EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS LIGHTWEIGHT STIFFENING SYSTEMS UNDER EXTERNAL HYDROSTATIC PRESSURE

by

John J. Healey

Distribution of This Document is Unlimited

STRUCTURAL MECHANICS LABORATORY RESEARCH AND DEVELOPMENT REPORT

August 1965

Report 2073
EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS
LIGHTWEIGHT STIFFENING SYSTEMS UNDER
EXTERNAL HYDROSTATIC PRESSURE

by

John J. Healey

Distribution of This Document is Unlimited

August 1965

Report 2073
S-F013 03 02
Task 1956
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>ADMINISTRATIVE INFORMATION</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>DESIGN AND DESCRIPTION OF MODELS</td>
<td>2</td>
</tr>
<tr>
<td>TEST PROCEDURE</td>
<td>8</td>
</tr>
<tr>
<td>RESULTS AND DISCUSSION</td>
<td>10</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>15</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>15</td>
</tr>
<tr>
<td>APPENDIX - HYDROSTATIC TESTS OF TWO CONTINUOUS CORE SANDWICH SHELLS</td>
<td>16</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>19</td>
</tr>
</tbody>
</table>

## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Axial Sections of Models CS-3 through CS-14</td>
<td>3</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Layered Shell Notation</td>
<td>7</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Modified Bryant Coefficients for Various Bulkhead Spacings</td>
<td>7</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Models prior to Testing</td>
<td>9</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Models after Collapse</td>
<td>13</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Axial Sections of Models CS-1 and CS-2</td>
<td>17</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Typical Stress-Strain Curve for 7075-T6 Aluminum</td>
<td>17</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Typical $E_T/E$ versus $\sigma$ Curves for 7075-T6 Aluminum</td>
<td>18</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Models CS-1 and CS-2 after Collapse</td>
<td>18</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Properties of Model Materials</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Summary of Results for Layered Models</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Summary of Results for Ring-Stiffened Models</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>Stress Sensitivity Results for Layered Models</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Stress Sensitivity Results for Ring-Stiffened Models</td>
<td>12</td>
</tr>
</tbody>
</table>
NOTATION

\( b \)  Faying width of ring frame in contact with shell

\( D \)  Diameter to midsurface of shell

\( E \)  Young's modulus of elasticity

\( E_T \)  Tangent modulus

\( F_1 \)  Function of shell geometry

\[ F_1 = \frac{2}{\theta} \left[ \frac{\cosh \theta - \cos \theta}{\sinh \theta + \sin \theta} \right] \]

\( I \)  Moment of inertia of shell cross section

\( I_e \)  Moment of inertia about the centroid of a section comprising one frame plus an effective length of shell

\( L_e \)  Effective length of shell

\( L_B \)  Bulkhead spacing

\( L_F \)  Center-to-center spacing of uniform ring frames

\( n \)  Number of circumferential waves in shell at collapse

\( p \)  Uniform external pressure

\( R \)  Radius to midsurface of shell

\( R_{CG} \)  Radius to neutral axis of frame, effective shell combination

\( R_0 \)  Radius to outside surface of shell

\( t \)  Shell thickness

\( \theta \)  Shell flexibility parameter

\[ \theta = \frac{\sqrt{3} (1 - \mu^2)(L_F - b)}{\sqrt{Rt}} \]

\( \lambda \)  \( \pi R/L_B \)

\( \mu \)  Poisson's ratio

\( \sigma_\theta \)  Circumferential stress

\( \sigma_x \)  Longitudinal stress
Twelve-machined cylindrical models with length-to-diameter ratios of approximately 5.0 were collapsed under external hydrostatic pressure to study the elastic buckling strength of metallic cylinders stiffened with low modulus, low density, relatively low strength materials. Three different configurations were studied: two-layered shells, continuous-core sandwich shells, and ring-stiffened shells. The test results demonstrated that the use of low modulus materials in various stiffening systems can lead to significant increases in the elastic buckling strength of metallic cylindrical shells.

The work described in this report was sponsored by the Bureau of Ships under Subproject S-F013 03 02, Task 1956.

The development of the deep-depth capability of manned and unmanned underwater vehicles has naturally led to the use of new high strength materials such as high strength steels, titanium alloys, and glass-reinforced plastics. One possibility for increasing structural efficiency is the use of low density, low modulus, relatively low strength materials in various stiffening schemes to provide a large increase in stability with a proportionately small increase in overall weight. The stability attained in this manner would enable the high yield strength of the basic shell material to be fully utilized. Because of the low modulus of elasticity of the low density material, the stress carried by this material remains low.

