MASSACHUSETTS INSTITUTE OF TECHNOLOGY
AEROELASTIC AND STRUCTURES RESEARCH LABORATORY

Technical Report 107-1

August, 1964

PROGRAMS FOR MACHINE COMPUTATION OF ROTOR BLADE DOWNWASH

by

Nancy Ghareeb

REPORT UNDER

U.S. Navy
Bureau of Naval Weapons
Contract NOw 62-0100-d

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PROGRAM IA - COMPUTATION OF DOWNWASH DUE TO TRAILING WAKE

This program computes the downwash at the rotor blade using Equation 15 of Ref. 1.* In this program the symbol, V_1, is used for w_1 of Ref. 1 and x for η. The chord variation, x, of Ref. 1 is here assumed to be zero. The computational techniques involved in the solution of this equation by numerical methods are chiefly those of integration and harmonic analysis. It was decided to perform the numerical integration by Simpson's rule (an approximation of the curve by a series of second degree parabolas) since this method seemed most appropriate to the nature of the function. Expressed symbolically, this means that given a function, $y = f(x)$, then

$$\int_a^b y\,dx = \frac{\Delta x}{3}\left[y_o + y_n + 4\sum_{i=1}^{n-1} y_i + 2\sum_{j=2}^{n-2} y_j\right]$$

where $y_o = f(a)$ and $y_n = f(b)$, i = odd integer, j = even integer

For ease in programming, a separate subroutine was written to perform the harmonic analysis. This was done as follows:

Let g(x) represent the function to be analyzed,
then $g(x) = a_o + \sum_{k=1}^{n} (a_k \cos k\,x + b_k \sin k\,x)$ where the a_k and b_k are defined in the following relations:

$$a_o = \frac{1}{2\pi}\int_Q^{Q+2\pi} g(x)\,dx$$

$$a_k = \frac{1}{\pi}\int_Q^{Q+2\pi} g(x)\cos k\,x\,dx$$

$$b_k = \frac{1}{\pi}\int_Q^{Q+2\pi} g(x)\sin k\,x\,dx \qquad (k \neq 0)$$

---

* R. H. Miller. "Theoretical Determination of Rotor Blade Harmonic Airloads." M. I. T. ASRL TR 107-2, 1964.

This analysis is, of course, applicable only for a range of 2π ($Q \rightarrow Q + 2\pi$). The integration again was performed using Simpson's rule.

The programs are written in Fortran II and should be operable at any IBM 709 or 7090 installation with one exception. The Massachusetts Institute of Technology Computation Center at which these programs were developed assigns logical tape numbers to physical tapes as follows:

Physical	Logical	
A_1	1	
A_2	4	input
A_3	2	output for printing
A_4	8	
B_1	5	
B_2	6	
B_3	7	
B_4	3	output for punching

Interval sizes of 7.5° for ψ and 7.5° and 2.5° for ϕ used with an m of 3 and QT of $-\frac{\pi}{2}$ have produced results of sufficient accuracy for the present purposes.

<u>CAUTION:</u> Care must be taken in selecting interval sizes so that an odd number of points is to be used in each integration.

<u>The Function $V_1(\psi, x)$ - Trailing Wake</u>

$$V_1 = \int_{\psi+\gamma}^{2\pi m+\psi+\gamma} K_1(\phi) \frac{\left\{\ell\left[\ell + d\cos\phi - x\cos(\psi-\phi) + \mu\sin\phi\right] - \mu x\sin\psi\right\} d\phi}{\left[\ell^2 + x^2 + d^2 + z^2 - 2x\,\ell\cos(\psi-\phi) + 2d(\ell\cos\phi - x\cos\psi)\right]^{3/2}}$$

where $K_1 = K_c \cos n\phi + K_s \sin n\phi$

$$d = \mu[(2\pi m + \psi + \gamma) - \phi]$$

$$z = \lambda[(2\pi m + \psi + \gamma) - \phi](1.0 + a \cos\phi) - b_o(\ell - x)$$

This program computes the value of the function for $K_c = K_s = 1$ for each $\ell - x$ pair used at from one to four values of γ for a given number of values of ψ. A value of zero is assumed to be among the γ's read into the computer for each $\ell - x$ pair, so that if only one value of γ is to be used that γ must be zero. The integration for the $\gamma = 0$ case <u>only</u> is split into two sections and integration performed between limits of $\psi + \gamma \rightarrow 2\pi m + \psi + \gamma + QT$ and $2\pi m + \psi + \gamma + QT \longrightarrow 2\pi m + \psi + \gamma$. The results of the latter integration are termed ZAPSI ($z_a(\psi)$) for the cos component and ZBPSI ($z_b(\psi)$) for the sin component. The results of integration for the first section and the results of integration for any other γ's are termed YiPSI (i = 1,2,3,4,5,6,7,8). The odd integers are assigned to the cos components and the even integers to the sin components. The i's will correspond to the γ's in the order in which the γ's were read into the computer, i.e., Y1PSI and Y2PSI will be assigned to the results for the first γ read into the computer.

Next the YiPSI terms are summed at each ψ. The sum for the cos components is called CHI (χ) and the sum for the sin components is called SIGMA (σ). Harmonic analysis is then performed on each of the four functions, χ, σ, $z_a(\psi)$, and $z_b(\psi)$.

The time required to obtain results from the IBM 7090 computer for one $\ell - x$ combination with 3 γ's, m = 3 and interval sizes of 7.5°, 7.5° and 2.5° is approximately two minutes.

Subroutines required with this program will be described in the following pages.

The listing of the program in Fortran II together with the subroutines is given in Appendix A.

Typical results are listed in Appendix B.

Physical Explanation of Symbols

$\psi \approx$ rotor azimuth measured from blade downwind position

$\gamma \approx$ blade spacing (cf. ζ)

$m \approx$ number of wake spirals

$V_1(\psi, x) \approx w_1(\psi, x)$

$\ell \approx$ rotor span parameter

$\phi \approx$ azimuth of wake measured from downwind position

$x \approx$ rotor span parameter (η of Ref. 1)

$n \approx$ harmonic of rotor speed

$\mu \approx$ advance ratio

$\lambda \approx \lambda_o$

$a \approx \dfrac{\lambda_1}{\lambda_o}$

z – vertical distance travelled by rotor hub

d – horizontal distance travelled by rotor hub

b_o – coning angle

Input Format

Card No. 1	L	KL	NP	II	MM	NN	QT
	I2	I2	I2	I2	I2	I2	E 13. 8

Card No. 1A BO (FORMAT E8. 3)

Card No. 2	EMU	AMBDA	EM	DEPSI	DELPHI	DELTA	K	IJ
	E 8. 3	E 8. 3	E 8. 3	E 13. 8	E 13. 8	E 13. 8	I3	I2

Card No. 3	A	IK
	E 8. 3	I2

Card No. 4	ELK	IL
	E 8. 3	I2

Card No. 5	EN	IM
	E 8. 3	I2

Card No. 6	X	IN
	E 8. 3	I2

Card No. 7	GAMMA
	E 13. 8

Explanation of Symbols

L	number of harmonics to be computed in the Fourier analysis
KL	$+1 \rightarrow$ print optional table No. 2, $-1 \rightarrow$ suppress this print out
NP	interval at which table No. 2 is printed (i.e., +1 will cause printout of integral at every ψ used, +2 at every other ψ etc.)
II	number of times card No. 2 is to be repeated
MM	$+1 \rightarrow$ print optional table No. 1, $-1 \rightarrow$ suppress this print out
NN	interval at which table No. 1 is printed (see NP)
QT	determines limits of integration for $\gamma = 0$ case (i.e., $QT = -\frac{\pi}{2}$ will cause integration from $\psi + \gamma$ to $\psi + \gamma + 2\pi m - \pi/2$ and from $\psi + \gamma + 2\pi m - \pi/2$ to $\psi + \gamma + 2\pi m$)
BO	b_o
EMU	μ
AMBDA	λ
EM	m
DEPSI	interval size used for ψ
DELPHI	interval size used for ϕ [in $\gamma = 0$ case, in region $\psi + \gamma \rightarrow \psi + \gamma + 2\pi m + QT$]
DELTA	interval size used for ϕ for $\gamma = 0$ case in region $\psi + \gamma + 2\pi m + QT$ to $\psi + \gamma + 2\pi m$
K	number of points of ψ to be used in harmonic analysis
IJ	number of values of a to follow
A	a
IK	number of values of ℓ to follow
ELK	ℓ
IL	number of values of n to follow
EN	n
IM	number of values of x to follow
X	x
IN	number of values of γ to follow
GAMMA	γ

Output Format

MU = LAMBDA = M = N = L = A = X =

DELTA PSI = DELTA PHI (1) = DELTA PHI (2) =

CHI

A - ZERO =

A - k = B - k =

↓ ↓

SIGMA

A - ZERO =

A - k = B - k =

↓ ↓

ZAPSI

A - ZERO =

A - k = B - k =

↓ ↓

ZBPSI

A - ZERO =

A - k = B - k =

↓ ↓

[Optional Table No. 1]

CHI SIGMA PSI ZAPSI ZBPSI

↓ ↓ ↓ ↓ ↓

[Optional Table No. 2]

PSI Y1PSI Y2PSI Y3PSI Y4PSI etc

↓ ↓ ↓ ↓ ↓

DELTA PHI (1) $\approx$ DELPHI [see explanation of symbols (input)]
DELTA PHI (2) $\approx$ DELTA

A - ZERO $\approx$ constant term resulting from Fourier analysis

A - k $\approx$ coefficients of the cos k ψ terms resulting from the Fourier analysis

B - k $\approx$ coefficients of the sin k ψ terms resulting from the Fourier analysis

Subroutine Harnal (PSI, FPSI)

This subroutine performs the actual harmonic analysis as previously described.

Information is transmitted to and from this subroutine by means of a common statement.

COMMON PSI, FPSI, I, L, K, DEPSI, AQUAY, BQUAY, A-ZERO

where

PSI - variable

FPSI - a function of PSI

I - an indicator to show type of function so that excess computation is avoided, i.e., +1 → an even function, +2 → an odd function, +3 → function is neither even nor odd. This is automatically set in (MAIN)

L - number of harmonics to be computed (maximum = 20)

K - number of values of PSI to be used, must be an odd integer ≥ 3

DEPSI - interval size used for ψ

AQUAY - output - the coefficients a-k of the cos kψ terms

BQUAY - output - the coefficients b-k of the sin kψ terms

A-ZERO - output - the constant term

It is assumed that the table of ψ will describe a range of 2π if the function is neither even nor odd and otherwise a range of π.

Subroutine Prince (L, AQUAY, BQUAY, A ZERO)

This subroutine prints out the results of the harmonic analysis in the following form:

A-ZERO =	(← constant term)
A - 1 =	B - 1 =
A - 2 =	B - 2 =
↓	↓
A - L =	B - L =
(coefficients of cos k ψ terms)	(coefficients of sin k ψ terms)

An optional version also punches these results.

Subroutine TRIP (CHI, SIGMA, PSI, ZAPSI, ZBPSI, NN, K)

This subroutine prints optional table No. 1.

Subroutine PRICK (NP)

Information is transmitted to this subroutine via COMMON. It is used to print optional table No. 2. The subroutine must conform to the number of γ's used in MAIN. If 3 γ's are used PRICK must output PSI and YiPSI (i = 1 to 6). The present version is written for three γ's, hence some modification will be necessary if a different number of γ's is to be used.

Sample Data for Program IA

Card No. 1	10 + 1 + 1 + 2 + 1 + 1 - . 15707963 E + 1
Card No. 1A	+. 069 E+0
Card No. 2	+ . 100 E + 0 + . 500 E - 1 + . 300 E + 1 + . 13089969 E + 0 + . 13089969 E + 0 + . 43633233 E - 1 + 49 + 1
Card No. 3	+ . 500 E + 0 + 1
Card No. 4	+ . 100 E + 1 + 1
Card No. 5	+ . 300 E + 1 + 1
Card No. 6	+ . 550 E + 0 + 3
Card No. 7	+ . 0000 0000 E + 0
Card No. 8	+ . 20943950 E + 1
Card No. 9	+ . 41887901 E + 1
Card No. 10	+ . 100 E + 0 +.200 E - 1 + . 300 E + 1 + . 13089969 E + 0 + . 13089969 E + 0 + . 43633233 E - 1 + 49 + 1
Card No. 11	+ . 000 E + 0 + 1
Card No. 12	+ . 100 E + 0 + 1
Card No. 13	+ . 000 E + 0 + 1
Card No. 14	+ . 975 E + 0 + 3
Card No. 15	+ . 0000 0000 E + 0
Card No. 16	+ . 20943950 E + 1
Card No. 17	+ . 41887901 E + 1

Note that the data block (cards No. 3→ 9) following card No. 2 must be repeated (cards No. 11→17) even though some of the parameters are unchanged.

PROGRAM 1B - COMPUTATION OF DOWNWASH DUE TO SHED WAKE

This program computes the downwash at the rotor blade using Equation 21 of Ref. 1 with the symbol, V_2, substituted for w_2 and x for η as before.

$$V_2 = \int_{\psi+\gamma}^{\psi+\gamma+2\pi m} K_2(\phi)\left[\frac{x\sin(\psi-\phi)+d\sin\phi}{z^2+(x\sin(\psi-\phi)+d\sin\phi)^2}\right]$$

$$\left[\frac{\ell+d\cos\phi-x\cos(\psi-\phi)}{\sqrt{\ell^2+x^2+z^2+d^2-2x\ell\cos(\psi-\phi)+2d(\ell\cos\phi-x\cos\psi)}}-\frac{d\cos\phi-x\cos(\psi-\phi)}{\sqrt{x^2+z^2+d^2-2xd\cos\psi}}\right]d\phi$$

$$d = \mu\,[(2\pi m+\psi+\gamma)-\phi]$$

$$z = \lambda\,[(2\pi m+\psi+\gamma)-\phi]$$

$$K_2(\phi) = K_c\cos n\phi + K_s\sin n\phi \qquad (K_c \text{ and } K_s \text{ are set to } 1.0)$$

This program evaluates the integral in two steps. For the first step (from $\psi+\gamma \rightarrow 2\pi m+\psi+\gamma+QT$) integration is performed by Simpson's rule using K_2 as defined above. The results of this integration are labelled S1PSI for the cos component and S2PSI for the sin component. For the second step $\cos n\phi$ and $\sin n\phi$ are expanded to

$\sin n(\psi-\phi)\sin n\psi + \cos n(\psi-\phi)\cos n\psi$ and
$\cos n(\psi-\phi)\sin n\psi - \sin n(\psi-\phi)\cos n\psi$ respectively.

The integration is then carried out from $(\psi+\gamma+2\pi m+QT)$ to $(\psi+\gamma+2\pi m)$ using $K_2 = K_c\sin n(\psi-\phi)\sin n\psi + (-K_s)\sin n(\psi-\phi)\cos n\psi$. The coefficient of K_c is called S4PSI and the coefficient of $(-K_s)$ is called S3PSI. The value of the function at the upper limit is approximated by setting $\phi = \phi - 1/2\,\Delta\phi$. This gives a very good approximation when the interval size used in this region is on the order of 2.5°. Harmonic analysis is then performed on all four of these functions (S1PSI, S2PSI, S3PSI, S4PSI). This program does not consider those terms involving $\cos n(\psi-\phi)$.

The time required for one case on the IBM 7090 computer (one x, one ℓ, one γ) is approximately .5 minutes for a 7.5° interval size and m = 3.

Subroutines HARNAL and PRINCE are used with this program. The program used for V_2 is listed in Appendix C with typical results in Appendix D.

Input Format

Card No. 1	L	NP	KL	II	QT			
	I2	I2	I2	I2	E 13.8			
Card No. 2	EMU	AMBDA	EM	DEPSI	DELPHI	DELTA	K	IJ
	E8.3	E8.3	E8.3	E13.8	E13.8	E13.8	I3	I2
Card No. 3	EN	IK						
	E8.3	I2						
Card No. 4	GAMMA	IL						
	E13.8	I2						
Card No. 5	ELK	IM						
	E8.3	I2						
Card No. 6	X							
	E8.3							

Explanation of Symbols

L	number of harmonics to be calculated
NP	interval of printing (+1 $\rightarrow$ print integral for every ψ used, +2 for every other ψ, etc)
KL	-1 $\rightarrow$ print table of SiPSI, +1 $\rightarrow$ suppress this table
II	number of cards No. 2 to follow
QT	determines limits of integration (QT = $-\frac{\pi}{2}$ will cause integration from $\psi + \gamma$ to $\psi + \gamma + 2\pi m - \pi/2$ and from $\psi + \gamma + 2\pi m - \pi/2$ to $2\pi m + \psi + \gamma$)
EMU	μ
AMBDA	λ
EM	m
DEPSI	interval size to be used for ψ
DELPHI	interval size to be used for ϕ in region $\psi + \gamma \rightarrow 2\pi m + \psi + \gamma$
DELTA	interval size to be used for ϕ in region $2\pi m + \psi + \gamma + QT \rightarrow 2\pi m + \psi + \gamma$
K	number of values of ψ to be used in the harmonic analysis
IJ	number of values of n to follow
EN	n
IK	number of values of γ to follow
GAMMA	γ
IL	number of values of ℓ to follow
ELK	ℓ
IM	number of values of x to follow
X	x

Output Format

MU = LAMBDA = M = N = GAMMA = L = X =

(optional table)

PSI S1PSI S2PSI S3PSI S4PSI

S1PSI

A-ZERO =

A - k = B - k =

S2PSI

A-Zero =

A - k = B - k =

S3PSI

A-ZERO =

A - k = B - k =

S4PSI

A-ZERO =

A - k = B - k =

Sample Data for Part I, B

Card No. 1 10 + 1 - 1 + 1 - .15707963 E + 1

Card No. 2 + .100 E + 0 + .500 E - 1 + .300 E + 1 + .13089969E + 0 + .13089969 E + 0 + .43633233 E - 1 + 49 + 1

Card No. 3 + .300 E + 1 + 2

Card No. 4 + .0000 0000 E + 0 + 1

Card No. 5 +.100 E + 1 + 1

Card No. 6 +.800 E + 0

Card No. 7 + .20943950 E + 1 + 1

Card No. 8 + .100 E + 1 + 1

Card No. 9 + .800 E + 0

PROGRAM II. ORIGINAL STRAIGHT LINE APPROXIMATION TO WAKE

This program attempts to define the trailing wake, "w", by means of the very much simplified expressions given below (see Ref. 1):

$$w = \frac{\Gamma}{4\pi R} \cdot \frac{2(y-\eta)\cos\delta}{Z^2 + (y-\eta)^2\cos^2\delta} \qquad \text{except for } \zeta = 0 \qquad (1)$$

where $y = d\cos\psi + \sqrt{\ell^2 - d^2\sin^2\psi}$ (1a)

$d = \mu[\zeta + \psi - \phi]$ (1b)

$Z = \lambda[\zeta + \psi - \phi] - a_o(\ell - \eta)$ (1c)

$d\sin\psi = \ell\sin(\phi - \psi)$ (1d)

$$\tan(\phi - \psi - \delta) = \frac{-\mu\cos\phi}{\ell + \mu\sin\phi} \qquad (1e)$$

Values for ϕ were found by combining equations (1b) and (1d) to give an expression of the form $f(\phi) = 0.0$. An initial guess was made for ϕ and that guess improved by a Newton-Raphson iteration where the improved ϕ equalled the original ϕ minus $f(\phi)/f^1(\phi)$. [$f^1(\phi)$ represents the derivative of $f(\phi)$ with respect to ϕ].

