Aerospace Research Laboratories

PREDICTING EQUILIBRIUM PRESSURES FROM TRANSIENT PRESSURE DATA

DAVID L. BROWN
UNIVERSITY OF CINCINNATI
CINCINNATI, OHIO

OFFICE OF AEROSPACE RESEARCH
United States Air Force
PREDICTING EQUILIBRIUM PRESSURES FROM TRANSIENT PRESSURE DATA

DAVID L. BROWN
UNIVERSITY OF CINCINNATI
CINCINNATI, OHIO

JANUARY 1965

Contract AF 33(616)-8453
Project 7064

AEROSPACE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO
FOREWORD

This interim technical report was prepared by the University of Cincinnati Hypersonic Aerodynamics Research Staff. This investigation was sponsored by the Aerospace Research Laboratories of the Office of Aerospace Research, U. S. Air Force, under Contract No. AF 33(616)8453 for research on Experimental Aerothermodynamic Investigations, Project No. 7064, "Aerothermodynamic Investigations in High Speed Flow", under the technical cognizance of Colonel Andrew Boreske, Jr., Acting Chief, Hypersonic Research Laboratory, of ARL.

This work was carried out during the period February 1964 to August 1964.
ABSTRACT

It was the purpose of this study to investigate the time-lag phenomena associated with wind tunnel pressure measurements, and find a method for predicting the equilibrium pressure from the transient pressure conditions. In order to do this a mathematical model of the actual physical case was formulated. Actual transient pressure data was fitted by least squares to this model and the equilibrium pressure predicted.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II  MATHEMATICAL MODEL</td>
<td>2</td>
</tr>
<tr>
<td>CURVE FITTING TECHNIQUE</td>
<td>9</td>
</tr>
<tr>
<td>LEAST SQUARES METHOD</td>
<td>10</td>
</tr>
<tr>
<td>COMPUTATIONAL PROCEDURE</td>
<td>14</td>
</tr>
<tr>
<td>III EXPERIMENTAL PROGRAM</td>
<td>16</td>
</tr>
<tr>
<td>TEST MODELS</td>
<td>17</td>
</tr>
<tr>
<td>DATA REDUCTIONS</td>
<td>17</td>
</tr>
<tr>
<td>TEST RESULTS</td>
<td>18</td>
</tr>
<tr>
<td>IV  CONCLUSIONS</td>
<td>21</td>
</tr>
<tr>
<td>V   REFERENCES</td>
<td>23</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

TABLE

1  Data for Constant Orifice Pressure Experiment  24

FIGURE

1  Variation of "A" with Tube Radius and Temperature  25
2  Schematic Diagram of IBM Computer Program  26
3  Schematic Diagram of Time-Lag Experiment in High Pressure Range  27
4  Schematic Diagram of Time-Lag Experiment in Low Pressure Range  28
5  Schematic Diagram of Variable Pressure Experiment  29
6  Schematic Diagram of Bridge Balancing Unit  30
7  Typical Variable Pressure Technique Predictions  31
8  Typical Variable Pressure Technique Predictions  32
9  Typical Variable Pressure Technique Predictions  33
10 Typical Unsteady Flow Region  34
11 Typical Unsteady Flow Region  35
12 Variation of Time-Lag Constant with Temperature  36
I INTRODUCTION

One of the major problem areas when testing in a hypersonic blow-down facility of short run duration is obtaining reliable pressure measurements. Usually at high Mach numbers the pressure readings are very low (0-1 psia range with the majority of readings at or below 0.1 psia) which creates two major problem areas. The first concerns the selection of an accurate, reliable pressure transducer and the second concerns the time-lag phenomenon created when using pressure transducers remotely located from the model.

This study was concerned with a method of solving the time-lag problem which is created because geometric and environmental conditions necessitate mounting the pressure transducer remotely from the point where a pressure reading is desired. The physical model was simulated by connecting a length of tubing from the orifice to the transducer, thereby restricting the flow which equalizes the pressure between the transducer and the orifice.

When a time-lag condition exists, the usual technique for measuring pressure is to wait until the pressure measuring system reaches equilibrium. In some cases this time period is prohibitive.

It was the purpose of this study to develop a method of extrapolating the equilibrium pressure from the transient data. In order to accomplish this, it was necessary to formulate a mathematical model of the physical problem. For ease of mathematical manipulation, this model had to be relatively simple yet still accurately describe the physical system.

Manuscript released October 1964 by the author for publication as an ARL Technical Documentary Report.
II MATHEMATICAL MODEL

A suitable mathematical model had to be formulated before the equilibrium pressure could be extrapolated from the transient data. The physical case which this model must describe consists of a small internal volume which is connected to a pressure orifice by a length of tubing. (See following diagram).

