2}$ inches because of mechanical interference considerations. The wavelength is about 50 inches, and a $\frac{1}{2}$-inch stub is too short (relative to the wavelength) to be an efficient radiator. Such a short element would be very difficult to feed, since the required impedance matching network would be lossy, narrow band, and sensitive to environmental changes.

The chance of mechanical interference would be reduced and the elements could be made longer by moving them close to the top of the satellite. Since they would also be closer to one another, fewer elements might be satisfactory. The computer program was modified to permit calculation of radiation patterns when the elements were not located on a great circle of the sphere but on a circle of any latitude.
When calculating patterns in a plane other than that containing the radiating elements, the orthogonally polarized field components must be determined at each point of observation. Thus, for the chosen spherical coordinate system, the component field in the $\Theta$ direction ($E_{\Theta}$) and that in the $\varphi$ direction ($E_{\varphi}$) would be the logical choice. These components are readily found from Eq. (1) by a transformation of the coordinate system which allows an element to be located anywhere on the sphere instead of being limited to the position shown in Fig. 2.

Geometry of the transformation is shown in Fig. 5. Note that $\Theta'$, which is measured from the center of the annular-slot element to any point $P(\Theta, \varphi)$, corresponds to $\Theta$ in Eq. (1). This change in notation permits use of the unprimed coordinate ($\Theta$) in the new, transformed coordinate system which applies to the more general case where the slot can be located anywhere on the sphere. The coordinates of the center of the slot are $\Theta_c$ and $\varphi_c$, as indicated in Fig. 5. From this figure, the following equations are obtained by spherical trigonometry.

$$\Theta' = \arccos \left[ \cos \Theta_c \cos \Theta + \sin \Theta_c \sin \Theta \cos (\varphi_c - \varphi) \right]$$

$$\varphi = \arcsin \frac{\sin \Theta_c \sin (\varphi_c - \varphi)}{\sin \Theta'}$$

or

$$\varphi = \arccos \frac{\cos \Theta_c - \cos \Theta \cos \Theta'}{\sin \Theta \sin \Theta'}$$

Having found the angle $\varphi$ from Eqs. (3) or (4), we may split the known relative field $E(\Theta')$ from Eq. (1) into orthogonal components; thus,

$$E(\Theta, \varphi) = (i_\Theta \cos \varphi \pm i_\varphi \sin \varphi) E(\Theta')$$

where $0 \leq \Theta' \leq 180^\circ$ and $i$ is the unit vector. When $180^\circ < \varphi_c - \varphi < 360^\circ$, use the plus sign; when $0^\circ < \varphi_c - \varphi < 180^\circ$, use the minus sign. For the special case of $\varphi_c = \varphi$,

$$E(\Theta, \varphi) = E(\Theta') \quad \text{when } \Theta > \Theta_c$$

$$E(\Theta, \varphi) = -E(\Theta') \quad \text{when } \Theta < \Theta_c$$

Equations (2) and (4) through (7) were programmed for the computer (see Appendix). Equation (4) is preferred over Eq. (3) for the computer, since it gives unambiguous principal values of the angle $\varphi$. This modified program permits calculation of the relative field at any point in the space around a sphere, with any number of radiators located anywhere on the sphere and fed with voltages of any phase and magnitude. It was made general for possible future use on other array configurations.

The configuration considered most practical for LES telemetry application, at this time, was the 4-element array finally adopted and described in Sec. II above. Four small annular slots, equally spaced around a sphere with $ka = 1.62$ and $\Theta_c = 64.5^\circ$, constitute the equivalent antenna. One of the elements is taken to be the reference element at $\varphi = 0$; the elements are fed in phase rotation, with the excitation voltage to each delayed an amount equal to the element's $\varphi$-coordinate.

In general, the resultant field is elliptically polarized; but, with perfect excitation, it is circularly polarized at $\Theta = 0^\circ$ and $180^\circ$. Basically, the configuration is analogous to a turnstile antenna. However, the performance is modified by the conducting sphere which is equivalent to the polyhedron satellite body.
In the equatorial plane ($\theta = 90^\circ$), the principal polarization ($E_\varphi$ as a function of $\varphi$) calculates to be omnidirectional to within $\pm 0.6$ db. $E_\varphi (\varphi)$ is not so omnidirectional, deviating by $\pm 4.3$ db. Patterns presented in Figs. 6(a) through (q) were calculated and plotted by the computer for constant values of $\theta$ in $10^\circ$ steps from $10^\circ$ through $170^\circ$. On Fig. 6(a) is a sketch defining the coordinate system; Fig. 6(i) is the plot for the equatorial plane ($\theta = 90^\circ$) from which the values cited previously were taken. At $\theta = 0^\circ$ and $180^\circ$, orthogonal field components are equal in magnitude and constant with $\varphi$.

In addition to the magnitudes, the computer gives the phase of each component at each calculated point so that the resultant field can be found, if desired. Since the intention is to use linearly polarized receiving antennas at the ground telemetry terminal, variation of the linearly polarized components around the spin axis of the satellite is important. "Conical-cut" patterns, such as those presented in Figs. 6(a) through (q), indicate the variation that can be expected in the signal received by horizontally and vertically polarized antennas at any viewing angle as the satellite spins on its axis. Patterns are rotationally symmetric and repeat every $90^\circ$ in $\varphi$; therefore, only one quadrant is presented in each figure. Reference level is the maximum which occurs at $\theta = 180^\circ$, as shown by patterns calculated in planes of constant $\varphi$. These pattern cuts, through the spin axis, are shown in Figs. 7(a) through (c). Patterns in Fig. 7(a) are typical of the radiation in planes containing two elements and the spin axis, while those in Fig. 7(b) are for $\varphi = 45^\circ$, the plane exactly between two elements. Minimum $E_\theta$ does not occur in this plane, however, but in the $\varphi = 35^\circ$ plane at $\theta = 80^\circ$ [see Fig. 6(h)]. Patterns are shown for $\varphi = 35^\circ$ in Fig. 7(c) as they emphasize an interesting phenomenon. As careful examination of the conical pattern cuts reveals [Figs. 6(a) through (q)], radiation is not symmetrical about the planes containing the antenna elements. Therefore, in general, maxima of $E_\theta$ do not occur in the planes of the elements and minima do not occur in the planes exactly between elements, as might be expected.

Radiation pattern information is summarized in Figs. 8 and 9 for use in estimating the effect of radiation characteristics on the performance of the telemetry link. Figure 8 is the ratio between the maximum and the minimum field, for each polarization, as a function of $\theta$. If, for example, the angle between the ground station and the spin axis is $80^\circ$, Fig. 8 shows that $E_\varphi$ would vary as much as $9.1$ db as the satellite rotates about its axis, while $E_\theta$ would vary less than $1.6$ db. Figure 9 shows that the average power level would be about $-6.5$ db for $E_\varphi$ and $-4.2$ db for $E_\theta$, with the field at $\theta = 180^\circ$ as the reference. Actually, Fig. 9 gives the average of maximum and minimum fields which (in db) is close to the average power level since the variation is almost sinusoidal. In the practical telemetry circuit, the variation observed would depend also on the method of detection.

Sufficient information is contained in the patterns to calculate the approximate directivity of the antenna by point-to-point integration. Thus, the maximum pattern directivity over isotropic for each field ($E_\varphi$ or $E_\theta$) is given by

$$D = \frac{E_{\max}^2 \sum \sin \theta}{\sum E^2 \sin \theta}$$

with the summation carried out over all of the area around the antenna for which the patterns were calculated, taking advantage of symmetry to reduce the number of points. The equation
is an approximation because a finite number of points are used; the larger the number of points and the closer together they are, the more accurate the value for the directivity. $E_{\text{max}}$ is the magnitude of the largest field at any point in the pattern. For a pattern normalized to the maximum, it becomes unity, of course. $E$ is the magnitude of either the $\phi$ or $\Theta$ field (depending on the field for which the directivity is being calculated) at each point $(\Theta, \phi)$.

The computer was programmed (see Appendix) to store all the field information and use it to solve Eq. (8) for the maximum directivity at the reference $(\Theta = 180^\circ)$. For $E_{\Theta}$, directivity is 3.86 times that of an isotropic antenna; for $E_{\phi}$, it is 2.54. As stated before, this is the radiation-pattern directivity which is a function only of the shape of each three-dimensional radiation field. At $\Theta = 180^\circ$, the resultant ideal field is known to be circularly polarized, as computed values of $E_{\Theta}$ and $E_{\phi}$ are equal in magnitude and in phase quadrature. This means that the power must be divided between the two fields in inverse proportions to their directivities, and the directivity of each linearly polarized component field is given by

$$D_L = \frac{D_{\Theta}D_{\phi}}{D_{\Theta} + D_{\phi}}.$$  (9)

For the specific case under consideration,

$$D_L = \frac{3.86 \times 2.54}{3.86 + 2.54} = 1.53 = 1.84 \text{ db}$$

and the directivity of the resultant circularly polarized field is twice that of each component, or 4.85 db.