When $\zeta = 0$ Then $w = \frac{\Gamma}{4\pi R} \frac{1 - \sin\delta}{(\ell - \eta)\cos\delta}$

where $\delta = \tan^{-1}\left(\frac{\mu\cos\psi}{\ell + \mu\sin\psi}\right)$

since $\phi = \psi$, $d = Z = 0$ $y = \ell$

This program also performs harmonic analysis of the function as summed over the blades.

The function $4\pi Rw/\Gamma$ and its harmonic coefficients are also punched out in a format suitable for input to Program III.

This program was written in FORTRAN II. It requires approximately .05 minute on an IBM 7094 computer to handle a case of one α, one η, one μ, and λ and 4 ζ at intervals of 15° in ψ. The program was developed at the Massachusetts Institute of Technology Computation Center in Cambridge, Massachusetts.

This program requires two subroutines:

OUD1 - to compute d and $\phi - \psi$
HANEW - to perform harmonic analysis

NOTE: INPUT AND OUTPUT are taken care of by

WRITE OUTPUT TAPE 2----------
READ INPUT TAPE 4 ----------

Therefore, it may be necessary to change an IOU table to correspond. A list of the logical to physical tape correspondences may be found under Program IA.

EXPLANATION OF SYMBOLS

Γ = circulation

R = rotor radius

d = horizontal distance travelled by rotor hub

z = vertical distance travelled by rotor hub

η = rotor span parameter (equivalent to x in program I)

μ = advance ratio

λ = inflow normal to rotor disc

ψ = rotor azimuth

ℓ = rotor span parameter

a_o = coning angle

S = blade spacing

δ = angle between vortex line and a perpendicular to the blade

ϕ = wake azimuth

INPUT FORMAT

Card No. 1	II	IK	NZ	NH	K	DEPSI	AO
	I2	I2	I2	I3	I3	E13.8	E8.3

Card No. 2	EMU	AMBDA	IJ
	E8.3	E8.3	I2

Card No. 3	EL	ETA
	E8.3	E8.3

EXPLANATION OF SYMBOLS

II = a counter to be set equal to the number of times Card No. 1 is to be repeated

IK = a counter to be set equal to the number of times Card No. 2 is to be repeated

NZ = the number of blades to be considered

NH = the number of harmonics to be considered when performing harmonic analysis

K = the number of values of ψ at which the function is to be computed, must be odd

DEPSI = the interval size to be used for ψ

AO = a_o

EMU = μ

AMBDA = λ

IJ = counter to be set equal to the number of times Card No. 3 is to be repeated

EL = ℓ

ETA = η

OUTPUT FORMAT

— BLADES MU = LAMBDA = L = ETA = DELTA PSI =

PSI TOTAL WAKE CONTRIBUTIONS FROM INDIVIDUAL BLADES

↓ ↓ ↓

HARMONIC ANALYSIS

A - zero =

A - i = B - i =

Here MU = μ LAMBDA = λ , L = ℓ , ETA = η , PSI = ψ DELTA PSI = interval size used in ψ .

CONTRIBUTIONS FROM INDIVIDUAL BLADES - These blades will be W $\frac{4 \pi R}{\Gamma}$ calculated at specific blade spacings or values of ζ in order of increasing ζ.

TOTAL WAKE - This column will be the sum of W $\frac{4 \pi R}{\Gamma}$ over all blades at each ψ.

A - zero $\simeq$ constant term resulting from harmonic analysis.

A - i $\simeq$ coefficients of the ith cosine terms resulting from harmonic analysis.

B - i $\simeq$ coefficients of the ith sine terms resulting from harmonic analysis.

There may also be an error message.

NO CONVERGENCE AT ZETA = . This message will be printed out if there is no convergence after repeating the iteration ten times.

In this case, the component of the trailing wake under consideration is set to zero.

PROGRAM III - ROTOR LOAD ANALYSIS

1. Description of Procedure

The nondimensionalized trailing wake, $V_1(\psi)$* is defined over a range of $0 \rightarrow 2\pi$ in the azimuth angle ψ. Values of ψ generally vary by 75° or 15°. For any given configuration, "m" values of η and "m + 1" values of ℓ, where ℓ and η are rotor span parameters, are considered, so that there are "$m^2 + m$" sets of $V_1(\psi)$. The members of these sets are designated as $A_{jk}(\psi)$ where "j" refers to a specific value of η and k refers to a specific value of ℓ. Harmonic analysis with respect to ψ is performed on each of these sets, so that there are also "$m^2 + m$" series of "2n + 1" harmonic coefficients P and Q such that $A_{jk}(\psi) \simeq P_{o_{jk}} + \sum_{i=1}^{n} (P_{i_{jk}} \cos i\psi + Q_{i_{jk}} \sin i\psi)$. In the following write-up, i will always indicate a harmonic, j a specific value of η, and k a specific value of ℓ.

A. To find the load coefficients, a system of "m" equations is set up:

$$\lambda_{o_i} = \frac{b}{2R} \sum_{t=1}^{m} \left[(\eta_t \theta_t - \lambda_{o_t})(P_{o_{it}} - P_{o_{it+1}}) \right] - \mu \tan i$$

and the λ_{o_i} are determined. These may then be compared with the "λ" or "λ"'s used in calculating the P's and Q's. If desired an iterative process may be set up, replacing the "λ" with the λ_{o_i}, until reasonable agreement is obtained between λ and the range of λ_{o_i}.

$$* \quad V_1(\psi) = \int_{\psi+\zeta}^{\psi+\zeta+2\pi M} \frac{[\ell(\ell + d\cos\phi - \eta\cos(\psi-\phi) + \mu\sin\phi) - \mu\eta\sin\psi]\,d\phi}{[\ell^2 + d^2 + \eta^2 + Z^2 - 2\eta\ell\cos(\psi-\phi) + 2\ell d\cos\phi - 2\eta d\cos\psi]^{3/2}}$$

A λ_{ic_j} and a λ_{is_j} may also be obtained.

$$\lambda_{ic_j} = \frac{b}{2R} \sum_{t=1}^{m} (\eta_t \theta_t - \lambda_{o_t})(P_{i_{jt}} - P_{i_{jt+1}})$$

$$\lambda_{is_j} = \frac{b}{2R} \sum_{t=1}^{m} (\eta_t \theta_t - \lambda_{o_t})(Q_{i_{jt}} - Q_{i_{jt+1}})$$

B. The time history of downwash $\lambda_j(\psi)$ is calculated in the following manner:

$$\lambda_j(\psi) = \frac{b}{2R}\left[\sum_{t=1}^{m} (\eta_t \theta_t - \lambda_{o_t})(A_{jt}(\psi) - A_{jt+1}(\psi))\right] - \mu \tan i$$

An alternate method $[\lambda_j(\psi) = \lambda_{o_j} + \sum_{i=1}^{n} (\lambda_{ic_j} \cos i\psi + \lambda_{is_j} \sin i\psi)]$

was rejected, because it did not appear to give a very accurate reconstruction, unless some rather high harmonics were included.

C. The distributed airloads $L_{n(c,s)_j}$ for $n > 2$ are calculated for each η_j as follows:

Let $\gamma_{ic_j} = F_j \lambda_{ic_j} - G_j \lambda_{is_j}$ where $i = 2,3,4,5,6,7$ and F and G are generally, but not necessarily, constant over i and j.

and $\gamma_{is_j} = F_j \lambda_{is_j} + G_j \lambda_{ic_j}$

Then $L_{ic_j} = C\left\{\eta_j \gamma_{ic_j} + \frac{\mu}{2}[\gamma_{i+1,s_j} - \gamma_{i-1,s_j}]\right\} L_{i(c,s)_j}$ for $i = 3,4,5,6$

$$L_{is_j} = C\left\{\eta_j \gamma_{is_j} + \frac{\mu}{2}[\gamma_{i-1,c_j} - \gamma_{i+1,c_j}]\right\}$$

C is a conversion factor, depending upon the dimensions of the test model under consideration.

D. The load function $L_i(\psi)$ is computed in two parts:

$$(1)\quad L_{I_i}(\psi) = -C(\eta_i + \mu \sin\psi)\,\lambda_i(\psi)$$

$$(2)\quad L_{G_i}(\psi) = +C\Big\{\theta_i(\eta_i^2 + \mu^2/2) - \eta_i \mu a_1 + [2\mu\theta_i\eta_i - a_1(\eta_i^2 + (3/4)\mu^2)]\sin\psi + [b_1(\eta_i^2 + \mu^2/4) - \mu a_o \eta_i]\cos\psi + (\mu b_1 \eta_i - \frac{\mu^2}{2} a_o)\sin 2\psi + (\mu a_1 \eta_i - \frac{1}{2}\mu^2\theta_i)\cos 2\psi + \frac{\mu^2}{4} a_1 \sin 3\psi - \frac{\mu^2}{4} b_1 \cos 3\psi\Big\}$$

$$L_i(\psi) = L_{I_i}(\psi) + L_{G_i}(\psi)$$

C is again a conversion factor, and a_o, a_1, b_1 are geometric parameters (blade coning angle and first harmonic flapping).

$L_i(\psi)$ is also computed with the zero-th, first, and second harmonics eliminated. (The third harmonics in $L_{G_i}(\psi)$ are considered negligible.)

$$\tilde{L}_i(\psi) = C\Big\{(\eta_i + \mu\sin\psi)\,\lambda_i(\psi) - \eta\lambda_{o_i} - \frac{\mu}{2}\lambda_{1s_i} - (\eta_i\lambda_{1c_i} + \frac{\mu}{2}\lambda_{2s_i})\cos\psi - (\eta_i\lambda_{1s_i} + \mu\lambda_{o_i} - \frac{\mu}{2}\lambda_{2c_i})\sin\psi - (\eta_i\lambda_{2c_i} - \frac{\mu}{2}\lambda_{1s_i} + \frac{\mu}{2}\lambda_{3s_i})\cos 2\psi - (\eta_i\lambda_{2s_i} + \frac{\mu}{2}\lambda_{1c_i} - \frac{\mu}{2}\lambda_{3c_i})\sin 2\psi\Big\}$$

NOTE: If it is desired to do a case where the number of I's is not one greater than the number of η, then a different procedure must be used. Suppose Program II has been run for five values of η and two values of ℓ.

$$\lambda_{o_i} = \rho\,[P_{o_{i1}} - P_{o_{i2}}] - \mu\tan i$$

$$\lambda_{ic_i} = \rho\,[P_{i_{i1}} - P_{i_{i2}}]$$

$$\lambda_{is_i} = \rho\,[Q_{i_{i1}} - Q_{i_{i2}}] \qquad \text{and} \qquad [i = 1 \rightarrow 5]$$

$$\lambda_i(\psi) = \rho\,[A_{i1}(\psi) - A_{i2}(\psi)] - \mu\tan i$$

Where ρ is an appropriate conversion factor. A rough approximation would be $\frac{b}{2R}$ times the average $(\eta_i \theta_i - \lambda)$.

These "λ" must then be put into the appropriate card format for input data to Program III C. If a small computer is available it is easy to write a simple program to do this.

2. Description of Variables

$V_1(\psi)$ = non-dimensionalized trailing wake for

ψ = azimuth angle for rotor

δ = determined by blade spacing

M = number of wake spirals

l, η = rotor span parameters (η is the same as x in program I)

d, z = horizontal and vertical distances travelled by rotor hub

μ = advance ratio

ϕ = azimuth angle for wake

m = number of values of η under consideration

n = number of harmonics taken in harmonic analysis of $V_1(\psi)$

ic or is = indicates an ith cosine or sine harmonic resulting from harmonic analysis

j = indicates a specific value of $\eta = \eta_j$

k = indicates a specific value of $l = l_k$

F = lift deficiency function - from data obtained from shed and harmonic trailing wakes.

G = lift phasing function - from data obtained from shed and harmonic trailing wakes.

3. Program Mechanics

This program is written in FORTRAN II. It requires approximately .3 minutes to handle a case involving five values of η on the IBM 7094. One subroutine "CROUT" is used to solve the system of simultaneous equations. A maximum of 9 η, θ, F or G, a maximum of 50 values ψ, and a maximum of 20 harmonics may be used. This program was developed with the aid of the facilities at the Massachusetts Institute of Technology Computation Center.

INPUT FORMAT

Card No. 1	DEPSI	BOER	TANMU	EMU	CF	II	M	KP	NH	NGP	I2P
	E13.8	E8.3	E8.3	E 8.3	E8.3	I3	I3	I3	I3	I3	I3

Card No. 2	ETA (I)	THETA (I)	F(I)	G(I)
	E8.3	E8.3	E8.3	E8.3

This card format must be repeated "m" times in succession

Card No. 3

A. If I2P = 1

$$\left.\begin{array}{ll} 1.\ A(I,\ J,\ K) & 5E14.7 \\ 2.\ B(I,\ J,\ K) & 6E12.5 \end{array}\right\} \text{group } \beta$$

These cards are to be grouped by η then by ℓ i.e., hold η constant; enter 1. and 2. for ℓ_1 followed by 1 and 2 for ℓ_2 etc., up to 1. and 2. for ℓ_{m+1} then consider $\eta = \eta_2$ and repeat the process.

B. If I2P = 2

In this case the ordering will be the same, except that each subgroup of 1 and 2 will be augmented by 3 and 4

1.	A(I, J, K)	5E14.7
2.	B(I, J, K)	6E12.5
3.	C(K)	5E14.7
4.	D(K)	6E12.5

in this case 1 and 3 are added together as are 2 and 4.

Card No. 4	AZERO	AONE	BONE
	3E8.3		

Explanation of Variables

DEPSI = interval size used for ψ

BOER = $\frac{b}{2R}$

TANMU = μ tan i

EMU = μ

CF = a conversion factor depending on the geometry of the model being considered

II = number of times Card No. 1 is to be repeated

M = number of values of η

KP = number of points of ψ

NH = number of harmonics (o^{th} term + all cos coefficients + all sin coefficients)

NGP = number of card No. 4 to be input

I2P = +2 indicates C and D arrays are to be input, + 1 indicates only A and B are to be input

ETA (I) = η_i

THETA (I) = θ_i

F(I) = F_i

G (I) = G_i

A(I, J, K) = harmonic coefficients resulting from the harmonic analysis of B(I, J, K)

B(I, J, K) "KP" values of the trailing wake function (If the function has been separated into near and far wakes, this will be the far wake portion)

* C(K) harmonic coefficients resulting from the harmonic analysis of D(K)

* D(K) "KP" values of the near wake portion of the trailing wake to be added on to B

* These will be omitted if I2P = +1

AZERO = a_o

AONE = a_1 Blade flapping is defined as $a_o - a_1 \cos \psi - b_1 \sin \psi$.