In order for the pressure in the transducer to equal the pressure on the orifice, a certain amount of air must flow through the tube. The physical properties of the gas and the geometric configuration of the tube determines this flow rate.

The mass flow-rate through a circular tube is given by Gordon P. Brown and Albert DiNarda (Reference 1) for the continuum, slip, and free molecular flow ranges as:

\[ Q = \frac{\pi r^4}{16 \mu LRT} (P_e^2 - P^2)(1 + 4 \left(\frac{2}{\pi} - 1\right) \frac{\lambda_m}{r}) \]  

(1)
where

\[ Q = \text{mass flow rate -- slugs/sec} \]
\[ r = \text{radius of tube -- ft} \]
\[ \mu = \text{viscosity of fluid -- slugs/ft \cdot sec} \]
\[ L = \text{length of tube -- ft} \]
\[ R = \text{gas constant, 1716.22 lb}_f\text{-ft}/\text{slugs}^\circ R \]
\[ f = \text{fraction of molecules diffusely reflected from the walls of tube} = 0.8 \]
\[ (1-f) = \text{fraction of molecules specularly reflected from the walls of tube} \]
\[ T = \text{temperature -- }^\circ R \]
\[ \lambda_m = \text{mean free path -- ft} \]
\[ P_e = \text{equilibrium pressure -- lb}_f/\text{ft}^2 \]
\[ P = \text{pressure at transducer -- lb}_f/\text{ft}^2 \]
\[ \frac{\lambda_m}{r} = \text{Knudsen number} \]

The above equation is based upon the following assumptions:

1. The flow in the tube is isothermal and laminar.
2. No inertial effects are present and the pressure force is balanced only against the viscous force. The flow is always quasisteady.
3. Pressure is constant across any tube cross section.
4. Gas evolution from outgassing, permeation and leakage is neglected.
5. The fluid is Newtonian.

At high pressures the term \(4\left(\frac{2}{f} - 1\right)\frac{\lambda_m}{r}\) approaches zero and the equation reduces to Poiseuille's equation which assumes no slip. As the pressure decreases the above term increases and the slip flow region is defined. At extremely low pressures the term becomes predominant and the equation describes the free molecular range.
The pressure change, due to mass flow in or out of a fixed volume, is given by:

\[ P = \frac{m}{V} RT \]  \hspace{1cm} (2)

where

- \( P \) = pressure
- \( m \) = mass
- \( V \) = volume
- \( R \) = gas constant
- \( T \) = temperature

By differentiation of Eq. (2) with respect to time, holding \( V \) and \( T \) constant, we obtain:

\[ \frac{dP}{dt} = \frac{dm}{dt} \frac{RT}{V} \]

or

\[ \frac{dm}{dt} = Q = V \frac{dP}{RT dt} \]  \hspace{1cm} (3)

From Eq. (1) and (3), we obtain:

\[ \frac{dP}{dt} = K(P_e^2 - P^2)(1 + 4\left(\frac{2}{r} - 1\right) \frac{\lambda_m}{r}) \]  \hspace{1cm} (4)

where

\[ K = \frac{\pi r^4}{16 \mu LV} \]

The mean free path, \( \lambda_m \), is given in Reference 1 as:

\[ \lambda_m = \left(\frac{\pi R T}{2}\right)^{1/2} \frac{\mu}{P_m} \]

where \( P_m \) equals the mean pressure in the tube, \( \frac{P_e + P}{2} \).
Substituting for the mean free path Eq. (4) becomes:

\[
\frac{dP}{dt} = K \left\{ (P_e^2 - P^2) + A(P_e - P) \right\}
\]  

(5)

where

\[ A = \frac{8\mu (\frac{2}{r} - 1)(\frac{TR}{2})^{1/2}}{r} \]

However, in this report, for computational reasons, an alternate procedure was used where \( \lambda_m \) was redefined as given in Reference 2:

\[
\lambda_m = \frac{1}{2^{1/2} \pi N_0^2} \frac{R^* M T_m}{M_o P_m}
\]

where

\( T_m = \) molecular temperature -- °R

\( M = \) molecular weight of air -- lb/mole

\( M_o = \) molecular weight at sea level -- lb/mole

\( T = \frac{M}{M_o} T_m = \) absolute temperature -- °R

\( N = \) Avagadro's Number\(^1\) -- 2.73179 x 10\(^{26}\)/lb - mole

\( \sigma = \) effective collision diameter of mean air molecule\(^1\) -- 1.1975 x 10\(^{-9}\) ft

\( R^* = \) universal gas constant\(^1\) -- 1545.31 ft lb/lb - mole °R

\( P_m = \) mean pressure \( \frac{P_e + P}{2} \) -- lb/ft\(^2\)

Substituting into Eq. (4), we obtain:

\(^1\)The above constants are based on the scale \( C^{12} = 12.000 \).
\[ \frac{dP}{dt} = K \left[ \left( P_e^2 - P^2 \right) + A(P_e - P) \right] \]  

(6)

where

\[ A = \frac{8}{r} \left( \frac{2}{r} - 1 \right) \frac{R^* T}{2 1/2 \pi N e^2} \]

Equation (6) is the mathematical model used in the experiment to predict the equilibrium pressure.