The gain of the test model antenna was measured at $\Theta = 90^\circ$ and $\phi = -17^\circ$, as this was a more convenient reference for the test setup than $\Theta = 180^\circ$. Translating this to $\Theta = 180^\circ$ by means of measured radiation patterns, the gain to linear polarization is found to be about 2 db above isotropic for one field and 1 db below isotropic for the orthogonal field. (Due to imperfect excitation, the experimental field is not circularly polarized.) From this, the gain of the resultant elliptically polarized field is calculated to be 3.76 db. Since the difference between directivity and gain is the loss in the antenna radiators and transmission-line system, this loss calculates to be about $4.84 - 3.76 = 1.08$ db, which is reasonable.

With the theoretical maximum directivity of each linearly polarized component field now known, the directivity at any point in space can be read, or interpolated, from the patterns shown in Figs. 6 and 7. Adding 1.84 db to any value found from these plots gives the directivity over isotropic at that point.

IV. MODEL STUDY

As indicated in Sec. III above, the first design consisted of six elements on a great circle through the satellite. The elements were restricted to $1\frac{1}{2}$ inches in length in order not to shadow the solar panels. This system accomplished the desired result in that there was radiation at all angles from the satellite in some polarization; however, the system had two basic faults. First, the polarization varied as the satellite was rotated about the spin axis, i.e., at 180 rpm, while the ground station finally chosen consisted of two linearly polarized receiving antennas either one of which, but not both, could be receiving at the same time. Second, and greater, the impedance matching device on the antenna could not maintain a match over the temperature changes of its environment. The theoretical impedance and some measured data for various
length antennas are shown in Fig. 10. The calculated VSWR of the 1\frac{1}{2}-inch stub is about 2000:1, but the antenna was matched using a shorted shunted stub approximately 6 inches from the feed point. However, the positioning of this device was so critical that a 50°F change in temperature resulted in a change in VSWR from 1.0 to 1.5. The antenna must work over at least a 100°F range; therefore, the longer antenna shown in Fig. 1 was tested. Its input impedance is approximately 9–j104 ohms and the corresponding VSWR is about 70:1. This impedance was reduced to 50 ohms by inserting a tee in the coaxial line (Fig. 11) with an adjustable short circuit on one arm. The VSWR of each antenna was reduced to approximately 1.1:1. It was physically impossible to measure the impedance of one antenna with the other antennas excited, but the input VSWR of the feed harness terminated in the four matched elements is about 1.15:1 (average of five harnesses).

This feed harness was composed of electrically measured lengths of coaxial line and impedance transformers (Fig. 12) so that there was a successive 90° delay between the four stubs and a roughly 50-ohm impedance at the input. Since the phase delay was inserted by varying cable length, it was necessary to find a coaxial line which would retain phase stability over a large temperature range. This line would naturally change phase with temperature; however, it should not change electrical length permanently. Two types of cable were temperature cycled: RG-188/U, a Teflon-insulated flexible cable; and a 0.161 semi-rigid coaxial line with a helical cut irradiated polyolphin dielectric. These cables were alternated between liquid nitrogen and boiling water five times, and the Teflon cable was found to have changed electrical length permanently by approximately 2 percent while the helical cut dielectric cable change was unmeasurable.

TM connectors were chosen because they mechanically clamp to the semi-rigid coaxial cable. This design permitted the use of aluminum connectors, thus reducing weight. The weight of the entire harness and antennas is $3\frac{1}{2}$ ounces. At each tee, a section of 35-ohm coaxial transmission line was inserted to match the junction to 50 ohms. This cable was of the same design as the 50-ohm cable except for inner-conductor size. The 50-ohm connectors were used on this 35-ohm line. Each transformer was measured and cut electrically so that it and the tee, and two 50-ohm loads, presented an input VSWR of 1.05 or less. Each cable electrical length was also measured in an attempt to keep the misphasing under 2° or about \frac{1}{2} cm in cable.

Antenna patterns were measured using a sheet-metal model of the satellite with the stub antennas mounted on the triangular panels and the feed harness inside. The test model was then mounted about 9 feet above the ground on an approximately $8 \times 3 \times 1$-foot styrofoam pillar (Fig. 13). A single cable was run down the center of the pillar through a rotary joint to the receiver. Antenna patterns were measured using two orthogonal linearly polarized transmitted signals (vertical and horizontal). The satellite and transmitter were placed outside over relatively flat ground. At this frequency there are problems with reflections which must be tolerated as no anechoic chamber is available. Tests were made to determine the effect of the reflections by changing the distance between the transmitter and receiver in increments of 1 foot for about 5 steps (i.e., from 26 to 30 feet). The depth of nulls was affected but, generally speaking, the basic patterns did not change for either polarization. Most of the patterns were taken with the satellite in the position shown in Fig. 13; that is, the satellite was rotated in a constant $\Theta$-plane, varying $\varphi$. A typical pattern is shown in Fig. 14. For the conical cuts, the transmitting antenna was elevated and the satellite was in the position shown in Fig. 13, then rotated 180° on the styrofoam resulting in two cuts equi-angle from the equator. These are shown in Figs. 15(a)
through (f). Several patterns were taken varying θ, with φ held constant. It is almost impossible to physically mount the satellite in those positions corresponding to the theoretical patterns. This, in part, can explain the difference in results obtained.

Three flight models of the LES were moved to the Antenna Test Range for pattern and effective radiated power measurements (Fig. 3). The telemetry transmitter was operating (∼500-mw output) and the signal was received on two orthogonal linear polarizations. Figure 14 shows the effective radiated power and equatorial pattern of the LES-2 package. The gain calculated from this measurement is about 0.6 db below isotropic at θ = 90°. The measurements on the other flight items provided approximately the same results with gain at θ = 90° near isotropic.

On 11 February 1965, the first satellite (LES-1) was launched from Cape Kennedy. It went into a circular parking orbit with the third stage of the Titan III A launch vehicle, as planned. A rocket package, which was supposed to inject the satellite into an elliptical orbit, failed to fire and to separate itself from the satellite. Therefore, the satellite with the injection package attached is apparently tumbling as well as spinning about its axis. This seems evident from the nature of the signals received. Indications are that the telemetry antenna survived the launch and is operating as expected, even though the radiation characteristics are altered by the presence of the injection package.

On 6 May 1965, LES-2 was launched from Cape Kennedy on a Titan III A and this time it went into the planned elliptical orbit. All systems are operating, and performance of the telemetry antenna is satisfactory.

V. CONCLUSIONS

A practical VHF telemetry antenna has been developed for the Lincoln Experimental Satellite. It consists of four short-stub antenna elements mounted on triangular panels of the satellite and fed in phase rotation. All the performance criteria were met, and operation in the space environment is satisfactory.

There is reasonable agreement between theoretical and measured radiation characteristics. These show that, at any point in space around the satellite, the effective radiated power in one of two orthogonal linearly polarized fields is always large enough to be utilized by the ground station, which employs a polarization diversity antenna system.

Theoretically, maximum signal occurs at θ = 180° where the field is circularly polarized. Directivity, at this coordinate, was computed to be 4.84 db. Model measurements confirm that maximum field occurs off the bottom of the satellite, but the field is elliptically polarized because of imperfect excitation of the antenna elements. Gain, to matched polarization, was calculated (from measured data) to be 3.76 db. The difference, 1.08 db, is attributed to losses in the elements and in the transmission-line system.

All components were designed to survive the environmental extremes imposed by the launch into orbit, and to operate in space. After thorough testing on the ground, two satellites were placed in orbit where satisfactory operation has proven the design.
ACKNOWLEDGMENT

The authors thank Mr. L. J. Ricardi for his invaluable technical advice, Mr. R. J. Peck for expertly programming the computer, Mr. W. D. Casey for so ably testing the antenna models, and the personnel of Group 71 for their mechanical design of the flight models.

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Fig. 1. First Lincoln Experimental Satellite.

Fig. 2. Slotted-sphere geometry and coordinate system.
Fig. 3. Calculated relative field of single small annular slot on an equivalent sphere, and measured relative field of single short stub on 26-side polyhedron.

Fig. 4. Calculated and measured radiation patterns of two elements in space and phase opposition.
Fig. 5. Transformation geometry.

Fig. 6. LES telemetry antenna, calculated radiation patterns as a function of $\phi$.

(a) $\theta = 10^\circ$.
(b) $\theta = 20^\circ$. 
(c) $\theta = 30^\circ$.

(d) $\theta = 40^\circ$.

(e) $\theta = 50^\circ$.

Fig. 6. Continued.
Fig. 6. Continued.

(f) $\theta = 60^\circ$.

(g) $\theta = 70^\circ$.

(h) $\theta = 80^\circ$. 
(i) $\theta = 90^\circ$.

(j) $\theta = 100^\circ$.

(k) $\theta = 110^\circ$.  

Fig. 6. Continued.
(l) $\theta = 120^\circ$.

(m) $\theta = 130^\circ$.

(n) $\theta = 140^\circ$.

Fig. 6. Continued.
(o) $\theta = 150^\circ$.

(p) $\theta = 160^\circ$.

(q) $\theta = 170^\circ$.

