POTENTIAL HULL STRUCTURES FOR RESCUE AND SEARCH VEHICLES OF THE DEEP-SUBMERGENCE SYSTEMS PROJECT

M. Krenzke, et al.

David Taylor Model Basin

March 1965
POTENTIAL HULL STRUCTURES FOR RESCUE AND SEARCH VEHICLES OF THE DEEP-SUBMERGENCE SYSTEMS PROJECT

by

M. Krenzke, K. Hom, and J. Proffitt

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

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ABSTRACT

Tradeoff studies between pressure hull material, configuration, and buoyancy are performed for both the Rescue and the Search Vehicles of the Deep-Submergence Systems Project. Spherical, cylindrical, and spheroidal shells of high strength steels, titanium alloys, aluminum alloys, glass-reinforced plastics, glass, and ceramics are considered. Failure criteria are developed based on recent research findings. Potential pressure hulls for both vehicles are discussed and the need to develop light-weight floatation systems for the Search Vehicle is emphasized.

ADMINISTRATIVE INFORMATION

By Special Projects Office, Department of the Navy, Project Order 5-0003 of 3 December 1964, the Structural Mechanics Laboratory was given the following assignment in the development of the Rescue and Search Vehicles of the Deep-Submergence Systems Project:

1. Develop and provide structural design information, including material and fabrication analysis, for the pressure hulls.

2. Design, fabricate, and test small-scale models to improve and/or verify pressure hull designs.

3. Verify strength of full-scale pressure hulls.

This report presents a portion of the work conducted under the first phase of the assignment and is aimed at providing the designer with the tradeoffs between pressure hull materials, configurations, and buoyancy. The material and fabrication analysis for metallic hulls, which is used extensively to support this study, is presented in Taylor Model Basin Report 1987. The analysis of structural details is not included in this report but was provided directly to the Bureau of Ships, who is conducting feasibility studies for both the Rescue and the Search Vehicles, in Taylor Model Basin letter Serial 0266 dated 8 March 1965.

INTRODUCTION

The Deep-Submergence Systems Review Group (DSSRG) was established in April 1963 to (1) review the Navy's plans for the development and procurement of components and systems related to location, identification, rescue from and recovery of deep-submerged large objects from the ocean floor, (2) recommend changes to such plans which will result in
expeditiously obtaining sufficient capabilities which could be used to recover large objects from the ocean, (3) develop a five-year program for implementing recommendations, and (4) recommend means and organization of responsibilities for implementation.

In February 1964 the Group reported its findings and recommendations. After review of the report, the Secretary of the Navy assigned the responsibility for implementation of the program, which was renamed the Deep-Submergence Systems Project (DSSP), to the Navy's Special Projects Office.

Two primary objectives of the DSSP are to develop the capability of rescuing personnel from distressed submarines and to locate and recover small objects from the ocean depths. Two distinct vehicles, the Rescue Vehicle and the Search Vehicle, will be developed to achieve these objectives. Present plans are to have six rescue vehicles and four search vehicles operational by 1970.

Of critical importance in the development of small vehicles for deep ocean operation is the selection of the pressure hull material and configuration. Therefore, tradeoff studies between hull material, configuration, and buoyancy were performed to provide the designer with a basis for selecting the pressure hull for both the rescue and search vehicles. Results of these studies are presented in this report.

**PROBLEM DEFINITION**

There are two primary and two secondary missions for the rescue vehicle, and two primary and one secondary mission for the search vehicle. No design compromises are to be made to permit the accomplishment of the secondary missions which will be detrimental to the accomplishment of the primary missions. The two primary missions for the rescue vehicle are to rescue personnel from a sunken submarine at depths as great as combatant submarine collapse depths and to effect transfer of personnel from one submerged submarine to another. The secondary missions of the rescue vehicle are to perform limited search and recovery operations and to accomplish oceanographic research tasks at depths to 6000 ft by utilizing alternate payloads. Only operating depths of 6000 ft are considered in this study.

The primary missions of the search vehicle are to search the ocean bottom to depths of 20,000 ft for either large or small objects and to recover small objects either by actually lifting them or by making an attachment to the object so that a surface lift, or buoyancy lift, can be accomplished. The secondary mission of the search vehicle is to perform limited work and maintenance operations at depths to 20,000 ft by utilizing alternate payloads.

Certain other requirements have an indirect bearing on the structural design of the pressure hulls. The rescue vehicle must be air transportable, and it is desirable also that the search vehicle be air transportable. This requires that the largest individual part, complete with its associated handling equipment, pass through a rectangular door 10 ft wide by
9 ft high and not weigh in excess of about 50,000 lb.* The rescue vehicle will have an operating crew of 2 men, and, in addition, must have a carrying capacity for at least 12 additional personnel. The search vehicle must have carrying capacity for at least 3 men. Both vehicles must be capable of mating with submersed submarines. It is evident, therefore, that relatively large hull penetrations will be required. Since the secondary mission of the rescue vehicle is to conduct oceanographic research, it appears realistic to assume that each vehicle will undergo an appreciable number of excursions from the surface to various depths.

In view of the above requirements, which affect the structural design of the vehicles, the following general assumptions were made:

1. The maximum operating depths shall be 6000 ft for the rescue vehicle and 20,000 ft for the search vehicle.
2. The design factor of safety shall be a minimum of 1.5; this safety factor shall exist after 1000 excursions from the surface to maximum operating depth over a 10-year time span.
3. The pressure hull shall have an outside diameter not less than 6 ft and not greater than 8 ft.
4. The pressure hull displacement shall not exceed 40,000 lb.
5. The largest pressure hull penetrations shall be at least 21 in. in diameter.
6. For the rescue vehicle, full-scale fabrication must proceed on 1 January 1966; for the search vehicle, on 1 January 1968.

ANALYSIS

The analysis of the potential pressure hulls may be characterized as an investigation of various hull configurations by applying established or modified elastic stress analyses and failure criteria based on an adequate knowledge of the properties of the materials under consideration. Several hull configurations and materials have potential application for the subject vehicles. The most promising configurations appear to be the unstiffened spherical shell, the ring-stiffened cylindrical shell, and the stiffened or unstiffened prolate spheroidal shell.

Other hull configurations have been proposed and studied over the years, including stiffened spherical shells, sandwich cylinders, and various shells of noncircular cross section. The stiffened spherical shell offers some strength advantage over the unstiffened shell in the unstable region. However, reliable design criteria do not exist today, and it appears unlikely that they can be developed in time for use in the rescue vehicle. Except for brittle materials such as glass and ceramics, an unstiffened sphere for the search vehicle would normally be more efficient than a stiffened sphere since these shells would be rather stable.

*This weight may approach 70,000 lb at the sacrifice of aircraft range.
Stiffening spherical shells of brittle materials would complicate existing design and fabrication problems. Furthermore, unstiffened glass and ceramic spheres offer very attractive strength-to-weight ratios if obvious problems can be overcome. Sandwich cylinders have received considerable attention over the past several years as a potential hull configuration for hydrospace vehicles. However, there appears to be little, if any, strength advantage over conventional ring-stiffened cylinder hulls for these applications. Sandwich construction may offer an advantage for the search vehicle in that it permits utilization of thinner plating. If the need arises for a geometry which minimizes shell thickness, very good estimates of the weight of sandwich cylinders may be obtained by using the weights for corresponding ring-stiffened cylinders. Noncircular shells, such as the oval cylinder and the oblate spheroid, are less efficient than shells of circular cross section and were not considered in this study.

Of all the metallic materials of interest in this study, HY-100 steel is the only one which is not strain-hardening, i.e., possessing a nonlinear stress-strain relationship above the proportional limit. To obtain realistic hydrostatic strength estimates, an inelastic buckling analysis must be applied. Such an analysis relies upon the determination of elastic stresses, elastic stability strength in each possible mode, applicable plasticity reduction factors, and a knowledge of the stress-strain curve of the material. The nonmetallic materials considered, glass-reinforced plastic, glass, and ceramics, possess an essentially linear stress-strain relationship up to the ultimate strength of the material and the inelastic analysis was not applicable.

Failure criteria used in the analysis of unstiffened spherical shells, ring-stiffened cylindrical shells, and stiffened or unstiffened spheroidal shells are delineated in the following discussion. Appendix A presents the background on the selection and establishment of these criteria.

**SPHERICAL SHELLS**

The collapse strength of both near-perfect shells and shells with initial imperfections may be predicted\(^1\) by the use of these equations:

\[
P_{3''} = 0.84kE \left( \frac{h_o}{R_{10}} \right) \quad \text{for } \nu = 0.3 \tag{1}
\]

\[
P_{E'} = 0.84k \sqrt{E_s E_t} \left( \frac{h_o}{R_{10}} \right)^2 \quad \text{for } \nu = 0.3 \tag{2}
\]
where $E$ is Young's modulus,

$\lambda_a$ is the average thickness over a critical arc length, as defined in Appendix A,

$k$ is an empirical coefficient based on test results and presented graphically in Figure 1,

$R_{10}$ is the local radius to the outside surface of the shell over a critical arc length,

$E_s$ is the secant modulus,

$E_t$ is the tangent modulus,

$p$ is the hydrostatic pressure,

$R_1$ is the local radius to the midsurface of the shell over a critical arc length, and

$\nu$ is Poisson's ratio.

Equation [1] is utilized for the elastic buckling case; Equation [2] for the inelastic stress region. Equation [3] is used to calculate the stress in the shell and is needed to solve Equation [2]. The primes in Equations [1] through [3] indicate that the local geometry, described in Appendix A, is used to calculate pressures and stresses. For stress-relieved shells, the $k$ value used in Equations [1] and [2] is described by the lower envelope of test results of the stress-relieved HY-80 steel shells presented in Figure 1. Similarly, the lower envelope of test results of the HY-80 steel shells tested in the "as fabricated" condition (Figure 1) was used to determine the $k$ value for spherical shells with residual fabrication stresses.