It should be noted that A is inversely proportional to the radius, r, and as r decreases A becomes more important in Eq. (6). If the volume of tubing is the same order of magnitude as the volume of the transducer, then the volume term used in constant K should be corrected as indicated in Reference 3. The correction is:

\[ V = V_T + 1/2 V_{\text{Tube}} \]

where

V = volume term in constant K

V_T = volume transducer

V_{\text{Tube}} = volume tube

This correction is of little importance in this application since K is determined by a curve fitting process.

The mathematical model developed in the above sections is only applicable to single tube systems. In actual wind tunnel testing it is sometimes desirable to construct the pressure tubing in sections of varying diameters. This is usually done to reduce the time-lag, and in some cases to ease model manufacturing difficulties.
In the high pressure region, where slip-flow effects can be neglected, a mathematical model for a multitube system can very easily be developed. A typical schematic of a multitube system is shown below.

\[ P_e \]

\[ P_1 \quad P_2 \quad P_3 \]

From Eq. (1), neglecting losses due to entrance or joint effects, the mass flow through any tube can be expressed as:

\[ Q_1 = K_1 \left( P_e^2 - P_1^2 \right) \quad (A) \]

\[ Q_2 = K_2 \left( P_1^2 - P_2^2 \right) \quad (B) \]

\[ \vdots \]

\[ Q_n = K_n \left( P_{n-1}^2 - P^2 \right) \quad (C) \]

Since the mass flow through all the tubes is equal, we can write:

\[ Q = K_1 \left( P_e^2 - P_1^2 \right) = \ldots = K_n \left( P_{n-1}^2 - P^2 \right) \]

By elimination of the intermediate pressure from Eq.'s (A) through (C) we obtain:
\[ Q = \frac{V}{RT} \frac{dP}{dt} = \frac{P_e^2 - P^2}{\sum_{i=1}^{n} \frac{1}{K_i}} = K_e (P_e^2 - P^2) \]  

(6a)

where \( K_e \) is the effective constant.

The above procedure is analogous to that used to calculate a series resistance network in electricity, where \( Q \) would correspond to current, \( (P_e^2 - P^2) \) to the E.M.F. and \( \frac{1}{K_e} \) to the resistance.

It is evident that Eq. (6a) is parametrically equivalent to Eq. (6) when \( A \) is equal to zero. Therefore, since \( K_e \) is determined by the curve fitting process, Eq.'s (6a) and (6) can be used in the same manner to predict the equilibrium pressure.

In the slip-flow range the equation for mass flow is non-linear (contains \( P^2 \) and \( P \) terms) and no simple expression can be developed for a multitube system. Therefore, in this range a single tube system should be used.

Since temperature gradients exist along the tube in most wind tunnels, it is advisable that the mathematical model be suitably modified.

The original model was based upon isothermal flow, however, the theory was modified to include temperature gradients by assuming that the flow is only isothermal across the tube at any station. This assumption seemed reasonable because of the small diameter tubing used and the low velocities of flow in the tubing.

A variation of temperature along the tube has the effect of varying the resistance to the flow along the tube. Therefore, this is analogous to the multitube concept where the resistance was varied by different diameter tubing. As a result, temperature gradients can be handled in a similar manner by using the same
parametric form of the mathematical model.

CURVE FITTING TECHNIQUE

In the preceding section a mathematical model of a typical pressure measuring system was derived in differential form (See Eq. 6). For a constant orifice pressure Eq. (6) can be integrated using the boundary condition that \( P = P_o \) at \( t = t_o \). The integrated form of the equation becomes:

\[
t = \frac{1}{K(2P_e + A)} \ln \left( \frac{(P_e + A + P)(P_e - P_o)}{(P_e - P)(P_e + A + P_o)} \right) + t_o
\]

The above equation describes the pressure response to a step function increase in orifice pressure from an initial value \( P_o \) to a final value \( P_e \).

The ideal solution would be to fit a curve which has the form of Eq. (7) to the transient data and determine the parameters \( K \) and \( A \) and the equilibrium pressure, \( P_e \), from a least squares fit. This method, however, could not be applied since it would involve solving three non-linear transcendental equations to determine \( K \), \( A \) and \( P_e \). This procedure would be extremely difficult, thereby making the method impractical.