Fig. 6. Continued.
Fig. 7. LES telemetry antenna, calculated radiation patterns as a function of $\theta$. 

(a) $\phi = 0^\circ$.

(b) $\phi = 45^\circ$.

(c) $\phi = 35^\circ$. 

18
Fig. 8. Ratio of maximum to minimum field.

Fig. 9. Average of maximum and minimum field.
Fig. 10. Theoretical and measured impedance of a single monopole, varying length.

Fig. 11. Tee matching device (scale 2X).
Fig. 12. Schematic of feed harness.

Fig. 13. LES mounted for antenna pattern measurement.

Fig. 14. Typical equatorial plane antenna pattern.
Fig. 15. Antenna patterns as a function of $\phi$ for various values of $\theta$. 

(a) $\theta = 50^\circ$. 

(b) $\theta = 60^\circ$. 

Fig. 15. Antenna patterns as a function of $\phi$ for various values of $\theta$. 
(c) $\theta = 70^\circ$.

(d) $\theta = 110^\circ$.

Fig. 15. Continued.
(e) $\theta = 120^\circ$.

(f) $\theta = 130^\circ$.

Fig. 15. Continued.
APPENDIX
COMPUTER PROGRAMS

TABLE A-1
ELEMENT RADIATION FIELD

| C | E FOR THETA = 0 - 180 DEG. - CALC. REAL + J(IMAG), VMAG, PHASE, DB |
| C | INPUT ARG, THETA1, ALPHA, DTHERA, NL, THZ, DTH, NTH |
| C | ARG = KA, POSITIVE, LESS THAN 50., REAL ONLY |
| C | THETA1 = ANG. POSITIVE, DEGREES |
| C | ALPHA = ANG. POSITIVE, DEGREES |
| C | DTHERA = INTERVAL OF 2 PT. GAUSS INTEGRAL |
| C | NL = N LIMIT, POSITIVE INTEGER, LESS THAN 100 |
| C | THZ = ANG. IN FAR-FIELD, DEGREES |
| C | DTH = ANG. INTERVAL OF FAR-FIELD ANGLE |
| C | NTH = NO. OF POINTS IN PATTERN |

C OUTPUT ARG, THETA, REAL, VMAG, MAGNITUDE, PHASE, VOLTS, NORMAL, DB
C OUTPUT ON A-5 WILL BE PUNCHED (HOLLERITH). DATA WILL BE USED FOR
C PROGRAM THAT CALCULATES X NO. OF RADIATORS ON A SPHERE.
C DIMENSION THTBL(181), PI(2), SMREAL(181), SMIMAG(181), VMAG(181),
C 1PHASE(181), VLNORM(181), VLD(181), HEAD(12)

C TABLE OF CONSTANTS
B PI(1) = 202622077325
B PI(2) = 147042055061
C C1 = PI/180.

C READ IN HEADING CARD
10 READ INPUT TAPE 2,1,(HEAD(I), I = 1, 12)
C READ IN CONSTANTS FOR EACH CASE (ARG, THETA1, ALPHA, NL)
C READ INPUT TAPE 2,2, (ARG, THETA1, ALPHA, NL)
C READ IN DTHERA, (THZ, DTH, NTH)
C WRITE OUTPUT TAPE 3,1, (HEAD(I), I = 1, 12)
C WRITE OUTPUT TAPE 3,3, (ARG, THETA1, ALPHA, NL)
C WRITE OUTPUT TAPE 3,4, (DTHERA, THZ, DTH, NTH)

C LOOP FOR FAR-FIELD ANGLE, THETA
C THETA = THZ
DO 28 J = 1, NTH
THTBL(J) = THETA
C LOOP TO CALCULATE SUMMATION, N=1, NL
SUMRE = 0.
SUMIM = 0.
DO 16 N = 1, NL
C LOOP TO CALC. BSUBN(THETA1, ALPHA, DTHERA, N, BNANS)
CALL BSUBN(THETA1, ALPHA, DTHERA, N, BNANS)
BSUBNA = BNANS
ALPHAR = C1*ALPHA
BND2AR = BSUBNA/(2.*ALPHAR)
C CALC. 1ST DERIVITIVE OF X*H sub N*(X)
CALL DDXHAN(ARG, N, REAL, GAM1)
C CALC. D/DTH PSUB N(COS THETA)
CALL DPNCOS(THETA, N, ANS)
DPNANS = ANS
C ABA = ARG*BND2AR*DPNANS
DEN = REAL**2+GAM1**2
ABADEN = ABA/DEN

25
```plaintext
C TEST FOR VALUE OF (J) EXP N 
NN = XMODF(N,4) + 1 
GO TO (11,12,13,14)*NN 
C NUMERATOR IS REAL, POSITIVE 
11 TRMRE = REAL*ABADEN 
TRMIM = -GAMI*ABADEN 
GO TO 15 
C NUMERATOR IS IMAGINARY, POSITIVE 
12 TRMRE = GAMI*ABADEN 
TRMIM = REAL*ABADEN 
GO TO 15 
C NUMERATOR IS REAL, NEGATIVE 
13 TRMRE = -REAL*ABADEN 
TRMIM = GAMI*ABADEN 
GO TO 15 
C NUMERATOR IS IMAGINARY, NEGATIVE 
14 TRMRE = -GAMI*ABADEN 
TRMIM = -REAL*ABADEN 
15 SUMRE = SUMRE+TRMRE 
WRITE OUTPUT TAPE 3,5,THETA,N,TRMRE,TRMIM 
16 SUMIM = SUMIM+TRMIM 
SMREAL(J) = SUMRE 
SMIMAG(J) = SUMIM 
C FIND VOLT. MAGNITUDE AND NORMALIZE TO MAXIMUM VALUE 
C CALCULATE PHASE ANGLE AND Db VALUE 
VLMAG(J) = SQRTF(SUMRE**2+SUMIM**2) 
C CALC. PHASE ANGLE -PI/2 TO 3PI/2 
PHASE(J) = ATANF(SUMIM/SUMRE) 
IF(SUMRE)21,24,28 
21 IF(SUMIM)22,23,22 
22 PHASE(J) = PHASE(J)+PI 
GO TO 28 
23 PHASE(J) = PI 
GO TO 28 
24 IF(SUMIM)25,26,27 
25 PHASE(J) = -PI/2. 
GO TO 28 
26 PHASE(J) = 0. 
GO TO 28 
27 PHASE(J) = PI/2. 
28 THETA = THETA+DTH 
C FIND MAXIMUM VALUE OF VLMAG 
VLMAX = VLMAG(1) 
DO 30 J=2,NTH 
IF(VLMAG=VLMAG(J))29,29,30 
29 JMAX = J 
VLMAX = VLMAG(J) 
30 CONTINUE 
C NORMALIZE VLMAG VALUES TO VLMAX AND CALC. Db VALUES 
DO 31 J=1,NTH 
PHASE(J) = PHASE(J)/C1 
VLNORM(J) = VLMAG(J)/VLMAX 
31 VLDB(J) = 20.*LOG10F(VLNORM(J)) 
C WRITE OUTPUT VALUES 
WRITE OUTPUT TAPE 3,6,(THETABL(J),SMREAL(J),SMIMAG(J),VLMAG(J),PHASE 1(J),VLNORM(J),VLDB(J),J=1,NTH) 
END FILE 3 
WRITE OUTPUT TAPE 5,7,(THETABL(J),SMREAL(J),SMIMAG(J),VLMAG(J),PHASE 1(J),J=1,NTH) 
END FILE 5 
CALL EXIT 
```
### TABLE A-1 (Continued)

<table>
<thead>
<tr>
<th>C</th>
<th>FORMAT STATEMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FORMAT(12A6)</td>
</tr>
<tr>
<td>2</td>
<td>FORMAT (3F10.6, I3)</td>
</tr>
<tr>
<td>3</td>
<td>FORMAT (55H0 INPUT VALUES -ARGUMENT THETA 1 ALPHA N LIMIT 1/1H+60X58H DTHETA THETA 0 DTH NO. OF POINTS IN FAR-FIELD 2LD//1H 12XF8.4, 5XF7.2, 5XF6.2, 6XI2)</td>
</tr>
<tr>
<td>4</td>
<td>FORMAT (1H+60XF5.2, 5XF7.2, 6XF6.2, 12X13///1H020X43H LISTING OF THE TERMS IN THE SUMMATION FOR E 1H05X5THETA4X1HN5X9H REAL TERMX214H IMAGINARY TERM)</td>
</tr>
<tr>
<td>5</td>
<td>FORMAT (1HF11.2, 15, 2E16.6)</td>
</tr>
<tr>
<td>6</td>
<td>FORMAT (95H1 THETA-DEG., SUM OF REAL SUM OF IMAG VOLT.-M 2AG. PHASE VOLT. NORMALIZED DB//((F10.2, 3E16.6, F10.2, E17. 36+F10.2))</td>
</tr>
<tr>
<td>7</td>
<td>FORMAT (F10.5, 3E16.6, F10.5)</td>
</tr>
</tbody>
</table>