BONE = b_1

OUTPUT FORMAT

Page 1 LOAD COEFFICIENTS FOR ETA = , , ,

HARMONIC NO. i ($\eta = \eta_1$) ($\eta = \eta_2$) ($\eta = \eta_m$)

i + 1

(These will follow sequentially so that the first sine harmonic will be assigned a number equal to that of the last cosine harmonic plus one)

Page 2 LAMBDA (PSI) FOR ETA =

psi $\lambda_i(\psi)$

Page 3 ETA L3-C L3-5 L4-C L4-S L5-C L5-S L6-C L6-S

Page 4 ETA =

PSI L(I) L(G) L(I) + L(G) L(G + I) - HARMONICS EXTRACTED

Explanation of Symbols

ETA = η PSI = ψ

LOAD COEFFICIENTS = λ_{o_i}, λ_{ic_i}, λ_{is_i}

LAMBDA (PSI) = $\lambda_i(\psi)$

Ln - C, Ln-S = L_{ic_i}, L_{is_i}

L(I) = $L_{I_i}(\psi)$

L(G) = $L_{G_i}(\psi)$

L(G + I) = HARMONICS EXTRACTED - $\tilde{L}_i(\psi)$

APPENDIX A

Program for V_1 - Trailing Wake

```
      DIMENSION Y1PSI(361),Y2PSI(361),Y3PSI(361),Y4PSI(361),Y5PSI(361),
    1 Y6PSI(361),Y7PSI(361),Y8PSI(361),Z4PSI(361),Z8PSI(361),PSI(361),
    2 CHI(361),SIGMA(361),AQUAY(20),BQUAY(20)
       COMMON PSI,Y1PSI,I,L,K,DEPSI,AQUAY,BCUAY,AZERO,Y2PSI,Y3PSI,
    1  Y4PSI,Y5PSI,Y6PSI,Y7PSI,Y8PSI
   1   READ INPUT TAPE 4,2,L,KL,NP,II,MM,NN,QT
  2     FORMAT(6I2,E13.8)
      READ INPUT TAPE 4,6,BO
  3     READ INPUT TAPE 4,4,EMU,AMBDA,EM,DEPSI,DELPHI,DELTA,K,IJ
  4      FORMAT(3E8.3,3E13.8,I3,I2)
5     READ INPUT TAPE 4,6,A,IK
6     FORMAT (E8.3,I2)
7     READ INPUT TAPE 4,6,ELK,IL
8     READ INPUT TAPE 4,6,EN,IM
9     READ INPUT TAPE 4,6,X,IN
        M = 5
10   WRITE OUTPUT TAPE 2,11,EMU,AMBDA,EM,EN,ELK,A,X,DEPSI,DELPHI,DELTA
11    FORMAT(6H1MU = F6.3,13H     LAMBDA = F7.4,9H       M = F6.3,
    19H       N = F6.3,9H       L = F6.3,9H       A = F6.3,9H       X = F6.3//
    2 13HODELTA PSI = F8.5,20H      DELTA PHI(1) = F8.5,20H      DELTA PH
    3I(2) = F8.5)
        DO 12 J = 1,K
        CHI(J) = 0.0
       SIGMA(J) = 0.0
12      CONTINUE
13      READ INPUT TAPE 4,14,GAMMA
14      FORMAT (E13.8)
         M = M - 1
       PSI(1) = 0.0
         DO 74 J = 1,K
         ALPHA = 0.0
        OMEGA = 0.0
        BETA = 0.0
        ZETA = 0.0
        PHI = PSI(J) + GAMMA
        SPHI = PHI + 6.2831852*EM
19        ASSIGN 25 TO JK
20        D = EMU*(SPHI - PHI)
     Z = AMBDA*(SPHI-PHI)*(1.0+A*COSF(PHI))-BO*(ELK-X)
        RHO = ELK**2+X**2+D**2+Z**2-2.0*X*ELK*COSF(PSI(J)-PHI)+2.0*D*
    1 (ELK*COSF(PHI)-X*COSF(PSI(J)))
        IF (RHO)  99,99,23
23    TAU=(ELK*(ELK+D*COSF(PHI)-X*COSF(PSI(J)-PHI))+EMU*(ELK*SINF(PHI)
    1 -X*SINF(PSI(J))))/(RHO**1.5)
         GO TO JK,(25,33,40,51,55,59)
25      ALPHA = ALPHA + TAU*COSF(EN*PHI)
        OMEGA = OMEGA + TAU*SINF(EN*PHI)
        PHI = PHI + DELPHI
         IF(GAMMA -.00001) 29,29,43
29      IF (PHI-SPHI-QT)  30,30,50
30      ASSIGN 33 TO JK
        GO TO 20
33       ALPHA = ALPHA + 4.0*TAU*COSF(EN*PHI)
        OMEGA = OMEGA + 4.0*TAU*SINF(EN*PHI)
        PHI = PHI + DELPHI
        IF ( PHI + 0.5*DELPHI-SPHI)  35,35,19
```

```
 35     IF(GAMMA-.00001)  37,37,36
36      ASSIGN 40 TO JK
        GO TO 20
37      IF(PHI +0.5*DELPHI-SPHI-QT)  36,36,19
40      ALPHA = ALPHA + 2.0*TAU*COSF(EN*PHI)
        OMEGA = OMEGA + 2.0*TAU*SINF(EN*PHI)
        PHI = PHI + DELPHI
        ASSIGN 33 TO JK
        GO TO 20
43       IF (PHI - SPHI)      30,30,45
45            GO TO (71,69,67,65),M
50      PHI = PHI - DELPHI
 51    BETA = BETA + TAU*COSF(EN*PHI)
         ZETA = ZETA + TAU*SINF(EN*PHI)
           PHI = PHI + DELTA
         IF (PHI- SPHI)  52,52,63
52        ASSIGN 55 TO JK
         GO TO 20
55        BETA = BETA + 4.0*TAU*COSF(EN*PHI)
        ZETA = ZETA + 4.0*TAU*SINF(EN*PHI)
         PHI = PHI + DELTA
         IF (PHI + 0.5*DELTA - SPHI)  57,57,58
57       ASSIGN 59 TO JK
         GO TO 20
58      ASSIGN 51 TO JK
         GO TO 20
59       BETA = BETA + 2.0*TAU*COSF(EN*PHI)
        ZETA = ZETA + 2.0*TAU*SINF(EN*PHI)
        PHI = PHI + DELTA
        ASSIGN 55 TO JK
        GO TO 20
63       ZAPSI(J) = (DELTA*BETA)/3.0
        ZBPSI(J) = (DELTA*ZETA)/3.0
        GO TO 45
65       Y1PSI(J) = (DELPHI*ALPHA)/3.0
        Y2PSI(J) = (DELPHI*OMEGA)/3.0
         CHI(J) = CHI(J) + Y1PSI(J)
        SIGMA(J) = SIGMA(J) + Y2PSI(J)
         GO TO 73
67      Y3PSI(J) = (DELPHI*ALPHA)/3.0
        Y4PSI(J) = (DELPHI*OMEGA)/3.0
        CHI(J) = CHI(J) + Y3PSI(J)
        SIGMA(J) = SIGMA(J) + Y4PSI(J)
        GO TO 73
69      Y5PSI(J) = (DELPHI*ALPHA)/3.0
         Y6PSI(J) = (DELPHI*OMEGA)/3.0
        CHI(J) = CHI(J) +Y5PSI(J)
         SIGMA(J) = SIGMA(J) + Y6PSI(J)
         GO TO 73
71       Y7PSI(J) = (DELPHI*ALPHA)/3.0
        Y8PSI(J) = (DELPHI*OMEGA)/3.0
         CHI(J) = CHI(J) + Y7PSI(J)
         SIGMA(J) = SIGMA(J) + Y8PSI(J)
73        PSI(J+1) = PSI(J) + DEPSI
74       CONTINUE
         IN = IN - 1
```

```
      IF(IN) 77,77,13
 77         L = 3
 79       CALL HARNAL (PSI,CHI)
        WRITE OUTPUT TAPE 2,80
 80     FORMAT (1H0////4H CHI////)
        CALL PRINCE(L,AQUAY,BQUAY,AZERO)
        CALL HARNAL (PSI,SIGMA)
         WRITE OUTPUT TAPE 2,82
 82     FORMAT (1H0////6H SIGMA////)
        CALL PRINCE(L,AQUAY,BQUAY,AZERO)
        CALL HARNAL (PSI,ZAPSI)
        WRITE OUTPUT TAPE 2,84
 84    FORMAT (1H0////6H ZAPSI////)
        CALL PRINCE (L,AQUAY,BQUAY,AZERO)
        CALL HARNAL (PSI,ZBPSI)
        WRITE OUTPUT TAPE 2,85
  85    FORMAT (1H0////6H ZBPSI////)
        CALL PRINCE( L,AQUAY,BQUAY,AZERO)
        IF (MM)  88,88,87
 87        CALL TRIP (CHI,SIGMA,PSI,ZAPSI,ZBPSI,NN,K)
 88      IF (KL)  90,90,89
 89      CALL PRICK (NP)
 90      IM = IM - 1
         IF (IM)  92,92,9
 92        IL = IL - 1
        IF (IL)  94,94,8
 94      IK = IK - 1
         IF (IK)   96,96,7
 96      IJ = IJ - 1
        IF (IJ)  98,98,5
  98     II = II - 1
        IF (II)    101,101,3
 99      WRITE OUTPUT TAPE 2,100,PSI(J),PHI
100    FORMAT (30H1SINGULARITY OCCURS AT PSI OF E16.8,8H ,PHI = E16.8)
         GO TO 90
101       CALL EXIT
     END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

```
      SUBROUTINE HARNAL (PSI,FPSI)
      DIMENSION PSI(361),FPSI(361),AQUAY(20),BQUAY(20)
      COMMON PSI,FPSI,I,L,K,DEPSI,AQUAY,BQUAY,AZERO
      KN=K-3
      R=1.0
      GO TO (5,35,5),I
    5   GRAL = FPSI(1) + FPSI(K) +4.0*FPSI(K-1)
      DO 12 M=2,KN,2
   12  GRAL = GRAL +4.0*FPSI(M) +2.0*FPSI(M+1)
      AZERO = (DEPSI*GRAL)/9.4247778
      Q=0.0
      DO 25 J=1,L
      Q=Q +1.0
      GRAND = FPSI(1)*COSF(Q*PSI(1)) +FPSI(K)*COSF(Q*PSI(K))
     1 +4.0*FPSI(K-1)*COSF(Q*PSI(K-1))
      DO 20 M=2,KN,2
   20 GRAND=GRAND +4.0*COSF(Q*PSI(M))*FPSI(M)
     1  +2.0*FPSI(M+1)*COSF(Q*PSI(M+1))
      AQUAY(J) = ((2.0*DEPSI)/9.4247778)*GRAND
      IF (I-3) 25,22,22
   22  AQUAY(J)=AQUAY(J)/2.0
   25  CONTINUE
      IF(I-3)49,34,34
   34  AZERO =AZERO/2.0
      R =0.5
   35  P =0.0
      DO 43 J =1,L
      P =P +1.0
      TGRAL = FPSI(1)*SINF(P*PSI(1)) + FPSI(K)*SINF(P*PSI(K))
     1 +4.0*FPSI(K-1)*SINF(P*PSI(K-1))
      DO 42 M =2,KN,2
   42  TGRAL =TGRAL +4.0*FPSI(M)*SINF(P*PSI(M))
     1  +2.0*FPSI(M+1)*SINF(P*PSI(M+1))
      BQUAY(J) =((2.0*DEPSI)/9.4247778)*TGRAL*R
   43 CONTINUE
   49  RETURN
      END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

```
      SUBROUTINE PRICK(NP)
    DIMENSION Y1PSI(361),Y2PSI(361),Y3PSI(361),Y4PSI(361),Y5PSI(361),
   1     Y6PSI(361),Y7PSI(361),Y8PSI(361),PSI(361),AQUAY(20),BQUAY(20)
     COMMON PSI,Y1PSI,I,L,K,DEPSI,AQUAY,BQUAY,AZERO,Y2PSI,Y3PSI,
   1 Y4PSI,Y5PSI,Y6PSI,Y7PSI,Y8PSI
       WRITE OUTPUT TAPE 2,3
 3     FORMAT(102H1PSI                Y1PSI              Y2PSI              Y3P
   1SI              Y4PSI              Y5PSI              Y6PSI///)
      DO 9 J = 1,K,NP
      WRITE OUTPUT TAPE 2,6,PSI(J),Y1PSI(J),Y2PSI(J),Y3PSI(J),
   1   Y4PSI(J),Y5PSI(J),Y6PSI(J)
6     FORMAT (E13.5,6E16.5)
9       CONTINUE
       RETURN
    END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

```
      SUBROUTINE TRIP(CHI,SIGMA,PSI,ZAPSI, BPSI,NN,K)
      DIMENSION CHI(361),SIGMA(361),PSI(361),ZAPSI(361),ZBPSI(361)
        WRITE OUTPUT TAPE 2,1
1      FORMAT(78H1CHI                    SIGMA                PSI
  1   ZAPSI                ZBPSI///)
        DO 9  J = 1,K,NN
        WRITE OUTPUT TAPE 2,5,CHI(J),SIGMA(J),PSI(J),ZAPSI(J),ZBPSI(J)
 5      FORMAT (E13.5,4E18.5)
9       CONTINUE
        RETURN
   END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

```
      SUBROUTINE PRINCF(L,AQUAY,BQUAY,AZERO)
    DIMENSION       AQUAY(20),BQUAY(20)
      WRITE OUTPUT TAPE 2,5,AZERO
5     FORMAT(1H0///10HCA-ZERO = E18.8)
    DO 9 J = 1,L
     WRITE OUTPUT TAPE 2,7,J,AQUAY(J),J,BQUAY(J)
7   FORMAT (3HCA-I2,3H = E18.8,10H          B-I2,3H = E18.8)
9    CONTINUE
     RETURN
  END(1,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

APPENDIX B

Typical Results from Program of Appendix A

```
MU =  0.100    LAMBDA =  0.0500     M =  3.000     N =  3.000     L =  1.000     A =  0.500     X =  0.550

DELTA PSI =  0.13090     DELTA PHI(1) =  0.13090     DELTA PHI(2) =  0.04363


CHI



A-ZERO =      0.28184911E 01
A- 1 =      0.23435717E-00          B- 1 =      0.18979249E 01
           -0.64222778E 01          B- 2 =     -0.30947544E-00
            0.21907055E 01          B- 3 =     -0.18879348E-02
A- 4 =     -0.32869507E-00          B- 4 =      0.17816691E-00
A- 5 =      0.74906559E-01          B- 5 =     -0.24031100E-00
A- 6 =      0.77531750E-01          B- 6 =      0.30199294E-00
A- 7 =     -0.23727806E-00          B- 7 =     -0.35005791E-00
A- 8 =      0.31815831E-00          B- 8 =      0.21435103E-00
A- 9 =     -0.33253226E-00          B- 9 =     -0.77003625E-01
A-10 =      0.30744545E-00          B-10 =     -0.72742325E-01