Equation (6) was fitted by least squares and initially all three parameters \( K \), \( A \) and \( P_e \) were determined. This method gave very poor and inconsistent results and it became evident that there were too many degrees of freedom in the problem. Thus, the degrees of freedom were reduced from three to two by determining \( K \) and \( P_e \) by least squares and computing \( A \) by theoretical means from the relation given in Eq. (6).
Since $A$ is a function of temperature, it becomes difficult to compute an effective value when temperature gradients exist along the tube. Fortunately, the mathematical model is rather insensitive to this parameter except at low pressures, therefore, only a rough estimate is needed. For example, in the experimental program values of $A$ were taken nearly three times the theoretical values. This would correspond to errors of approximately $1000^\circ R$ in the temperature, yet this gave maximum errors of only 0.1 mm HG in the pressure prediction when the pressure was above 1 mm Hg. These errors could be further reduced by using more transient data. At pressures under 1 mm Hg, the errors can become appreciable and more care must be taken in selecting the value of $A$ (See Figure 1 for variation of $A$ with temperature). When temperature gradients exist, $A$ can be determined from the mean temperature. Since the model is normally rather insensitive to $A$, it is only necessary to roughly estimate the temperature gradients.

Using Eq. (6) as the mathematical model and determining only $K$ and $P_e$ by least squares, excellent results were obtained.

LEAST SQUARES METHOD

When the orifice pressure is constant the transient pressure data starts at some initial condition ($P_0$ and $t_0$) and exponentially approaches the equilibrium pressure (See following diagram).
Using Eq. (6) at two points on the transient curve, it is possible to determine the parameters $K$ and $P_e$. However, this procedure induces large errors and a more logical process would be to use a technique based upon a least squares selection of $K$ and $P_e$.

The error at any point, $i$, along the curve is given by:

$$E_i = \left[ \frac{dP_i}{dt} - K(P_e^2 + AP_e) + K(P_1^2 + AP_1) \right]^2$$  \hspace{1cm} (8)

where $\frac{dP_i}{dt}$ and $P_1$ are measured from the transient data.

Since the above term is non-linear with respect to $P_e$, the resulting equations will be non-linear. To linearize the method the following transformations are used:

$$D = K(P_e^2 + AP_e)$$  \hspace{1cm} (9)

and

$$C_i = P_1^2 + AP_1$$  \hspace{1cm} (10)

where $C_i$ is only a function of the transient data and the computed value of $A$. Therefore, the equation for the error at point $i$ becomes:

$$E_i = \left[ \frac{dP_i}{dt} - D + KC_i \right]^2$$  \hspace{1cm} (11)

Summing the errors terms the total error becomes:

$$\text{ERROR} = E_1 + E_2 + E_3 \ldots \ldots E_n$$
where

\[ E_1 = \left( \frac{dP_1}{dt} - D + KC_1 \right)^2 \]

\[ E_2 = \left( \frac{dP_2}{dt} - D + KC_2 \right)^2 \quad \text{etc.} \]

The next step is to minimize the total error with respect to the parameters \( D \) and \( K \).

Therefore,

\[ \frac{\partial \text{ERROR}}{\partial D} = 0 \quad (12) \]

and

\[ \frac{\partial \text{ERROR}}{\partial K} = 0 \quad (13) \]

determine a set of equations (12 and 13) which can be solved for \( D \) and \( K \).

Eq.'s (12) and (13) become:

\[
2 \left\{ \left( \frac{dP_1}{dt} - D + KC_1 \right) + \left( \frac{dP_2}{dt} - D + KC_2 \right) \\
+ \ldots \left( \frac{dP_n}{dt} - D + KC_n \right) \right\} = 0
\]

and

\[
2 \left\{ \left( \frac{dP_1}{dt} - D + KC_1 \right)c_1 + \left( \frac{dP_2}{dt} - D + KC_2 \right)c_2 \\
+ \ldots \left( \frac{dP_n}{dt} - D + KC_n \right)c_n \right\} = 0
\]
or simplifying

\[ \sum_{i=1}^{n} \frac{dP_i}{dt} - n D + K \sum_{i=1}^{n} C_i = 0 \quad (14) \]

and

\[ \sum_{i=1}^{n} \frac{dP_i}{dt} C_i - D \sum_{i=1}^{n} C_i + K \sum_{i=1}^{n} C_i^2 = 0 \quad (15) \]

Solving Eqs. (14) and (15) for $K$ the following result is obtained:

\[ K = \frac{n \sum_{i=1}^{n} \frac{dP_i}{dt} C_i - \sum_{i=1}^{n} \frac{dP_i}{dt} \sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} C_i - \sum_{i=1}^{n} C_i - n \sum_{i=1}^{n} C_i^2} \quad (16) \]

Solving Eq. (14) for $D$ find:

\[ D = \frac{K \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} \frac{dP_i}{dt}}{n} \quad (17) \]

The equilibrium pressure $P_e$ can be found by substituting for $K$ and $D$ in the transformation given by Eq. (9). If this is done,
the relation for $P_e$ becomes:

$$P_e = -\frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 + \frac{D}{K}}$$

(18)

Therefore, using the least squares technique and the mathematical model, a set of equations was developed which can be used to predict the equilibrium pressure from the transient conditions.