END
TABLE A-I1
RADIATION FIELD OF ANY NUMBER OF ELEMENTS
ON A GREAT CIRCLE OF A SPHERE

C MR2SEP64 TOTAL RADIATION PATTERN FOR ANY NO. OF RADIATORS ON A SPHERE
C ************** Includes FPDRN and INNDRN SUBROUTINES
C
C THE(I) = THETA (DEGREES)
C ER(I) = SUM OF REAL
C EI(I) = SUM OF IMAGINARY
C VOL(I) = MAGNITUDE (VOLTAGE)
C PH(I) = PHASE (DEGREES)
C PHI(J) = PHASE CORRECTION (DEGREES)
C STD = STANDARD DEVIATION FOR RANDOM PHASE (DEG.)
C STDV = STANDARD DEVIATION FOR VOLTAGE MAGNITUDE
C INITRN = INITIAL NUMBER TO START RANDOM NUMBERS ROUTINE
C "0 OR - , NO RANDOM NUMBERS IN PHASE OR VOLTAGE"
C ICONT = CONTROL CARD FOR RANDOM NUMBERS. 0 RANDOM PHASE, 1 RANDOM VOLTAGE MAGNITUDE,
C "+1 * RANDOM PHASE AND VOLTAGE"
C NTOTAL = TOTAL NUMBER OF PATTERNS PER SET OF STD, STDV OR BOTH
C NEXC = TOTAL NO. OF EXCITORS ON A SPHERE MINUS 1
C
C DIMENSION THE(50),ER(50),EI(50),VOL(50),PH(50),PHI(10),TH(10),EV(510),EN(50),EDB(50),PHAS(50),SURE(50),SUIM(50),AN(10),RANV(10),ABDB(2100),FN(100)
C 1*1745329E-2
READ INPUT TAPE 2,1
READ INPUT TAPE 2,2,(THE(I),ER(I),EI(I),VOL(I),PH(I)),I=1,37
READ INPUT TAPE 2,3,(PHI(J)),J=1,NEXC
READ INPUT TAPE 2,3,(TH(J)),J=1,NEXC
WRITE OUTPUT TAPE 3,1
WRITE OUTPUT TAPE 3,1,(THE(I),ER(I),EI(I),VOL(I),PH(I)),I=1,37
WRITE OUTPUT TAPE 3,3,(PHI(J)),J=1,NEXC
WRITE OUTPUT TAPE 3,5,(TH(J)),J=1,NEXC
IC=1
SUMD=0.
READ INPUT TAPE 2,12,INITRN
100 READ INPUT TAPE 2,12,ICONT,NTOTAL
IF(INITRN)18,18,140
140 INITRN=INNDRN(INITRN)
IF(ICONT)15,16,17
15 READ INPUT TAPE 2,3,STD
GO TO 18
16 READ INPUT TAPE 2,3,STDV,DB
GO TO 18
17 READ INPUT TAPE 2,3,STD,STDV,DB
C LOOP FOR NTOTAL
18 DO 300 M=1,NTOTAL
WRITE OUTPUT TAPE 3,1
IF(INITRN)22,22,180
180 IF(ICONT)19,20,21
19 WRITE OUTPUT TAPE 3,11,STD
GO TO 22
20 WRITE OUTPUT TAPE 3,13,STDV,DB
GO TO 22
21 WRITE OUTPUT TAPE 3,14,STD,STDV,DB
WRITE OUTPUT TAPE 3,9
C LOOP FOR THETA
22 DO 65 I=1,37
28
SUMR=ER(I)
SUMI=EI(I)

C
C LOOP FOR OTHER NEXC EXCITORS
DO 45 J=1,NEXC
E2D=THE(I)+TH(J)
IF(E2D)<30*25,350
25 I=1
GO TO 40
30 E2D=360+E2D
GO TO 35
350 IF(E2D<360.,35,37)
37 E2D=E2D-360.
35 I=I+1
IF(INITRN)41,411,40
40 IF(icont)440,411,440
411 AN(J)=PHI(J)
GO TO 42
440 IF(THE(I))42,41,42
41 FMEAN=PHI(J)
AN(J)=FPNDRN(STD*FMEAN+INITRN)
42 PHIT=(PHI(I)+AN(J))*C1
IF(INITRN)441,441,440
440 IF(icont)444,444,444
441 RANV(J)=1.
GO TO 450
444 IF(THE(I))450,445,450
445 RANV(J)=FPNDRN(STDV,1.)*INITRN
450 SUMR=SUMR+VOL(I)*COSF(PHIT)*RANV(J)
45 SUMI=SUMI+VOL(I)*SINF(PHIT)*RANV(J)
SURE(I)=SUMR
SUMI(I)=SUMI
EV(I)=SQRTF(SUMR**2+SUMI**2)
PHA=ATANF(SUMI/SUMR)
IF(SUMR)<50,54,50
50 IF(SUMI)<51,53,51
51 PHA=PHI+C1
GO TO 60
53 PHA=3.14159
GO TO 60
54 IF(SUMI)55,56,57
55 PHA=-1.570745
GO TO 60
56 PHA=0.
GO TO 60
57 PHA=1.570745
60 PHAS(I)=PHA/C1
65 CONTINUE
IF(INITRN)680,680,650
650 IF(icont)66,67,68
66 WRITE OUTPUT TAPE 3,8,(AN(J),J=1,NEXC)
GO TO 680
67 WRITE OUTPUT TAPE 3,150
WRITE OUTPUT TAPE 3,8,(RANV(J),J=1,NEXC)
GO TO 680
68 WRITE OUTPUT TAPE 3,8,(AN(J),J=1,NEXC)
WRITE OUTPUT TAPE 3,150
WRITE OUTPUT TAPE 3,8,(RANV(J),J=1,NEXC)
680 EVMAX=EV(I)
DO 70 I=1,37
IF(EVMAX-EV(I))69,70,70
69 EVMAX=EV(I)
70 CONTINUE
DO 80 I=1,37
EN(I)=EV(I)/EVMAX
80 EDB(I)=20*LOG10F(EN(I))
WRITE OUTPUT TAPE 3,10
WRITE OUTPUT TAPE 3,6*(THE(I),SURE(I),SUIM(I),EV(I),PHAS(I),EN(I))
1EDB(I)=I=1,37
FNUL=EN(I)
DO 90 I=1,37
IF(EN(I))=FNUL I=1,37,90
85 FNUL=EN(I)
90 CONTINUE
WRITE OUTPUT TAPE 3,156,FNUL
FNU=1/FNUL
SUMD=SUMD+FNU
FN(IC)=FNU
300 IC=IC+1
IF(NTOTAL-1)100,100,110
110 IC=IC-1
FMEAN=SUMD/FLOATF(IC)
SUT=0.
DO 120 I=1,IC
SUT=SUT+(FNU(I)-FMEAN)*FNU(I)*FNU(I)
120 IC=IC-1
FMEAN=SUMD/FLOATF(IC)
STD=SQRTF(SUT/FLOATF(IC))
WRITE OUTPUT TAPE 3,1
3.154,FNUL
FNU=1/FNUL
(FNU)(IC)=FNU
300 IC=IC+1
IF(NTOTAL-1)100,100,110
110 IC=IC-1
3.160,FMEAN,STD
FORMAT STATEMENTS
1 FORMAT (72H)
2 FORMAT (F5.0,5X3E16.6,F10.2)
3 FORMAT (F5.0)
4 FORMAT (1H05X23HPHI CORRECTIONS (DEG) -6F10.2)
5 FORMAT (1H05X25HTHETA CORRECTIONS (DEG) -6F10.2)
6 FORMAT (F10.2,3E16.6,F10.2.E16.6.F10.2)
7 FORMAT (1H010X19HRANDOM PHASE (DEG)/5X4HN0.26X4HN0.36X4HN0.46X4HN0.56X4HN0.6)
8 FORMAT (1H07X21HSTANDARD DEV. (DEG) = F15.1//8X21HSTANDARD OEW (VOL) = F10.5»15X4HDB
9 FORMAT (1H010X19HRANDOM PHASE (DEG)/5X4HN0.26X4HN0.36X4HN0.46X4HN0.56X4HN0.6)
10 FORMAT (12H THETA-DEG.-3X11HSUM OF REAL5X11HSUM OF IMAG6X10HVOLT.-1IMAG5X5PHASE4X11HVOL.-NORM 10X2HDB)
11 FORMAT (1HO 7X21HSTANDARD DEV. (DEG) = F5.1)
12 FORMAT (2I5)
13 FORMAT (1H07X21HSTANDARD DEV. (VOL) = F10.5*15X4HDB = F6.1)
14 FORMAT (1H07X21HSTANDARD DEV. (DEG) = F15.1//8X21HSTANDARD DEV. (VOL) = F10.5*15X4HDB = F6.1)
150 FORMAT (1H010X19HRANDOM MAGN. (VOL)/5X4HN0.26X4HN0.36X4HN0.46X4HN0.56X4HN0.6)
154 FORMAT (15X5H1/VOL)
155 FORMAT (1HO 9X43HCALCULATING THE MEAN AND THE ST. DEV. FROM 13+18H RADIATION PATTERNS)
156 FORMAT (1HO20X11HMIN (VOL) =E15.6)
157 FORMAT (5XE15.6)
160 FORMAT (1H010X11HMEAN (VO) =F8.3*15X20HSTANDARD DEV. (VO) =F8.3)
END
RELATIVE RADIATION FIELD OF ANY NUMBER OF ELEMENTS LOCATED ANYWHERE ON A SPHERE