**CYLINDRICAL SHELLS**

For the determination of metallic ring-stiffened cylinder geometries, the following criteria were established:

1. Axisymmetric Shell Failure — The inelastic axisymmetric shell buckle strength for those shells which had a ratio of elastic to inelastic collapse pressure of less than six was calculated by the equation:

$$P_{ML} = P_{OL} + 0.2\left(\frac{P_{EL}}{P_{OL}} - 1.0\right)(P_L - P_{OL})$$  \[4\]
where \( p_L \) is Lunchick's inelastic axisymmetric shell buckle pressure,\(^2\)

\[ p_{EL} \] is Lunchick's elastic axisymmetric shell buckle pressure,\(^2\) and

\[ p_{OL} \] is Lunchick's inelastic axisymmetric shell buckle pressure modified by using
midbay outside stresses rather than membrane stresses.

Lunchick's inelastic shell buckle theory was used without modification for those shells having a ratio of elastic to inelastic strength greater than six. The effects of residual stresses on the collapse strength were estimated from Figure 2 by calculating the ratio of elastic to inelastic pressures and determining the collapse pressure reduction factor from the lower envelope of test results for welded shells.

2. Asymmetric Shell Failure — Both elastic and inelastic shell failures were calculated according to the theories of Reynolds.\(^3\) The effects of residual stresses were estimated using Figure 2, in the manner described above.

3. General Instability Failure — The elastic and inelastic general instability pressures were calculated\(^4\) using the equation

\[
p_{st} = (E_s E_t)^{1/2} \frac{h}{R} \frac{\lambda^4}{(n^2 + \lambda^2/2 - 1)} \left( \frac{n^2 + \lambda^2}{n^2 + 1} \right)^2 + \frac{E_t I_e}{L_f R_0 R_{cg}^2} (n^2 - 1) [5]
\]

where \( h \) is the shell thickness,

\( I_e \) is the moment of inertia about the centroid of a section comprising one frame plus an effective length of shell,

\( L_b \) is the bulkhead spacing,

\( L_f \) is the typical frame spacing,

\( n \) is the number of circumferential waves,

\( R \) is the radius to the midsurface of the shell,

\( R_0 \) is the outside radius of the shell,

\( R_{cg} \) is the radius to the neutral axis of the frame, effective shell combination, and

\[ \lambda = \frac{\pi R}{L_b} \]

The effects of residual stresses and imperfections on the general instability mode of welded cylinders were assumed to be the same as those for the shell failure mode as shown in Figure 2. Due to the uncertainties in this area, failure in the general instability mode was set at 1.05 times the design collapse pressure. The deep frames were designed to provide a
ratio of elastic stability pressure in the overall mode to design collapse pressure of 1.5. The contribution of hemispherical end closures in resisting general instability failure was neglected. Strength of the deep frames was determined by the equation

\[ p_{DF} = \frac{3 EI_{DF}}{L_0 R_0 R_{DF}^2} \]

where \( l_{DF} \) is the moment of inertia about the centroid of a section comprising one deep frame and an effective length of shell, and \( R_{DF} \) is the radius to the neutral axis of such a section.

4. Shell Stresses at the Frame - The maximum stresses in the shell at the frame and in the frame flange of typical bay geometry as calculated by the analysis of Salerno and Pulos\(^5\) were not allowed to exceed 75 percent of the yield strength of the material at operating depth in order to:

a. Permit a reasonable chance of keeping the maximum stresses in the areas of structural details below the yield point of the material at operating depth and thus to account for, in a very general way, low-cycle fatigue, creep, stress corrosion, etc.

b. Ensure a reasonable stress level in the frame flange prior to collapse for those shells with initial out-of-roundness.

5. Frame Web Buckling Stress - The elastic buckling stress in the frame webs was calculated\(^6\) according to

\[ \sigma_w = \frac{\pi^2 E}{2.73} \left( \frac{t}{d} \right)^2 \text{ for } \nu = 0.3 \]

where \( d \) is the web depth and \( t \) is the web width. This stress was set at a minimum of 3 times the yield strength of the material.

PROLATE SPHEROIDAL SHELLS

1. Unstiffened Shells - The elastic stresses were calculated using Flugge's expressions.\(^7\) The circumferential and longitudinal stresses are, respectively:

\[ \sigma_\theta = -\frac{p b}{2 h} \left[ \frac{2a}{b} - \frac{2a^2}{ab} - \frac{b}{a} + \frac{2b^2}{a^3} \right] \frac{1}{\sqrt{\left(\frac{x}{a}\right)^2 - \left(\frac{x}{b}\right)^2 + \left(\frac{a}{b}\right)^2}} \]

\[ [8a] \]
where \( a \) is the semimajor axis,

\( b \) is the semiminor axis, and

\( x \) is measured along the major axis.

Elastic collapse strength was based on Healey's equation

\[
p_{EH} = AE \left( \frac{A}{R_2} \right)^B \quad \text{for } \nu = 0.3
\]

where \( A \) and \( B \) are functions of the length-to-diameter ratio as presented in Appendix A and \( R_2 \) is the circumferential radius to the midplane of the shell at the center.

For the inelastic collapse strength a simple plasticity reduction factor of \( \sqrt{E_s/E_t} \) was assumed.

2. Stiffened Shells — The elastic shell stresses and shell strength were determined by modifying the cylindrical shell theory as outlined in Appendix A. The inelastic shell strength was calculated by applying the plasticity reduction factor \( \sqrt{E_s/E_t} \) to the elastic shell buckle pressure. Estimates of the collapse pressure in the general instability mode were made using the equation

\[
p_c = \frac{Eh}{R_2} \left( \frac{\beta^4}{(n^2 + \beta^2/2 - 1)(n^2 + \beta^2/2)^2} \right) + \frac{E_c (n^2 - 1)}{L_f R_0 R_c g^2}
\]

where

\[
\frac{\beta^4}{(n_H^2 + \beta^2/2 - 1)(n_H^2 + \beta^2)^2} = \frac{R_2}{h} \left[ \frac{p_{EH}}{E} - \frac{h^3}{12} \frac{(n_H^2 - 1)}{R_0 R_c g^2} \right]
\]

and \( n_H \) is the number of circumferential waves observed experimentally for an unstiffened shell of the same length-to-diameter and thickness-to-radius ratios.

The inelastic general instability pressure was determined by applying reduction factors of \( \sqrt{E_s/E_t} \) and \( E_c/E \) to the first and second terms of Equation [10], respectively.
3. Shell Stress at Frames of Stiffened Shells — The maximum stresses in the shell at the frame were not allowed to exceed 75 percent of the yield strength at operating depth.

POTENTIAL HULL STRUCTURES FOR THE RESCUE VEHICLE

WEIGHT-TO-DISPLACEMENT RATIOS FOR SHELLS OF VARIOUS MATERIALS AND CONFIGURATIONS

The common factor which relates the strength-weight characteristics for hulls of various materials and configurations for a given design depth is the ratio of the weight of the hull to the weight of the sea water displaced by the hull. In this report, this ratio will be referred to as the \( W/D \) ratio. \( W/D \) ratios were calculated for hulls composed of high-strength steels, titanium alloys, aluminum alloys, glass-reinforced plastics (GRP), glass, glass-ceramics, and alumina. The assumed properties of the materials considered for both vehicles are given in Table 1.

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<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus ( \text{lb/in}^2 )</th>
<th>Poisson's Ratio</th>
<th>Density ( \text{lb/ft}^3 )</th>
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<tr>
<td>High-Strength Steels</td>
<td>( 30.0 \times 10^6 )</td>
<td>0.30</td>
<td>490</td>
</tr>
<tr>
<td>Titanium Alloys</td>
<td>( 18.0 \times 10^6 )</td>
<td>0.30</td>
<td>276</td>
</tr>
<tr>
<td>Aluminum Alloys</td>
<td>( 10.8 \times 10^6 )</td>
<td>0.30</td>
<td>173</td>
</tr>
<tr>
<td>GRP - Solid Fibers</td>
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<td></td>
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<tr>
<td>Cylindrical and Spheroidal Shells</td>
<td>( 6.4 \times 10^6 )</td>
<td>0.15</td>
<td>128</td>
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<tr>
<td>Spherical Shells</td>
<td>( 5.8 \times 10^6 )</td>
<td>0.15</td>
<td>128</td>
</tr>
<tr>
<td>GRP - Hollow Fibers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical and Spheroidal Shells</td>
<td>( 3.0 \times 10^6 )</td>
<td>0.15</td>
<td>94</td>
</tr>
<tr>
<td>Spherical Shells</td>
<td>( 2.8 \times 10^6 )</td>
<td>0.15</td>
<td>94</td>
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<tr>
<td>Glass</td>
<td>( 9.0 \times 10^6 )</td>
<td>0.20</td>
<td>139</td>
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<td>Glass-Ceramics</td>
<td>( 17.3 \times 10^6 )</td>
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<td>163</td>
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<tr>
<td>Alumina</td>
<td>( 45.0 \times 10^6 )</td>
<td>0.20</td>
<td>233</td>
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</tbody>
</table>
The calculation of $W/D$ ratios depends on the shape of the compressive stress-strain curves. The assumed shapes of the stress-strain curves of the high-strength steels, titanium alloys, and aluminum alloys are presented in Figure 3. They are representative measured compressive stress-strain curves of 1-in. 721 titanium plate, 2-in. HY-150 steel plate, and 1.5-in. 7079-T6 aluminum plate. It is assumed that the shape of the compressive stress-strain curve and associated values of the secant and tangent modulus of other alloys can be approximated by use of these curves. Glass-reinforced plastics, glass, glass-ceramics, and alumina are each assumed to possess linear stress-strain curves.

The $W/D$ ratios for shells of the various materials under consideration are presented in Figures 4 through 7. It should be emphasized that these ratios represent only the shells indicated. For example, the $W/D$ ratios for spherical shells are based on calculations for complete spheres with no structural details. Thus, obtaining $W/D$ ratios for connected or nested spheres requires the addition of the weight penalty resulting from the juncture reinforcement.

The ratios for ring-stiffened cylinders are for simply supported cylinders with a bulkhead, or deep frame, spacing of one diameter. The ratios do not include the weight of the deep frames or end closures. The ratios for cylinders with deep frames and end closures are presented later in the text for several hulls which are considered to have potential application. Only one cylinder length is shown since a good estimate of the $W/D$ ratios for cylinders of other lengths can be made by using the values given and by realizing that compartment length can be successfully broken up by deep frames at a weight penalty which is normally less than 10 percent.