COMPUTATIONAL PROCEDURE

In order to use the relations discussed in the preceding section, it is necessary to know the value of $\frac{dP}{dt}$ and $P$ at various points along the transient curve. The output of the pressure transducer gives pressure, $P$, versus time, $t$, data directly, and the $\frac{dP}{dt}$ data must be computed from the pressure-time curve. Since a differentiation process tends to amplify extraneous noise on the original data, a smoothing technique was used.

The method used was to take $n$ points off the transient data at equal time intervals (See following diagram).

From this data, the slope at any point (1) was computed by the
following relation

\[ \left( \frac{dP}{dt} \right)_i = \frac{P(i + m) - P(i - m)}{2m\Delta t} \]

where

\[ \Delta t = \text{time between successive pressure readings} \]
\[ m = \text{integer (smoothing function)} \]
\[ P_i = \text{pressure reading at point "i"} \]

The product of \( m\Delta t \) determines the smoothing ability of the preceding relation. As this becomes larger both the smoothing process and the potential error in \( \frac{dP}{dt} \) become larger. Since the pressure curve is generally a slow varying curve in our case, \( m\Delta t \) can be made rather large without introducing serious errors in \( \frac{dP}{dt} \).

Since it was necessary to use a large number of points to define the transient region, a computer became almost a necessity in solving this problem. Therefore, the IBM 1620 and 7094 were programmed for this problem (See Figure 2). These programs will be briefly outlined in the following paragraph.

The pressure-time data was read and stored in the computer along with a value for the smoothing function. Using this stored data, the slope at various points along the curve was computed and stored with its corresponding value of \( C \) (defined in Eq. 10).

From this new data, the summations

\[ \sum_{i=1}^{n} \frac{dP_i}{dt}, \quad \sum_{i=1}^{n} C_i, \]

\[ \sum_{i=1}^{n} C_i^2, \quad \sum_{i=1}^{n} \frac{dP_i}{dt} C_i \]

were determined. The values of \( K, D \) and \( P_e \) were then determined from Eqs. (16), (17) and (18) using the values of these summations.
III EXPERIMENTAL PROGRAM

The experimental portion of the time-lag program was divided into the following two basic experiments:

1. Constant Orifice Pressure - the pressure on the orifice of a typical pressure measuring system was held constant and the method of analysis developed in the preceding section was used.

2. Varying Orifice Pressure - in a second series of tests, the pressure on the orifice was slowly varied with time. Equation (6) was then used to predict the orifice pressure based on the pressure-time variation measured by the transducer.

In the first set of experiments a model was constructed which was compatible with the mathematical model developed in the preceding sections. The model consisted of a long piece of stainless steel tubing of small diameter connected between a reference tank and a 0-1 psia Statham transducer. A valve separated the tubing from the reference tank (See Figures 3-4). The pressure in the system was set at some predetermined value. The valve was closed and the reference tank was readjusted to a new pressure reading. The valve was suddenly opened and the pressure history at the transducer was recorded on a Minneapolis-Honeywell model 1508 Visicorder and later on the Consolidated Systems Corporation Microsadic Data Acquisition System until the system reached equilibrium. From the transient portion of the data the equilibrium pressure was estimated by the method developed in this report.

In the second series of tests the same model was used with the exception that a 0-1 psia Statham transducer was added to measure the variation of pressure in the reference tank. The pressure variation in the reference tank and the transducer was recorded on the Visicorder (see Figures 5-6).

Initially, the tube was calibrated for its value of K by the
method used in the first set of tests. Once this value was determined, the pressure in the reference tank was varied. From the pressure-time curve at the transducer, the orifice pressure could be predicted from Eq. (6) as follows:

\[ P_e = - \frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 + \left(P^2 + AP + \frac{dP}{dt}\right)} \]  

The varying orifice pressure experiment was designed primarily to test the feasibility of using the mathematical model developed in this report to predict dynamic pressure variations in a system with considerable time-lag.

TEST MODELS

Two configurations, typical of the installations used in the ARL hypersonic wind tunnel facilities, were tested. One model consisted of a 0.042 I.D. hypodermic tube 8 feet long and the second of 0.038 inch I.D. tube 4 feet long. Both tubes were connected to a 0-1 psia Statham transducer, using a special fitting which effectively increased the internal volume of the transducer. This was done to increase the time-lag of each tube.