C4MR15JAN65 TOTAL RADIATION PATTERN FOR AN ARRAY OF SLOTS ON THE SPHERE
C THE CTR. LOCATION OF THE CONE AT THETA SUB C - PHI SUB C
C PROGRAM IS DIMENSIONED FOR A MAX. OF 6 RADIATORS (NRAD1) AND 25
C TOTAL RADIATION PATTERNS (NCUTS)
C INCLUDE SUBROUTINES DPNCOS - DDXHAN - BSUBN - FACT - ERPR
C DIMENSION HEA(12),PI(2),SMREAT(100,6),SMMAT(100,6),VLMAT(100,6),
C 1PHAS(100,6),PSL(100),PH(100),SMREP(100,6),SMIMP(100,6),VLM(100,6)
C 2,PMAS(100,6),THDET(100),ANGLE(6),PHIN(6),THN(6),PDB(50),TDB(50),BU
C 3FFER(800),VNBTT(50),VNDBP(50),TCWR(9),PCWR(9),BCC(1),BCD(2),BCE(2)
C DIMENSION TSRETP(37,25),TSMTP(37,25),TVLMP(37,25),TPHP(37,25)
C COMMON TSRET,TSITM,TVLMT,TPHT,TSREP,TSMTP,TVLMP,TPHP,C1,PI,KEY,KL,
C 1VLMAX
C
TABLE OF CONSTANTS
C PI(1) = 202622077325
C PI(2) = 147042055061
C C = PI/180.
C READ INPUT TAPE 2,11,(HEAD(1),I=1,12)
C 1 FORMAT(12A6)
C ARG = KG...POSITIVE, LESS THAN 50. , REAL ONLY
C THETA1 = HALF WIDTH OF CONE (DEG)
C ALPHA = HALF WIDTH OF SLOT (DEG)
C DTHETA = INTERVAL OF 2 PT. GAUSS INTEGRAL
C NL = N LIMIT. POSITIVE INTERGER, LESS THAN 10
C READ INPUT TAPE 2,8,ARG,THETA1,ALPHA,DTHETA,NL
C 8 FORMAT (4F10.6,I3)
C CONTROL CARD TO OMIT PLOTTING ON A6.
C NTP = 0 NO PLOT
C NTP = 1 PROGRAM WILL PLOT
C NRAD1 TOTAL NUMBER OF RADIATORS ON THE SPHERE
C ICONT = CONTROL CARD TO DETERMINE WHICH TO VARY.
C VOLMAX = MAX VOLTAGE ON SPHERE * IT CAN BE 0 AND PROGRAM WILL NORM
C TO PEAK
C NCUTS = TOTAL NUMBER OF RADIATION PATTERNS FOR ONE COMPUTER RUN
C READ INPUT TAPE 2,111,NCUTS,NTP,NRAD1,ICONT,VOLMAX
C 111 FORMAT (4I3,E15.7)
C K1=0
C IC=0
C NRAD=NRAD1-1
C 550 IF(NRAD)551,555,550
C 551 IF(NTP)656,501,656
C 656 CALL PLOTTLE(BUFFER,800)
C 501 IF(icont)513,500,513
C NPHI = NO. OF POINTS IN PATTERN
C NWREL = CONTROL CARD TO WRITE ELEMENT PATTERN 0=WRITE ELEMENT
C PATTERN,= 1=DO NOT WRITE ELEMENT PATTERN
C 500 READ INPUT TAPE 2,2,NWREL,PHIN,THN,DT,NPHI
C 2 FORMAT (12,3F10.6,I3)
C 890 XLEN=10*
C BCE(1)=6HPHI (D
C BCE(2)=6HEG) =
C BCD(1)=6MTHETA

31
TABLE A-III (Continued)

<table>
<thead>
<tr>
<th>BCD(2) = 6H(DEG)</th>
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<tbody>
<tr>
<td>BCC(1) = 6H THETA</td>
</tr>
<tr>
<td>IC = IC + 1</td>
</tr>
<tr>
<td>THDET(1C) = PHI</td>
</tr>
<tr>
<td>IF(PHI = 700), 701, 700</td>
</tr>
<tr>
<td>PHI = 1.0E-3</td>
</tr>
<tr>
<td>GO TO 700</td>
</tr>
<tr>
<td>C THETA = ANGLE FROM ZENITH (DEG)</td>
</tr>
<tr>
<td>C PHIS = ANGLE AROUND THE EQUATOR OF THE SPHERE (DEG)</td>
</tr>
<tr>
<td>C DP = INTERVAL IN PHI (DEG)</td>
</tr>
</tbody>
</table>

513 READ INPUT TAPE 2,2, NWREL, THETA, PHIS, DP, NPHI

891 XLENG = 5,
| BCE(1) = 6H THETA |
| BCE(2) = 6H(DEG) |
| BCD(1) = 6H PHI |
| BCD(2) = 6H(DEG) |
| BCC(1) = 6H PHI |
| IC = IC + 1 |
| THDET(1C) = THETA |
| 700 IF(NWREL) = 100, 657, 100 |

657 WRITE OUTPUT TAPE 3, 10

10 FORMAT (35H1 ELEMENT PATTERN FOR EACH RADIATOR)

WRITE OUTPUT TAPE 3, 3, ARG, THETA, ALPHA, NL

3 FORMAT (55H0 INPUT VALUES - ARGUMENT THETA 1 ALPHA LIMIT
| 1/1H+60X6HDTETHETA4X26HNO. OF POINTS IN FAR-FIELD/1H 12XF8.4, 5XF7.2, |
| 25XF6.2, 6X12 |

WRITE OUTPUT TAPE 3, 4, DTTHETA, NPHI

4 FORMAT (1H1+60XF5.2, 12X13)

PROGRAM THAT CALCULATES THE RADIATION PATTERN FOR X NO. OF

100 DO 3101 J = 1, NRAD1

READ INPUT TAPE 2, 8, TC, PC

TCWR(J) = TC
PCWR(J) = PC

C ---- LOOP FOR FAR-FIELD ANGLE, THETA - P

SIGN = 1

C CALCULATION LOOP FOR PHI

TCR = TC*C1
CTC = COSF(TCR)
STC = SINF(TCR)

IF(ICONT) = 503, 502, 503

502 THETA = THET

GO TO 661

503 PHI = PHIS

661 IF(NWREL) = 504, 659, 504

659 WRITE OUTPUT TAPE 3, 658, TC, PC

658 FORMAT (1H0, 4HPOSITION OF RADIATORS THETA SUB C (DEG) = F7.2, 5X16H |
| 1PHI SUB C (DEG) = F7.2 |

WRITE OUTPUT TAPE 3, 17, BCE, THDET(K1)

17 FORMAT (1H020X2A6, F10.2)

WRITE OUTPUT TAPE 3, 660, BCC

660 FORMAT (1H024X11HE SUB THETA = 40X9HE SUB PHI//1X6 |
| 4X11HSUM OF | 4REAL*4X11HSUM OF IMAG*3X10HVOLT* - MAG*3X5PHASE* 4X1H* |
| 2X11HSUM OF R | 2X11HSUM OF IMAG*5X10HVOLT* - MAG*3X5PHASE*4X9H THCD*3X3HPSI)

504 DO 510 K = 1, NPHI

TR = THETA*C1

ST = SINF(TR)

CT = COSF(TR)

PCP = C1*(PC - PHI)

SPCP = SINF(PCP)
TABLE A-III (Continued)