This type of construction has other inherent advantages. Less welding of the difficult-to-weld high strength metals would be required and thinner metal shell plating could be used. In addition, the multilayered shells are expected to have superior acoustical properties.

Prior to any consideration of specific material combinations or optimum shell configurations, the basic question of whether or not the low modulus, low strength materials can significantly increase the stability of metallic shells must be answered. The purpose of this exploratory study was, therefore, to determine the correlation of model test results with the calculated elastic buckling pressure for a few material combinations in several different stiffening systems. The types considered included two-layered shells consisting of one metallic layer and one layer of low modulus material, sandwich shells with a continuous core of low modulus material, shells stiffened with rectangular frames of low modulus materials, and shells stiffened with T-frames with a low modulus materials for the web and a metallic flange.

This report describes the model design, fabrication, and testing and evaluates the test results.
DESIGN AND DESCRIPTION OF MODELS

The model materials were selected on the basis of machinability, availability, and proper material characteristics. The metal used for the shell and flange material was 7075-T6 aluminum. Two different low modulus materials were used: (1) an epoxy resin formed by the mixture of equal parts of Versamid 140 polyamide resin and Epon 828 resin and (2) Inlyte (manufactured by the Inland Division of General Motors) a syntactic foam consisting of glass microspheres embedded in an epoxy resin matrix. The properties of these materials are presented in Table 1 along with those of another syntactic foam material used in the preliminary work described in the Appendix.

### TABLE 1

Properties of Model Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (lb/cu ft)</th>
<th>Young's Modulus ($10^6$ psi)</th>
<th>Poisson's Ratio</th>
<th>Nominal Proportional Limit psi</th>
<th>Nominal Yield Strength 0.2 Percent Offset psi</th>
<th>Nominal Ultimate Strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy Resin Versamid and Shell</td>
<td>67</td>
<td>0.325</td>
<td>0.4</td>
<td>4,600</td>
<td>7,100</td>
<td>8,700</td>
</tr>
<tr>
<td>Inlyte Syntactic Foam</td>
<td>44</td>
<td>0.535</td>
<td>0.3</td>
<td>7,500</td>
<td>13,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Mat Lab Syntactic Foam</td>
<td>46</td>
<td>0.600</td>
<td>0.3</td>
<td>7,500</td>
<td>14,000</td>
<td>17,000</td>
</tr>
<tr>
<td>7075-T6 Aluminum</td>
<td>173</td>
<td>10.80</td>
<td>0.3</td>
<td>64,000</td>
<td>78,000</td>
<td>&gt;80,000</td>
</tr>
</tbody>
</table>

Schematic diagrams of the models are shown in Figure 1. The models were designed for failure by elastic general instability with length-to-diameter ratios of about 5.0.

The shell layers of all the layered and continuous core sandwich models were not bonded. As a result of the assumption that the layers would act together, it was possible to estimate the buckling strength of these shells by the Bresse-Bryan equation for a long monolithic cylinder,\(^1\) that is

---

\(^1\)References are listed on page 19.
Figure 1 – Axial Sections of Models CS-3 through CS-14

Figure 1a – Model CS-3

Figure 1b – Model CS-4

Figure 1c – Models CS-5 and CS-12
Figure 1d — Model CS-8

Figure 1e — Model CS-9

Figure 1f — Model CS-14
Figure 1g - Models CS-6 and CS-10

Figure 1h - Models CS-7, CS-11, and CS-13

NOTE: IN THIS MODEL, THE WEB WAS BONDED TO BOTH THE SHELL AND THE FLANGE.
\[ p_B = \frac{2E}{1 - \mu^2} \left( \frac{t}{D} \right)^3 \]  \[ 1 \]

which can also be expressed as

\[ p_B = \frac{3EI}{(1 - \mu^2) L_B R^3} \]  \[ 2 \]

The buckling pressure was then determined by calculating the moment of inertia for the layered shell acting as a unit, based on the thickness and material properties of the individual layers. The notation used in these calculations is given in Figure 2. The neutral axis of the cross section was determined by the following expression:

\[ \bar{y} = \frac{\sum_{i=1}^{N} E_i t_i (\bar{y}_i)}{\sum_{i=1}^{N} E_i t_i} \]  \[ 3 \]

where \( N \) is the number of layers and \( i \) refers to layers 1, 2, etc. The buckling pressure \( p_t \) was then calculated from this equation,

\[ p_t = \frac{3}{R^2 C_G R_0} \sum_{i=1}^{N} \frac{E_i}{1 - \mu_i^2} \left[ \frac{t_i^3}{12} + t_i (\bar{y} - \bar{y}_i)^2 \right] \]  \[ 4 \]

The buckling pressure based on the assumption of no interaction between the layers \( (p_n) \) can be calculated from Equation [4] by omitting the second term in the brackets.