SIGMA



A-ZERO =      0.45014069E-00
A- 1 =     -0.44811134E-00          B- 1 =      0.23248084E 01
A- 2 =      0.81733908E 00          B- 2 =     -0.26325015E 01
A- 3 =     -0.18907241E-00          B- 3 =      0.34310731E 01
A- 4 =      0.33956618E-00          B- 4 =     -0.16649289E-00
```

A- 5 =	-0.20658962E-00	B- 5 =	0.40824365E-00
A- 6 =	-0.23854294E-01	B- 6 =	-0.25071085E-00
A- 7 =	0.38533878E-00	B- 7 =	0.12272582E-00
A- 8 =	-0.41684166E-00	B- 8 =	-0.14012528E-01
A- 9 =	0.26763588E-00	B- 9 =	-0.74038545E-01
A-10 =	-0.21372104E-01	B-10 =	0.16637603E-00

ZAPSI

A-ZERO = -0.93950700E-03

A- 1 =	0.26513033E-01	B- 1 =	0.59074956E-02
A- 2 =	-0.69089228E-01	B- 2 =	-0.24875500E-00
A- 3 =	0.61862555E 00	B- 3 =	0.15431789E 01
A- 4 =	-0.21788670E-01	B- 4 =	0.57252838E-02
A- 5 =	0.17025375E-02	B- 5 =	0.27875036E-02
A- 6 =	0.42724407E-03	B- 6 =	-0.79680853E-04
A- 7 =	0.16942464E-04	B- 7 =	-0.53993321E-04
A- 8 =	-0.52219255E-05	B- 8 =	-0.53964967E-05
A- 9 =	-0.11450093E-05	B- 9 =	0.26211555E-06
A-10 =	-0.22061996E-06	B-10 =	0.17850348E-06

ZBPSI

A-ZERO = -0.27789952E-02

A- 1 =	-0.57612904E-02	B- 1 =	0.27118210E-01
A- 2 =	0.24882170E-00	B- 2 =	-0.69089957E-01
A- 3 =	-0.15431759E 01	B- 3 =	0.61861937E 00
A- 4 =	-0.57254104E-02	B- 4 =	-0.21786939E-01
A- 5 =	-0.27870707E-02	B- 5 =	0.17027783E-02
A- 6 =	0.80190378E-04	B- 6 =	0.42748532E-03
A- 7 =	0.54488785E-04	B- 7 =	0.17114811E-04
A- 8 =	0.58944438E-05	B- 8 =	-0.50005811E-05
A- 9 =	0.21461810E-06	B- 9 =	-0.95512301E-06
A-10 =	0.35473040E-06	B-10 =	-0.13038515E-07

CHI	SIGMA	PSI	ZAPSI	ZBPSI
-0.14443E 01	0.89760E 00	0.	0.55547E 00	-0.13113E 01
-0.11941E 01	0.18391E 01	0.13090E-00	0.10443E 01	-0.98809E 00
-0.76018E 00	0.25004E 01	0.26180E-00	0.13677E 01	-0.49453E-00
-0.12395E-00	0.27826E 01	0.39270E-00	0.14708E 01	0.91397E-01
0.71141E 00	0.26691E 01	0.52360E 00	0.13335E 01	0.67611E 00
0.16927E 01	0.22028E 01	0.65450E 00	0.97399E 00	0.11651E 01
0.27830E 01	0.14463E 01	0.78540E 00	0.44572E-00	0.14779E 01
0.40208E 01	0.48433E-00	0.91630E 00	-0.17012E-00	0.15609E 01
0.54198E 01	-0.53830E 00	0.10472E 01	-0.77732E 00	0.13957E 01
0.69236E 01	-0.13406E 01	0.11781E 01	-0.12795E 01	0.10032E 01
0.84441E 01	-0.17467E 01	0.13090E 01	-0.15951E 01	0.43979E-00
0.97849E 01	-0.17127E 01	0.14399E 01	-0.16705E 01	-0.21006E-00
0.10868E 02	-0.96056E 00	0.15708E 01	-0.14885E 01	-0.84722E 00
0.11421E 02	-0.65259E-01	0.17017E 01	-0.10714E 01	-0.13728E 01
0.11685E 02	0.16317E 01	0.18326E 01	-0.47842E-00	-0.17036E 01
0.10882E 02	0.34286E 01	0.19635E 01	0.20351E-00	-0.17851E 01
0.98817E 01	0.42460E 01	0.20944E 01	0.87248E 00	-0.16002E 01
0.85090E 01	0.48889E 01	0.22253E 01	0.14273E 01	-0.11721E 01
0.71490E 01	0.48419E 01	0.23562E 01	0.17828E 01	-0.56127E 00
0.52338E 01	0.65697E 01	0.24871E 01	0.18835E 01	0.14352E-00
0.14907E 01	0.85724E 01	0.26180E 01	0.17115E 01	0.83875E 00
-0.38389E 01	0.10007E 02	0.27489E 01	0.12901E 01	0.14218E 01
-0.77273E 01	0.74171E 01	0.28798E 01	0.67972E 00	0.18064E 01
-0.50550E 01	0.37151E 01	0.30107E 01	-0.30868E-01	0.19361E 01
-0.55942E 01	0.16783E 01	0.31416E 01	-0.73825E 00	0.17921E 01
-0.46113E 01	-0.20345E-00	0.32725E 01	-0.13400E 01	0.13965E 01
-0.64174E 01	-0.32291E 01	0.34034E 01	-0.17500E 01	0.80824E 00
-0.35379E 01	-0.49287E 01	0.35343E 01	-0.19114E 01	0.11413E-00
-0.14294E 01	-0.55730E 01	0.36652E 01	-0.18045E 01	-0.58468E 00
0.48191E-00	-0.61685E 01	0.37961E 01	-0.14492E 01	-0.11879E 01
0.21510E 01	-0.71092E 01	0.39270E 01	-0.90139E 00	-0.16111E 01
0.48529E 01	-0.58772E 01	0.40579E 01	-0.24385E-00	-0.17975E 01
0.65008E 01	-0.33759E 01	0.41888E 01	0.42639E-00	-0.17262E 01
0.71178E 01	-0.18400E 01	0.43197E 01	0.10126E 01	-0.14140E 01
0.75489E 01	-0.10496E 01	0.44506E 01	0.14323E 01	-0.91254E 00
0.75093E 01	-0.24684E-00	0.45815E 01	0.16292E 01	-0.29957E-00
0.71550E 01	0.44548E-00	0.47124E 01	0.15804E 01	0.33241E-00
0.63113E 01	0.10630E 01	0.48433E 01	0.12993E 01	0.88990E 00
0.52503E 01	0.11142E 01	0.49742E 01	0.83325E 00	0.12919E 01
0.40534E 01	0.81138E 00	0.51051E 01	0.25629E-00	0.14813E 01
0.28464E 01	0.27679E-00	0.52360E 01	-0.34184E-00	0.14336E 01
0.16618E 01	-0.43618E-00	0.53669E 01	-0.86886E 00	0.11600E 01
0.59144E 00	-0.11775E 01	0.54978E 01	-0.12438E 01	0.70621E 00
-0.26154E-00	-0.17743E 01	0.56287E 01	-0.14093E 01	0.14539E-00
-0.85951E 00	-0.20126E 01	0.57596E 01	-0.13406E 01	-0.43253E-00
-0.12398E 01	-0.17726E 01	0.58905E 01	-0.10498E 01	-0.93490E 00
-0.14528E 01	-0.11042E 01	0.60214E 01	-0.58404E 00	-0.12808E 01
-0.15240E 01	-0.15184E-00	0.61523E 01	-0.18457E-01	-0.14138E 01
-0.14443E 01	0.89760E 00	0.62832E 01	0.55546E 00	-0.13113E 01

PSI	Y1PSI	Y2PSI	Y3PSI	Y4PSI	Y5PSI	Y6PSI
0.	0.44979E-01	0.26885E-00	0.52610E-01	0.47286E-00	-0.15419E 01	0.15589E-00
0.13090E-00	0.18267E-00	0.45487E-00	-0.31113E-01	0.87167E 00	-0.13456E 01	0.51260E 00
0.26180E-00	0.36249E-00	0.56912E 00	-0.14079E-00	0.12234E 01	-0.98188E 00	0.70781E 00
0.39270E-00	0.53428E 00	0.65078E 00	-0.21153E-00	0.14733E 01	-0.44670E-00	0.65860E 00
0.52360E 00	0.68742E 00	0.75318E 00	-0.16008E-00	0.15246E 01	0.18406E-00	0.39138E-00
0.65450E 00	0.83514E 00	0.88598E 00	0.57973E-01	0.12354E 01	0.79960E 00	0.81334E-01
0.78540E 00	0.10230E 01	0.94040E 00	0.45242E-00	0.54307E 00	0.13076E 01	-0.37190E-01
0.91630E 00	0.13126E 01	0.67134E 00	0.98632E 00	-0.30082E-00	0.17219E 01	0.11381E-00
0.10472E 01	0.17617E 01	-0.12197E-00	0.16195E 01	-0.81320E 00	0.20386E 01	0.39687E-00
0.11781E 01	0.24226E 01	-0.10462E 01	0.22690E 01	-0.69734E 00	0.22320E 01	0.40294E-00
0.13090E 01	0.32073E 01	-0.12944E 01	0.28131E 01	-0.12384E-00	0.24237E 01	-0.32846E-00
0.14399E 01	0.38509E 01	-0.57027E 00	0.30856E 01	0.43924E-00	0.28485E 01	-0.15816E 01
0.15708E 01	0.41986E 01	0.85241E 00	0.30364E 01	0.34411E-00	0.36333E 01	-0.21571E 01
0.17017E 01	0.40902E 01	0.26031E 01	0.29162E 01	-0.11088E 01	0.44145E 01	-0.15596E 01
0.18326E 01	0.33610E 01	0.42560E 01	0.34031E 01	-0.26665E 01	0.49209E 01	0.42147E-01
0.19635E 01	0.18486E 01	0.42729E 01	0.41785E 01	-0.30863E 01	0.48550E 01	0.22420E 01
0.20944E 01	0.95947E 00	0.16702E 01	0.48300E 01	-0.21503E 01	0.40922E 01	0.47262E 01
0.22253E 01	0.13939E 01	-0.18260E 01	0.47391E 01	-0.41135E-00	0.23760E 01	0.71263E 01
0.23562E 01	0.32120E 01	-0.49330E 01	0.40271E 01	0.12032E 01	-0.90076E-01	0.85716E 01
0.24871E 01	0.50312E 01	-0.50851E 01	0.28785E 01	0.24992E 01	-0.26758E 01	0.91555E 01
0.26180E 01	0.55509E 01	-0.39490E 01	0.13646E 01	0.33223E 01	-0.54249E 01	0.91991E 01
0.27489E 01	0.60020E 01	-0.29964E 01	-0.27168E-00	0.35088E 01	-0.95692E 01	0.94947E 01
0.28798E 01	0.64659E 01	-0.18655E 01	-0.17616E 01	0.30895E 01	-0.12432E 02	0.61931E 01
0.30107E 01	0.62488E 01	-0.51717E 00	-0.29370E 01	0.21871E 01	-0.83667E 01	0.20452E 01
0.31416E 01	0.57608E 01	0.47342E-00	-0.36673E 01	0.93441E 00	-0.76878E 01	0.27050E-00
0.32725E 01	0.55280E 01	0.13082E 01	-0.38378E 01	-0.48967E-00	-0.63015E 01	-0.10220E 01
0.34034E 01	0.52754E 01	0.22495E 01	-0.34063E 01	-0.18497E 01	-0.82864E 01	-0.36289E 01
0.35343E 01	0.46790E 01	0.30123E 01	-0.24424E 01	-0.28997E 01	-0.57745E 01	-0.50412E 01
0.36652E 01	0.41575E 01	0.35736E 01	-0.11286E 01	-0.34509E 01	-0.44583E 01	-0.56956E 01
0.37961E 01	0.37572E 01	0.43051E 01	0.28882E-00	-0.34377E 01	-0.35641E 01	-0.70359E 01
0.39270E 01	0.29722E 01	0.48499E 01	0.15989E 01	-0.29156E 01	-0.24201E 01	-0.90435E 01
0.40579E 01	0.20926E 01	0.46664E 01	0.26657E 01	-0.19684E 01	0.94582E-01	-0.85752E 01
0.41888E 01	0.14085E 01	0.42390E 01	0.33521E 01	-0.71291E 00	0.17402E 01	-0.69020E 01
0.43197E 01	0.67317E 00	0.29861E 01	0.35894E 01	0.60353E 00	0.28553E 01	-0.54296E 01
0.44506E 01	0.49654E-00	0.85042E 00	0.34492E 01	0.17997E 01	0.36032E 01	-0.36997E 01
0.45815E 01	0.78522E 00	-0.97211E 00	0.29480E 01	0.26240E 01	0.37761E 01	-0.18987E 01
0.47124E 01	0.12866E 01	-0.19207E 01	0.23149E 01	0.27497E 01	0.35535E 01	-0.38347E-00
0.48433E 01	0.15375E 01	-0.19242E 01	0.16848E 01	0.22299E 01	0.30890E 01	0.75738E 00
0.49742E 01	0.15664E 01	-0.15149E 01	0.11776E 01	0.11816E 01	0.25063E 01	0.14475E 01
0.51051E 01	0.14078E 01	-0.98318E 00	0.78903E 00	0.16471E-00	0.18566E 01	0.16298E 01
0.52360E 01	0.11814E 01	-0.56415E 00	0.46389E-00	-0.48919E-00	0.12011E 01	0.13301E 01
0.53669E 01	0.95436E 00	-0.33186E-00	0.19637E-00	-0.80935E 00	0.51106E 00	0.70503E 00
0.54978E 01	0.74392E 00	-0.26641E-00	0.16053E-01	-0.90179E 00	-0.16853E-00	-0.92733E-02
0.56287E 01	0.54747E 00	-0.30757E-00	-0.58691E-01	-0.87351E 00	-0.75032E 00	-0.59324E 00
0.57596E 01	0.34770E-00	-0.35458E-00	-0.50194E-01	-0.76121E 00	-0.11570E 01	-0.89677E 00
0.58905E 01	0.16889E-00	-0.31946E-00	0.54318E-02	-0.56347E 00	-0.14122E 01	-0.88963E 00
0.60214E 01	0.41712E-01	-0.18005E-00	0.61434E-01	-0.28073E-00	-0.15560E 01	-0.64340E 00
0.61523E 01	-0.53301E-02	0.36314E-01	0.84216E-01	0.73822E-0.	-0.16029E 01	-0.26197E-00
0.62832E 01	0.44979E-01	0.26885E-00	0.52611E-01	0.47286E-00	-0.15419E 01	0.15589E-00

MU = 0.200 LAMBDA = 0.0250 M = 3.000 N = 0. L = 0.500 A = 0. X = 0.850

DELTA PSI = 0.26180 DELTA PHI(1) = 0.26180 DELTA PHI(2) = 0.04363

CHI

A-ZERO = 0.24896850E 01

A- 1 =	0.76733813E 01	B- 1 =	-0.59209049E 01
A- 2 =	0.95971692E 01	B- 2 =	-0.45034871E 01
A- 3 =	0.92279459E 01	B- 3 =	-0.24482561E 01
A- 4 =	0.70018407E 01	B- 4 =	-0.63409612E 00
A- 5 =	0.38095658E 01	B- 5 =	0.59846934E 00
A- 6 =	0.55665474E 00	B- 6 =	0.11683953E 01
A- 7 =	-0.20751181E 01	B- 7 =	0.11370528E 01
A- 8 =	-0.37050755E 01	B- 8 =	0.63462853E 00
A- 9 =	-0.42231362E 01	B- 9 =	-0.17602880E-00
A-10 =	-0.37263282E 01	B-10 =	-0.10911534E 01

SIGMA

A-ZERO = 0.

A- 1 =	0.	B- 1 =	-0.
A- 2 =	-0.	B- 2 =	-0.
A- 3 =	-0.	B- 3 =	-0.
A- 4 =	-0.	B- 4 =	0.

A- 5 =	-0.	B- 5 =	0.
A- 6 =	0.	B- 6 =	0.
A- 7 =	0.	B- 7 =	0.
A- 8 =	0.	B- 8 =	-0.
A- 9 =	0.	B- 9 =	-0.
A-10 =	-0.	B-10 =	-0.

ZAPSI

A-ZERO = -0.14611866E 01

A- 1 =	-0.50505327E 00	B- 1 =	-0.56519654E 00
A- 2 =	-0.14080133E-00	B- 2 =	-0.58095719E-01
A- 3 =	-0.35666509E-01	B- 3 =	0.40008912E-02
A- 4 =	-0.73993352E-02	B- 4 =	0.54272138E-02
A- 5 =	-0.10136589E-02	B- 5 =	0.23007582E-02
A- 6 =	0.55484471E-04	B- 6 =	0.89713659E-03
A- 7 =	0.44217125E-03	B- 7 =	0.90215145E-03
A- 8 =	0.25126047E-02	B- 8 =	0.18273765E-02
A- 9 =	0.11903889E-01	B- 9 =	0.13314038E-02
A-10 =	0.46938109E-01	B-10 =	-0.19368302E-01

ZOPSI

A-ZERO = -0.

A- 1 =	-0.	B- 1 =	0.
A- 2 =	0.	B- 2 =	0.
A- 3 =	0.	B- 3 =	0.
A- 4 =	0.	B- 4 =	-0.
A- 5 =	0.	B- 5 =	-0.
A- 6 =	-0.	B- 6 =	-0.
A- 7 =	-0.	B- 7 =	-0.
A- 8 =	-0.	B- 8 =	0.
A- 9 =	-0.	B- 9 =	0.
A-10 =	0.	B-10 =	0.

CHI	SIGMA	PSI	ZAPSI	ZBPSI
0.35169E 02	0.	0.	-0.21508E 01	-0.
0.27842E 02	0.	0.26180E-00	-0.22650E 01	-0.
-0.47245E 01	0.	0.52360E 00	-0.22878E 01	-0.
-0.21978E 02	0.	0.78540E 00	-0.22422E 01	-0.
-0.13122E 02	0.	0.10472E 01	-0.21506E 01	-0.
-0.82261E 01	0.	0.13090E 01	-0.20307E 01	-0.
-0.56000E 01	0.	0.15708E 01	-0.18949E 01	-0.
-0.39901E 01	0.	0.18326E 01	-0.17515E 01	-0.
-0.29033E 01	0.	0.20944E 01	-0.16060E 01	-0.
-0.21134E 01	0.	0.23562E 01	-0.14624E 01	-0.
-0.15026E 01	0.	0.26180E 01	-0.13233E 01	-0.
-0.10037E 01	0.	0.28798E 01	-0.11910E 01	-0.
-0.57500E 00	0.	0.31416E 01	-0.10677E 01	-0.
-0.18872E-00	0.	0.34034E 01	-0.95602E 00	-0.
0.17512E-00	0.	0.36652E 01	-0.85945E 00	-0.
0.53181E 00	0.	0.39270E 01	-0.78287E 00	-0.
0.89274E 00	0.	0.41888E 01	-0.73343E 00	-0.
0.12629E 01	0.	0.44506E 01	-0.72136E 00	-0.
0.16290E 01	0.	0.47124E 01	-0.76070E 00	-0.
0.19150E 01	0.	0.49742E 01	-0.86827E 00	-0.
0.18124E 01	0.	0.52360E 01	-0.10581E 01	-0.
0.95603E 00	0.	0.54978E 01	-0.13276E 01	-0.
0.21277E 02	0.	0.57596E 01	-0.16415E 01	-0.
0.33543E 02	0.	0.60214E 01	-0.19354E 01	-0.
0.35169E 02	0.	0.62832E 01	-0.21508E 01	-0.

PSI	Y1PSI	Y2PSI	Y3PSI	Y4PSI	Y5PSI	Y6PSI
0.	0.52813E 01	0.	0.23796E 02	0.	0.60917E 01	0.
0.26180E-00	0.80571E 01	0.	0.11823E 02	0.	0.79620E 01	0.
0.52360E 00	0.41082E 01	0.	-0.18973E 02	0.	0.10141E 02	0.
0.78540E 00	-0.43200E 01	0.	-0.10339E 02	0.	-0.73193E 01	0.
0.10472E 01	-0.22420E 01	0.	-0.64288E 01	0.	-0.44514E 01	0.
0.13090E 01	-0.12714E 01	0.	-0.44177E 01	0.	-0.25370E 01	0.
0.15708E 01	-0.77806E 00	0.	-0.32279E 01	0.	-0.15940E 01	0.
0.18326E 01	-0.48537E-00	0.	-0.24545E 01	0.	-0.10502E 01	0.
0.20944E 01	-0.29126E-00	0.	-0.19166E 01	0.	-0.69546E 00	0.
0.23562E 01	-0.15021E-00	0.	-0.15219E 01	0.	-0.44134E-00	0.
0.26180E 01	-0.38865E-01	0.	-0.12188E 01	0.	-0.24497E-00	0.
0.28798E 01	0.56345E-01	0.	-0.97687E 00	0.	-0.831[illegible]E-01	0.
0.31416E 01	0.14438E-00	0.	-0.77713E 00	0.	0.57752E-01	0.
0.34034E 01	0.23212E-00	0.	-0.60753E 00	0.	0.18669E-00	0.
0.36652E 01	0.32583E-00	0.	-0.46054E-00	0.	0.30983E-00	0.
0.39270E 01	0.43222E-00	0.	-0.33231E-00	0.	0.43190E-00	0.
0.41888E 01	0.55941E 00	0.	-0.22323E-00	0.	0.55656E 00	0.
0.44506E 01	0.71805E 00	0.	-0.14136E-00	0.	0.68622E 00	0.
0.47124E 01	0.92239E 00	0.	-0.11356E-00	0.	0.82017E 00	0.
0.49742E 01	0.11908E 01	0.	-0.22414E-00	0.	0.94839E 00	0.
0.52360E 01	0.15448E 01	0.	-0.76600E 00	0.	0.10336E 01	0.
0.54978E 01	0.20115E 01	0.	-0.20770E 01	0.	0.10215E 01	0.
0.57596E 01	0.26561E 01	0.	0.16641E 02	0.	0.19794E 01	0.
0.60214E 01	0.36497E 01	0.	0.23979E 02	0.	0.59144E 01	0.
0.62832E 01	0.52813E 01	0.	0.23796E 02	0.	0.60917E 01	0.

APPENDIX C

Program for V_2 - Shed Wake

```
      DIMENSION PSI(361),S1PSI(361),S2PSI(361),S3PSI(361),S4PSI(361),
   1  AQUAY(20),BQUAY(20)
      COMMON PSI,S1PSI,I,L,K,DEPSI,AQUAY,BQUAY,AZERO
      READ INPUT TAPE  4,3,L,NP,KL,II,QT
3        FORMAT (4I2,E13.8)
4       READ INPUT TAPE 4,5,EMU,AMBDA,EM,DEPSI,DELPHI,DELTA,K,IJ
 5    FORMAT (3F8.3,3E13.8,I3,I2)
6     READ INPUT TAPE 4,7,EN,IK
 7     FORMAT (E8.3,I2)
8     READ INPUT TAPE 4,9,GAMMA,IL
 9     FORMAT (E13.8,I2)
10    READ INPUT TAPE 4,7,ELK,IM
11     READ INPUT TAPE 4,12,X
12       FORMAT (E8.3)
      PSI(1)  = 0.0
      WRITE OUTPUT TAPE 2,15,EMU,AMBDA,EM,EN,GAMMA,ELK,X
15   FORMAT(6H1MU = F6.3,12H   LAMBDA = F6.3,7H   M = F6.3,7H   N =
   1 F6.3,11H   GAMMA = F8.5,7H   L = F6.3,7H   X = F6.3/////)
      DO 53  J = 1,K
       ALPHA = 0.0
      BETA = 0.0
      ZETA = 0.0
      ETA = 0.0
      SPHI = 6.2831852*EM+PSI(J) + GAMMA
        EPHI = SPHI + QT
      PHI = PSI(J) + GAMMA
17    ASSIGN 22 TO JJ
18    D = EMU*(SPHI - PHI)
      Z = AMBDA*(SPHI-PHI)
      PI = Z**2+(X*SINF(PSI(J)-PHI) +D*SINF(PHI))**2
      IF (PI)  90,90,20
20    RHO = X**2+Z**2+D**2-2.0*D*X*COSF(PSI(J))
    SIGMA = RHO+ELK**2-2.0*X*ELK*COSF(PSI(J)-PHI)+2.0*D*ELK*COSF(PHI)
      THETA = D*COSF(PHI) -X*COSF(PSI(J)-PHI)
       TAU =((X*SINF(PSI(J)-PHI)+D*SINF(PHI))/PI)*((ELK+THETA)/SQRTF
   1 (SIGMA) - THETA/SQRTF(RHO))
      GO TO JJ , (22,25,30,35,40,45)
22      ALPHA = ALPHA + TAU*COSF(EN*PHI)
      BETA = BETA + TAU*SINF(EN*PHI)
       PHI = PHI + DELPHI
      ASSIGN 25 TO JJ
      IF (PHI-EPHI)  18,23,23
23    PHI = PHI - DELPHI
 24    ASSIGN 35 TO JJ
       GO TO 18
 25      ALPHA = ALPHA + 4.0*TAU*COSF(EN*PHI)
      BETA = BETA + 4.0*TAU*SINF(EN*PHI)
      PHI = PHI + DELPHI
       IF(EPHI -(PHI +0.5*DELPHI)) 17,17,27
 27      ASSIGN 30 TO JJ
       GO TO 18
 30     ALPHA = ALPHA + 2.0*TAU*COSF(EN*PHI)
      BETA = BETA + 2.0*TAU*SINF(EN*PHI)
       PHI = PHI + DELPHI
       ASSIGN 25 TO JJ
       GO TO 18
```