A length-to-diameter ratio of 2.0 was arbitrarily chosen for the varying thickness, prolate spheroid calculations. Other length-to-diameter ratios would have a large effect for the unstiffened case. In the case of the stiffened spheroid, however, the effect of length-to-diameter ratios on the $W/D$ ratios would not be severe. The $W/D$ ratios for the shorter spheroids would approach those of the sphere and the ratios for the longer spheroids should never exceed those shown for the one-diameter-long cylinder by more than about 10 percent. A weight penalty of 5 percent was arbitrarily placed on the stiffened spheroidal shells to compensate for the difficulty in efficiently varying thickness, frame spacing, and frame size along their length.

The effect of fabrication tolerances and residual stresses on the required $W/D$ ratios for spherical and cylindrical shells of steel and titanium is emphasized in Figures 4 and 5. Calculations presented are for $\Delta_8$ values of 0, 1/16, 1/8, and 3/16 in., where $\Delta_8$ is the departure from sphericity for an 8-ft-diameter sphere and is defined in Figure 18 of Appendix A. An out-of-roundness value of 1/8 in. for an 8-ft-diameter cylinder was assumed in the welded internally stiffened cylinder calculations.
Due to the limited time available for this study, it was not feasible to use a computer to calculate the strengths of the cylindrical and spheroidal shells. However, a sufficient number of calculations were performed to ensure that the weights presented approach the minimum values that may be obtained based on the analysis used.

MATERIAL AND FABRICATION ASPECTS

The material and fabrication aspects of the pressure hull for the rescue vehicle will be only briefly summarized in this report. This summary is based on Reference 8 for the metallic materials and on a brief discussion presented in Appendix B for the nonmetallic materials.

In summarizing data of this nature, many different approaches may be taken. There is no apparent approach that does not have its shortcomings. The approach chosen for this study is to review available data in an effort to determine the probability of having an acceptable solution to each critical problem in time to proceed with fabricating the full-scale rescue vehicle by 1 January 1966. The basic problem with this approach is that of eliminating biased opinions of the authors.

In a broad sense, the critical problem areas are considered to be:

1. Designability — The ability to adequately define material response under a given set of design conditions.

2. Producibility — The ability to consistently produce a material to established specifications in the required shapes and sizes.

3. Toughness — The ability of a material to resist crack propagation when used under a given set of design conditions.

4. Permanence — The ability of a material to retain its integrity throughout its design life.

5. Weldability — The ability of a material to withstand design loads in the welded condition.

A summary of the material aspects is presented in Table 2.

POTENTIAL HULL STRUCTURES

A cursory look at the W/D ratios for shells of various shapes and for a given material (Figures 4 through 7) shows that the ratios are roughly the same for spherical, cylindrical, and spheroidal shells. Thus, the choice of a structural shape for the pressure hull will probably lie with the naval architect, and the shape will be selected on the basis of overall arrangement rather than on minimum weight considerations. However, from the structural designers’ viewpoint, the problem of designing large penetrations for these relatively small-diameter hulls would be much simpler for the spherical shells (because of symmetry) than for the cylindrical and spheroidal shells. This is not meant to infer that large penetrations cannot be successfully attained in cylindrical shells.
The material aspects, as summarized in Table 2, lead to a somewhat more clearly delineated choice of hull material than reached for hull configurations. First, if the naval architect can live with a pressure hull having a W/D ratio of about 0.5 to 0.6, HY-100 steel is a clear choice for the pressure hull material. Similarly, if the designer cannot live with the weight of an HY-100 steel hull but requires a hull with a W/D ratio of about 0.4 to 0.5, HY-150 steel with a design compressive yield strength of 140,000 psi appears to be the best choice of material. 721 Titanium alloy, with a compressive yield strength of 110,000 psi, appears to be the only material with a reasonably good chance of meeting all the design requirements and also providing a main pressure hull with a W/D ratio significantly below 0.4.
The method of fabrication and the tolerances achievable have a rather severe influence on the $W/D$ ratios for most of the spherical shells for the rescue vehicle. It does not appear realistic to rely on a spherical shell of uniform thickness to fulfill the standards of near-perfect geometry unless it is machined on all surfaces. If the details are such that the basic shells can be machined and that all areas requiring welding can be built up to account for initial imperfections and can subsequently be stress-relieved to eliminate the detrimental effects of residual stresses, then the $W/D$ ratios for near-perfect geometry will be realistic when corrected for the additional weight of built-up areas. If the fabrication procedures used for ALVIN* are applied to the rescue vehicle, an increase in $W/D$ ratio over that of the near-perfect shell will be required.* The completed ALVIN hull has a maximum initial departure from sphericity corresponding to about $\Delta_8 = 1/16$ in. when scaled up to an 8-ft sphere. Thus, it appears reasonable to assume that when going to a thinner sphere of new hull material and a sphere fabricated in the same manner as ALVIN, a specified maximum departure from sphericity of about $\Delta_8 = 1/8$ in. will be required.

Residual stresses must also be accounted for in the design if the shells contain residual forming and/or welding stresses and if the complete hull cannot be stress-relieved.

It should be noted that welded construction is not a specified requirement for either the rescue or the search vehicle. However, it is felt that since each of the top contending materials for the rescue vehicle are considered weldable, the advantages of welded construction outweigh the penalty which accompanies corresponding initial imperfections and possible residual stresses.

Many of the potential pressure hulls for the rescue vehicle are shown in Table 3. Since such a wide combination of shapes can be considered, it is recognized that Table 3 is not all inclusive. However, combining the $W/D$ ratios shown in Table 3 with those given in Figures 4 through 7 should permit a creditable estimate for other shapes if they are of the same general form as shown in Table 3.

Included in Table 3 are several nested sphere configurations. A brief discussion of the method used to determine the $W/D$ ratio is given in Appendix C. Estimates of $W/D$ ratios for other designs of this type can easily be obtained by the same method.

Pressure hull geometry is not indicated in Table 3 although it can easily be calculated for the spheres. The reluctance to present hull geometry at this time is based on very simple reasoning. It has been observed that a very little increase in $W/D$ ratios permits large changes in hull geometry for relatively stable stiffened structures. This point is illustrated in Figure 8, which presents the effect of stiffener spacing on the $W/D$ ratio for ring-stiffened titanium alloy cylinders with simple support edge conditions and with a length-to-diameter ratio

*The ALVIN pressure hull is composed of two spun HY-100 steel hemispheres. The inside contour of each hemisphere was machined to size and the two hemispheres were welded to form the complete sphere. The outside contour was machined after welding. Five large penetrations were made in the completed hull and reinforcement inserts were welded in place. After all welding, the hull was stress relieved.
<table>
<thead>
<tr>
<th>Material</th>
<th>Shape</th>
<th>Weight/Displacement</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Near-Perfect</td>
<td>$\Delta_w = 1/8$ in.</td>
<td>$\Delta_w = 1/8$ in.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stress-Relieved</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>As Fabricated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HY-100 Steel</td>
<td></td>
<td>0.47</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.49</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.49</td>
<td>0.36</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.51</td>
<td>0.58</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.52</td>
<td>---</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>HY-140 Steel</td>
<td>0.39</td>
<td>0.46</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.40</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.41</td>
<td>0.48</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.41</td>
<td>0.49</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.43</td>
<td>---</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.42</td>
<td>---</td>
<td>0.47</td>
</tr>
<tr>
<td>HY-110 Titanium</td>
<td></td>
<td>0.28</td>
<td>0.32</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>---</td>
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<tr>
<td></td>
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<td>0.30</td>
<td>0.34</td>
<td>0.38</td>
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<td></td>
<td></td>
<td>0.30</td>
<td>0.34</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.31</td>
<td>---</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.31</td>
<td>---</td>
<td>0.33</td>
</tr>
</tbody>
</table>
of one. Similar plots may be developed for longer ring-stiffened cylinders and for stiffened spheroids. Because of the flatness of the curves in Figure 8, it would be very misleading to show structurally optimum geometry at this time. Rather, it is suggested that after the final selection of pressure hull shape and material, a very thorough study be made of the effect of basic geometry on $W/D$ ratio as well as on the ease of successfully incorporating design details, on the arrangement, and on the ease and cost of fabrication before the final geometry is selected.

The comparison of potential pressure hulls in Table 3 is made in terms of $W/D$ ratios. Figure 9, which shows the relationship between displacement and outside diameter for several shapes, permits the designer to easily obtain displacement as well as positive buoyancy (by simply multiplying the displacement times the corresponding $W/D$ ratio) for many of the hulls under consideration.

In using the $W/D$ ratios presented in this report, the designer should be reminded that the additional weights of hull penetrations, skirts, etc., have not been included. It would be difficult, if not impossible, to make reasonable estimates of these weights at this time without a better knowledge of the details involved. This is particularly true of the escape skirt.

A study of buoyancy material, as such, is considered beyond the scope of this assignment. However, it appears that hollow body floats, such as a titanium sphere, offer considerable strength advantage over existing floatation material if additional buoyancy is required and if the vehicle arrangement will permit their incorporation into the design. The $W/D$ ratios developed for the pressure hulls are equally applicable to the structural floats.

**POTENTIAL HULL STRUCTURES FOR THE SEARCH VEHICLE**

**WEIGHT-TO-DISPLACEMENT RATIOS FOR SHELLS OF VARIOUS MATERIALS AND CONFIGURATIONS**

The $W/D$ ratios for shells of the various materials under consideration are presented in Figures 10 through 14. In calculating $W/D$ ratios for the ring-stiffened cylinders, it was determined that no appreciable difference existed between the weight of near-perfect cylinders and internally stiffened, welded cylinders. It was further discovered that both types could be designed on the basis of seminfinite bulkhead or deep-frame spacing without significant weight penalty. Thus, the ratios for ring-stiffened cylinders may be used for cylinders of any length and will not require deep frames when close by hemispherical or other doubly curved shells.

As expected, the effect of fabrication procedures on the collapse strength of the spherical shells for the search vehicle shows a noticeable decrease when compared to the effect on the strength of the spherical shells for the rescue vehicle. This is a result of the increased shell thicknesses and, in the case of the metallic hulls, the relative increase in stability.
MATERIAL AND FABRICATION ASPECTS

As for the rescue vehicle, the material and fabrication aspects will only be summarized in this report. The approach in this summary will be the same as that used for the rescue vehicle. It is assumed that an acceptable solution to each critical problem is needed by 1 January 1968. The summary is presented in Table 4.