The elastic buckling pressures of the ring-stiffened models were determined from the modified Bryant equation, \(^2\) i.e.,

\[ p_r = \frac{Et}{R} \left[ \frac{\lambda^4}{(\mu_2 + \lambda^2/2 - 1) (\mu_2 + \lambda^2)^2} \right] + \frac{E I_e (\mu^2 - 1)}{L F R_0 R^2 C_G} \]  \[ 5 \]

where \( I_e \) is the moment of inertia about the centroid of a section consisting of one frame plus an effective length of shell \( L_e \) defined by the following relation

\[ L_e = (L_F - b) F_1 + b \]  \[ 5a \]
Figure 2 – Layered Shell Notation

Figure 3 – Modified Bryant Coefficients for Various Bulkhead Spacings
The term in the brackets was determined for the given value of $L_B/D$ from the plot presented in Figure 3. In order to perform the calculations, it was convenient to convert the two-material shell-frame section to an equivalent all-metal section by multiplying the width of the low modulus rectangular frame by the ratio of the modulus of the lightweight material to the modulus of the metal. However, the effective length of shell $L_e$ was determined on the basis of the actual width of the low modulus material. Here again, the assumption was made that the tight fit between the frames and the shell enables the cross section to act as a unit. For the sake of comparison, the low modulus material in the frames of two models, CS-13 and CS-14, was glued to the metal on all surfaces of contact. The elastic interbay buckling pressure for each model, as calculated by means of Reference 3, was at least two and a half times the general instability buckling pressure.

The stresses in the stiffened models were calculated by the Salerno and Pulos analysis. The effective frame area was based on the equivalent all-metal frame and the Lame correction for the position of the frame. An estimate of the stresses in the layered models was obtained by assuming that the radial deflection is constant over the cross section, that the longitudinal contraction of each layer is the same, and that the magnitude of the circumferential and longitudinal stress in each layer is proportional to the modulus of elasticity of the individual layer.

The models were accurately machined to the dimensions shown in Figure 1. The Inlyte pieces were machined from bars 2 1/2 inches in diameter and 10 inches long. In the cases where it was necessary to cast the epoxy resin material in place (i.e., Models CS-4, CS-5, and CS-12), the inner aluminum shell was coated with a releasing agent to prevent the formation of a bond between the layers. As mentioned previously, all pieces were made to a Class 4 tightest fit which specified zero clearance between the pieces. Photographs of some of the models prior to collapse are shown in Figure 4.

**TEST PROCEDURE**

The models were tested to collapse under external hydrostatic pressure in a 10-in.-diameter tank with oil as the pressurizing medium. The ends of the models were sealed with a standard 0-ring and flat closure plate arrangement. The closure plates were equipped with an insert machined to a tight fit with the inner wall of the cylinder to prevent radial deflections at the ends. The end closures for the layered models were so designed that in every case the end load was transmitted to each of the layers. The models were loaded incrementally with final pressure increments less than 2 percent of the buckling pressure in every case.

Strain measurements were taken in both the circumferential and longitudinal directions at various locations on the models by means of foil resistance strain gages. These readings made possible the determination of the magnitude of the stress levels at collapse, the distribution of stress in the shells, and the number of circumferential lobes in the shells at collapse.
Figure 4a — Models CS-3, CS-5, CS-7, CS-6, and CS-4

Figure 4b — Models CS-11, CS-10, and CS-9

Figure 4c — Models CS-14 and CS-8

Figure 4 — Models prior to Testing
RESULTS AND DISCUSSION

Tables 2 and 3 compare results for the two-layered, the sandwich, and the ring-stiffened models with the pressures predicted by the methods outlined previously and, in the case of the two-layered models, with the analyses presented in References 6 and 7. The stress sensitivities calculated from the measured strains for Models CS-3 through CS-14 are compared with the theoretical stress sensitivities in Tables 4 and 5. Figure 5 shows some of the models after collapse.