CPCP*COSF(ABSF(PCP))
IF (ICONT) = 505
505 PH(K) = THETA
GO TO 507
506 PH(K) = PHI
C CALCULATION FOR THETA PRIME
507 THPR = ACOSF(CTC*CT + STC*ST*CPCP)
THCD = THPR / C1
C CHANGING SIGN FOR E SUB PHI FIELD W
C TEST AT 0 - 180 - 360 DEG. FOR CORRECT ANGLE PSI
NT = THETA
IF (PCP) = 760
760 IF (PCP + PI) = 609
609 IF (PCP + PI) = 761
761 SIG = -1.
GO TO 6090
762 IF (PCP - PI) = 609
609 SIG = 1.
6090 IF (INT) = 610
610 IF (INT - 180) = 5072
5072 IF (INT - 360) = 5072
5072 PSIR = ABSF(PI - ABSF(PCP))
GO TO 5076
806 PSIR = ACOSF(PCPC)
5076 ART = SINF(PSIR)
CPSI = COSF(PSIR)
GO TO 5073
C CALCULATION FOR PSI
5072 CPSI = (CTC - CT*COSF(THPR)) / (ST*SINF(THPR))
IF (ABSF(CPSI) = 1.)
74 CPSI = 1.
GO TO 75
74 CPSI = -1.
75 PSIR = ACOSF(CPSI)
ART = SINF(PSIR)
5073 PSI(K) = PSIR / C1
C LOOP TO CALCULATE SUMMATION N = 1, NL
SUMRE = 0.
SUMIM = 0.
DO 16 N = 1, NL
C LOOP TO CALCULATE BSUBN(THETA1, ALPHA, DTHETA, N, BNANS)
CALL BSUBN(THETA1, ALPHA, DTHETA, N, BNANS)
BSUBNA = BNANS
ALPHAR = C1*ALPHA
BND2AR = BSUBNA / (12.*ALPHAR)
C CALCULATE 1ST DERIVATIVE OF X#H2SUBN(X)
CALL DDXHAN(ARG, N, REAL, GAM1)
C CALCULATE D/DTH ETA PSUBN(COS THETA)
CALL DPNCOS(THCD, N, ANS)
DPNANS = ANS
C ABA = ARG*BND2AR*DPNANS
DEN = REAL**2 + GAM1**2
ABA DEN = ABA / DEN
C TEST FOR VALUE OF (J) EXP N
NN = XMDF(N + 4) + 1
GO TO (11, 12, 13, 14, N)
C TEST FOR VALUE OF (J) EXP N
11 TRMRE = REAL*ABADEN
TRMIM = -GAM1*ABADEN
GO TO 15

33
C NUMERATOR IS IMAGINARY, POSITIVE
12 TRMRE = GAM1*ABADEN
TRMIM = REAL*ABADEN
GO TO 15
C NUMERATOR IS REAL, NEGATIVE
13 TRMRE = -REAL*ABADEN
TRMIM = GAM1*ABADEN
GO TO 15
C NUMERATOR IS IMAGINARY, NEGATIVE
14 TRMRE = -GAM1*ABADEN
TRMIM = -REAL*ABADEN
15 SUMRE = SUMRE+TRMRE
16 SUMIM = SUMIM+TRMIM
C TESTING FOR CORRECT SIGN ON REAL AND IMAGINARY
301 IF(PCP)>310,301,310
302 IF(THETA-TC)>300,305,320
300 SUMRE=SUMRE
SUMIM=SUMIM
ERP=0.
EIP=0.
GO TO 315
305 SUMRE=CPSI*SUMRE
SUMIM=CPSI*SUMIM
ERP=0.
EIP=0.
GO TO 315
310 ERP=-ART*SUMRE*SIG
EIP=-ART*SUMIM*SIG
SUMIM=CPSI*SUMIM*SIGN
SUMRE=CPSI*SUMRE*SIGN
GO TO 315
320 ERP=0.
EIP=0.
TCP18=TC+180.
IF(THETA-TC)<315,315,300
315 SMREAT(K,J)=SUMRE
SMIMAT(K,J)=SUMIM
SMREP(K,J)=ERP
SMIMP(K,J)=EIP
C FIND VOLT. MAGNITUDE AND NORMALIZE TO MAXIMUM VALUE
CALCULATE PHASE ANGLE AND DB VALUE
VLM(K,J)=SQRTF(ERP**2+EIP**2)
PHAS(K,J)=ATANF(EIP/ERP)
IF(ERP)>210,240,280
210 IF(EIP)>220,230,220
220 PHAS(K,J)=PHAS(K,J)+PI
GO TO 280
230 PHAS(K,J)=PI
240 IF(EIP)>250,260,270
250 PHAS(K,J)=-PI/2.
GO TO 280
260 PHAS(K,J)=0.
GO TO 280
270 PHAS(K,J)=PI/2.
280 VLMAT(K,J)=SQRTF(SUMRE**2+SUMIM**2)
C CALC. PHASE ANGLE -PI/2 TO 3PI/2
PHAST(K,J)=ATANF(SUMIM/SUMRE)
IF(SUMRE)>21,24,28
21 IF(SUMIM)>22,23,22
22 PHAST(K,J)=PHAST(K,J)+PI
GO TO 28
TABLE A-III (Continued)

23 PHAST(K,J)=PI
24 IF(SUMIM(25,26,27)
25 PHAST(K,J)=-PI/2,
26 PHAST(K,J)=0,
27 PHAST(K,J)=PI/2,
C CONV1ERTING TO DEGREES
28 PHAS(K,J)=PHAS(K,J)/C1
30 PHAST(K,J)=PHAST(K,J)/C1
IF(ICON)509,508,509
508 THETA=THETA+DT
509 PHI=PHI+DP
510 IF(NWREL)=510,653,510
653 WRITE OUTPUT TAPE 3,651,PH(K),SUMRE,SMIM,SMMAT(K,J),PHAST(K,J),ER
1P+E1P+VLM(K,J),PHAS(K,J),THCD,PSI(K)
651 FORMAT (F7,1,1E15,7,F8,2,2X1H*,3E15,7,F6,2,F8,1,F7,1)
510 CONTINUE
310 CONTINUE
510 CONTINUE
6540 KEY=1
C E SUB THETA
663 VLEMT=VLMAT(1,1)
664 VLEMT=VLMAT(K,1)
665 CONTINUE
C E SUB PHI
666 PDB(K)=20.*LOG10F(VLMAT(K,1)/VLEMT)
669 TDB(K)=20.*LOG10F(VLM(K,1)/VLEMT)
671 WRITE OUTPUT TAPE 3,673,BCC(1),(PH(K),PDB(K),TDB(K),K=1,NPHI)
673 FORMAT (110,9X11HE SUB THETA,2X9HE SUB PHI,4X6*8X2HDB9X2HDB/(F
110,1,2F11.2))
GO TO 801
6540 KEY=1
C E SUB THETA
CALL TRADPA(PH,SMREAT,SMMAT,VLMAT,PHAST,NPHI,NRAD,ANGLE,VOlMAX)
C E SUB PHI
KEY=2
CALL TRADPA(PH,SMREP,SMIMP,VLM,PHAS,NPHI,NRAD,ANGLE,VOlMAX)
IF(VOLMAX)=800,800,8011
800 IF(NCUTS=K1)801,801,501
8011 KI=0
NCU=1
801 DO 850 L=1,NCU
SESITT=0
SESITP=0
SSINF=0
IF(NRAD)=1,899,899,8010
8010 WRITE OUTPUT TAPE 3,1,(HEAD(1),I=1,12)
WRITE OUTPUT TAPE 3,3,ARG,THETA,ALPHA,NL
WRITE OUTPUT TAPE 3,4,DTHETA,NPHI
WRITE OUTPUT TAPE 3,705,(TCWR(J),PCWR(J),J=1,NRAD1)
705 FORMAT (22H0POSITION OF RADIATORS//5X11HTHETA SUB THETA SUB C//
1(4X2F12.2))
512 WRITE OUTPUT TAPE 3,9,BCE,THDET(L)
9 FORMAT (1H101X)1HE SUB THETA10X2A6,F7.2)
709 WRITE OUTPUT TAPE 3,50,NRAD1S(ANGLE(J)+I=1,NRAD)*BCC
50 FORMAT (1H101HTHERE ARE I1,24H RADIATORS ON THE SPHERE//20H PHASE
1CORRECTIONS =(3F10.2)//4X16.5X11HSUM OF REAL5X11HSUM OF IMAG6X10HV
2OLT,-MAG5X5PHASE3X16HVOLT. NORMALIZED6X2HDB)
DO 714 J=1,NPHI
VNDBT(J)=TVLMT(J,L)/VLMAXT
VNDBP(J)=TVLMP(J,L)/VLMAXT
TDB(J)=20.4LOG10F(VNDBT(J))
PDB(J)=20.4LOG10F(VNDBP(J))
IF (ICONT) 711,711,712
711 ST=SINF(PH(J)*C1)
GO TO 713
712 ST=SINF(THDET(L)*C1)
713 SSINF=SSINF+ST
SESITT=SESITT+ST*VNDBT(J)**2
714 SESITP=SESITP+ST*VNDBP(J)**2
WRITE OUTPUT TAPE 3,70,(PH(J),TSREP(J,L),TSIMP(J,L),TVLMT(J,L),TPH
1T(J,L),VNDBT(J),TDB(J),J=1,NPHI)
20 FORMAT (F10.2,3E16.6,F10.2,E17.6,F10.2)
LOOP FOR E SUB PHI
WRITE OUTPUT TAPE 3,1100,BCC
1100 FORMAT (1H010X)9HE SUB PHI//4XA6.5X11HSUM OF REAL5X11HSUM OF IMAG6X
110HVOLT,-MAG5X5PHASE3X16HVOLT. NORMALIZED6X2HDB
WRITE OUTPUT TAPE 3,70,(PH(J),TSREP(J,L),TSIMP(J,L),TVLMT(J,L),TPH
1P(J,L),VNDBP(J),PDB(J),J=1,NPHI)
WRITE OUTPUT TAPE 3,1101,SSINF,SESITT,SESITP
1101 FORMAT (1H05X)16HSUM
SIN(THETA)=E15.7//6X16HSUM ST**2.TH=E15.7/
1/6X16HSUM ST*EN**2,PH=E15.7)
899 IF (NTP) 901,850,901
901 CALL AXISK0..0..2HDB.2.5..90..-20..4..0.0.1.
CALL AXISK0..0..BCD.-12.XLENG.0..0..18..0.0.1.
DO 999 I=1,NPHI
IF (ABS(TDB(J))-20.) 997,997,996
996 TDB(J)=-20.4
997 IF (ABS(PDB(J))-20.) 999,999,998
998 PDB(J)=-20.4
999 CONTINUE
CALL SCLGPH (PH,TDB,NPHI,0.*0.,0.*18.,-20.*4.)
CALL SCLGPH (PH,PDB,NPHI,1.-2.0.*18.,-20.*4.)
894 CALL SYMBL5 (5.*5..5..15.BCE,0..-12)
895 CALL NUMBRI (6.5..5..5..5..15.THDET(L),0..-0)
CALL SYMBL5 (8.3..2.0..5..0..8..-20..-1)
CALL SYMBL5 (8.5..2.0..15.HSUB PHIni0..9)
CALL SYMBL5 (8.5..1.75..15.HSUB THETA,0..11)
XMOVE=XLENG+4
CALL PLOT1(XMOVE,0..-3)
END FILE 6
850 CONTINUE
IF (NCUTS-IC) 2001,2001,501
2001 IF (NRAD=1-1) 501,501,2000
C E SUB PHI
2000 GAIN=SSINF,SESITP
C E SUB THETA
GAINT=SSINF,SESITP
WRITE OUTPUT TAPE 3,5,GAIN,GAINT
5 FORMAT (38HIDIRECTIVITY FOR TOTAL NO OF PATTERNS//6X9HE SUB PH15X
111HE SUB THETA/2E16.7)
CALL EXIT
END
**TABLE A-IV**  
**SUBROUTINES**