The $W/D$ ratios for all spheroidal shells except those of glass-reinforced plastics were calculated for varying thickness geometry. Varying the thickness along the length of an un-stiffened spheroidal shell having a length-to-diameter ratio of two reduces the $W/D$ ratio by about 15 percent. Of course, this is based on the assumption that the material properties are the same throughout the shell. Due to anticipated fabrication problems, the glass-reinforced plastic spheroids were assumed to have constant thickness throughout.

### TABLE 4

Summary of Material Aspects for the Pressure Hull of the Search Vehicle

<table>
<thead>
<tr>
<th>$W/D$</th>
<th>Material</th>
<th>Compressive Yield Strength ksi</th>
<th>Nominal Shell Thickness for 8-Ft Diameter Sphere in.</th>
<th>Designability</th>
<th>Productivity</th>
<th>Toughness</th>
<th>Permanence</th>
<th>Weldability</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0.9</td>
<td>1.02* Q and T Steel, Maraging Steel</td>
<td>150</td>
<td>2 1/4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>0.9 to</td>
<td>0.84* Maraging Steel</td>
<td>200</td>
<td>1 7/8</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2†</td>
</tr>
<tr>
<td>0.8</td>
<td>0.78* Aluminum</td>
<td>60</td>
<td>5 1/4</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.7</td>
<td>0.54* Solid Glass Fiber-Reinforced Plastics, Titanium</td>
<td>60††</td>
<td>5 3/4</td>
<td>3-2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>†</td>
</tr>
<tr>
<td>0.6</td>
<td>0.62* Solid Glass Fiber-Reinforced Plastics, Titanium</td>
<td>150</td>
<td>2 1/2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>†</td>
</tr>
<tr>
<td>0.5</td>
<td>0.32* Solid Glass Fiber-Reinforced Plastics, Titanium</td>
<td>180</td>
<td>2 1/8</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&lt;0.5</td>
<td>0.4* Glass</td>
<td>100††</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>Glass-Ceramics</td>
<td>150††</td>
<td>2 1/4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>0.3</td>
<td>Alumina</td>
<td>225††</td>
<td>1 3/4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>Solid Glass Fiber-Reinforced Plastics, Alumina</td>
<td>300††</td>
<td>1 1/4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

*Based on stress-relieved sphere with $\Delta u_1 = 1/8$ in.
**Based on near-perfect geometry.
†Must be post heat-treated.
††Stress away from structural details in a spherical shell at a depth of 30,000 ft.

Key - Chances of having an acceptable answer by 1 January 1968:
4 - excellent
3 - good
2 - fair
1 - poor
- - not applicable
POTENTIAL HULL STRUCTURES

The shape of the pressure hull for the search vehicle will probably be influenced by its $W/D$ ratio. As a general rule, a very deep pressure hull will be only as large as required to provide sufficient internal volume if a floatation material or system exists which possesses a $W/D$ ratio significantly below that of the hull. Syntactic foam, which is composed of glass microballoons embedded in a plastic matrix, is currently undergoing thorough Navy investigation for possible use as a float material. It appears that syntactic foam of sufficient strength for use in the search vehicle will have a $W/D$ ratio slightly less than 0.7. Therefore, if a hull with a $W/D$ ratio greater than 0.7 is selected, it seems reasonable to assume that it will be a sphere of 7 to 8 ft in diameter. This sphere should be large enough to accommodate the crew and associated equipment. If the hull has a weight-to-displacement ratio equal to or less than the lightest floatation system then spherical, spheroidal, and cylindrical shapes each have potential application. It is unfortunate that most, if not all, of the materials which permit the attainment of low $W/D$ ratios are difficult to fabricate. Thus, the pressure hull shape that may be most easily fabricated may finally be the shape that is dictated if the corresponding hull material is to be selected.

The material and fabrication aspects are very difficult to evaluate for the search vehicle. The primary reason for this difficulty is the need to project research and development progress over the next 3 years in areas where relatively little work has been accomplished to date. The large payoff which would accompany the development of pressure hulls using brittle materials certainly is an important factor in favor of recommending a concentrated effort in this area, although it would be foolish to assume at this time that such a hull can definitely be developed in less than a 3-year period. Thus, it appears that two or three different hulls should be considered in the early stages of the program.

The summary in Table 4 indicates that the choice of two materials, HY-150 steel and HY-110 titanium, for use in the hull would provide good, if not excellent, chances of having acceptable solutions to each critical problem by 1 January 1968. Actually, the chances are good that the strength of steel can be pushed as high as 180,000 psi in the time available. Unfortunately, in the case of the high-strength steels, the weight of the pressure hull approaches its displacement when an allowance for the weight of structural details is included. The outlook for HY-110 titanium hulls is not much better. The displacement of ALUMINAUT and the preliminary design of the TRIDENT work vehicle suggest that high-strength steel or HY-110 titanium hulls, when used together with the present syntactic foam with a $W/D$ ratio of about 0.7, would result in vehicle displacements in excess of that which is presently considered air transportable.
Materials which permit obtaining W/D ratios significantly less than 0.7 include titanium with a yield strength above about 140,000 psi, glass-reinforced plastics, glass, glass-ceramics, and alumina. The summary presented in Table 4 indicates that considerable difficulty is anticipated in developing pressure hulls of any of these materials. However, the most promising materials appear to be titanium with a nominal yield strength of about 150,000 psi, glass-reinforced plastics with a maximum stress level of 50,000 psi at operating depth when used in spherical form, and glass. Titanium at strength levels above 150,000 psi, glass ceramics, and alumina all have very desirable strength-to-weight characteristics, but it does not appear that they will be suitable for this application by 1 January 1968.

Several potential pressure hulls for the search vehicle are presented in Table 5. The hulls are broken into two groups. The chances of developing the upper group of HY-150 steel, HY-180 steel, and HY-110 titanium are considered to be reasonably good to excellent. The chances of developing any member of the lower group of HY-150 titanium, glass-reinforced plastics, and glass are only fair at best. Unfortunately, to obtain a relatively small vehicle size, development of the lower group of pressure hulls is most likely to be required. This is particularly true of the glass and the glass-reinforced plastic hulls.

As for the rescue vehicle, the W/D ratios given for the search vehicle hulls may also be used to determine W/D ratios for structural floats. It is in the role of a structural float that relatively large glass and glass-reinforced plastic spheres as well as glass-ceramic and alumina spheres look very promising. For example, the development of about 4- to 6-ft-diameter glass spheres for use as structural floats would alleviate the need to develop a pressure hull with a low W/D ratio. Combining a 7-ft-diameter HY-110 titanium hull with two 5- to 6-ft-diameter glass spheres would result in a net W/D ratio of about 0.5 and a total displacement of about 10 tons. The results of the planned tests of 44-in.-diameter fusion-sealed glass spheres, which are briefly described in Appendix B, should shed considerable light on the potential of glass spheres for this application as well as for the main pressure hull. Recent tests of seamless alumina spheres fabricated by Coors Porcelain have demonstrated that they also have considerable promise for use as structural floats. However, present industrial capacity limits their size to less than 30 in. in diameter. Segmented glass, glass-ceramic, and alumina spheres are also worthy of further investigation for use as floats.

The problem of developing 4- to 6-ft-diameter spheres of brittle materials, although considerably less difficult than the problem of developing main pressure hulls of these materials, is still considered to be a very challenging job. Rather obvious reasons why floatation spheres may be developed more easily than main pressure hulls are that the floats would be of smaller size and thickness and would not require hull penetration and other structural details. Furthermore, these structural floats could be encapsulated with a protective jacket and could be supported inside an outer hull to minimize the danger of damage due to impact.
### TABLE 5
Potential Pressure Hulls for the Search Vehicle

<table>
<thead>
<tr>
<th>Material</th>
<th>Shape</th>
<th>Weight/Displacement</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Near-Perfect</td>
<td>$\Delta_b = 1/8$ in. Stress-Relieved</td>
<td>$\Delta_b = 1/8$ in. As Fabricated</td>
</tr>
<tr>
<td>HY-150 Steel</td>
<td><img src="image" alt="Shape" /></td>
<td>0.95</td>
<td>1.02</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>HY-180 Steel</td>
<td><img src="image" alt="Shape" /></td>
<td>0.84</td>
<td>0.92</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>HY-110 Titanium</td>
<td><img src="image" alt="Shape" /></td>
<td>0.72</td>
<td>0.75</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>HY-150 Titanium</td>
<td><img src="image" alt="Shape" /></td>
<td>0.58</td>
<td>0.62</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>GRP Solid Glass Fibers</td>
<td><img src="image" alt="Shape" /></td>
<td>0.52</td>
<td>---</td>
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<td></td>
</tr>
<tr>
<td>GRP Hollow Glass Fibers</td>
<td><img src="image" alt="Shape" /></td>
<td>0.37</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td><img src="image" alt="Shape" /></td>
<td>0.27 to 0.31</td>
<td>0.29 to 0.33</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>
or other dynamic loading. The major obstacle in the way of developing glass-reinforced plastic floatation spheres is the general problem of proving that they have acceptable permanence. In the case of glass floats, which can probably be fabricated by fusion-sealing two hemispheres, the major obstacle is the development of rational hull toughness criteria which hopefully will permit the use of brittle materials. In fact, the lack of adequate hull toughness criteria appears to be one of the major problems which must be faced in the development of both a pressure hull and a floatation system for the search vehicle.

SUMMARY

Failure criteria were developed for spherical, cylindrical, and spheroidal shells based on a review of existing theoretical and experimental studies. Using these criteria, weight-to-displacement ratios were calculated for shells of high-strength steels, titanium alloys, aluminum alloys, glass-reinforced plastics, glass, glass-ceramics, and alumina for both the rescue and the search vehicles.

A review of material and fabrication aspects of the rescue vehicle indicated that HY-100 steel, HY-140 steel, and HY-110 titanium are the leading contenders for the main pressure hull material. Specifically, HY-100 steel is clearly the first choice if a $W/D$ ratio of about 0.5 to 0.6 is acceptable. If the designer cannot tolerate the $W/D$ ratio associated with HY-100 steel but requires a hull with a $W/D$ ratio of 0.4 to 0.5, HY-150 steel with a design compressive yield strength of 140,000 psi appears to be the best choice. 721 Titanium alloy with a compressive yield strength of 110,000 psi seems to be the only material which has a reasonably good chance of meeting all of the design requirements and also providing a hull with a $W/D$ ratio significantly below 0.4. The choice of pressure hull shapes for the rescue vehicle will probably lie with the naval architect and will be based on overall arrangement rather than on minimum weight considerations.