Results for the two-layered and sandwich models indicate that the unbonded layers did act together although not to the same degree in every case. The analysis of Reference 7 actually provided a better estimate of the buckling strength of the two-layered shells than that obtained from Equation [41, as can be seen in Table 2a. The two-layered models which had the low modulus material on the outside of the metallic layers behaved differently than did the models with the low modulus material on the inside. Models CS-3 and CS-9 with inner layers of epoxy resin and syntactic foam, respectively, each failed at pressures more than twice those calculated for the layers acting independently but at only 57 percent of the pressure predicted for the layers acting together. However, the models with these same low modulus materials on the outside (Models CS-4 and CS-8) failed at pressures between four and five times the calculated pressures for the layers acting independently and at 88 percent and 113 percent, respectively, of the pressure predicted by Reference 7.

Although not in exact agreement with the results of the approximate theoretical analysis, the experimental strain data for the two-layered models indicate that all stresses in both the metallic and low modulus layers of all models were below the proportional limit of the materials at collapse. The mode of collapse for the two-layered models, as determined from the strain data, was \( n = 2 \), as expected. However, the measured strains indicate that the layers did not deflect radially as a unit and that, in general, the low modulus layers displayed a great deal more nonlinear behavior prior to collapse than did the aluminum layers. This latter observation indicates that perhaps the low modulus layers buckled initially and offers a possible explanation of the discrepancy in the results between models with the low modulus layer on the outside and those with it on the inside.

Although the two identical sandwich models failed at pressures approximately ten times the values predicted for the layers acting independently, each model reached only about 30 percent of the pressure for the layers acting as a unit. These poor results for the elastic sandwich models are in sharp contrast with the results for sandwich models discussed in the Appendix which failed by plastic general instability within 15 percent of the calculated pressures for the layers acting together.

The strains measured prior to buckling on the outer shell of the sandwich models indicate that the number of circumferential lobes at collapse was \( n = 3 \) and that here again the shell layers did not deflect as a unit. Inspection of the collapsed shape of Model CS-5 supported this result. This observation indicates that the layers of these models acted
### TABLE 2
Summary of Results for Layered Models

#### a. Two Layered Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Experimental Collapse Pressure psi</th>
<th>( P_{exp} )</th>
<th>( P_{exp}^* )</th>
<th>( P_{EXP} )</th>
<th>( P_{EXP}^* )</th>
<th>Low-Modulus Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS-3</td>
<td>325</td>
<td>2.74</td>
<td>0.65</td>
<td>0.58</td>
<td>0.57</td>
<td>Epoxy Resin Inside</td>
</tr>
<tr>
<td>CS-4</td>
<td>475</td>
<td>4.40</td>
<td>1.08</td>
<td>1.15</td>
<td>0.88</td>
<td>Epoxy Resin Outside</td>
</tr>
<tr>
<td>CS-8</td>
<td>548</td>
<td>4.94</td>
<td>1.43</td>
<td>1.41</td>
<td>1.13</td>
<td>Syntactic Foam Outside</td>
</tr>
<tr>
<td>CS-9</td>
<td>455</td>
<td>2.07</td>
<td>0.64</td>
<td>0.55</td>
<td>0.57</td>
<td>Syntactic Foam Inside</td>
</tr>
</tbody>
</table>

*Reference 6.
**Reference 7.

#### b. Sandwich Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Experimental Collapse Pressure psi</th>
<th>( P_{exp} )</th>
<th>( P_{EXP} )</th>
<th>Core Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS-5</td>
<td>250</td>
<td>10.00</td>
<td>0.31</td>
<td>Epoxy Resin</td>
</tr>
<tr>
<td>CS-12</td>
<td>240</td>
<td>9.60</td>
<td>0.29</td>
<td>Epoxy Resin</td>
</tr>
</tbody>
</table>

### TABLE 3
Summary of Results for Ring-Stiffened Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Experimental Collapse Pressure psi</th>
<th>( P_{exp} )</th>
<th>Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS-6</td>
<td>885</td>
<td>0.86</td>
<td>Epoxy Resin Rectangular Unbonded</td>
</tr>
<tr>
<td>CS-7</td>
<td>790</td>
<td>0.58</td>
<td>Epoxy Resin Aluminum Unbonded</td>
</tr>
<tr>
<td>CS-10</td>
<td>1175</td>
<td>1.12</td>
<td>Syntactic Foam Rectangular Unbonded</td>
</tr>
<tr>
<td>CS-11</td>
<td>1260</td>
<td>0.91</td>
<td>Syntactic Foam Aluminum Unbonded</td>
</tr>
<tr>
<td>CS-13</td>
<td>960</td>
<td>0.70</td>
<td>Epoxy Resin Aluminum Flanged Bonded</td>
</tr>
<tr>
<td>CS-14</td>
<td>418</td>
<td>1.08</td>
<td>Syntactic Foam Rectangular Bonded</td>
</tr>
</tbody>
</table>
TABLE 4