| CBSUBN | SUBROUTINE TO CALC. BSUBN — INPUT — THETA1, ALPHA, DTHETA, N  
C | INCLUDE DPNCOS SUBROUTINE  
C | OUTPUT — BNANS  
C | CALCULATE INTEGRAL FROM THETA1-ALPHA TO THETA1+ALPHA  
C | SUBROUTINE BSUBN(THETA1, ALPHA, DTHETA, N, BNANS)  
DIMENSION PI(1)  
B | PI(1) = 202622077325  
C1 = 0.57735027  
C2 = PI/180.  
HDT = DTHETA/2.0*C2  
T = C1*HDT  
TZ = THETA1-ALPHA  
THE = TZ  
NT = 4.0*(ALPHA/DTHETA)  
SUM = 0.  
DO 20 I = 1, NT+2  
TH = THE+HDT  
THE = TH+T  
TRAD = THETA*C2  
CALL DPNCOS(THETA, N, ANS)  
SUM = SUM+ANS*SINF(TRAD)*HDT  
20 THE = THE+DTHETA  
FN = N  
TWN = 2*N  
BNANS = -(TWN+1.0)/(FN+1.0)*SUM/TWN  
RETURN  
END |

| CDDXHAN | CALC. 1ST DERIVITIVE OF X*BESSEL(N) USING BESSEL  
SUBROUTINE CDDXHAN(X, N, R, G)  
DIMENSION ANS(101), ANSN(101)  
C | INPUT IS ARGUMENT (X) AND ORDER (N)  
C | X CANNOT BE GREATER THAN 50.  
C | N CANNOT BE GREATER THAN 100  
C | OUTPUT IS REAL AND IMAGINARY PARTS OF COMPLEX ANSWER  
C | PIDT = 1.5707963  
S = -1.0  
FN = N  
N1 = N+1  
N2 = N+2  
M = N-1  
C | ARG = SQRTF(PIDT/X)  
CALL BESSEL(1, X, N, 0.5, ANS, 0.0)  
CALL BESSEL(1, X, N1, 0.5, ANSN, -0.0) |

| R = ARG*(X*ANS(N)-FN*ANS(N1))  
G = ARG*(S*M FN*ANS(N2)-S*N X*ANS(N1))  
RETURN  
END |
TABLE A-IV (Continued)

CDPNCOS D/PNCOS SUBROUTINE
CDPNCOS D/DTHETA * P SUBN (COS THETA) - DOUBLE-PRECISION
C
C SUMMATION FROM M=0 TO M=M1
C
(-1)**M (2N-2M)F (N-2M) (SIN THETA) (COS THETA)**(N-1-2M)
SUM = - ----------
C
C WHERE M1=N/2 OR (N-1)/2 WHICHER EVER IS AN INTERGER
C
C DPNOS CAN BE COMPUTED FOR N GREATER THAN 20 BY AN APPROXIMATION
C
C
SUBROUTINE DPNCOS(T,N,A)
DIMENSION F(39), PI(1), THETA(1), COEFNT(20,11), FN(1)
DIMENSION NTEST(20), N3(20,11), AB(3), F3F(3)
C
TEST SWITCH AND SET OR JUMP
IF(ISW=12345)1,2,1
1 ISW = 12345
DO 100 I=1,32
100 F(I) = DFACT(I)
B
PI(1) = 202622077325
PI(2) = 147042055061
C
S = -1.0
C
FN(1) WERE CALCULATED USING TRIPLE PRISCION ROUTINE
C
-N=17 I=1
B
FN(1) = 623447446013
B
FN(2) = 570014000000
D
COEFNT(17,1) = FN
C
-N=18 I=1,2
B
FN(1) = 624460211013
B
FN(2) = 571261777776
D
COEFNT(18,1) = FN
B
FN(1) = 226447446013
B
FN(2) = 173014000000
D
COEFNT(18,2) = FN
C
-N=19 I=1,2,3
B
FN(1) = 225470560304
B
FN(2) = 572432400000
D
COEFNT(19,1) = FN
B
FN(1) = 227503221454
B
FN(2) = 174035077776
D
COEFNT(19,2) = FN
B
FN(1) = 630425063512
B
FN(2) = 575253200000
D
COEFNT(19,3) = FN
C
-N=20 I=1,2,3,4
B
FN(1) = 626500745674
B
FN(2) = 573200200000
D
COEFNT(20,1) = FN
B
FN(1) = 230537636335
B
FN(2) = 175075637774
D
COEFNT(20,2) = FN
B
FN(1) = 631503221454
B
FN(2) = 576350777776
D
COEFNT(20,3) = FN
B
FN(1) = 231503221454
B
FN(2) = 176351000000

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<tr>
<td>D</td>
<td>COEFNT(20,4) = FN</td>
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<td>N3(17,1) = 16</td>
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<tr>
<td>N3(18,1) = 17</td>
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<tr>
<td>N3(18,2) = 15</td>
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<tr>
<td>N3(19,1) = 18</td>
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<tr>
<td>N3(19,2) = 16</td>
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<tr>
<td>N3(19,3) = 14</td>
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<tr>
<td>N3(20,1) = 19</td>
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<tr>
<td>N3(20,2) = 17</td>
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<tr>
<td>N3(20,3) = 15</td>
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<tr>
<td>N3(20,4) = 13</td>
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<tr>
<td>2</td>
<td>M = N/2</td>
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<tr>
<td>M1 = M+1</td>
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<tr>
<td>THETA(1) = T</td>
<td></td>
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<tr>
<td>THETA(2) = 0*</td>
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<tr>
<td>D</td>
<td>TRAD = C1*THETA</td>
</tr>
<tr>
<td>D</td>
<td>SINT = SINF(TRAD)</td>
</tr>
<tr>
<td>D</td>
<td>COST = COSF(TRAD)</td>
</tr>
<tr>
<td>C</td>
<td>IF N IS GREATER THAN 20 AN APPROXIMATION IS USED</td>
</tr>
<tr>
<td>IF(N-20) = 20<em>20</em>15</td>
<td></td>
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<tr>
<td>D</td>
<td>PN = N</td>
</tr>
<tr>
<td>PN1 = S*SQRTF(2.<em>PN/(PI</em>SINT))</td>
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</tr>
<tr>
<td>D</td>
<td>PN2 = (PN+5)*TRAD-PI/4.</td>
</tr>
<tr>
<td>D</td>
<td>SUM = PN1*SINF(PN2)</td>
</tr>
<tr>
<td>GO TO 30</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>TEST IF N DONE BEFORE</td>
</tr>
<tr>
<td>IF(NTEST(N) = 12345) = 3, 7, 3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>CALC* TABLE OF COEF(N,1) AND N3(N,1)</td>
</tr>
<tr>
<td>3</td>
<td>NTEST(N) = 12345</td>
</tr>
<tr>
<td>NM1 = N-1</td>
<td></td>
</tr>
<tr>
<td>N2 = 2*N</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>T WN = 2*O**N</td>
</tr>
<tr>
<td>IF(N-16) = 32, 32, 33</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>MS = 0</td>
</tr>
<tr>
<td>MT = 1</td>
<td></td>
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<tr>
<td>GO TO 41</td>
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</tr>
<tr>
<td>33</td>
<td>NN M7 = N-16</td>
</tr>
<tr>
<td>GO TO (34, 36, 38, 40), NNM7</td>
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</tr>
<tr>
<td>34</td>
<td>MS = 1</td>
</tr>
<tr>
<td>MT = 2</td>
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<tr>
<td>GO TO 41</td>
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</tr>
<tr>
<td>36</td>
<td>MS = 2</td>
</tr>
<tr>
<td>MT = 3</td>
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<tr>
<td>GO TO 41</td>
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<tr>
<td>38</td>
<td>MS = 3</td>
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<tr>
<td>MT = 4</td>
<td></td>
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<tr>
<td>GO TO 41</td>
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</tr>
<tr>
<td>40</td>
<td>MS = 4</td>
</tr>
<tr>
<td>MT = 5</td>
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</tr>
<tr>
<td>41</td>
<td>DO 6 I = MT, M1</td>
</tr>
<tr>
<td>MST = 2*MS</td>
<td></td>
</tr>
<tr>
<td>N1 = N-MS</td>
<td></td>
</tr>
<tr>
<td>N3(N+1) = NM1-MST</td>
<td></td>
</tr>
<tr>
<td>N4 = N-MST</td>
<td></td>
</tr>
<tr>
<td>N5 = N2-MST</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>FN4 = N4</td>
</tr>
<tr>
<td>IF(MS) = 4, 4, 4, 5</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>COEFNT(N+1) = -S**MS/TWN*F(N5)/F(N4)*FN4/F(N1)</td>
</tr>
<tr>
<td>GO TO 6</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>COEFNT(N+1) = -S**MS/TWN*F(N5)/F(MS)*FN4/F(N1)*1.0/F(N4)</td>
</tr>
<tr>
<td>6</td>
<td>MS = MS+1</td>
</tr>
</tbody>
</table>