A review of the material and fabrication aspects of the search vehicle indicate that high-strength steel, with a compressive yield strength approaching 180,000 psi, and titanium, with a compressive yield strength of 110,000 psi, are the materials with the highest strength-to-density ratios whose choice for use in the pressure hull would provide a good chance of having acceptable solutions to each critical problem by the projected target date of 1 January 1968. Unfortunately, the $W/D$ ratios for hulls of these materials are so large that their use would probably result in vehicle displacements in excess of those which are presently considered air transportable unless new, lighter weight floatation systems are developed. The most promising materials for obtaining low $W/D$ ratios for the search vehicle capsule are titanium with a compressive yield strength of about 150,000 psi, glass-reinforced plastics, and glass. At best, the chances of developing acceptable pressure hulls of these materials within the projected time scale are only fair.
Since the size of the crew and associated equipment are such that a 7- to 8-ft-diameter spherical pressure hull will most likely provide sufficient internal volume, the use of relatively large glass-reinforced plastic, glass, or ceramic spheres as structural floats together with an HY-110 titanium spherical hull offers an attractive approach to obtaining a low effective $W/D$ ratio. The problem of developing spherical floats of this type would be simpler than the problem of developing main pressure hulls of these materials because of their smaller size and thickness and because of the elimination of large hull penetrations and other structural details. Furthermore, these structural floats could be encapsulated with a protective jacket and could be supported inside an outer hull to minimize the danger of damage due to impact or other dynamic loading. A major problem in developing both pressure hulls and floatation systems for the search vehicle is the development of rational hull toughness criteria which, it is hoped, will permit the use of materials less tough than those currently in use.

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Figure 1 — Summary of Model Basin Data on Unstiffened Spherical Shells

Figure 2 — A Comparison of Model Basin Data on Machined Ring-Enhanced Cylinders with Welded Cylinders
Figure 3 – Assumed Material Characteristics of High-Strength Steels, Titanium Alloys, and Aluminum Alloys

Figure 3a — Stress-Strain Curve for High-Strength Steel

Figure 3b — Plastic Moduli for High-Strength Steel

\[
\sigma \text{ is stress intensity} \\
\sigma_y \text{ is yield strength} \\
E \text{ is the elastic modulus} \\
E_t \text{ is the tangent modulus} \\
E_s \text{ is the secant modulus}
\]
Figure 3c – Stress-Strain Curve for Titanium Alloy

Figure 3d – Plastic Moduli for Titanium Alloy
Figure 3e – Stress-Strain Curve for Aluminum Alloy

Figure 3f – Plastic Moduli for Aluminum Alloy

\[ \sigma \text{ IS STRESS INTENSITY} \]
\[ \sigma_y \text{ IS YIELD STRENGTH} \]
\[ E \text{ IS THE ELASTIC MODULUS} \]
\[ E_t \text{ IS THE TANGENT MODULUS} \]
\[ E_s \text{ IS THE SECANT MODULUS} \]
Figure 4 — Weight-to-Displacement Ratios for High-Strength Steel Shells for the Rescue Vehicle

Figure 4a — High-Strength Steel Spheres

Figure 4b — High-Strength Steel Cylinders and Spheroids
Figure 5 — Weight-to-Displacement Ratios for Titanium Alloy Shells for the Rescue Vehicle

Figure 5a — Titanium Spheres

Figure 5b — Titanium Cylinders and Spheroids
Figure 6 – Weight-to-Displacement Ratios for Aluminum Alloy Shells for the Rescue Vehicle

Figure 7 – Weight-to-Displacement Ratios for Near-Perfect Glass-Reinforced Plastic, Glass, and Ceramic Spherical Shells for the Rescue Vehicle
Figure 8 – Effect of Stiffener Spacing on Weight-to-Displacement Ratio for a Titanium Ring-Stiffened Cylinder

Figure 9 – Displacement versus Outside Diameter for Various Shapes
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Figure 11 – Weight-to-Displacement Ratios for Titanium Alloy Shells for the Search Vehicle
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Figure 13 — Weight-to-Displacement Ratios for Near-Perfect Glass-Reinforced Plastic Shells for the Search Vehicle
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Figure 15 – Experimental Buckling Data for Machined Deep Spherical Shells with Ideal Boundaries
Figure 16 - Experimental Inelastic Buckling Data for Machined Shallow and Deep Spherical Shells with Clamped Edges

\[ \theta = \left[ \frac{3}{4} (1 - \nu^2) \right]^{1/4} \frac{L_a}{\sqrt{R_h}} = 0.91 \left( \frac{L_a}{\sqrt{R_h}} \right) \nu = 0.3 \]

Figure 17 - Assumed Relationship between Out-of-Roundness \( \Delta \) and Local Radius \( R_1 \)
Figure 18 - Relationship between $R_1/R$ and $\Delta/h_o$

Figure 19 - Experimental Buckling Data for Machined Models with Imperfections
Figure 20 — Lower Bound on Weight-to-Displacement Ratios for Nested Spheres
APPENDIX A

A BRIEF BACKGROUND ON THE STRUCTURAL ANALYSIS OF SPHERICAL, CYLINDRICAL, AND SPHEROIDAL SHELLS

This appendix discusses the analysis used in this study to determine the strength-weight characteristics of the spherical, cylindrical, and spheroidal shells. Most of the back-up data used to develop the analysis of cylindrical shells have been published in the open literature and will not be repeated in this report. However, since much of the data on which the analysis for the spherical and spheroidal shells was developed have not yet been published, a more thorough background in these areas is presented.

SPHERICAL SHELLS

The elastic behavior and collapse strength of unstiffened spherical shells has been studied rather extensively by various investigators. The stress analysis is well developed for axisymmetric problems associated with structural details in spherical shells and will not be discussed herein. However, the collapse strength of spherical shells is not, in general, clearly understood.

The classical elastic buckling pressure\(^{10}\) was first developed by Zoelly in 1915 and may be expressed as

\[
p_1 = \frac{2E(h/R)^2}{\sqrt{3(1-\nu^2)}} = 1.21 E (h/R)^2 \text{ for } \nu = 0.3 \tag{12}\]

Experimental elastic collapse pressures are frequently as low as one-fourth or less of the pressure obtained from this classical theory. Recent tests have demonstrated that this large discrepancy is a result of the failure of the models to meet the idealized geometry and material assumptions of theory. The strength of those shells which fail in the inelastic region is also affected by these factors. Specific factors which decrease the strength of spherical shells include the presence of initial departures from sphericity, variations in thickness, non-isotropic material properties, adverse boundary effects and penetrations, mismatch, residual stresses, dynamic disturbances, and local loadings. Various theoretical attempts to quantitatively predict the effects of these factors have been relatively unsuccessful.

The lack of agreement between theory and experiment prompted the initiation of an extensive research program at the Model Basin, under the sponsorship of the Bureau of Ships, to develop reliable criteria for predicting the collapse strength of spherical shells. First, machined hemispheres with ideal, membrane boundaries were tested to study the experimental collapse pressure of shells which approach the idealized assumptions of classical theory.
These tests indicated that the collapse pressures given by Equation [12] may be obtained for the ideal spherical shell. However, the tests also demonstrated that for small, almost unmeasurable imperfections, the strength falls off very rapidly to about 70 percent of the pressure obtained from Equation [12]. Based on these results, the following formula was developed for predicting the elastic buckling strength of near-perfect spherical shells:

\[ p_3 = 0.84 \frac{E}{h/R_0}^2 \text{ for } \nu = 0.3 \quad [13] \]

Here a shell is considered near-perfect when its initial departure from sphericity is less than about 2 to 3 percent of a shell thickness and when the effects of all other factors mentioned previously are negligible. Initially perfect shells may buckle at pressures approaching 43 percent greater than the pressure given by the empirical Equation [13]. It is unrealistic, however, to rely on this additional strength.

Based on the results of the elastic buckle specimens, an empirical formula was also developed which adequately predicted the collapse of near-perfect machined hemispherical shells with ideal boundaries and which failed at stress levels above the proportional limit. This formula may be expressed as

\[ p_E = 0.84 \frac{E_i}{h/R_0} (h/R_0)^2 \text{ for } \nu = 0.3 \quad [14] \]

For stress levels below the proportional limit, Equation [14] reduces to Equation [13]. From simple equilibrium, the average stress may be expressed as

\[ \sigma_{\text{AVG}} = \frac{p R_0^2}{2 h R} \quad [15] \]

Equation [14] can then be solved by an iteration process using Equation [15] and the stress-strain curve for the material used in the test specimen. The agreement between this empirical equation and the model tests is shown in Figure 15. Equation [14] therefore provides a baseline for predicting the elastic or inelastic collapse of a near-perfect, initially stress-free, deep spherical shell with ideal boundaries.

Tests were also conducted to determine the relationship between unsupported arc length and the elastic and inelastic collapse strength of machined shallow spherical caps with clamped edges. Although previous data in the literature showed wide disagreement in experimental
results, these tests followed a very definite pattern. The test results for the inelastic case, which is of particular interest, are plotted in Figure 16. The ordinate is the ratio of the experimental collapse pressure to the empirical pressure and the abscissa is the nondimensional parameter $\theta$ defined as

$$\theta = \left[\frac{3}{4} (1 - \nu^2)^{1/4}\right] \frac{L_a}{\sqrt{R h}} = \frac{0.91 L_a}{\sqrt{R h}} \quad \text{for } \nu = 0.3 \quad [16]$$

where $L_a$ is the unsupported arc length of the shell. The results are plotted in families of curves which basically represent varying degrees of stability; shells with the lowest values of $p_c/p_1$ are the most stable. The results demonstrate that for $\theta$ values greater than approximately 2.2, the effect of clamping the edges diminished as the shells become more stable.