```
 35    ZETA = ZETA+TAU*SINF(EN*(PSI(J)-PHI))*COSF(EN*PSI(J))
       ETA = ETA + TAU*SINF(EN*(PSI(J)-PHI))*SINF(EN*PSI(J))
       PHI = PHI + DELTA
       ASSIGN 40 TO JJ
       IF (PHI -SPHI)    18,50,50
40     ZETA = ZETA +4.0*TAU*SINF(EN*(PSI(J)-PHI))*COSF(EN*PSI(J))
       ETA = ETA +4.0*TAU*SINF(EN*(PSI(J)-PHI))*SINF(EN*PSI(J))
       PHI = PHI + DELTA
        IF (SPHI -(PHI +0.5*DELTA))  44,44,43
44      PHI = PHI - 0.5*DELTA
       ASSIGN 35 TO JJ
        GO TO 18
 43    ASSIGN 45 TO JJ
       GO TO 18
45     ZETA = ZETA +2.0*TAU*SINF(EN*(PSI(J)-PHI))*COSF(EN*PSI(J))
       ETA = ETA +2.0*TAU*SINF(EN*(PSI(J)-PHI))*SINF(EN*PSI(J))
       PHI = PHI + DELTA
        ASSIGN 40 TO JJ
        GO TO 18
50   S1PSI(J)= (DELPHI/3.0)*ALPHA
    S2PSI(J) = (DELPHI/3.0)*BETA
    S3PSI(J) = (DELTA/3.0)*ZETA
      S4PSI(J)= (DELTA/3.0)*ETA
       PSI(J+1) = PSI(J) + DEPSI
 53    CONTINUE
       IF (KL)    55,60,60
55        WRITE OUTPUT TAPE 2,56
56   FORMAT(82H0PSI                    S1PSI                    S2PSI
   1   S3PSI                  S4PSI)
        DO 59 J = 1,K,NP
    WRITE OUTPUT TAPE 2,57,PSI(J),S1PSI(J),S2PSI(J),S3PSI(J),S4PSI(J)
59        CONTINUE
57        FORMAT(1H E14.6,4E18.6)
60        I = 3
       CALL HARNAL (PSI,S1PSI)
       WRITE OUTPUT TAPE 2,61
61        FORMAT (1H0////6HOS1PSI//////)
       CALL PRINCE(L,AQUAY,BQUAY,AZERO)
       CALL HARNAL (PSI,S2PSI)
        WRITE OUTPUT TAPE 2,63
63        FORMAT(1H0////6HOS2PSI//////)
       CALL PRINCE (L,AQUAY,BQUAY,AZERO)
       CALL HARNAL (PSI,S3PSI)
        WRITE OUTPUT TAPE 2,65
65        FORMAT(1H0////6HOS3PSI//////)
       CALL PRINCE(L,AQUAY,BQUAY,AZERO)
       CALL HARNAL (PSI,S4PSI)
        WRITE OUTPUT TAPE 2,67
67        FORMAT(1H0////6HOS4PSI//////)
       CALL PRINCE(L,AQUAY,BQUAY,AZERO)
 80    IM = IM -1
       IF (IM)  82,82,11
82     IL = IL - 1
       IF (IL)  84,84,10
84      IK = IK - 1
        IF (IK)   86,86,8
```

```
 86     IJ = IJ - 1
        IF (IJ)     88,88,6
 88    II = II - 1
          IF (II)  92,92,4
90      WRITE OUTPUT TAPE 2,91,PHI,PSI(J)
91    FORMAT(1H0/////22H SINGULARITY AT PHI = E15.6,10H PSI(J) = E15.6)
         GO TO 80
92        CALL EXIT
    END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE PRINCE(L,AQUAY,BQUAY,AZERO)
      DIMENSION AQUAY(20),BQUAY(20)
       WRITE OUTPUT TAPE 2,5,AZERO
5     FORMAT(1H0///10H0A-ZERO = E18.8)
      DO 9 J = 1,L
      WRITE OUTPUT TAPE 2,7,J,AQUAY(J),J,BQUAY(J)
7     FORMAT(3H0A-I2,3H = E18.8,10H          B-I2,3H = E18.8)
9     CONTINUE
     RETURN
   END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE HARNAL (PSI,FPSI)
      DIMENSION PSI(361),FPSI(361),AQUAY(20),BQUAY(20)
      COMMON PSI,FPSI,I,L,K,DEPSI,AQUAY,BQUAY,AZERO
      KN=K-3
      R=1.0
      GO TO (5,35,5),I
 5      GRAL = FPSI(1) + FPSI(K) +4.0*FPSI(K-1)
      DO 12 M=2,KN,2
12     GRAL = GRAL +4.0*FPSI(M) +2.0*FPSI(M+1)
      AZERO = (DEPSI*GRAL)/9.4247778
      Q=0.0
      DO 25 J=1,L
      Q=Q +1.0
      GRAND = FPSI(1)*COSF(Q*PSI(1)) +FPSI(K)*COSF(Q*PSI(K))
     1 +4.0*FPSI(K-1)*COSF(Q*PSI(K-1))
      DO 20 M=2,KN,2
20    GRAND=GRAND +4.0*COSF(Q*PSI(M))*FPSI(M)
     1  +2.0*FPSI(M+1)*COSF(Q*PSI(M+1))
      AQUAY(J) = ((2.0*DEPSI)/9.4247778)*GRAND
      IF (I-3) 25,22,22
22     AQUAY(J)=AQUAY(J)/2.0
25     CONTINUE
      IF(I-3)49,34,34
34     AZERO =AZERO/2.0
      R =0.5
35     P =0.0
      DO 43 J =1,L
      P =P +1.0
      TGRAL = FPSI(1)*SINF(P*PSI(1)) + FPSI(K)*SINF(P*PSI(K))
     1 +4.0*FPSI(K-1)*SINF(P*PSI(K-1))
      DO 42 M =2,KN,2
42     TGRAL =TGRAL +4.0*FPSI(M)*SINF(P*PSI(M))
     1  +2.0*FPSI(M+1)*SINF(P*PSI(M+1))
      BQUAY(J) =((2.0*DEPSI)/9.4247778)*TGRAL*R
43    CONTINUE
49     RETURN
      END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

APPENDIX D

Typical Results from Program of Appendix C

MU = .100 LAMBDA = .050 M = 3.000 N = 3.000 GAMMA = 0. L = 1.000 X = .800

PSI	S1PSI	S2PSI	S3PSI	S4PSI
.000000E 00	.105105E 01	-.226098E-01	.370212E 01	.000000E 00
.130900E 00	.165695E 01	.789388E 00	.335883E 01	.139127E 01
.261799E 00	.137994E 01	.173813E 01	.252405E 01	.252405E 01
.392699E 00	.345929E 00	.212482E 01	.134130E 01	.323817E 01
.523599E 00	-.829141E 00	.175237E 01	.307796E-06	.344264E 01
.654498E 00	-.166683E 01	.864239E 00	-.129471E 01	.312572E 01
.785398E 00	-.199575E 01	-.188466E 00	-.235285E 01	.235285E 01
.916298E 00	-.185742E 01	-.113131E 01	-.302641E 01	.125358E 01
.104720E 01	-.138674E 01	-.180721E 01	-.322872E 01	.769787E-06
.117810E 01	-.748817E 00	-.215793E 01	-.294419E 01	-.121952E 01
.130900E 01	-.835152E-01	-.219689E 01	-.222760E 01	-.222759E 01
.143990E 01	.495588E 00	-.198787E 01	-.119383E 01	-.288217E 01
.157080E 01	.927763E 00	-.161939E 01	-.130290E-05	-.309508E 01
.170170E 01	.119757E 01	-.117344E 01	.117746E 01	-.284265E 01
.183260E 01	.131290E 01	-.714832E 00	.216741E 01	-.216741E 01
.196350E 01	.129684E 01	-.292644E 00	.282724E 01	-.117108E 01
.209439E 01	.118251E 01	.634169E-01	.306199E 01	-.165399E-05
.222529E 01	.100283E 01	.341306E 00	.283693E 01	.117509E 01
.235619E 01	.785140E 00	.539211E 00	.218232E 01	.218232E 01
.248709E 01	.551892E 00	.660338E 00	.118967E 01	.287212E 01
.261799E 01	.321984E 00	.711423E 00	.291089E-05	.313809E 01
.274889E 01	.110859E 00	.702372E 00	-.121469E 01	.293252E 01
.287979E 01	-.699645E-01	.645488E 00	-.227453E 01	.227454E 01
.301069E 01	-.213100E 00	.554413E 00	-.301686E 01	.124963E 01
.314159E 01	-.315217E 00	.443177E 00	-.331997E 01	.387114E-05
.327249E 01	-.376929E 00	.325388E 00	-.312249E 01	-.129337E 01
.340339E 01	-.402706E 00	.213348E 00	-.243533E 01	-.243533E 01
.353429E 01	-.400582E 00	.116950E 00	-.134402E 01	-.324473E 01
.366519E 01	-.381425E 00	.425131E-01	-.557887E-05	-.358269E 01
.379609E 01	-.357728E 00	-.814991E-02	.139859E 01	-.337650E 01
.392699E 01	-.342135E 00	-.382770E-01	.263514E 01	-.263515E 01
.405789E 01	-.345761E 00	-.563965E-01	.350799E 01	-.145306E 01
.418879E 01	-.376096E 00	-.761894E-01	.386409E 01	-.693832E-05
.431969E 01	-.434385E 00	-.115297E 00	.362726E 01	.150245E 01
.445059E 01	-.513372E 00	-.192330E 00	.281524E 01	.281523E 01
.458149E 01	-.596514E 00	-.322670E 00	.154151E 01	.372151E 01
.471239E 01	-.657894E 00	-.513527E 00	.887411E-05	.406506E 01
.484329E 01	-.664534E 00	-.755766E 00	-.156552E 01	.377953E 01
.497419E 01	-.587656E 00	-.101663E 01	-.290264E 01	.290265E 01
.510509E 01	-.416934E 00	-.124375E 01	-.379415E 01	.157160E 01
.523599E 01	-.169540E 00	-.137611E 01	-.409642E 01	.991923E-05
.536689E 01	.105063E 00	-.136084E 01	-.376432E 01	-.155922E 01
.549779E 01	.331660E 00	-.117405E 01	-.285801E 01	-.285799E 01
.562869E 01	.477621E 00	-.840795E 00	-.153065E 01	-.369529E 01
.575959E 01	.333392E 00	-.452993E 00	-.111090E-04	-.394975E 01
.589049E 01	.110717E 00	-.166774E 00	.148993E 01	-.359705E 01
.602138E 01	-.333754E-01	-.122222E 00	.270988E 01	-.270989E 01
.615228E 01	.263967E 00	-.726484E 00	.348129E 01	-.144201E 01
.628318E 01	.105104E 01	-.226143E-01	.370212E 01	-.112952E-04

S1PSI

A-ZERO =	-.21411513E-01		
A- 1 =	-.25145815E-01	B- 1 =	.29183535E 00
A- 2 =	.10111784E 00	B- 2 =	-.63340063E 00
A- 3 =	.62900431E 00	B- 3 =	-.27584056E 00
A- 4 =	.28164475E 00	B- 4 =	.23914435E 00
A- 5 =	.91548303E-01	B- 5 =	.35043467E 00
A- 6 =	.22174516E-01	B- 6 =	.26881925E 00
A- 7 =	.18064930E-02	B- 7 =	.17496643E 00
A- 8 =	-.47383640E-02	B- 8 =	.11087642E 00
A- 9 =	-.60423293E-02	B- 9 =	.70610853E-01
A-10 =	-.50286448E-02	B-10 =	.45569707E-01

S2PSI

A-ZERO =	-.22339575E 00		
A- 1 =	-.23005702E 00	B- 1 =	.11047615E-01
A- 2 =	.90764877E 00	B- 2 =	.13577742E 00
A- 3 =	.51593588E 00	B- 3 =	.65914144E 00
A- 4 =	-.91504155E-01	B- 4 =	.30250693E 00

A- 5 =	-.26919181E 00	B- 5 =	.10419498E 00
A- 6 =	-.22367084E 00	B- 6 =	.29961998E-01
A- 7 =	-.14919041E 00	B- 7 =	.66204493E-02
A- 8 =	-.95658010E-01	B- 8 =	-.22677273E-02
A- 9 =	-.60439077E-01	B- 9 =	-.51788870E-02
A-10 =	-.37174831E-01	B-10 =	-.50278204E-02

S3PSI

A-ZERO = -.14096730E-02

A- 1 =	-.17254469E-01	B- 1 =	.94688194E-02
A- 2 =	.96945646E-01	B- 2 =	.24101576E 00
A- 3 =	.35455516E 01	B- 3 =	.31801561E-05
A- 4 =	.96940739E-01	B- 4 =	-.24099350E 00
A- 5 =	-.17257415E-01	B- 5 =	-.92455038E-02
A- 6 =	-.14103765E-02	B- 6 =	.15222281E-02
A- 7 =	.17204632E-05	B- 7 =	.22320531E-03
A- 8 =	.36669273E-05	B- 8 =	.21800398E-04
A- 9 =	-.11167592E-05	B- 9 =	.29984447E-05
A-10 =	-.13137857E-05	B-10 =	.50332809E-06

S4PSI

A-ZERO = .15222061E-02

A- 1 =	-.90223721E-02	B- 1 =	-.17259163E-01
A- 2 =	-.24097212E 00	B- 2 =	.96937053E-01
A- 3 =	.31818117E-05	B- 3 =	.35455525E 01
A- 4 =	.24099442E 00	B- 4 =	.96941967E-01
A- 5 =	.92456488E-02	B- 5 =	-.17256195E-01
A- 6 =	-.15222091E-02	B- 6 =	-.14090571E-02
A- 7 =	-.22318876E-03	B- 7 =	.29909941E-05
A- 8 =	-.21850483E-04	B- 8 =	.48826136E-05
A- 9 =	-.29959611E-05	B- 9 =	.14238887E-06
A-10 =	-.43792856E-06	B-10 =	.29802321E-07

MU = .100 LAMBDA = .050 M = 3.000 N = 3.000 GAMMA = 2.09439 L = 1.000 X = .800

PSI	S1PSI	S2PSI	S3PSI	S4PSI
.000000E 00	-.176251E 01	-.460854E 01	.427255E 00	-.000000E 00
.130900E 00	.852773E 00	-.442948E 01	.437500E 00	.181218E 00
.261799E 00	.277902E 01	-.315950E 01	.365599E 00	.365599E 00
.392699E 00	.362160E 01	-.129776E 01	.212658E 00	.513401E 00
.523599E 00	.333976E 01	.526938E 00	.525265E-07	.587499E 00
.654498E 00	.226321E 01	.180143E 01	-.233641E 00	.564059E 00
.785398E 00	.854271E 00	.235259E 01	-.440703E 00	.440703E 00
.916298E 00	-.521477E 00	.224739E 01	-.577057E 00	.239025E 00
.104720E 01	-.162598E 01	.164394E 01	-.614214E 00	.146440E-06
.117810E 01	-.232530E 01	.730749E 00	-.547350E 00	-.226720E 00
.130900E 01	-.257300E 01	-.303859E 00	-.396272E 00	-.396272E 00
.143990E 01	-.239068E 01	-.129252E 01	-.198952E 00	-.480312E 00
.157080E 01	-.184965E 01	-.210134E 01	-.199168E-06	-.473132E 00
.170170E 01	-.105060E 01	-.263519E 01	.161743E 00	-.390483E 00
.183260E 01	-.112093E 00	-.283232E 01	.262288E 00	-.262288E 00
.196350E 01	.822766E 00	-.266998E 01	.295768E 00	-.122511E 00
.209439E 01	.159031E 01	-.220080E 01	.271990E 00	-.146920E-06
.222529E 01	.207141E 01	-.158685E 01	.210337E 00	.871242E-01
.235619E 01	.230094E 01	-.102799E 01	.132819E 00	.132819E 00
.248709E 01	.245052E 01	-.557843E 00	.584514E-01	.141114E 00
.261799E 01	.257906E 01	.797394E-02	.113471E-06	.122327E 00
.274889E 01	.252257E 01	.763380E 00	-.368696E-01	.890114E-01
.287979E 01	.215263E 01	.155176E 01	-.526378E-01	.526379E-01
.301069E 01	.152238E 01	.218071E 01	-.519451E-01	.215164E-01
.314159E 01	.744409E 00	.257479E 01	-.413118E-01	.481702E-07
.327249E 01	-.118706E 00	.271895E 01	-.271140E-01	-.112309E-01
.340339E 01	-.100682E 01	.258600E 01	-.141600E-01	-.141600E-01
.353429E 01	-.181043E 01	.219059E 01	-.500830E-02	-.120910E-01
.366519E 01	-.245371E 01	.162817E 01	-.129732E-07	-.833124E-02
.379609E 01	-.296411E 01	.958141E 00	.214604E-02	-.518100E-02
.392699E 01	-.334736E 01	.101378E 00	.340816E-02	-.340817E-02
.405789E 01	-.346031E 01	-.999479E 00	.546964E-02	-.226562E-02
.418879E 01	-.314044E 01	-.221805E 01	.891130E-02	-.160010E-07
.431969E 01	-.234584E 01	-.333330E 01	.127889E-01	.529731E-02
.445059E 01	-.113711E 01	-.414199E 01	.147153E-01	.147153E-01
.458149E 01	.355061E 00	-.448046E 01	.114640E-01	.276763E-01
.471239E 01	.194124E 01	-.424013E 01	.905179E-07	.414645E-01
.484329E 01	.339579E 01	-.338100E 01	-.212603E-01	.513272E-01
.497419E 01	.447106E 01	-.193849E 01	-.512535E-01	.512538E-01
.510509E 01	.491222E 01	-.456308E-01	-.854744E-01	.354049E-01
.523599E 01	.450134E 01	.203883E 01	-.115925E 00	.280706E-06
.536689E 01	.313576E 01	.392783E 01	-.132068E 00	-.547041E-01
.549779E 01	.918791E 00	.515602E 01	-.122871E 00	-.122871E 00
.562869E 01	-.177378E 01	.529309E 01	-.797667E-01	-.192572E 00
.575959E 01	-.429857E 01	.411101E 01	-.694342E-06	-.246869E 00
.589049E 01	-.587307E 01	.177368E 01	.110492E 00	-.266753E 00
.602138E 01	-.589066E 01	-.105259E 01	.235764E 00	-.235765E 00
.615228E 01	-.431674E 01	-.342061E 01	.351090E 00	-.145427E 00
.628318E 01	-.176254E 01	-.460854E 01	.427254E 00	-.130356E-05