Stress Sensitivity Results for Layered Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Theory</th>
<th>Experiment</th>
<th>$\sigma_{\theta_0}^*$ (psi/psi)</th>
<th>$\sigma_{\theta_i}^{**}$ (psi/psi)</th>
<th>$\sigma_{x_0}^*$ (psi/psi)</th>
<th>$\sigma_{x_i}^{**}$ (psi/psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS-3</td>
<td></td>
<td></td>
<td>42.6</td>
<td>1.3</td>
<td>22.1</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>45.3</td>
<td>-</td>
<td>16.2</td>
<td>-</td>
</tr>
<tr>
<td>CS-4</td>
<td></td>
<td></td>
<td>1.3</td>
<td>43.5</td>
<td>0.8</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6</td>
<td>56.5</td>
<td>0.8</td>
<td>21.6</td>
</tr>
<tr>
<td>CS-8</td>
<td></td>
<td></td>
<td>3.6</td>
<td>77.6</td>
<td>2.1</td>
<td>43.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.6</td>
<td>62.2</td>
<td>1.5</td>
<td>28.1</td>
</tr>
<tr>
<td>CS-9</td>
<td></td>
<td></td>
<td>40.0</td>
<td>2.0</td>
<td>20.3</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>37.5</td>
<td>2.8</td>
<td>15.0</td>
<td>1.3</td>
</tr>
<tr>
<td>CS-5</td>
<td></td>
<td></td>
<td>45.8</td>
<td>45.8</td>
<td>23.6</td>
<td>23.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>53.8</td>
<td>32.6</td>
<td>21.0</td>
<td>15.2</td>
</tr>
</tbody>
</table>

*Subscript 0 refers to outer layer.
**Subscript $i$ refers to innermost layer.

TABLE 5

Stress Sensitivity Results for Ring-Stiffened Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Theory</th>
<th>Experiment</th>
<th>$\sigma_{\theta_{OF}}^*$ (psi/psi)</th>
<th>$\sigma_{x_{OF}}^*$ (psi/psi)</th>
<th>$\sigma_{\theta_{OM}}^{**}$ (psi/psi)</th>
<th>$\sigma_{x_{OM}}^{**}$ (psi/psi)</th>
<th>$\sigma_{\theta_{FR}}^{***}$ (psi/psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS-6</td>
<td></td>
<td></td>
<td>41.2</td>
<td>16.7</td>
<td>46.3</td>
<td>28.8</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>41.2</td>
<td>16.5</td>
<td>46.5</td>
<td>22.9</td>
<td>-</td>
</tr>
<tr>
<td>CS-7</td>
<td></td>
<td></td>
<td>34.6</td>
<td>9.3</td>
<td>45.5</td>
<td>34.6</td>
<td>33.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>32.7</td>
<td>11.7</td>
<td>50.1</td>
<td>31.2</td>
<td>-</td>
</tr>
<tr>
<td>CS-10</td>
<td></td>
<td></td>
<td>38.4</td>
<td>13.2</td>
<td>45.5</td>
<td>30.3</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38.3</td>
<td>14.4</td>
<td>48.6</td>
<td>26.7</td>
<td>2.2</td>
</tr>
<tr>
<td>CS-11</td>
<td></td>
<td></td>
<td>31.5</td>
<td>6.6</td>
<td>43.0</td>
<td>34.0</td>
<td>32.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>32.8</td>
<td>12.5</td>
<td>47.1</td>
<td>29.6</td>
<td>32.5</td>
</tr>
<tr>
<td>CS-13</td>
<td></td>
<td></td>
<td>34.6</td>
<td>9.3</td>
<td>45.5</td>
<td>34.6</td>
<td>33.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38.6</td>
<td>18.5</td>
<td>47.8</td>
<td>33.6</td>
<td>30.1</td>
</tr>
<tr>
<td>CS-14</td>
<td></td>
<td></td>
<td>70.0</td>
<td>21.7</td>
<td>87.0</td>
<td>63.4</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>69.1</td>
<td>25.0</td>
<td>-</td>
<td>-</td>
<td>2.3</td>
</tr>
</tbody>
</table>