The eight foot model was constructed from a piece of surplus tubing. As a result its cross-section varied considerably along its length due to bending. This was done deliberately because actual tubing used in wind tunnel testing is frequently in this condition. The four foot model was an ideal model constructed out of tempered stainless steel tubing with both ends machined to reduce end or entrance effects. This was done to provide a comparison with the more typical case.

DATA REDUCTION

The data from the eight foot tube was taken from the
Visicorder plots at 0.1 second intervals and punched in the proper format on IBM data cards. Four seconds of data was used to predict $P_e$. To neglect unsteady effects, this data was taken two seconds after the pressure step was applied. The data cards were then processed on the IBM 1620 and later on the IBM 7094 by the program described in the section on Computational Procedures.

The data from the four foot tube was processed directly by the IBM 7094 by editing the Microsadic tape and rewriting a new tape in Fortran Compatiable language. The new tape was then processed according to the aforementioned computer program.

The variable pressure data was processed in two steps. The calibration step was carried out in the same manner as the constant orifice pressure experiment. Once the calibration data was known, the variable pressure data was reduced as follows. The data was transcribed from the recordings at 0.1 second intervals to IBM cards in a manner analagous to the previously described test. An IBM 1620 was programmed to numerically compute the slope at various points along the curve. From this data Eq. (19) was solved for the orifice pressure.

TEST RESULTS

The test results for the constant orifice pressure experiment are listed in Table I. Also listed in the table is the amount of transient data used in the prediction along with the resulting time-lag (time necessary for the pressure to approach within 1% of the equilibrium pressure). This was done to illustrate the amount of time saved by using the procedure outlined in this report. It should be noted that the time-lag was calculated from Eq. (17) and not measured during the experimental program.

As can be seen from Table I, the results were very good. The maximum error for the eight foot tube was little over 0.1 mm Hg when based on only four seconds of transient data. For the four foot tube (ideal model), which was tested at pressures under
1 mm of Hg, the maximum error was only 0.013 mm Hg. This is equivalent to the repeatability of the transducer used to measure the pressure.

Some typical results from the variable pressure test are shown in Figures 7 - 9. As can be seen from these figures the predicted pressure is fairly good. However, there is a reasonable amount of scatter (0.2 mm Hg) in the prediction, which was caused by noise on the original data. Since no ultra-smoothing process like least squares can be used, noise constitutes a more serious problem. However, by using the process previously described for smoothing the $\frac{dP}{dt}$ data and by fairing a smooth curve through the predicted data, very good results were obtained.

A second problem area encountered was due to unsteady flow in the tube. When this condition exists, the mathematical model breaks down because the quasi-steady flow assumption is violated. As a result the predicted pressure lags behind the actual orifice pressure.

Any rapidly changing pressure differential across the tube will cause this unsteady flow condition. Therefore, the variable pressure technique is limited to slowly varying pressures. No serious attempt was made in this study to consider these effects, except to determine their possible cause and to propose a semi-empirical correction to the mathematical model.

Initially, no inertial effects were considered in the derivation of Eq. (1). To correct this situation the inertia terms were later included in the derivation (in a manner similar to Reference 3) and the following mathematical model was formulated:

$$\left(\frac{dP}{dt} + B \frac{d^2P}{dt^2}\right) = K \left[ (P_e^2 - P^2) + A(P_e - P) \right]$$  \hspace{1cm} (20)
where

\[ B = \frac{d^2 \int P \, dx}{32\mu RT} = \frac{d^2(P_e + P)}{64\mu RT} \]

is the inertia coefficient. If \( B \) is evaluated for the models tested, the inertia term becomes negligible and Eq. (20) reduces to the mathematical model used in this report. At first this conclusion was quite confusing, but a closer study indicated that the problem must be one of boundary layer growth, not one of inertia. To check this hypothesis, the time necessary for the boundary layer to fill the tube was computed and compared to the time period of unsteady flow as measured from the experimental data. These were equivalent, which would tend to support the above hypothesis.

Time did not permit a detailed analysis of the boundary layer growth in the tube. Therefore, no analytical solution was formulated. However, a semi-empirical solution was devised which considerably improved the pressure predictions in these unsteady flow regimes.