S1PSI

A-ZERO =	-.42709232E-01		
A- 1 =	-.21139412E 00	B- 1 =	.21096176E 00
A- 2 =	-.37111007E 00	B- 2 =	-.11780591E 01
A- 3 =	-.11801338E 01	B- 3 =	.27526765E 01
A- 4 =	-.21306370E 00	B- 4 =	.19521114E 01
A- 5 =	.49026115E-01	B- 5 =	.80651021E 00
A- 6 =	.87392528E-01	B- 6 =	.26319832E 00
A- 7 =	.57828449E-01	B- 7 =	.18170567E 00
A- 8 =	.17061530E-01	B- 8 =	.74928516E-02
A- 9 =	.23163780E-01	B- 9 =	.97607718E-02
A-10 =	.32263728E-02	B-10 =	.12639331E-01

S2PSI

A-ZERO =	-.23103523E 00		
A- 1 =	-.11731506E 00	B- 1 =	-.12839404E 00
A- 2 =	.13110791E 01	B- 2 =	-.34214275E 00
A- 3 =	-.25415402E 01	B- 3 =	-.12022938E 01
A- 4 =	-.18638599E 01	B- 4 =	-.21291893E 00

A- 5 =	-.77360434E 00	B- 5 =	.31124090E-01
A- 6 =	-.22494183E 00	B- 6 =	.77201430E-01
A- 7 =	-.18027032E 00	B- 7 =	.54323212E-01
A- 8 =	.56673586E-02	B- 8 =	.11827571E-01
A- 9 =	-.53487776E-02	B- 9 =	.23232483E-01
A-10 =	-.98697685E-02	B-10 =	.40925261E-02

S3PSI

A-ZERO = -.52749978E-02

A- 1 =	-.61597183E-02	B- 1 =	-.33950984E-01
A- 2 =	.10176542E 00	B- 2 =	-.10989538E 00
A- 3 =	.24660170E 00	B- 3 =	.22481971E-04
A- 4 =	.10166075E 00	B- 4 =	.10986031E 00
A- 5 =	-.57524160E-02	B- 5 =	.33329481E-01
A- 6 =	-.52747615E-02	B- 6 =	.19136677E-02
A- 7 =	-.40681027E-03	B- 7 =	-.62015753E-03
A- 8 =	.99350462E-04	B- 8 =	-.33505829E-04
A- 9 =	-.79803994E-06	B- 9 =	.22507117E-04
A-10 =	-.53294932E-05	B-10 =	-.15441845E-05

S4PSI

A-ZERO = .19134452E-02

A- 1 =	.32710506E-01	B- 1 =	-.53463580E-02
A- 2 =	.10982835E 00	B- 2 =	.10156655E 00
A- 3 =	.22478504E-04	B- 3 =	.24660312E 00
A- 4 =	-.10986342E 00	B- 4 =	.10167123E 00
A- 5 =	-.33331998E-01	B- 5 =	-.57536605E-02
A- 6 =	-.19132220E-02	B- 6 =	-.52752415E-02
A- 7 =	.62032366E-03	B- 7 =	-.40656083E-03
A- 8 =	.33542513E-04	B- 8 =	.99517556E-04
A- 9 =	-.22433904E-04	B- 9 =	-.62098517E-06
A-10 =	.15768843E-05	B-10 =	-.51412109E-05

APPENDIX E

Program II with Typical Results

```
      DIMENSION PSI(50),ANG(50),ZETA(6),TW(50),T(6,50),CO(50),SI(50),
     1A(20),B(20)
      COMMON EL,EMU,AMBDA,ETA
1     READ INPUT TAPE 4,2,II,IK,NZ,NH,K,DEPSI,AO
2     FORMAT(3I2,2I3,E13.8,E8.3)
      ZN = NZ
      CON = 6.2831852/ZN
      ZETA(1)=0.0
      DO 10 I = 2,NZ
10    ZETA(I)=ZETA(I-1)+CON
      PSI(1) = 0.0
      DO 3 J = 1,K
      ANG(J)=57.295781*PSI(J)
      CO(J)=COSF(PSI(J))
      SI(J) = SINF(PSI(J))
3     PSI(J+1)=PSI(J)+DEPSI
4     READ INPUT TAPE 4,5,EMU,AMBDA,IJ
5     FORMAT(2E8.3,I2)
6     READ INPUT TAPE 4,5,EL,ETA
      BLE=AO*(EL-ETA)
      WRITE OUTPUT TAPE 2,12,NZ,EMU,AMBDA,EL,ETA,DEPSI
12    FORMAT(1H1 I2,14H BLADES    MU = F6.3,11H    LAMBDA = F6.3,6H    L =
     1F6.3,8H    ETA = F6.3,13H  DELTA PSI = F8.5/1H0/66H    PSI       TOTA
     2L WAKE          CONTRIBUTIONS FROM INDIVIDUAL BLADES/1H0)
      DO 30 J = 1,K
      TW(J) = 0.0
      DO 25 I = 1,NZ
      PZA=PSI(J)+ZETA(I)
      IF(I-1) 13,13,16
13    THNU=EMU*CO(J)
      THDE=EL+EMU*SI(J)
      IF(ABSF(THDE)-.00001) 20,14,14
14    DELTA=ABSF(ATANF(THNU/THDE))
      IF(THDE)  15,115,115
15    DELTA=3.1415926-DELTA
115   IF(THNU) 116,117,117
116   DELTA = - DELTA
117   T(I,J)=(1.0-SINF(DELTA))/((EL-ETA)*COSF(DELTA))
      GO TO 22
16    CALL OUD1(SI(J),PSI(J),PZA,KER,PHI)
      PZP=PZA-PHI
      IF(KER-2) 17,60,17
60    WRITE OUTPUT TAPE 2,61,ZETA(I)
      GO TO 20
61    FORMAT(25H0NO CONVERGENCE AT ZETA = F10.7)
17    D = EMU*PZP
      Z = (AMBDA*PZP-BLE)**2
      ERN = EL**2-(D*SI(J))**2
      IF(ERN) 20,21,21
20    T(I,J) = 0.0
      GO TO 22
21    ERN = SQRTF(ERN)
      YE=D*CO(J)+ERN-ETA
      THNU=-EMU*COSF(PHI)
      THDE=EL+EMU*SINF(PHI)
      IF(ABSF(THDE)-.00001) 20,70,70
```

```
70    PSD=ABSF(ATANF(THNU/THDE))
     IF(THDE) 71,72,72
71    PSD=3.1415926-PSD
72    IF(THNU) 73,74,74
73    PSD=-PSD
74    DELTA=PHI-PSI(J)-PSD
      UTT=YE*COSF(DELTA)
      T(I,J)=2.0*UTT/(Z+UTT**2)
22    TW(J)=T(I,J)+TW(J)
25    CONTINUE
30    WRITE OUTPUT TAPE 2,33,ANG(J),TW(J),(T(I,J),I=1,NZ)
33    FORMAT(1H F8.1,F13.5,F15.5,5F12.5)
      CALL HANEW(PSI,TW,K,NH,DEPSI,AZERO,A,B)
      WRITE OUTPUT TAPE 2,44,AZERO,(N,A(N),N,B(N),N=1,NH)
44    FORMAT(18H1HARMONIC ANALYSIS/1HO/9HOA-ZERO =E12.5/(3HOA-I2,1H= E
    112.5,5X,3H B-I2,1H= E12.5))
      PUNCH 101,AZERO,(A(J),J=1,NH),(B(I),I=1,NH)
       PUNCH 100,(TW(J),J=1,K)
100   FORMAT(6E12.5)
101   FORMAT(5E14.7)
      IJ = IJ - 1
      IF(IJ) 53,53,6
53    IK = IK - 1
      IF(IK) 54,54,4
54    II = II - 1
      IF(II) 56,56,1
56    CALL EXIT
     END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE OUD1(SI,PSI,PZA,KER,PHI)
      COMMON EL,EMU
      NC = 1
      KER = 1
      C1=EMU*SI
      PHI = PSI
2     EFOD=C1*(PHI-PZA)+EL*SINF(PHI-PSI)
      EFPOD=C1+EL*COSF(PHI-PSI)
      AD = EFOD/EFPOD
      IF(ABSF(AD)-.00001) 10,10,4
4     NC = NC + 1
      IF(NC-10) 8,8,6
6     KER = 2
      GO TO 10
8     PHI = PHI - AD
      GO TO 2
10    RETURN
     END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE HANEW(PSI,T,K,NH,DEPSI,AZERO,A,B)
      DIMENSION PSI(50),T(50),A(20),B(20),W(50)
      Q = DEPSI/9.4247778
      AZERO = T(1)+T(K)+4.0*T(2)
      DO 2 II = 1,NH
      A(II) =0.0
2     B(II) = 0.0
      W(1)=1.0
      W(K)=1.0
      W(2)=4.0
      KKK = K - 1
      DO 4 JJ = 3,KKK,2
      W(JJ)=2.0
      W(JJ+1)=4.0
       AZERO = AZERO +2.0*T(JJ)+4.0*T(JJ+1)
4     CONTINUE
      AZERO = 0.5*Q*AZERO
      DO 8 II = 1,NH
      FLL = II
      DO 10 NB = 1,K
      A(II)=A(II)+W(NB)*T(NB)*COSF(FLL*PSI(NB))
10    B(II)=B(II)+W(NB)*T(NB)*SINF(FLL*PSI(NB))
      A(II)=Q*A(II)
      B(II) = Q*B(II)
8     CONTINUE
      RETURN
     END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

4 BLADES MU = .200 LAMBDA = .025 L = 1.000 ETA = .950 DELTA PSI = .26180

PSI	TOTAL WAKE	CONTRIBUTIONS FROM INDIVIDUAL BLADES			
0.	26.93724	16.39608	5.54507	2.96902	2.02708
15.0	28.52674	16.66103	6.08984	3.37927	2.39660
30.0	31.75955	17.09723	7.15731	4.24545	3.25956
45.0	37.94855	17.67494	9.07674	5.96833	5.22854
60.0	51.02715	18.36179	12.79333	9.89772	9.96832
75.0	45.90268	19.15115	21.52012	15.14319	-9.91178
90.0	24.75162	20.00000	27.70366	-15.42452	-7.52752
105.0	-22.80051	20.88647	-29.71257	-8.97872	-4.99569
120.0	-5.02990	21.77725	-16.90221	-6.17026	-3.73468
135.0	2.99334	22.63091	-11.81830	-4.80810	-3.01116
150.0	7.34065	23.39560	-9.41482	-4.06156	-2.57856
165.0	9.85217	24.00812	-8.16925	-3.64970	-2.33700
180.0	11.10684	24.39608	-7.57625	-3.46602	-2.24696
195.0	11.22926	24.48546	-7.46246	-3.47990	-2.31385
210.0	10.05849	24.21600	-7.82017	-3.71366	-2.62368
225.0	6.81985	23.56381	-8.80556	-4.25384	-3.68455
NO CONVERGENCE AT ZETA = 4.7123889					
240.0	6.36870	22.56473	-10.91124	-5.28480	0.
255.0	16.89822	21.32429	-15.67933	-7.02600	18.27926
NO CONVERGENCE AT ZETA = 4.7123889					
270.0	-8.32543	20.00000	-19.19655	-9.12888	0.
285.0	14.77329	18.75796	20.03869	-9.07433	-14.94902
NO CONVERGENCE AT ZETA = 4.7123889					
300.0	38.60882	17.72678	10.64250	10.23955	0.
315.0	35.21124	16.97518	7.17173	5.05356	6.01077
330.0	28.38611	16.51800	5.84963	3.37712	2.64136
345.0	26.71420	16.33622	5.43521	2.92165	2.02112
360.0	26.93724	16.39608	5.54507	2.96902	2.02708

HARMONIC ANALYSIS

A-ZERO = .18087E 02

A- 1= .15947E 02	B- 1= .91212E 00
A- 2= .29015E 01	B- 2= .62804E 01
A- 3= -.11550E 02	B- 3= -.11793E 01
A- 4= -.35911E 01	B- 4= -.73613E 01
A- 5= .44264E 01	B- 5= .16926E 01
A- 6= .19471E 01	B- 6= .74949E 01
A- 7= -.18754E 01	B- 7= -.19209E 01
A- 8= .28111E 00	B- 8= -.56557E 01
A- 9= .34981E 01	B- 9= .24231E 01
A-10= -.12456E 01	B-10= .34025E 01

```
NO CONVERGENCE AT ZETA = 3.1415926
   300.0       7.77835        -2.95908      0.            0.            10.73743

NO CONVERGENCE AT ZETA = 3.1415926

NO CONVERGENCE AT ZETA = 4.7123889
   315.0     -13.50982        -2.72228    -10.78755      0.            0.

NO CONVERGENCE AT ZETA = 4.7123889
   330.0      20.00500        -2.62685     14.51674      8.11511       0.
   345.0      20.80732        -2.63173     15.87829      4.73445       2.82631
   360.0      23.23921        -2.70813     17.66847      5.29967       2.97920
```

HARMONIC ANALYSIS

A-ZERO = -.73798E 01

A- 1=	.97773E 01	B- 1=	-.91342E 01
A- 2=	.73915E 01	B- 2=	.48276E 01
A- 3=	.10012E 02	B- 3=	.40065E 00
A- 4=	.76078E 01	B- 4=	.36063E 00
A- 5=	.30506E 01	B- 5=	.16712E 01
A- 6=	.47120E 00	B- 6=	.13112E 01
A- 7=	-.38262E 01	B- 7=	.13126E 01
A- 8=	-.63643E 01	B- 8=	.59635E 00
A- 9=	-.53324E 01	B- 9=	-.40836E 00
A-10=	-.30983E 01	B-10=	-.61389E 00

APPENDIX F

Program III with Typical Results

```
      DIMENSION ETA(10),THETA(10),F(10),G(10),PSI(50),ANG(50),SI(50),CO
     1(50),CT(50),ST(50),CTT(50),STT(50),C(21),D(50),A(10,10,21),B(10,10
     2,50),Q(20,21),Z(20),TIMES(10),ELAM(10,21),PL(10,50),GC(10),GS(10),
     3ELC(8),ELS(8)
 1    READ INPUT TAPE 4,2,DEPSI,BOER,TANMU,EMU,CF,II,M,KP,NH,NGP,I2P
 2    FORMAT(E13.8,4E8.3,6I3)
      N2= NH/2
      MN = M +1
      ROB = 1.0/BOER
      DEMU = 0.5*EMU
      E2M=0.5*EMU*EMU
 3    READ INPUT TAPE 4,4,(ETA(I),THETA(I),F(I),G(I),I=1,M)
 4    FORMAT (4E8.3)
      PSI(1)=0.0
      ID = 0
      DO 6 J = 1,KP
      ANG(J)=57.2957781*PSI(J)
      CO(J) = COSF(PSI(J))
      SI(J) = SINF(PSI(J))
      CT(J) = COSF(2.0*PSI(J))
      ST (J) = SINF(2.0*PSI(J))
      CTT(J)=COSF(3.0*PSI(J))
      PSI(J+1) = PSI(J) + DEPSI
 6    STT(J) = SINF(3.0*PSI(J))
      DO 7 IJI = 1,M
 7    TIMES(IJI) = ETA(IJI)*THETA(IJI)
      DO 20 I = 1,M
      DO 13 J = 1,MN
      READ INPUT TAPE 4,8,(A(I,J,K),K=1,NH)
      READ INPUT TAPE 4,9,(B(I,J,K),K=1,KP)
 8    FORMAT(5E14.7)
 9    FORMAT (6E12.5)
      GO TO (13,10),I2P
 10   READ INPUT TAPE 4,8,(C(L),L=1,NH)
      READ INPUT TAPE 4,9,(D(L),L=1,KP)
      DO 11 K = 1,NH
 11   A(I,J,K) = A(I,J,K) + C(K)
      DO 12 L = 1,KP
 12   B(I,J,L) = B(I,J,L) + D(L)
 13   CONTINUE
      Q(I,MN)=ROB*TANMU
      DO 18 N = 1,M
     DO 15 K = 1,NH
 15   A(I,N,K) = A(I,N,K) -A(I,N+1,K)
      DO 16 L = 1,KP
 16   B(I,N,L)= B(I,N,L)-B(I,N+1,L)
      Q(I,N)=A(I,N,1)
      IF(I-N) 18,17,18
 17      Q(I,N)=Q(I,N) + ROB
 18   Q(I,MN)=Q(I,MN)+A(I,N,1)*TIMES(N)
 20   CONTINUE
      CALL CROUT(JJ,M,Q,Z)
      IF(JJ) 24,24,22
 22   WRITE OUTPUT TAPE 2,23
 23   FORMAT(38H1NO SOLUTION--DIAGONAL ELEMENT = ZERO.)
      GO TO 64
```