*Subscript OF refers to outside of shell at a frame.
**Subscript OM refers to outside of shell at midbay locations.
***Subscript FR refers to inner edge of frame.
Figure 5a - Models CS-4, CS-3, and CS-9

Figure 5b - Models CS-7 and CS-12

Figure 5 - Models after Collapse
independently since the mode of collapse for each aluminum shell by itself is \( n = 3 \) according to Reference 1. The anticipated mode for the layers acting as a unit was \( n = 2 \). The difference in behavior between the elastic buckling sandwich models and the sandwich models discussed in the Appendix which failed by inelastic buckling may have been due to the fact that all the layers in the elastic models were unbonded whereas the core layer in the inelastic models was bonded to the inner aluminum shell.

The failure of these two-layered and sandwich models at pressures considerably above those predicted for the layers acting independently indicates that tightly fitted unbonded layers of low modulus and metallic materials can act somewhat as a unit. The results for a given shell configuration were consistent for both the low modulus materials used. However, it was generally not possible to predict the actual buckling pressure of the shells or to explain the difference in behavior between the various configurations studied. Further model tests will be required to determine the extent of interaction possible for unbonded shells, to establish whether existing analyses can consistently predict their buckling pressures, and to determine whether, in general, bonding of the layers is necessary for these types of construction to be considered for practical applications.

The results for the ring-stiffened models were generally in better agreement with the calculated buckling pressures than were the layered models. In this case, however, there was a significant difference between the results for the models using epoxy resin frames (CS-6, CS-7, and CS-13), and those using the higher modulus, higher strength syntactic foam material (CS-10, CS-11, and CS-14). Regardless of the type of frame—rectangular or flanged—or the presence or absence of bonding, the buckling pressures for the models with syntactic foam frames failed within about 10 percent of the pressure for full interaction as calculated from Equation [5]. The models with epoxy resin frames, however, failed at pressures between about 60 and 85 percent of the estimated pressures.

The limited test results obtained in this investigation also indicated to some extent the effect of bonding on the buckling strength of this type of shell. A bond between the low modulus frame and the shell was not observed to have an effect on the buckling strength of shells with rectangular frames, as evidenced by a comparison of the results for Models CS-10 and CS-14. However, a more severe test of the ability of this type of construction to transfer inertia occurs when a metallic flange is added to the frame. This effect is shown by comparing the test results for CS-6 with CS-7 and for CS-10 with CS-11. In this case, the bonding of the interfaces between the metallic shell, the low modulus web, and the metallic flange caused an increase in the buckling strength as illustrated by the difference in the test results for Models CS-7 and CS-13.

The experimental and calculated stress sensitivities for the ring-stiffened models agreed within about 10 percent in every case. The main discrepancy was observed in the longitudinal stresses on the outside surface at the frame and at midbay. The experimental data indicate that the frame induced less bending in the shell than was expected and that a more accurate
means of determining the effective web thickness is necessary. The strain-gage measurements also indicated that the mode of collapse for all the ring-stiffened models was \( n = 2 \) and that all stresses in the models were elastic at collapse.

Comparison of the model results with the predicted buckling pressures of the metallic shells alone illustrates that the addition of low modulus material can substantially increase the buckling strength of metallic shells. The buckling pressures predicted by Equation [1] for the aluminum shells with thicknesses of 0.02 and 0.01 in. on a 2-in. diameter are 24 and 3 psi, respectively. In general, therefore, the results for both the layered and stiffened models were encouraging despite the fact that it was not possible to consistently predict the buckling pressure of the models. This lack of agreement is not surprising because many factors were necessarily neglected in the use of approximate analyses based on existing theory for one-material stiffened and unstiffened cylinders. Theoretical analyses and systematic experimental programs which take into account such factors as the effect of bonding, friction between shell layers, relative modulus and thicknesses of the layers are necessary before the potential of the various types of construction considered here can be evaluated.

**CONCLUSIONS**

1. The use of low modulus, low density, relatively low strength materials in various stiffening systems can significantly increase the elastic stability of cylindrical shells, as demonstrated by the test results for the two-layered, the sandwich, and the ring-stiffened models of this exploratory investigation.

2. The model collapse pressures and actual stresses were not always in good agreement with the calculated values because theoretical analyses directly applicable to the types of shells under consideration were not available.

3. Further experimental investigations and the development of improved methods of analysis will be necessary to fully evaluate the potential of these types of shell construction for practical applications.