Although all of the test results discussed thus far have been for near-perfect models, they do provide the basis for the analysis of spherical shells with initial imperfections. Equations [13] and [14] adequately predict the collapse of near-perfect models. Figure 16 indicates that the collapse of a spherical shell with a $\theta$ value greater than about 2.2 is relatively independent of the boundary conditions. Thus, the collapse strength of shells with initial imperfections depends primarily on the local geometry over a critical arc length. These two observations form the basis of the imperfection analysis developed in Reference 1. In this analysis, the general form of Equations [13] through [15] becomes Equations [1] through [3]. In Reference 1 it is assumed that the critical arc length $L_c$ may be determined by

$$L_c = \frac{2.2}{0.91} \sqrt{R_1 h_a} \quad \text{for } \nu = 0.3 \quad [17]$$

Thus, for the case of the spherical shell with imperfections, it is necessary to determine the local radius over a critical arc length.* The simplest way to do this is to express the local radius in terms of deviations from a nominal radius or in terms of out-of-roundness. Figure 17 shows the assumed relationship between deviations from a nominal radius, out-of-roundness $\Delta$, and local radius $R_1$ over a critical arc length. With a given out-of-roundness, the ratio of $R_1/R$ may be calculated in terms of $\Delta/h_a$ and $h_a/R$ from geometry. The results of these calculations are presented graphically in Figure 18.

To verify the validity of the analysis just given, models with known flat spots and thin spots were machined and tested. A detailed description of the models and analysis of the results is presented in Reference 1. Briefly, the collapse of these models was predicted using the local geometry over a critical arc length as described above. A comparison of the experimental results and the pressure predicted by Equation [2] is presented in Figure 19.

*In this analysis it is assumed that the critical arc length is associated with a $\theta$ value of 2.2.
In addition to the machined models with imperfections, 30- and 66-in.-diameter HY-80 steel models fabricated according to feasible large-scale fabrication techniques are being studied. Although the tests have not been completed, the preliminary results permit a creditable approach to the strength analysis of practical spherical shells.

The HY-80 models consist of welded, segmented hemispheres with ring-stiffened boundary cylinders. Some of the hemispheres have penetrations; some are stress-relieved after fabrication. Precise contours and shell thicknesses of the hemispheres are measured at close intervals. These data are fed into a computer which calculates the mean radius, the center of the hemisphere, and the corrected departure from sphericity. The departures from true sphericity are plotted on contour maps. The maps are examined through the use of proper scales to determine the worst flat spot. This method was used in evaluating the strength of the ALVIN capsules and is described in more detail in Reference 9.

The preliminary results obtained by Kiernan and Nishida of the Model Basin staff on the HY-80 welded hemispheres are presented in Figure 1. All calculations are based on measured local geometry. The ordinate is the ratio of experimental collapse pressure $p_{\text{exp}}$ to the empirical collapse pressure obtained by Equation [2]. The abscissa is divided into two parts. The first part represents the ratio of the calculated stress $a'^*_{p3'}$, which corresponds to the stress obtained using Equation [3] for the pressure $p'_3$, to the proportional limit of the material $a''_{p_{pl}}$. The second part of the abscissa represents the ratio of elastic to inelastic collapse strength for those shells whose calculated elastic buckle stresses are above the proportional limit.

Very briefly, the data represented in Figure 1 may be explained as follows: The ratio of $p_{\text{exp}}/p'_E = 1$ is associated with the pressure calculated using Equations [2] and [3] and has been found to be in good agreement with tests of machined aluminum models, both of near-perfect geometry and of initially imperfect geometry, except in the region of low ratios of $p'_3/p'_E$. The curve for the STS shells also indicates that there is a slight reduction in pressure below that obtained from Equations [2] and [3] for those shells which are associated with relatively low ratios of $p'_3/p'_E$. It is interesting to note that the agreement between the STS model results and the empirical formula (Equations [2] and [3]) is quite good although the empirical formula was developed on the basis of tests of shells of aluminum, which has considerable strain-hardening characteristics, while STS is essentially an elastic, ideally plastic material. The differences between the results of the STS shells and the stress-relieved HY-80 shells can be attributed primarily to the effects of secondary moments which were present due to imperfections and which were not considered in the development of the empirical formula. The segmented shells whose test results are represented in Figure 1 had varying degrees of initial departures from sphericity, frequently corresponding to ratios of $R_1/R$ of about 1.3 to 1.4 and more. The difference between the two lower curves in Figure 1 represents the detrimental effects of residual stresses, both welding and forming, on collapse.
strength. When shells of near-perfect geometry are involved, the use of the lower envelope of results of the HY-80 steel shells tested in the “as fabricated” condition to predict the collapse strength of spherical shells with residual fabrication stresses should be slightly conservative in the low stability region. In addition, admittedly it is in error to assume that the effect of residual stresses on the collapse strength of HY-80 steel shells fabricated by welding together pressed segments is the same as their effect on the collapse strength of spherical shells of a variety of materials most likely fabricated by joining two stress-relieved hemispheres by a girth weld. However, the data cited are the only known data on the effect of residual stresses on collapse in which sufficient measurements of initial imperfections were made to permit a detailed analysis. Programs have recently been initiated at the Model Basin, under the sponsorship of the Bureau of Ships, to study these effects on both HY-150 steel and HY-110 titanium spherical shells. Unfortunately, it will be some time before useful data will be available in this important area. In view of the data at hand and the obvious fact that residual stresses, even if they are only those due to welding, must have a detrimental effect on collapse in the relatively unstable region, it appears that use of the data from Figure 1 is the most realistic approach at this time for obtaining estimates of thickness-to-radius ratios required.

**CYLINDRICAL SHELLS**

The elastic behavior and collapse strength of ring-stiffened cylindrical shells have been studied rather thoroughly through both experiment and theoretical treatment and have been documented rather completely in the literature. Thus, here no attempt will be made to give a complete background or bibliography on this subject. Rather, the assumptions used in this study to obtain geometrical configurations for basic ring-stiffened cylinders will be outlined.

The elastic stresses for typical bay geometry can be obtained from the nonlinear analysis of Salerno and Pulos\(^5\) when the effective frame area is adjusted to take into account its internal or external position.\(^\text{13}\) This analysis has been verified by a wide variety of experiments. The stresses in way of deep frames and other axisymmetric discontinuities can be obtained by the analysis of Short and Bart.\(^\text{14}\) Due to the complexity of this analysis, however, the design of these discontinuities for a parametric study is impractical and usually will be accomplished by engineering judgment. Adequate theoretical analysis of stresses around hull penetrations in stiffened cylindrical shells does not exist, and these structural details can only be designed by semi-empirical means which combine previous experimental results and engineering judgment.
A cylindrical shell can fail in three basic modes: axisymmetric shell failure; asymmetric shell failure; and overall, or general instability. Failure in each of the three modes may occur in the elastic or the inelastic region.

The elastic axisymmetric shell buckle failure of initially near-perfect, ring-stiffened cylindrical shells may be obtained from Lunchick's analysis. Unfortunately, it is very difficult to obtain this mode of collapse in practice and no data are available to check the validity of this analysis. The inelastic axisymmetric failure of near-perfect shells may best be calculated by modifying the analysis of Lunchick. Recent unpublished experiments conducted at the Model Basin have demonstrated that the analysis of Lunchick gives results which are not conservative when shells are relatively unstable. For margins of stability of 6.0 or less, better agreement is obtained with experiment when the semi-empirical Equation [4] is used.

Both the elastic and the inelastic shell buckle pressure for near-perfect, ring-stiffened cylinders can be calculated by using an analysis developed by Reynolds. Good agreement has been obtained between this theory and experimental results.

Elastic general-instability strength between deep frames for near-perfect shells can be calculated by Kendrick or Modified Bryant. The Kendrick theory is generally accepted as the best available theory for elastic general instability and is normally conservative when compared with experimental results. It is rather time-consuming, however, to calculate collapse pressure by this method and, therefore, Modified Bryant is often used in parametric studies. The difference between the pressures obtained from Kendrick and from Modified Bryant is seldom greater than 10 to 20 percent. The inelastic general instability strength can easily be calculated by a method presented in Reference 4. Good agreement has been obtained between pressures calculated by this method and experiment as long as bending stresses in the shell are kept at a reasonable level. It appears that this method is conservative for large frame spacings and reasonable levels of bending stress although this observation is based on limited experimental evidence.

The effect of deep frames on the elastic general instability of ring-stiffened cylinders is not fully understood. Until recently, it appeared that a Modified Levy (Equation [6]) approach to the design of deep frames would be conservative. In this analysis it is assumed that the entire load between deep frames must be resisted by the deep frames only when considering the buckling strength in the overall mode. However, recent tests have shown that collapse pressures may be obtained which are considerably below the pressure calculated in this manner, particularly as the buckling pressure of a long cylinder with deep frames approaches its buckling pressure between deep frames. Reynolds has recently developed an analysis which yields pressures which agree with experiment. More experimentation is required, however, before full confidence can be placed in this analysis.
Because the effect of end conditions provided by end closures is not understood, the contribution of the hemispherical end closures in resisting general instability failure is normally neglected. This is very conservative for short cylinders but no other approach appears justifiable at this time. Very recent, unpublished tests by Blumenberg have shown that hemispherical end closures do not provide end conditions which approach simple support. In fact, the collapse pressures obtained to date have sometimes been closer to those calculated for a semi-infinite cylinder than the pressures calculated for a cylinder of the finite length represented by the models.

A summary of data obtained over the years at the Model Basin on the effect of residual stresses and imperfections on inelastic shell failure of stiffened cylinders is presented in Figure 2. Note that the effect is similar to that observed in the spherical shell tests. Unfortunately, the results for welded cylindrical shells were also obtained using materials which had essentially elastic, ideally plastic stress-strain characteristics. An investigation of the effects of residual stresses and imperfections on the strength of internally stiffened cylinders of HY-150 steel has recently been initiated at the Model Basin under the sponsorship of the Bureau of Ships but data will not be available in time for this study. Little data are available on the effects of residual stresses and out-of-roundness on collapse strength in the plastic general instability mode. The stresses in the frame flange due to out-of-roundness can be calculated and are in good agreement with experiment. These calculated stresses may be used to predict that pressure at which yielding in the frame flange initiates.

PROLATE SPHEROIDAL SHELLS

Relatively little work has been accomplished in the analysis of the elastic behavior and collapse strength of spheroidal shells. This is particularly true for stiffened spheroidal shells, which are of prime interest in this study.