39
TABLE A-IV (Continued)

C            CALC. SUMMATION
D  7 SUM = 0
   DO 10 I=1,M1
   N3NI = N3(N+1)
   IF(N3NI<9) GO TO 10
D  8 SUM = SUM + COEFNT(N+1)*SINT
       GO TO 10
D  9 SUM = SUM + COEFNT(N+1)*SINT*COST**N3NI
10 CONTINUE
30 A = SUM
   RETURN
   END

*INDRN
* FAP
   COUNT 10
   ENTRY INNDRN
INNDRN CLA* 1,4
   ARS 18
   LBT
   ADD =1
   ADD* 1,4
   STO* 1,4
  TRA 2,4
   END

* FAP
   COUNT 100
   ENTRY FPNDRN
FPNDRN PXD 0+0
NDRN SXD NDRN+86,4
   CLA 1,4
   STA NDRN+64
   CLA 2,4
   STA NDRN+65
   CLA 3,4
   STA NDRN+9
   STA NDRN+11
   STA NDRN+12
   LDQ COMPUTE RI
   MPY NDRN+68
   STQ
   CLA 013 AANDRN
   ARS 8 COMPUTE UI
   ADD NDRN+69
   FAD NDRN+70
   STO NDRN+80
   LRS 35 COMPUTE VI
   FMP NDRN+71
   FAD NDRN+72
   SSM 021 AANDRN
   FAD NDRN+72
   LRS 35
   FMP NDRN+73

40
| TSX     | $LOG_{10}$, 4 |
| NOP    |              |
| LRS 35 |              |
| FMP NDRN+71 |          |
| STO NDRN+82 |            |
| TSX     | $SQRT_{10}$, 4 |
| NOP    |              |
| STO NDRN+81 |          |
| LRS 35 |              |
| FMP NDRN+82 |          |
| STO NDRN+83 |            |
| CLA NDRN+72 |          |
| STO NDRN+84 |            |
| LXA NDRN+20 |              |
| LDQ NDRN+77 |            |
| FMP NDRN+84 |          |
| FAD NDRN+84 |            |
| STO NDRN+84 |            |
| TIX NDRN+38 |              |
| CLA NDRN+77 |          |
| STO NDRN+85 |            |
| LXA NDRN+54 |              |
| LDQ NDRN+80 |            |
| FMP NDRN+83 |          |
| FAD NDRN+85 |            |
| STO NDRN+85 |            |
| TIX NDRN+46 |              |
| FDP NDRN+84 |            |
| STQ NDRN+84 |            |
| CLA NDRN+84 |            |
| CHS    |              |
| FAD NDRN+81 |            |
| STO NDRN+85 |            |
| CLA NDRN+80 |            |
| FSB NDRN+73 |            |
| TPL NDRN+62 |            |
| CLS NDRN+85 |            |
| TRA NDRN+63 |            |
| CLA NDRN+85 |            |
| LRS 35 |              |
| FMP    |              |
| FAD    |              |
| LXD NDRN+86 |            |
| TRA    | 4, 4         |
| OCT 011060471625 | 200000000000000.0 |
| DEC -2.911555555+5 | 1.4327888818926999001308 |
| DEC 2.515517802853010328 |            |
| BSS 7  |              |

**END**

* FAP
  - COUNT 50
  - ENTRY DFACT
  - DFAC X
  - CLA* 1.4
  - SXA EXIT+4
  - TMI ERM
  - PDX +4
  - TXH ERH+4, 33
  - CLA FAC+33, 4
  - LDQ FACL+33, 4
  - STO 32767
  - STQ 32766

DFCT 002
DFCT 003
DFCT 004
DFCT 005
DFCT 006
DFCT 007
DFCT 008
DFCT 009
DFCT 010
DFCT 011
DFCT 012
DFCT 013
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* **FAP**
* **ERPR**
  **COUNT** | 100 | **ERPR0002**
  **ERROR PRINT OUT ** | VERSION USING LOWER MEMORY LOCATION FOR SIGN ON | **ERPR0004**
  **ENTRY** | (ERPR) | **ERPR0005**
  **CALLING SEQUENCE**- | **ERPR0006**
  **TSX** | (ERPR) | **ERPR0007**
  **PZE** | ERRM+N, N** ADDRESS OF ERROR MESSAGE AND NUMBER WORD** | **ERPR0010**
  **PZE** | A** LOCATION WHERE INDEX 4 ON ENTRY IS STORED** | **ERPR0012**
  **PZE** | **IN ADDRESS** | **ERPR0013**
  **PZE** | **IN ADDRESS** | **ERPR0014**
<table>
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<tr>
<td><strong>(FPTC)</strong> BOOL 117</td>
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<tr>
<td><strong>(ERPR)</strong> SYN *</td>
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<tr>
<td>SXA</td>
</tr>
<tr>
<td>EXIT+4</td>
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<tr>
<td>EXIT+1,2</td>
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<td>STO ARG</td>
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<tr>
<td>IF IT WAS AN F TYPE FUNCTION ARG IN AC</td>
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<td>STA MQ</td>
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<td>ADD =01000000</td>
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<td>STA =15B9</td>
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<td>TNZ ++2</td>
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<td>TXI *-4,4,=-1</td>
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<tr>
<td>WITH LINKAGE DIRECTOR ADDRESS AND STATEMENT NUMBER OR LOCATION</td>
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<td>IF IT ISENT MAIN PROGRAM FIND ADDRESS OF WORD HAVING SYMBOLIC NAME, ELSE SET MAIN</td>
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<td>CLA AMAIN</td>
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<td>ADD +1</td>
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<td>CAL 1,4</td>
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<td>S1 LDQ **-**2</td>
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<td>STR</td>
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<tr>
<td>TIX <em>-2</em>2,1</td>
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<td>TSX $(FIL),4</td>
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<td>S2 LDQ **</td>
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<tr>
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<tr>
<td>FMT1 BCI 2,(1H0,19A6)</td>
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<tr>
<td>FMT2 BCI 5,116H WHEN CALLED BY A6,3H AT17</td>
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<tr>
<td>BCI 6*14H (INTERNAL NO.176H) WITH E16.8</td>
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<tr>
<td>AMAIN PZE BMAIN-1</td>
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<td>BMAIN BCI 1,<em>MAIN</em></td>
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<td>LOCEXT PZE</td>
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<td>LOC PZE 0</td>
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<td>ARG PZE</td>
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<td>MQ PZE</td>
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43
**DOCUMENT CONTROL DATA – R&D**

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<th>I. ORIGINATING ACTIVITY (Corporate author)</th>
<th>2a. REPORT SECURITY CLASSIFICATION</th>
<th>2b. GROUP</th>
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<td>Telemetry Antenna for Lincoln Experimental Satellites LES-1 and LES-2</td>
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<th>4. DESCRIPTIVE NOTES (Type of report and inclusive dates)</th>
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<td>Technical Report</td>
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<tr>
<td>Devane, Mark E. and Rosenthal, Milton L.</td>
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<td>The telemetry antenna used on the first two Lincoln Experimental Satellites consists of four short stubs equally spaced around, and parallel to, the spin axis of the satellite. A detailed description of the antenna and its transmission-line system is presented. Theoretical and model studies leading to the design of this antenna are discussed. Calculated and measured performance data are presented and compared.</td>
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