**ACKNOWLEDGMENTS**

The technical assistance of Mr. Martin A. Krenzke, Head, Design Analysis Branch, and Mr. Thomas E. Reynolds is gratefully acknowledged.
Prior to the work described in this report, two preliminary continuous core sandwich models (designated CS-1 and CS-2) were tested to collapse. This preliminary study was undertaken to determine the feasibility of using low modulus materials to increase the strength of cylindrical shells.

With the exception of their core materials, the models were identical sandwich shells consisting of inner and outer layers of 7075-T6 aluminum and a length-to-diameter ratio of 5.0. The core materials for Models CS-1 and CS-2 were, respectively, (1) an epoxy resin formed by the mixture of equal parts of Versamid 140 plyamide resin and Epon 828 resin and (2) a syntactic foam material developed by the Naval Applied Science Laboratory and referred to as the Mat Lab formula. The properties of these materials are given in Table 1. Drawings of the models showing nominal dimensions are presented in Figure 6. It should be noted that no effort was made to prevent the bonding of the core materials to the inner aluminum shells during the casting process. The core materials were not bonded to the outside shells.

When the layers were assumed to act independently, the calculated elastic buckling pressures were 620 and 1080 psi for Models CS-1 and CS-2, respectively. On the other hand, using Equation [4], the calculated elastic buckling pressures for the layers acting together were 14,800 and 15,200 psi for Models CS-1 and CS-2. At these pressures, however, the stresses in the aluminum layers would be considerably beyond the yield strength of the material. An estimate of the inelastic buckling strength of these models was obtained from the following equation

$$P_{e_i} = \frac{E_T}{E} (P')$$

where $P'$ is the elastic buckling pressure defined by Equation [4]. The stress used in conjunction with this equation is the Hencky-von Mises stress intensity defined as

$$\sigma_i = [\sigma_\theta^2 + \sigma_x^2 - \sigma_\theta \sigma_x]^{1/2}$$

where $\sigma_\theta$ and $\sigma_x$ are the calculated circumferential and longitudinal stress sensitivities in the aluminum shells. Typical stress-strain and $E_T/E$ versus $\sigma$ curves are presented in Figures 7 and 8, respectively. The pressures calculated by this procedure were 4030 and 4350 psi for Models CS-1 and CS-2, respectively.

Collapse occurred at 3465 psi for Model CS-1 and at 4730 psi for Model CS-2. Photographs of the models after collapse are shown in Figure 9. The failure of the models within 15 percent of the pressure predicted by Equation [7] indicated that the layers acted together as a unit. As a consequence of these encouraging results, it was decided to investigate this area further, and the work summarized in this report was initiated.
Figure 6 — Axial Sections of Models CS-1 and CS-2

Figure 7 — Typical Stress-Strain Curve for 7075-T6 Aluminum
Figure 8 – Typical $E_T/E$ versus $\sigma$ Curves for 7075-T6 Aluminum

Figure 9 – Models CS-1 and CS-2 after Collapse
REFERENCES


INITIAL DISTRIBUTION

Copies

16 CHBUSIIPS
   2 Sci & Res Sec (Code 442)
   1 Lab Mgt (Code 320)
   3 Tech Info Br (Code 210L)
   1 Struc Mech, Hull Mat & Fab (Code 341A)
   1 Prelim Des Br (Code 420)
   2 Prelim Des Sec (Code 421)
   1 Ship Protec (Code 423)
   1 Hull Des Br (Code 440)
   1 Struc Sec (Code 443)
   1 Sub Br (Code 525)
   1 Hull Arrgt, Struc, & Preserv (Code 633)
   1 Hull Arrgt, Struc, & Preserv (Code 651F)

2 CHONR
   1 Struc Mech Br (Code 439)
   1 Undersea Programs (Code 466)

4 CNO
   1 Tech Anal & Adv Gr (Op 07T)
   1 Plans, Programs & Req Br (Op 311)
   1 Sub Program Br (Op 713)
   1 Tech Support Br (Op 725)

2 CHBUWEP, SP-001

20 DDC
   1 CO & DIR, USNMEL
   1 CDR, USNOL
   1 DIR, USNRL (Code 2027)
   1 CO & DIR, USNUSL
   1 CDR, USNOTS, China Lake
   1 CDR, USNOTS, Pasadena
   1 CO, USNUOS
   2 NAVSHIPYD PTSMH
   2 NAVSHIPYD MARE
   1 NAVSHIPYD CHASN
   1 SUPSHIP, Groton
   1 EB Div, Gen Dyn Corp
   1 SUPSHIP, Newport News
   1 NNSB & DD Co