The membrane analysis of elastic stresses in unstiffened spheroidal shells is presented in Flügge and has been found to be in good agreement with the limited experimental data. No analysis is available for the stresses in an unstiffened spheroidal shell of varying thickness although an estimate of the stress may be made by modifying the membrane analysis. The analysis of elastic stresses in a ring-stiffened prolate spheroidal shell has not been accomplished. These stresses may be estimated by the Salerno and Pulos analysis for ring-stiffened cylinders modified according to the analysis of unstiffened spheroids to account for the reduction in circumferential stress due to the curvature in the longitudinal direction. As for ring-stiffened cylinders, the theoretical analysis of stresses around hull penetrations does not exist and, thus, this design must be accomplished on the basis of engineering judgment.
The elastic buckling strength of unstiffened prolate spheroidal shells has been treated by Mushtari\textsuperscript{18} and others. Recent experiments by Healey of the Model Basin staff have shown that existing analyses are inadequate. A preliminary survey of his tests indicates that the elastic buckling strength of unstiffened spheroidal shells may be expressed as Equation [9].

Experimental values for $A$ and $B$ of Equation [9] obtained by Healey for a length-to-diameter ratio of 2.0 were about 0.37 and 2.17, respectively. The number of lobes associated with these coefficients is a function of thickness-to-radius ratio and was 7, 5, and 4, for ratios of 0.02, 0.04, and 0.06, respectively.

The elastic buckling strength of stiffened prolate spheroidal shells has not been treated theoretically. An approximation can be made to the strength in the overall mode by using the experimental results on unstiffened spheroids and the simple general-instability formula developed by Bryant for ring-stiffened cylinders. The formula for calculating the elastic general-instability collapse pressure in this manner is given by Equation [10]. The elastic shell buckle strength of stiffened spheroids may be conservatively estimated by the analysis used for ring-stiffened cylinders.

No analysis of the inelastic strength of prolate spheroids has been developed. Collapse strength in these modes may be estimated by use of the elastic stress and strength analysis discussed above with the plasticity reduction factors used for cylindrical shell analysis.
APPENDIX B

NON-METALLIC MATERIALS FOR VEHICLES OF THE DEEP-SUBMERGENCE SYSTEMS PROJECT

During the past six years, considerable interest has been focused on the potential use of nonmetallic materials in deep-submergence pressure hulls. In the main, the interest has been due to the attractive strength-to-weight characteristics of many nonmetallic materials, which include glass-reinforced plastics, glass, and ceramics.

A review of the state of the art and future projections for nonmetallic as well as metallic materials for hydrospace applications was recently made and reported in Reference 19. The following discussion will not attempt to repeat this study but will briefly discuss those aspects which have a direct bearing on evaluation of the use of nonmetallic materials in the pressure hulls or structural floats of the rescue and search vehicles.

GLASS-REINFORCED PLASTICS

In the past, major effort dealt with the investigation of properties and fabrication techniques of glass-filament-reinforced plastics. A great quantity of data related to the mechanical and physical properties of glass-reinforced plastics (GRP) has been generated by subjecting unstiffened cylinders to hydrostatic pressure loading. These specimens were of extremely thick-walled construction so as to preclude instability failure and exhibited little bending and shear stresses. Compressive strengths of 180,000 psi have recently been observed from the tests of such specimens. By virtue of these tests, which indicated high compressive strength-to-weight characteristics, glass-reinforced plastic appeared to have promise as a structural material for pressure hulls of deep-diving submersibles. However, research was needed to utilize GRP in realistic hull structures. Efforts within the past two years have been devoted specifically to studies of stiffened pressure hulls and associated structural details such as closures, joints, and penetrations. These studies were also orientated to provide an insight as to the effects of cyclic fatigue and long-term exposure to deep-submergence pressures on the load-carrying capacity of filament-wound structures. A "state-of-the-art" review of these effects would give an insight as to possible trends in technology and development of GRP materials and identification of R and D needs.

The studies have shown that in order to utilize GRP material efficiently in cylindrical pressure hulls, a stiffening system must be incorporated in the pressure hull to circumvent the instability mode of failure. Tests of ring-stiffened cylinders have consistently shown that at a collapse depth of 30,000 ft, compressive strengths on the order of only 100,000 psi were developed in these structures prior to failure. This is in contrast with stress levels of 180,000 psi realized from tests of thick-walled, unstiffened cylinders. Such a comparison
indicated that design based on compressive strength will not adequately define material response of a realistic structure and that consideration must be given to the fact that the allowable shear stresses of GRP cannot be designed to more than about 1/20 of the compressive design limit. These studies have indicated the need to develop analytical tools and conduct experimental investigations to more fully understand the failure mechanism of laminated orthotropic structures. The problem becomes further magnified by the requirements of relatively large hull penetrations for both the rescue and search vehicles; the penetrations act as "hard spots" in the pressure hull and induce localized bending and shear stresses. Due to the lack of symmetry, the penetration problem in cylindrical pressure hulls is analytically less tractable by many orders of magnitude than the stiffener-system problem. In fact, analyses for structural behavior around hull penetrations do not exist; present design methods for hulls fabricated of isotropic materials are based on engineering judgment. Coupled with the fact that the failure mechanics of laminated orthotropic structures are not fully understood, the design of light-weight reinforced openings for any given set of design conditions appears to be highly unlikely within the time frame allotted for the rescue vehicle. Previous experience related to openings in GRP pressure hulls has been with the use of metallic fittings designed to carry in-plane shell loads about the opening in hoop compression and bending. Flanges around the fitting were used to provide a method to clamp down and to locally support the cut fibers around the opening of the shell and thereby assist in the transfer of high compressive bearing loads into the fitting. For large-diameter openings, this method of reinforcement leads to appreciable increase in overall structural weight. Studies so far have demonstrated the capability of resolving structural problems such as penetrations in pressure hulls fabricated from nonweldable materials such as glass-reinforced plastics. However, additional refinements in these areas are necessary to obtain more efficient structures.

Fatigue tests conducted under Bureau of Ships Contract NObs 88351 by H. I. Thompson Fiber Glass Company with GRP models having realistic structural details such as closures, large openings, and joints have consistently obtained 10,000 cycles without failure. These models were pressure cycled to 50,000-psi stress levels in the typical regions away from structural details and were tested at extremely high cyclic rates, 9 cycles per minute.

Similar results were obtained when a pressure cycle with a 1-min dwell time at maximum and minimum pressures was incorporated in recent fatigue tests of ring-stiffened cylinders without structural details. With the introduction of a large-diameter opening, however, premature failure occurred after 3898 cycles. This result is limited, in the sense that only one model was tested; further fatigue tests are being conducted with identical models to obtain additional results to determine the designability of structural details in GRP hulls. Studies are also underway with models subjected to pressure cycles similar to those which might be experienced by future deep-depth vehicles. Small-scale models are being and will
be subjected to two loading cycles a day for a period of 2 years. One such model with two small-diameter penetrations and a titanium jacket to protect the GRP hull from the deep-sea environment has sustained, so far, 127 days (254 cycles) without any apparent loss in structural integrity. The major problem with GRP pressure hulls is to prove that they have acceptable permanence. It is conceivable that with extensive research, knowledge of the design limitations of filament-wound structures and solutions to design details such as hull penetrations would be within the time frame allotted for the search vehicle.

Discussion on the effects of water absorption in glass-reinforced plastic subjected to high-pressure water has always been controversial due to contradictory experimental evidence. Most of the experiments were conducted with filament-wound materials in which there were basically no cut fibers exposed to the pressure medium, and the degree of strength degradation appears to be a function of the quality of the fabricated laminate. Recent studies conducted by U. S. Rubber Company under Bureau of Ships Contract NObs 90246 have indicated that the effect of water absorption is a more serious problem for laminates having cut fibers exposed directly to the pressure medium. Similar strength reductions have also been noted in cylindrical structural models tested at the Taylor Model Basin where cut fibers on the surface of the laminate were exposed directly to the pressure medium. However, when the cut-fiber surfaces on the cylinder models were protected by a rubber-base sealing compound, no apparent strength reduction occurred. The objective of the U. S. Rubber study is to develop a suitable coating system capable of resisting water from attacking the GRP laminate.

Other possibilities toward alleviating the apparent shortcoming of fiber-reinforced plastics to attack of high-pressure water and to assist in the ability of the material to retain its integrity throughout its design life may exist in metallic sheathings. Studies have been initiated along these lines. Sheath construction consists of GRP cylinders or rings, placed side by side and not physically joined together, surrounded by a thin metallic jacket. The cylinders or rings provide the major resistance to hydrostatic loading while the jacket ensures watertight integrity, provides longitudinal strength, and prevents sea water corrosion of the GRP elements. This type of construction would permit the manufacturer to produce the elements in shorter, easier to fabricate, sections. Construction would become a matter of merely placing the rings together and surrounding them with the metal jacket. Hydrostatic tests with 1-ft-diameter models have shown that a titanium jacket provides a feasible method of protecting GRP laminates. Successful fatigue results so far have been limited to a previously mentioned model on which the test is still in progress and which has sustained 254 cycles at a rate of 2 cycles per day without failure. The only other fatigue test with models of this type of construction resulted in a premature failure caused by wrinkling of an excessively loose jacket.
All of the results mentioned so far have been based on tests with small-scale models, maximum of 1 ft in diameter. These models were fabricated under ideal conditions which lead to laminates that were essentially free of voids and delamination. Recently a trial section, 3 ft in diameter, was made and tested to study fabrication techniques and scale-up effects of thick-walled lamination. Hydrostatic tests of this section indicated that it was possible to obtain a high quality part, and the test results were very favorable when compared with results of the small-scale model tests. This trial section incorporated glass cloth end closures, 3 in. thick, which were attached to both ends of a 1-diameter-length ring-stiffened cylinder. The shell and ring frames were machined from two filament-wound cylinders, each 3 in. thick. It appears that within the time schedule of the search vehicle the chances of having an acceptable answer in terms of producibility, as defined in the text of this report, are very good for the cylindrical shaped pressure hull.

The filament winding of other shapes such as thick-walled spheres and prolate spheroids requires investigation prior to any forecast as to their suitability as deep-submergence pressure hulls. The structural performance of spherical and spheroidal shells appears to show great promise as indicated by the curves shown in Figures 7 and 13. These curves, however, are based on assumed values of material properties projected from experience with filament-wound cylinders. Basic information such as optimum fabrication techniques and physical and mechanical properties of filament-wound materials used in the spherical and spheroidal shapes is needed. Filament-wound materials cannot be characterized in the same manner as homogeneous materials because the materials do not behave as a composite material until the structural element has actually been fabricated, and the material properties depend on the winding pattern, shape, etc.