```
24    WRITE OUTPUT TAPE 2,25,(ETA(I),I=1,M)
25    FORMAT(28H1LOAD COEFFICIENTS FOR ETA =,9(F6.3,1H,)//)
      WRITE OUTPUT TAPE 2,26,ID,(Z(I),I=1,M)
26    FORMAT(13H0HARMONIC NO. ,I2,9E13.5)
      DO 28 K = 1,M
28    TIMES(K)=BOER*(TIMES(K)-Z(K))
      DO 35 J=2,NH
      ID = J - 1
      DO 30 I = 1,M
      ELAM(I,J) = 0.0
      DO 30 K = 1,M
30    ELAM(I,J)=ELAM(I,J)+TIMES(K)*A(I,K,J)
35    WRITE OUTPUT TAPE 2,26,ID,(ELAM(N,J),N=1,M)
      DO 41 I = 1,M
      WRITE OUTPUT TAPE 2,42,ETA(I)
42    FORMAT(22H1LAMBDA(PSI) FOR ETA = F6.3//)
      DO 41 J = 1,KP
      PL(I,J) = TANMU
      DO 43 K = 1,M
43    PL(I,J)=PL(I,J)+TIMES(K)*B(I,K,J)
      WRITE OUTPUT TAPE 2,44,ANG(J),PL(I,J)
44    FORMAT(1H F7.2,E15.6)
41    CONTINUE
      WRITE OUTPUT TAPE 2,45
45    FORMAT(4H1ETA,9X,4HL3-C,11X,4HL3-S,11X,4HL4-C,11X,4HL4-S,11X,4HL5
     1-C,11X,4HL5-S,11X,4HL6-C,11X,4HL6-S/)
      DO 50 J = 1,M
      DO 46 I = 2,7
      NNN = N2 + I
      GC(I)=F(J)*ELAM(J,I+1)-G(J)*ELAM(J,NNN)
46    GS(I)=F(J)*ELAM(J,NNN)+G(J)*ELAM(J,I+1)
      DO 47 K = 3,6
      ELC(K)=CF*(ETA(J)*GC(K)+DEMU*(GS(K+1)-GS(K-1)))
47    ELS(K)=CF*(ETA(J)*GS(K)+DEMU*(GC(K-1)-GC(K+1)))
      WRITE OUTPUT TAPE 2,49,ETA(J),(ELC(L),ELS(L),L=3,6)
49    FORMAT(1H0F6.3,8E15.6)
50    CONTINUE
52    READ INPUT TAPE 4,54,AZERO,AONE,BONE
54    FORMAT(3E8.3)
      DO 63 I = 1,M
      WRITE OUTPUT TAPE 2,56,ETA(I)
56    FORMAT(6H1ETA =F6.3/4HOPSI,11X,4HL(I),12X,4HL(G),10X,9HL(I)+L(G),
     15X,26HL(G+I)-HARMONICS EXTRACTED//)
      C = ETA(I)**2
      C1=THETA(I)*(C+E2M)-ETA(I)*EMU*AONE
      C2=2.0*EMU*THETA(I)*ETA(I)-AONE*(C+1.5*E2M)
      C3=BONE*(C+.5*E2M)-EMU*AZERO*ETA(I)
      C4=EMU*BONE*ETA(I)-AZERO*E2M
      C5=EMU*AONE*ETA(I)-THETA(I)*E2M
      C6= 0.5*E2M*AONE
      C7= -0.5*E2M*BONE
      B1=ETA(I)*Z(I)+DEMU*ELAM(I,N2+1)
      B2=ETA(I)*ELAM(I,2)+DEMU*ELAM(I,N2+2)
      B3=ETA(I)*ELAM(I,N2+1)+EMU*Z(I)-DEMU*ELAM(I,3)
      B4=ETA(I)*ELAM(I,3)-DEMU*(ELAM(I,N2+1)-ELAM(I,N2+3))
      B5=ETA(I)*ELAM(I,N2+2)+DEMU*(ELAM(I,2)-ELAM(I,4))
```

```
      DO 63 J = 1,KP
       R = CF*(ETA(I)+EMU*SI(J))
      ELEYE=-R*PL(I,J)
     ELG=CF*(C1+C2*SI(J)+C3*CO(J)+C4*ST(J)+C5*CT(J)+C6*STT(J)+C7*CTT(J)
    1)
      SUML=ELG+ELEYE
      HEL=+ELEYE+CF*(B1+B2*CO(J)+B3*SI(J)+B4*CT(J)+B5*ST(J))
      WRITE OUTPUT TAPE 2,58,ANG(J),ELEYE,ELG,SUML,HEL
58    FORMAT(1H F7.2,4E16.6)
63    CONTINUE
64    NGP = NGP - 1
      IF(NGP) 66,66,52
66    II = II - 1
      IF(II) 68,68,1
68    CALL EXIT
     END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

```
      SUBROUTINE CROUT(JJ,M,A,Z)
      DIMENSION A(20,21),Z(20)
      JJ=-2
      MN = M +1
       DO 27 I = 1,M
      NN = 1
      DO 25 J = 1,MN
      SUMA = 0.0
       IF(J-1)    25,25,9
9      IF (I-1)   19,19,12
12    DO 15 K = 1,NN
      SUMA = SUMA+A(I,K)*A(K,J)
15     CONTINUE
17     A(I,J)=A(I,J)-SUMA
      IF (I-J) 19,26,26
19     IF (A(I,I))  23,21,23
21    JJ = 4
       GO TO 35
23     A(I,J) = A(I,J)/A(I,I)
26    IF(I-1-NN)  25,25,24
 24     NN = NN +1
25       CONTINUE
27     CONTINUE
       IN = M
       Z(IN) = A(M,MN)
28     IN = IN -1
      IF(IN-1)  35,29,29
29     SUMB = 0.0
       KK = IN + 1
       DO 31 K = KK,M
       SUMB = SUMB + Z(K)*A(IN,K)
31   CONTINUE
       Z(IN) = A(IN,MN) - SUMB
       GO TO 28
35     RETURN
    END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)
```

LOAD COEFFICIENTS FOR ETA = .950, .850, .750, .600, .300,

HARMONIC NO. 0	.32747E-01	.26967E-01	.24287E-01	.26822E-01	.20601E-01
HARMONIC NO. 1	.22231E-01	.21895E-01	.16365E-01	-.19054E-03	-.21749E-02
HARMONIC NO. 2	.41460E-02	-.44885E-02	-.14419E-01	-.43768E-02	-.98755E-03
HARMONIC NO. 3	-.13205E-01	-.92275E-02	-.38956E-02	-.42504E-02	-.45973E-02
HARMONIC NO. 4	-.88347E-02	-.24848E-03	-.24182E-02	.15770E-02	-.71222E-03
HARMONIC NO. 5	.25496E-02	-.65105E-03	-.26847E-02	.21982E-02	-.29281E-02
HARMONIC NO. 6	.25599E-02	.41512E-03	.26852E-02	.10051E-02	-.32817E-02
HARMONIC NO. 7	-.22915E-02	.12095E-02	-.94535E-03	-.11379E-02	-.92176E-03
HARMONIC NO. 8	-.22469E-03	.48575E-05	.14419E-02	-.39938E-02	-.20479E-02
HARMONIC NO. 9	.33202E-02	-.69679E-04	-.15142E-02	-.43604E-03	-.14484E-02
HARMONIC NO.10	-.37242E-02	.14563E-03	.38901E-05	-.17468E-02	-.29227E-02
HARMONIC NO.11	.16680E-02	-.78302E-03	-.66816E-03	.10567E-02	-.49356E-03
HARMONIC NO.12	-.12904E-03	-.22413E-02	.20340E-02	-.29061E-02	.26315E-03
HARMONIC NO.13	-.94060E-03	.21760E-02	-.24570E-04	-.13821E-03	-.13191E-02
HARMONIC NO.14	.24553E-03	-.18430E-02	-.10530E-02	.54496E-04	.11778E-02
HARMONIC NO.15	.91562E-04	-.20882E-02	.40499E-02	-.68198E-03	-.12945E-02
HARMONIC NO.16	-.13683E-02	.32438E-02	-.30041E-02	.14621E-02	-.49949E-03
HARMONIC NO.17	-.10085E-02	.31465E-04	-.13381E-02	.41457E-03	.86297E-03
HARMONIC NO.18	.46477E-03	-.23984E-02	.22857E-02	-.16059E-02	-.10907E-02
HARMONIC NO.19	-.15741E-03	.22188E-02	-.24689E-02	-.60196E-05	.62770E-05

LAMBDA(PSI) FOR ETA = .950

0.	.393006E-01
7.50	.412342E-01
15.00	.401294E-01
22.50	.461387E-01
30.00	.575040E-01
37.50	.629770E-01
45.00	.627435E-01
52.50	.622757E-01
60.00	.581023E-01
67.50	.511076E-01
75.00	.397935E-01
82.50	.324955E-01
90.00	.350328E-01
97.50	-.105605E-01
105.00	-.121480E-02
112.50	.653595E-02
120.00	.992324E-02
127.50	.119370E-01
135.00	.135129E-01
142.50	.148859E-01
150.00	.160976E-01
157.50	.171584E-01
165.00	.180841E-01
172.50	.188912E-01
180.00	.195956E-01
187.50	.202024E-01
195.00	.207083E-01
202.50	.211029E-01
210.00	.213661E-01
217.50	.214681E-01
225.00	.213633E-01
232.50	.209663E-01
240.00	.200970E-01
247.50	.183917E-01
255.00	.151204E-01
262.50	.829840E-02
270.00	.119347E-02
277.50	.439814E-01
285.00	.456505E-01
292.50	.549819E-01
300.00	.609659E-01
307.50	.658873E-01
315.00	.677144E-01
322.50	.618553E-01
330.00	.540788E-01
337.50	.439627E-01
345.00	.476748E-01
352.50	.454590E-01
360.00	.393006E-01

LAMBDA(PSI) FOR ETA = .850

0.	.327020E-01
7.50	.366983E-01
15.00	.377216E-01
22.50	.354524E-01
30.00	.374273E-01
37.50	.476039E-01
45.00	.484284E-01
52.50	.507238E-01
60.00	.519439E-01
67.50	.465539E-01
75.00	.409969E-01
82.50	.345839E-01
90.00	.280900E-01
97.50	.194914E-01
105.00	.291090E-01
112.50	.340384E-01
120.00	.693205E-02
127.50	.284771E-02
135.00	.923531E-03
142.50	.145817E-03
150.00	.207453E-02
157.50	.496284E-02
165.00	.727524E-02
172.50	.874531E-02
180.00	.960222E-02
187.50	.100543E-01
195.00	.101577E-01
202.50	.991727E-02
210.00	.945993E-02
217.50	.903670E-02
225.00	.862390E-02
232.50	.754002E-02
240.00	.534030E-02
247.50	.200572E-02
255.00	-.264776E-02
262.50	.355883E-01
270.00	.364823E-01
277.50	.342315E-01
285.00	.392416E-01
292.50	.489399E-01
300.00	.504342E-01
307.50	.496072E-01
315.00	.503208E-01
322.50	.512479E-01
330.00	.456528E-01
337.50	.405146E-01
345.00	.399595E-01
352.50	.371673E-01
360.00	.327020E-01

LAMBDA(PSI) FOR ETA = .750

0.	.182515E-01
7.50	.201353E-01
15.00	.219033E-01
22.50	.311319E-01
30.00	.355044E-01
37.50	.372450E-01
45.00	.464060E-01
52.50	.440177E-01
60.00	.413394E-01
67.50	.419334E-01
75.00	.433239E-01
82.50	.331743E-01
90.00	.334649E-01
97.50	.317486E-01
105.00	.210990E-01
112.50	.209885E-01
120.00	.207185E-01
127.50	.240902E-01
135.00	.236190E-01
142.50	.199721E-01
150.00	-.940046E-02
157.50	-.155745E-01
165.00	.966265E-03
172.50	.250352E-02
180.00	.351395E-03
187.50	.220759E-02
195.00	.175090E-02
202.50	-.391148E-02
210.00	-.396602E-02
217.50	.305269E-02
225.00	.146388E-02
232.50	.760257E-02
240.00	.180651E-01
247.50	.421307E-01
255.00	.369715E-01
262.50	.333526E-01
270.00	.321327E-01
277.50	.338911E-01
285.00	.444184E-01
292.50	.454520E-01
300.00	.422217E-01
307.50	.467704E-01
315.00	.498476E-01
322.50	.362136E-01
330.00	.263378E-01
337.50	.245093E-01
345.00	.239612E-01
352.50	.213221E-01
360.00	.182515E-01

LAMBDA(PSI) FOR ETA = .600

0.	.145676E-01
7.50	.184266E-01
15.00	.233498E-01
22.50	.230004E-01
30.00	.207868E-01
37.50	.187643E-01
45.00	.155022E-01
52.50	.232511E-01
60.00	.478714E-01
67.50	.423764E-01
75.00	.342303E-01
82.50	.334754E-01
90.00	.320067E-01
97.50	.241611E-01
105.00	.276462E-01
112.50	.304692E-01
120.00	.273488E-01
127.50	.164054E-01
135.00	.117842E-01
142.50	.177455E-01
150.00	.220762E-01
157.50	.261717E-01
165.00	.247810E-01
172.50	.246358E-01
180.00	.251058E-01
187.50	.266327E-01
195.00	.310334E-01
202.50	.311155E-01
210.00	.286345E-01
217.50	.277257E-01
225.00	.280401E-01
232.50	.290208E-01
240.00	.285897E-01
247.50	.256500E-01
255.00	.251185E-01
262.50	.259390E-01
270.00	.306452E-01
277.50	.349176E-01
285.00	.362258E-01
292.50	.371660E-01
300.00	.386104E-01
307.50	.299190E-01
315.00	.230312E-01
322.50	.247650E-01
330.00	.262140E-01
337.50	.270837E-01
345.00	.281067E-01
352.50	.211327E-01
360.00	.145676E-01

LAMBDA(PSI) FOR ETA = .300

0.	.379364E-02
7.50	.357627E-02
15.00	.120355E-01
22.50	.220645E-01
30.00	.238651E-01
37.50	.225014E-01
45.00	.195868E-01
52.50	.180028E-01
60.00	.203301E-01
67.50	.196863E-01
75.00	.182966E-01
82.50	.183697E-01
90.00	.212359E-01
97.50	.202705E-01
105.00	.201786E-01
112.50	.187036E-01
120.00	.136833E-01
127.50	.449549E-02
135.00	.155909E-01
142.50	.231787E-01
150.00	.250816E-01
157.50	.258778E-01
165.00	.273119E-01
172.50	.267728E-01
180.00	.251940E-01
187.50	.250856E-01
195.00	.234620E-01
202.50	.217737E-01
210.00	.216486E-01
217.50	.201478E-01
225.00	.200811E-01
232.50	.218215E-01
240.00	.210326E-01
247.50	.226704E-01
255.00	.231486E-01
262.50	.239732E-01
270.00	.247541E-01
277.50	.250558E-01
285.00	.251764E-01
292.50	.250615E-01
300.00	.243169E-01
307.50	.241396E-01
315.00	.240643E-01
322.50	.249820E-01
330.00	.272602E-01
337.50	.280900E-01
345.00	.139043E-01
352.50	.782729E-02
360.00	.379364E-02

ETA	L3-C	L3-S	L4-C	L4-S	L5-C	L5-S	L6-C	L6-S
.950	-.296774E 01	.115353E 01	-.215651E 01	-.194460E-01	.539417E 00	-.387405E 00	.579229E 00	-.184772E-01
.850	-.195468E 01	-.411027E-01	.266835E-01	.247736E 00	-.344236E 00	-.322014E 00	.716943E-01	-.525317E 00
.750	-.657917E 00	.271740E 00	-.514613E 00	.111117E 00	-.470653E 00	-.200213E 00	.627585E 00	.557836E 00
.600	-.794455E 00	-.375145E 00	.246166E 00	-.257688E 00	.307075E 00	-.645967E-01	.171891E 00	-.685681E-01
.300	-.344403E 00	.111636E 00	-.673453E-01	-.125604E 00	-.166983E 00	.204816E 00	-.314287E 00	-.618270E-01

ETA = .950

PSI	L(I)	L(G)	L(I)+L(G)	L(G+I)-HARMONICS EXTRACTED
0.	-.126941E 02	.431043E 02	.304103E 02	.645536E 01
7.50	-.136846E 02	.440923E 02	.304077E 02	.591962E 01
15.00	-.136681E 02	.447652E 02	.310971E 02	.613929E 01
22.50	-.161035E 02	.450750E 02	.289715E 02	.363782E 01
30.00	-.205289E 02	.449884E 02	.244595E 02	-.112836E 01
37.50	-.229486E 02	.444904E 02	.215418E 02	-.415621E 01
45.00	-.232831E 02	.435862E 02	.203032E 02	-.534689E 01
52.50	-.234747E 02	.423027E 02	.188280E 02	-.661181E 01
60.00	-.221887E 02	.406869E 02	.184982E 02	-.657537E 01
67.50	-.197195E 02	.388036E 02	.190851E 02	-.548297E 01
75.00	-.154671E 02	.367318E 02	.212647E 02	-.269417E 01
82.50	-.126868E 02	.345587E 02	.218719E 02	-.137641E 01
90.00	-.136978E 02	.323748E 02	.186770E 02	-.382556E 01
97.50	.412299E 01	.302668E 02	.343898E 02	.126416E 02
105.00	.472172E 00	.283125E 02	.287847E 02	.776519E 01
112.50	-.252172E 01	.265759E 02	.240542E 02	.370886E 01
120.00	-.378958E 01	.251032E 02	.213137E 02	.156645E 01
127.50	-.449963E 01	.239217E 02	.194220E 02	.183378E 00
135.00	-.501439E 01	.230386E 02	.180242E 02	-.800460E 00
142.50	-.542435E 01	.224434E 02	.170190E 02	-.148379E 01
150.00	-.574683E 01	.221097E 02	.163629E 02	-.190161E 01
157.50	-.598867E 01	.220002E 02	.160115E 02	-.208598E 01
165.00	-.615944E 01	.220701E 02	.159106E 02	-.207706E 01
172.50	-.626954E 01	.222722E 02	.160027E 02	-.191862E 01
180.00	-.632939E 01	.225615E 02	.162322E 02	-.165457E 01
187.50	-.634607E 01	.228982E 02	.165521E 02	-.132311E 01
195.00	-.632431E 01	.232506E 02	.169263E 02	-.955410E 00
202.50	-.626710E 01	.235970E 02	.173299E 02	-.574394E 00
210.00	-.617480E 01	.239259E 02	.177511E 02	-.192549E 00
217.50	-.604550E 01	.242359E 02	.181904E 02	.188393E 00
225.00	-.587313E 01	.245345E 02	.186613E 02	.579296E 00
232.50	-.564102E 01	.248361E 02	.191950E 02	.100936E 01
240.00	-.530783E 01	.251607E 02	.198528E 02	.153888E 01
247.50	-.478509E 01	.255313E 02	.207462E 02	.227985E 01
255.00	-.389074E 01	.259722E 02	.220815E 02	.344017E 01
262.50	-.212092E 01	.265071E 02	.243862E 02	.554944E 01
270.00	-.304335E 00	.271575E 02	.268532E 02	.780207E 01
277.50	-.112408E 02	.279409E 02	.167001E 02	-.258370E 01
285.00	-.117466E 02	.288698E 02	.171232E 02	-.241299E 01
292.50	-.143050E 02	.299503E 02	.156453E 02	-.416662E 01
300.00	-.161017E 02	.311806E 02	.150789E 02	-.503761E 01
307.50	-.177271E 02	.325502E 02	.148231E 02	-.563343E 01
315.00	-.186158E 02	.340389E 02	.154230E 02	-.541542E 01
322.50	-.174187E 02	.356160E 02	.181973E 02	-.306965E 01
330.00	-.156288E 02	.372409E 02	.216121E 02	-.131068E 00
337.50	-.145408E 02	.388631E 02	.243223E 02	.205892E 01
345.00	-.145599E 02	.404242E 02	.258643E 02	.304686E 01
352.50	-.142798E 02	.418600E 02	.275803E 02	.419144E 01
360.00	-.126941E 02	.431043E 02	.304102E 02	.645536E 01