Copies

1 SUPSHIP, Pascagoula
   1 Ingalls Shipbldg Corp
   1 SUPSHIP, Camden
   1 New York Shipbldg Corp
   1 DIR DEF R&E, Attn: Tech Lib
   1 CO, USNROTC & NAVADMINU, MIT
   1 O in C, PGSCOL, Webb
   1 DIR, APL, Univ of Washington, Seattle
   1 NAS, Attn: Comm on Undersea Warfare
   1 WHOI
      1 Mr. J. Mavor
   1 Dr. R. DeHart, SWRI
   1 Prof J. Kempner, PIB
   1 Dean V.L. Salerno, Fairleigh Dickinson University
   1 Prof Bernard Bundiansky, Harvard University

21
EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS LIGHTWEIGHT STIFFENING SYSTEMS UNDER EXTERNAL HYDROSTATIC PRESSURE

Twelve machined cylindrical models with length-to-diameter ratios of approximately 5.0 were collapsed under external hydrostatic pressure to study the elastic buckling strength of metallic cylinders stiffened with low modulus, low density, relatively low strength materials. Three different configurations were studied: two-layered shells, continuous-core sandwich shells, and ring-stiffened shells. The test results demonstrated that the use of low modulus materials in various stiffening systems can lead to significant increases in the elastic buckling strength of metallic cylindrical shells.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical Shells</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ring-Stiffened</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-Layered</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandwich-Continuous Core</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Buckling Model Tests</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**INSTRUCTIONS**

1. **ORIGINATING ACTIVITY**: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION**: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP**: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE**: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES**: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S)**: Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE**: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES**: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES**: Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER**: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER**: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S)**: Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S)**: If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES**: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

   1. "Qualified requesters may obtain copies of this report from DDC."
   2. "Foreign announcement and dissemination of this report by DDC is not authorized."
   3. "U.S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through

      "_________________________"

   4. "U.S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through

      "_________________________"

   5. "All distribution of this report is controlled. Qualified DDC users shall request through

      "_________________________"

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES**: Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY**: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. **ABSTRACT**: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

   It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

   There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS**: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.
EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS LIGHTWEIGHT STIFFENING SYSTEMS UNDER EXTERNAL HYDROSTATIC PRESSURE, by John J. Healey. Aug 1965. iv, 21p. illus., graphs, tables, refs. UNCLASSIFIED

Two hundred cylindrical models with length-to-diameter ratios of approximately 5.0 were collapsed under external hydrostatic pressure to study the elastic buckling strength of metallic cylinders stiffened with low modulus, low density, relatively low strength materials. Three different configurations were studied: two-layered shells, continuous-core sandwich shells, and ring-stiffened shells. The test results demonstrated that the use of low modulus materials in various stiffening systems can lead to significant increases in the elastic buckling strength of metallic cylindrical shells.

EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS LIGHTWEIGHT STIFFENING SYSTEMS UNDER EXTERNAL HYDROSTATIC PRESSURE, by John J. Healey. Aug 1965. iv, 21p. illus., graphs, tables, refs. UNCLASSIFIED

Two hundred cylindrical models with length-to-diameter ratios of approximately 5.0 were collapsed under external hydrostatic pressure to study the elastic buckling strength of metallic cylinders stiffened with low modulus, low density, relatively low strength materials. Three different configurations were studied: two-layered shells, continuous-core sandwich shells, and ring-stiffened shells. The test results demonstrated that the use of low modulus materials in various stiffening systems can lead to significant increases in the elastic buckling strength of metallic cylindrical shells.

EXPLORATORY TESTS OF CYLINDERS WITH VARIOUS LIGHTWEIGHT STIFFENING SYSTEMS UNDER EXTERNAL HYDROSTATIC PRESSURE, by John J. Healey. Aug 1965. iv, 21p. illus., graphs, tables, refs. UNCLASSIFIED

Two hundred cylindrical models with length-to-diameter ratios of approximately 5.0 were collapsed under external hydrostatic pressure to study the elastic buckling strength of metallic cylinders stiffened with low modulus, low density, relatively low strength materials. Three different configurations were studied: two-layered shells, continuous-core sandwich shells, and ring-stiffened shells. The test results demonstrated that the use of low modulus materials in various stiffening systems can lead to significant increases in the elastic buckling strength of metallic cylindrical shells.