There exists today a very definite reluctance to use materials of low ductility for pressure hull applications. Before GRP can be judged to have acceptable toughness for use in the search vehicle, realistic hull toughness criteria must be developed. These criteria should be based on overall structural response to realistic loads rather than, as in the past, on the ability of the hull material to absorb energy or resist cracking. At this time, it cannot be determined whether such hull toughness criteria will be developed by January 1968; and if such criteria are developed, whether GRP hulls for the search vehicle will have sufficient toughness when judged by these criteria.

GLASS AND CERAMICS

The development of reliable pressure hulls of brittle materials such as glass and ceramics would enable attainment of all-depth capability for submersibles at small, if not minimum, size. True to form, however, is the fact that such a development is one of the most challenging problems which faces those engaged in submarine structural research.
It is often stated by representatives of the glass industry that there has never been a known failure in glass that has not initiated at the surface and has not been caused by tensile stresses. Bridgeman\textsuperscript{20} demonstrated that certain glasses have a compressive and shear strength in excess of one million psi. Although the tensile strength of annealed glass is known to be extremely high also, it is theorized that stress concentrations of surface micro-cracks under tension cause an extreme reduction in this value. In practice, the observed nominal tensile stress of annealed glass at fracture is frequently only a few thousand psi.

The tensile strength of glass may be increased by superimposing compressive stresses at and near its surface. It is normally increased by tempering or by chemically strengthening. Recent tests by Stachiw\textsuperscript{21} and Perry\textsuperscript{22} have demonstrated that the compressive reaction stresses in a low body under uniform pressure also increase the apparent tensile strength of brittle materials. The most effective method of superimposing compressive stresses in glass surfaces is by chemically strengthening. Currently, the tensile strength of chemically strengthened glass approaches 100,000 psi whereas strengths in excess of 300,000 psi are projected for the future.\textsuperscript{19}

Certain ceramics also possess extremely high compressive strength. Of these, alumina and glass-ceramics show the most immediate promise for use in hydrospace structures. Recent uniaxial compression tests conducted at the Model Basin on alumina rods have produced compressive membrane stresses approaching 500,000 psi. The tensile strength of alumina and glass-ceramics is also small when compared to their compressive strength, but normally it is higher than that of annealed glass. To date, chemically strengthening techniques have not been applied to ceramics to improve tensile strength.

The low tensile strength of glass and ceramics can be attributed to their lack of ductility or, in other words, their failure to yield in the presence of a notch. This basic characteristic is the source of the problems foreseen in developing hulls of these materials.

There is no chance of having acceptable answers in all of the critical areas of designability, fabricability, toughness, permanence, and weldability for glass and ceramic materials before the projected deadline for the rescue vehicle of January 1968. Thus, the following discussion will be focused on the possible use of these materials for the search vehicle pressure hull or structural float by January 1968. It is evident that except for permanence, each critical area in material aspects must be associated with a given structural configuration.

At present, the designability of glass and ceramic pressure hulls approximately 7 to 8 ft in diameter and about 1 to 2 in. thick must be considered very poor. It is true that the compressive and shear strength of glass is accepted to be sufficiently high. However, the tensile strength of large shells of annealed and strengthened glass must be demonstrated. Similar studies are required for ceramic shells.
It can be argued whether or not the bearing strength of these materials is a structural or a material designability problem. In any event, the tests conducted on the compressive or bearing strength of joints have shown considerable scatter. To date, a higher average bearing strength has been obtained in chemically strengthened glass than in tempered and annealed glass whereas the highest bearing strength, approaching 500,000 psi, has been obtained in alumina. No improvement has been obtained by using gasketing material between glass bearing surfaces, and no noticeable trend has been observed between the results on glass with ground surfaces and those on glass with polished surfaces. Thus, fairly encouraging trends in the bearing strength of glass and ceramics have been observed but the test results have shown considerable scatter.* There is no reason to believe that the scatter will decrease as the specimen size is scaled-up. It appears that a large experimental effort is required, particularly on scale-up effects, before design values can be confidently established.

The fabricability of 7- to 8-ft-diameter glass and ceramic hulls remains to be demonstrated although there are several promising approaches. With the development of proper equipment, it appears feasible to spin glass and glass-ceramic hemispheres of the required size and thicknesses. Introduction of other segmented shapes, such as geodesic shapes, permits use of smaller pieces and appears feasible for alumina as well as for glass and glass-ceramics if the joint problem can be solved. The quality control problem requires investigation for large glass and ceramic segments. Glass, due to its transparency, has a definite advantage over alumina and glass-ceramics from an inspection standpoint.

Several efforts in fabricating large pieces of glass have been made or are currently underway. The Pittsburgh Plate Glass Company has made a 2-in. thick, 60-deg segment of a 6-ft glass sphere of Herculite II, a chemically strengthened glass, for the Naval Ordnance Laboratory (NOL). Current development at Corning Glass Works indicates a capability of yielding glass hemispheres from 3 to 4 ft in diameter with thicknesses greater than 1 in. The Model Basin plans to procure a group of 44-in.-diameter hemispheres to study the effect of scale-up (from a 10-in. diameter) on static, cyclic, and dynamic strength.

Both alumina and glass-ceramics have been made in thicknesses of at least 2 in. However, little is known of their properties in these thicknesses.

At present, there is a reluctance to utilize brittle materials in the design of pressure hulls because they do not meet today’s accepted toughness requirements, which have been successfully applied in the past to steels with yield strengths as high as 100,000 psi. These requirements are commonly measured in terms of shell thicknesses of deformation without fractures, foot-pounds of energy absorption, or percent permanent strain in presence of a through-thickness flaw. It is obvious that glass and ceramics would never be judged an acceptable material under these standards.

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*This scatter is not unexpected due to the lack of ductility in the materials tested.
Before glass and ceramics can be judged to have acceptable toughness for use in the search vehicle, realistic hull toughness criteria must be developed. These criteria must be based on overall structural response to realistic loads rather than solely on the ability of the hull material to absorb a certain level of energy or display a certain level of permanent deformation without fracture. It cannot be determined at this time whether (1) such hull toughness criteria will be developed by January 1968 and (2) if such criteria are developed, whether glass or ceramic hulls of the search vehicle will have sufficient toughness when judged by these criteria.

Permanence will probably be one of the strong features of glass and ceramic hulls for the search vehicle. Unfortunately, sufficient test data do not exist to support this statement and must be accumulated. Currently, NOL has glass under test in the ocean, and the Model Basin will initiate fatigue tests of glass spheres in the near future.

The most reliable compression joints in glass shells that have been tested to date have been fusion-sealed. The Model Basin has recently tested a series of 10-in.-diameter No. 7740 glass spheres which were obtained by fusion-sealing two hemispheres having a nominal wall thickness of 0.281 and a calculated elastic buckling strength of about 20,000 psi. Those models in which the mismatch of the joint was held to a reasonably low level collapsed at pressures very close to the calculated elastic buckle pressure. Corning Glass Works representatives, who fabricated the 10-in.-diameter models, have stated that they have successfully joined glass with thicknesses in excess of 1 in. in this manner. The Model Basin is planning tests of 44-in.-diameter fusion-sealed spheres to study the scale-up effect on static, cyclic, and dynamic behavior. No test data are available on welded ceramic shells since little effort has been made to fabricate ceramics in this manner.
APPENDIX C

EFFECT OF OVERALL LENGTH ON WEIGHT-TO-DISPLACEMENT RATIOS FOR NESTED SPHERES

During the past eight years there has been considerable discussion of the potential of nested or connected sphere pressure hulls for deep-submergence applications. The principal debate pertains to the weight penalty of this type of construction over that of a complete sphere. Complicating this problem, from a parametric view, is the sensitivity of spherical shells to initial imperfections and the effect of the location and size of the reinforcement, which becomes extremely large for some geometries, on the magnitude of the weight penalty.

A reasonable estimate of the lower bound of the weight penalty of nested spheres over complete spheres can be obtained by assuming that all the material is undergoing an equal stress and then calculating the area of reinforcing ring required to keep the shell in equilibrium. The assumption that the reinforcement area is concentrated at the point of intersection permits calculation of the relationship of the $W/D$ ratio of the nested spheres to that of a complete sphere independent of the thickness-to-radius ratio. This relationship for configurations utilizing three-nested spheres is presented in Figure 20.

As a result of the assumptions, Figure 20 represents estimates for near-perfect geometry. However, the error in applying the corrections shown in Figure 20 to spheres with relatively small imperfections should be small. The assumption that the reinforcement material is at the same stress level (in the circumferential direction) as the shell material necessitates that the junctures deflect radially an amount in excess of the radial deflection of an undisturbed sphere. This excessive deflection requires tapering the thickness of the spherical shells at the juncture to keep the bending stresses, as well as the circumferential membrane stresses, in the shells at an acceptable level. Thus, the assumption that the reinforcement area is located at the point of intersection of the spherical shells is probably realistic for most geometries.

An upper bound on the weight penalty can be obtained by assuming that the effective reinforcement area is that which produces membrane deflection of the spherical shells at the juncture. Assuming that the reinforcement is located at the point of intersection yields an upper bound on the weight penalty approximately double the lower bound penalty shown in Figure 20. However, this penalty will increase if the reinforcement is effected by use of an external stiffener and will decrease if an internal stiffener is used. For large thickness-to-radius ratios, the effect of reinforcement location on weight penalty factors can be appreciable. Combating the weight savings gained in using an internal stiffener for the juncture reinforcement, of course, is the loss in clear internal diameter at the juncture. The above discussion is based on the assumption that the reinforcement required to satisfy a membrane condition will have sufficient inertia to prohibit collapse in the general-instability mode.
REFERENCES


**ABSTRACT**

Tradeoff studies between pressure hull material, configuration, and buoyancy are performed for both the Rescue and the Search Vehicles of the Deep-Submergence Systems Project. Spherical, cylindrical, and spheroidal shells of high strength steels, titanium alloys, aluminum alloys, glass-reinforced plastics, glass, and ceramics are considered. Failure criteria are developed based on recent research findings. Potential pressure hull for both vehicles are discussed and the need to develop light-weight flotation systems for the Search Vehicle is emphasized.
### Key Words

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