ETA = .850

PSI	L(I)	L(G)	L(I)+L(G)	L(G+I)-HARMONICS EXTRACTED
0.	-.945086E 01	.374110E 02	.279602E 02	.327050E 01
7.50	-.109315E 02	.386418E 02	.277103E 02	.226296E 01
15.00	-.115654E 02	.396244E 02	.280590E 02	.206685E 01
22.50	-.111683E 02	.403081E 02	.291398E 02	.284162E 01
30.00	-.120890E 02	.406543E 02	.285653E 02	.221199E 01
37.50	-.157281E 02	.406398E 02	.249116E 02	-.124888E 01
45.00	-.163244E 02	.402594E 02	.239350E 02	-.180404E 01
52.50	-.173956E 02	.395271E 02	.221315E 02	-.299162E 01
60.00	-.180708E 02	.384755E 02	.204048E 02	-.395535E 01
67.50	-.163788E 02	.371540E 02	.207752E 02	-.273197E 01
75.00	-.145409E 02	.356253E 02	.210844E 02	-.154256E 01
82.50	-.123263E 02	.339612E 02	.216348E 02	-.147581E 00
90.00	-.100281E 02	.322371E 02	.222090E 02	.117718E 01
97.50	-.694708E 01	.305260E 02	.235797E 02	.315613E 01
105.00	-.103245E 02	.288972E 02	.185728E 02	-.141974E 01
112.50	-.119755E 02	.274040E 02	.154285E 02	-.432792E 01
120.00	-.241159E 01	.260883E 02	.236767E 02	.396177E 01
127.50	-.976617E 00	.249752E 02	.239986E 02	.414847E 01
135.00	-.311307E 00	.240733E 02	.237620E 02	.363406E 01
142.50	-.481775E-01	.233759E 02	.2332[illegible]E 02	.282585E 01
150.00	-.670072E 00	.228637E 02	.221936E 02	.127610E 01
157.50	-.156341E 01	.225080E 02	.209446E 02	-.373015E 00
165.00	-.223059E 01	.222748E 02	.200442E 02	-.160314E 01
172.50	-.260502E 01	.221286E 02	.195236E 02	-.233567E 01
180.00	-.277504E 01	.220362E 02	.192612E 02	-.265603E 01
187.50	-.281645E 01	.219697E 02	.191533E 02	-.264603E 01
195.00	-.275681E 01	.219085E 02	.191517E 02	-.234749E 01
202.50	-.260802E 01	.218404E 02	.192324E 02	-.179369E 01
210.00	-.241228E 01	.217622E 02	.193499E 02	-.105372E 01
217.50	-.223753E 01	.216788E 02	.194412E 02	-.226129E 00
225.00	-.207764E 01	.216019E 02	.195243E 02	.662801E 00
232.50	-.177230E 01	.215489E 02	.197766E 02	.174128E 01
240.00	-.122886E 01	.215406E 02	.203117E 02	.307195E 01
247.50	-.453647E 00	.215999E 02	.211462E 02	.462229E 01
255.00	.591290E 00	.217500E 02	.223413E 02	.640918E 01
262.50	-.788571E 01	.220132E 02	.141275E 02	-.137413E 01
270.00	-.806259E 01	.224094E 02	.143466E 02	-.914236E 00
277.50	-.758507E 01	.229554E 02	.153703E 02	.140330E 00
285.00	-.876333E 01	.236636E 02	.149003E 02	-.515595E 00
292.50	-.110690E 02	.245413E 02	.134722E 02	-.234685E 01
300.00	-.116054E 02	.255890E 02	.139835E 02	-.244433E 01
307.50	-.116603E 02	.268000E 02	.151397E 02	-.208196E 01
315.00	-.121231E 02	.281592E 02	.160361E 02	-.213525E 01
322.50	-.126892E 02	.296419E 02	.169527E 02	-.228676E 01
330.00	-.116415E 02	.312140E 02	.195725E 02	-.809815E 00
337.50	-.106544E 02	.328319E 02	.221775E 02	.626483E 00
345.00	-.108450E 02	.344437E 02	.235987E 02	.905345E 00
352.50	-.104115E 02	.359908E 02	.255794E 02	.182287E 01
360.00	-.945086E 01	.374110E 02	.279602E 02	.327050E 01

ETA = .750

PSI	L(I)	L(G)	L(I)+L(G)	L(G+I)-HARMONICS EXTRACTED
0.	-.465412E 01	.312757E 02	.266216E 02	.208173E 01
7.50	-.531323E 01	.326656E 02	.273523E 02	.192393E 01
15.00	-.597083E 01	.338694E 02	.278986E 02	.192080E 01
22.50	-.874877E 01	.348355E 02	.260867E 02	-.854877E-01
30.00	-.102608E 02	.355214E 02	.252606E 02	-.756615E 00
37.50	-.119393E 02	.358974E 02	.248581E 02	-.681708E 00
45.00	-.140649E 02	.359494E 02	.218846E 02	-.290309E 01
52.50	-.135992E 02	.356802E 02	.220810E 02	-.174465E 01
60.00	-.129760E 02	.351095E 02	.221334E 02	-.598790E 00
67.50	-.133274E 02	.342723E 02	.209449E 02	-.648337E 00
75.00	-.138932E 02	.332170E 02	.193238E 02	-.117215E 01
82.50	-.123081E 02	.320006E 02	.196926E 02	.170612E 00
90.00	-.108092E 02	.306850E 02	.198758E 02	.113488E 01
97.50	-.102363E 02	.293316E 02	.190953E 02	.890518E 00
105.00	-.676611E 01	.279971E 02	.212310E 02	.328811E 01
112.50	-.667063E 01	.267295E 02	.200589E 02	.209866E 01
120.00	-.650332E 01	.255651E 02	.190618E 02	.825478E 00
127.50	-.744262E 01	.245270E 02	.170844E 02	-.164289E 01
135.00	-.715852E 01	.236248E 02	.164663E 02	-.290324E 01
142.50	-.591964E 01	.228556E 02	.169360E 02	-.314917E 01
150.00	.271673E 01	.222064E 02	.249231E 02	.413418E 01
157.50	.437878E 01	.216567E 02	.260335E 02	.463790E 01
165.00	-.263403E 00	.211822E 02	.209188E 02	-.908185E 00
172.50	-.650618E 00	.207580E 02	.200974E 02	-.192160E 01
180.00	-.896056E-01	.203617E 02	.202721E 02	-.165453E 01
187.50	-.543343E 00	.199759E 02	.194325E 02	-.209532E 01
195.00	-.415665E 00	.195894E 02	.191737E 02	-.165115E 01
202.50	.895641E 00	.191983E 02	.200940E 02	.249994E 00
210.00	.876491E 00	.188061E 02	.196826E 02	.104919E 01
217.50	-.652068E 00	.184222E 02	.177701E 02	.511152E 00
225.00	-.302900E 00	.180613E 02	.177584E 02	.195900E 01
232.50	-.152851E 01	.177417E 02	.162132E 02	.187288E 01
240.00	-.354276E 01	.174840E 02	.139412E 02	.972672E 00
247.50	-.809653E 01	.173093E 02	.921273E 01	-.255333E 01
255.00	-.699934E 01	.172384E 02	.102391E 02	-.566171E 00
262.50	-.625635E 01	.172910E 02	.110346E 02	.889985E 00
270.00	-.600882E 01	.174845E 02	.114757E 02	.164966E 01
277.50	-.635735E 01	.178340E 02	.114766E 02	.160407E 01
285.00	-.840916E 01	.183511E 02	.994194E 01	-.346072E 00
292.50	-.873478E 01	.190436E 02	.103088E 02	-.748259E 00
300.00	-.828010E 01	.199144E 02	.116343E 02	-.512315E 00
307.50	-.940327E 01	.209604E 02	.115572E 02	-.195031E 01
315.00	-.103143E 02	.221715E 02	.118572E 02	-.321982E 01
322.50	-.773537E 01	.235294E 02	.157940E 02	-.988328E 00
330.00	-.582065E 01	.250072E 02	.191865E 02	.642700E 00
337.50	-.561209E 01	.265687E 02	.209566E 02	.677939E 00
345.00	-.568839E 01	.281694E 02	.224810E 02	.575573E 00
352.50	-.524789E 01	.297574E 02	.245095E 02	.116150E 01
360.00	-.465412E 01	.312757E 02	.266216E 02	.208173E 01

ETA = .600

PSI	L(I)	L(G)	L(I)+L(G)	L(G+I)-HARMONICS EXTRACTED
0.	-.297178E 01	.218688E 02	.188970E 02	.150739E 01
7.50	-.392258E 01	.233400E 02	.194175E 02	.890947E 00
15.00	-.517430E 01	.247101E 02	.195358E 02	.251247E-01
22.50	-.529060E 01	.259281E 02	.206375E 02	.330529E 00
30.00	-.494726E 01	.269490E 02	.220018E 02	.111288E 01
37.50	-.460469E 01	.277365E 02	.231318E 02	.189153E 01
45.00	-.390794E 01	.282659E 02	.243580E 02	.300077E 01
52.50	-.599757E 01	.285260E 02	.225285E 02	.127976E 01
60.00	-.125849E 02	.285201E 02	.159352E 02	-.500050E 01
67.50	-.113070E 02	.282647E 02	.169577E 02	-.349219E 01
75.00	-.923133E 01	.277885E 02	.185572E 02	-.127356E 01
82.50	-.908583E 01	.271291E 02	.180432E 02	-.107892E 01
90.00	-.870583E 01	.263296E 02	.176238E 02	-.744821E 00
97.50	-.655777E 01	.254349E 02	.188771E 02	.126616E 01
105.00	-.745572E 01	.244876E 02	.170319E 02	.148582E 00
112.50	-.812990E 01	.235251E 02	.153952E 02	-.814908E 00
120.00	-.718971E 01	.225770E 02	.153873E 02	-.217458E 00
127.50	-.423174E 01	.216636E 02	.174319E 02	.236293E 01
135.00	-.297061E 01	.207958E 02	.178252E 02	.323140E 01
142.50	-.435466E 01	.199759E 02	.156212E 02	.145923E 01
150.00	-.525414E 01	.191992E 02	.139450E 02	.194524E 00
157.50	-.602009E 01	.164562E 02	.124361E 02	-.897830E 00
165.00	-.549147E 01	.177356E 02	.122441E 02	-.644236E 00
172.50	-.524437E 01	.170264E 02	.117820E 02	-.612258E 00
180.00	-.512159E 01	.163200E 02	.111984E 02	-.640609E 00
187.50	-.519669E 01	.156122E 02	.104155E 02	-.803324E 00
195.00	-.578463E 01	.149036E 02	.911892E 01	-.142016E 01
202.50	-.553786E 01	.141997E 02	.866185E 01	-.115217E 01
210.00	-.486787E 01	.135108E 02	.864295E 01	-.422537E 00
217.50	-.450831E 01	.128506E 02	.834226E 01	.213647E-01
225.00	-.437192E 01	.122349E 02	.786300E 01	.252064E 00
232.50	-.435463E 01	.116809E 02	.732626E 01	.359095E 00
240.00	-.414865E 01	.112054E 02	.705674E 01	.636996E 00
247.50	-.362116E 01	.108245E 02	.720338E 01	.120770E 01
255.00	-.347431E 01	.105531E 02	.707880E 01	.136133E 01
262.50	-.354280E 01	.104044E 02	.686162E 01	.125925E 01
270.00	-.416774E 01	.103904E 02	.622266E 01	.560526E 00
277.50	-.476911E 01	.105217E 02	.575257E 01	-.150505E 00
285.00	-.501065E 01	.108076E 02	.579695E 01	-.529407E 00
292.50	-.524696E 01	.112558E 02	.600889E 01	-.919328E 00
300.00	-.560277E 01	.118719E 02	.626916E 01	-.143106E 01
307.50	-.448940E 01	.126581E 02	.816872E 01	-.460536E 00
315.00	-.359095E 01	.136122E 02	.100212E 02	.323887E 00
322.50	-.402690E 01	.147260E 02	.106991E 02	-.182412E 00
330.00	-.445638E 01	.159842E 02	.115278E 02	-.625648E 00
337.50	-.482029E 01	.173634E 02	.125431E 02	-.936821E 00
345.00	-.523910E 01	.188316E 02	.135925E 02	-.123032E 01
352.50	-.412350E 01	.203489E 02	.162254E 02	.848577E-01
360.00	-.297178E 01	.218688E 02	.188970E 02	.150739E 01

ETA = .300

PSI	L(I)	L(G)	L(I)+L(G)	L(G+I)-HARMONICS EXTRACTED
0.	-.386951E 00	.585480E 01	.546785E 01	.138397E 01
7.50	-.396522E 00	.692331E 01	.652679E 01	.153280E 01
15.00	-.143944E 01	.802883E 01	.658939E 01	.648721E 00
22.50	-.282475E 01	.913314E 01	.630840E 01	-.580377E 00
30.00	-.324565E 01	.101961E 02	.695044E 01	-.850649E 00
37.50	-.322660E 01	.111787E 02	.795209E 01	-.689296E 00
45.00	-.293965E 01	.120461E 02	.910642E 01	-.270878E 00
52.50	-.280749E 01	.127700E 02	.996247E 01	-.203081E-01
60.00	-.327091E 01	.133305E 02	.100596E 02	-.380216E 00
67.50	-.324477E 01	.137169E 02	.104721E 02	-.267006E 00
75.00	-.306802E 01	.139277E 02	.108597E 02	-.207579E-01
82.50	-.311216E 01	.139694E 02	.108572E 02	-.137635E-01
90.00	-.361011E 01	.138550E 02	.102449E 02	-.479387E 00
97.50	-.343419E 01	.136018E 02	.101676E 02	-.290070E 00
105.00	-.338361E 01	.132293E 02	.984574E 01	-.244843E 00
112.50	-.308280E 01	.127571E 02	.967431E 01	.323188E-01
120.00	-.220151E 01	.122032E 02	.100017E 02	.872344E 00
127.50	-.701063E 00	.115331E 02	.108820E 02	.231479E 01
135.00	-.233994E 01	.109097E 02	.856977E 01	.602251E 00
142.50	-.332373E 01	.101932E 02	.686952E 01	-.469659E 00
150.00	-.341110E 01	.944231E 01	.603120E 01	-.658254E 00
157.50	-.331294E 01	.866479E 01	.535185E 01	-.672940E 00
165.00	-.326649E 01	.786901E 01	.460252E 01	-.749371E 00
172.50	-.296846E 01	.706457E 01	.409611E 01	-.582553E 00
180.00	-.256988E 01	.626280E 01	.369292E 01	-.321713E 00
187.50	-.233608E 01	.547675E 01	.314067E 01	-.230243E 00
195.00	-.198020E 01	.472062E 01	.274042E 01	-.192805E-01
202.50	-.165431E 01	.400884E 01	.235453E 01	.161253E 00
210.00	-.147211E 01	.335481E 01	.188270E 01	.199853E 00
217.50	-.122104E 01	.276964E 01	.154860E 01	.311353E 00
225.00	-.108271E 01	.226113E 01	.117842E 01	.316482E 00
232.50	-.104857E 01	.183308E 01	.784515E 00	.226093E 00
240.00	-.906720E 00	.148524E 01	.578517E 00	.254378E 00
247.50	-.888140E 00	.121378E 01	.325640E 00	.172562E 00
255.00	-.840687E 00	.101245E 01	.171764E 00	.134840E 00
262.50	-.829034E 00	.874059E 00	.450249E-01	.784010E-01
270.00	-.841641E 00	.792200E 00	-.494404E-01	.163800E-01
277.50	-.866473E 00	.762918E 00	-.103555E 00	-.379062E-01
285.00	-.914331E 00	.786003E 00	-.128327E 00	-.943431E-01
292.50	-.981814E 00	.865711E 00	-.116103E 00	-.149030E 00
300.00	-.104831E 01	.101073E 01	-.375757E-01	-.181294E 00
307.50	-.115995E 01	.123336E 01	.734064E-01	-.237677E 00
315.00	-.129747E 01	.154789E 01	.250426E 00	-.299779E 00
322.50	-.151402E 01	.196851E 01	.454490E 00	-.422100E 00
330.00	-.185369E 01	.250679E 01	.653098E 00	-.650498E 00
337.50	-.213421E 01	.316933E 01	.103512E 01	-.804849E 00
345.00	-.117353E 01	.395563E 01	.278211E 01	.294393E 00
352.50	-.728910E 00	.485676E 01	.412785E 01	.887186E 00
360.00	-.386951E 00	.585479E 01	.546784E 01	.138397E 01