The second derivative of pressure with respect to time is a measure of the rate of change of mass flow through the tube, and therefore, an indication of the unsteady condition (See Eq. 3). If \( \frac{d^2P}{dt^2} \) equals zero the flow is steady; if \( \frac{d^2P}{dt^2} \) is small the flow is quasi-steady; and if \( \frac{d^2P}{dt^2} \) is large the flow is unsteady. Therefore, the original mathematical model was modified by adding a \( \frac{d^2P}{dt^2} \) term as follows:

\[ \left( \frac{dP}{dt} + F \frac{d^2P}{dt^2} \right) = K \left[ (P_e^2 - P^2) + A(P_e - P) \right] \quad (21) \]
where

\[ F = \text{the unsteady flow coefficient} \]

The coefficient \( F \) was then determined from the experimental data assuming that it was constant over any small pressure range. When this was done, the unsteady flow regions were considerably improved (see Figures 10 and 11). The coefficient \( F \) is not actually constant, but some function of pressure and possibly pressure differential across the tube. However, over any small pressure range (3 to 10 mm Hg and 10 to 20 mm Hg in this study) it appeared to be fairly constant. At lower pressures it appeared that the variation was greater, but insufficient data was available to determine its pressure dependence.

If unsteady flow regimes are of interest, a detailed analytic study should be conducted to determine a suitable mathematical model. If it should be impossible to formulate a usable model from this analysis, then one similar to that proposed in Eq. 21 could be developed both from an analytic and experimental standpoint.

**IV CONCLUSIONS**

A method for predicting equilibrium pressures from transient conditions was developed and tested with very good results. The tests were concerned only with configurations under consideration for application in the ARL hypersonic wind tunnel facilities. In these tests no actual wind tunnel data was evaluated—only simulated data. Several conditions were not simulated by the tests; temperature gradients along the tube and multitube configurations, which, however, should not present any problems except at very low pressures.

When applying this method there are several requirements which must be followed:
1. After a pressure disturbance, several seconds of data must be discarded to assure no unsteady effects are present.

2. The pressure on the orifice must be constant during the time the data is taken.

3. Temperature gradients must not appreciably vary during the time when the data is taken. (Effect of temperature on time-lag constant $K$ is shown in Figure 12).

A second procedure for measuring variable pressure was also tested with reasonable results. This procedure, unlike the first, would be more difficult to adapt to wind tunnel testing. Variations of temperature with time would change the time-lag constant, $K$, which is used in the computation of the orifice pressure (See Figure 12). Since in most wind tunnels this facet would occur, it appears unlikely that this method could be used. One exception would be where the temperature varied slowly and the pressure was periodically constant on the orifice. If this were true, then the tube could be periodically calibrated for $K$, which would then change only slightly between calibration points.

The second method outlined in this report applied only to slowly varying pressures because the method is based upon a quasi-steady flow assumption. A brief investigation of the unsteady flow region was made and the results are included in the Section on Test Results.
V REFERENCES


### TABLE I - DATA FOR CONSTANT ORIFICE PRESSURE EXPERIMENT

**DATA FROM 8-FOOT TUBE**

<table>
<thead>
<tr>
<th>RUN NO.</th>
<th>K</th>
<th>TIME-LAG* (SECS)</th>
<th>EQUILIBRIUM PRESSURE (mm Hg)</th>
<th>PREDICTED PRESSURE (mm Hg)</th>
<th>TRANSIENT DATA (SECS)</th>
<th>APPROXIMATE TIME-LAG* (SECS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0256</td>
<td>5.91</td>
<td>5.95</td>
<td></td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>0.0248</td>
<td>5.95</td>
<td>5.98</td>
<td></td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>0.0254</td>
<td>5.92</td>
<td>5.96</td>
<td></td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>0.0250</td>
<td>5.83</td>
<td>5.85</td>
<td></td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>0.0244</td>
<td>4.81</td>
<td>4.81</td>
<td></td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>0.0240</td>
<td>4.76</td>
<td>4.77</td>
<td></td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>0.0244</td>
<td>4.73</td>
<td>4.78</td>
<td></td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>0.0240</td>
<td>4.79</td>
<td>4.76</td>
<td></td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>0.0242</td>
<td>3.92</td>
<td>3.88</td>
<td></td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>0.0242</td>
<td>3.68</td>
<td>3.78</td>
<td></td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>0.0250</td>
<td>3.78</td>
<td>3.87</td>
<td></td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>0.0252</td>
<td>3.70</td>
<td>3.79</td>
<td></td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>0.0246</td>
<td>2.90</td>
<td>2.92</td>
<td></td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>0.0252</td>
<td>2.80</td>
<td>2.89</td>
<td></td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>0.0244</td>
<td>2.84</td>
<td>2.85</td>
<td></td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>16</td>
<td>0.0248</td>
<td>2.83</td>
<td>2.86</td>
<td></td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>0.0244</td>
<td>1.98</td>
<td>2.07</td>
<td></td>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td>18</td>
<td>0.0238</td>
<td>1.89</td>
<td>1.87</td>
<td></td>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td>19</td>
<td>0.0244</td>
<td>1.85</td>
<td>1.86</td>
<td></td>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td>20</td>
<td>0.0242</td>
<td>1.75</td>
<td>1.748</td>
<td></td>
<td>4</td>
<td>105</td>
</tr>
<tr>
<td>21</td>
<td>0.0236</td>
<td>1.58</td>
<td>1.51</td>
<td></td>
<td>4</td>
<td>131</td>
</tr>
<tr>
<td>22</td>
<td>0.0252</td>
<td>1.08</td>
<td>1.16</td>
<td></td>
<td>4</td>
<td>75</td>
</tr>
</tbody>
</table>

**DATA FROM 4-FOOT TUBE**

<table>
<thead>
<tr>
<th>RUN NO.</th>
<th>K</th>
<th>TIME-LAG* (SECS)</th>
<th>EQUILIBRIUM PRESSURE (mm Hg)</th>
<th>PREDICTED PRESSURE (mm Hg)</th>
<th>TRANSIENT DATA (SECS)</th>
<th>APPROXIMATE TIME-LAG* (SECS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1146</td>
<td>0.075</td>
<td>.0684</td>
<td></td>
<td>6</td>
<td>75</td>
</tr>
<tr>
<td>1**</td>
<td>.1148</td>
<td>0.075</td>
<td>.0673</td>
<td></td>
<td>4</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>.1158</td>
<td>0.065</td>
<td>.0646</td>
<td></td>
<td>4</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>.1156</td>
<td>0.40</td>
<td>.413</td>
<td></td>
<td>4</td>
<td>34</td>
</tr>
<tr>
<td>3**</td>
<td>.1154</td>
<td>0.40</td>
<td>.403</td>
<td></td>
<td>2</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>.1152</td>
<td>0.60</td>
<td>.608</td>
<td></td>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>.1152</td>
<td>0.080</td>
<td>.074</td>
<td></td>
<td>4</td>
<td>73</td>
</tr>
<tr>
<td>6</td>
<td>.1148</td>
<td>0.064</td>
<td>.051</td>
<td></td>
<td>4</td>
<td>79</td>
</tr>
</tbody>
</table>

* Time-lag calculated from Eq. 7 for Pressures within 1% of equilibrium

** Same run with different amounts of transient data
\[ 0.05 \Delta A = \frac{8}{\pi^2} \left( \frac{\pi T}{2} - 1 \right) \left( R^2 T / 4 \pi \right) \]

**Figure 1** - Variation of \( \Delta A \) With Tube Radius and Temperature
FIGURE 2 -- SCHEMATIC DIAGRAM OF IBM COMPUTER PROGRAM

WHERE:

- \( P_c \) = Measured Equilibrium Pressure
- \( P_i \) = Value of Pressure @ Point \( i \) on Transient Curve
- \( T_i \) = Time at Point \( i \)
- \( \frac{dp}{dt}_i \) = Slope at Point \( i \)
- \( M \) = Smoothing Function
- \( A \) = Slip Flow Coefficient
- \( K \) = Time-lag Coefficient
- \( P_E \) = Calculated Equilibrium Pressure
- \( C_1 \) and \( D \) Transformations (see text)
FIGURE 3 --SCHEMATIC DIAGRAM OF TIME-LAG EXPERIMENT IN HIGH PRESSURE RANGE
FIGURE 4--SCHEMATIC DIAGRAM OF TIME-LAG EXPERIMENT IN LOW PRESSURE RANGE
Figure 5 -- Schematic diagram of variable pressure experiment.
FIGURE 8 -- TYPICAL VARIABLE PRESSURE TECHNIQUE PREDICTIONS
FIGURE 9 -- TYPICAL VARIABLE PRESSURE TECHNIQUE PREDICTIONS
Figure 12—Variation of Time-Lag Constant with Temperature

$K_0 =$ Time-lag Constant @ 460 °R
  or 0 °F

$\frac{K}{K_0}$ Ratio of Time-lag Constants

TEMP. °R

TEMP. °F

0 300 400 500 600 700 800 900 1000 1100 1200
-100 0 100 200 300 400 500 600
**Abstract**

It was the purpose of this study to investigate the time-lag phenomena associated with wind tunnel pressure measurements and to find a method for predicting the equilibrium pressure from the transient pressure conditions. In order to do this a mathematical model of the actual physical case was formulated. Actual transient pressure data was fitted by least squares to this model and the equilibrium pressure predicted.
### Security Classification

**AEROTHERMODYNAMIC INVESTIGATION**

**LEAST SQUARES METHOD**

**EQUILIBRIUM PRESSURE**

#### INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

   (1) "Qualified requesters may obtain copies of this report from DDC."

   (2) "Foreign announcement and dissemination of this report by DDC is not authorized."

   (3) "U.S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________, ________, ________.

   (4) "U.S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________, ________, ________.

   (5) "All distribution of this report is controlled. Qualified DDC users shall request through ________, ________, ________.

   If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

   It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (T3), (S), (C), or (U).

   There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.