MEASUREMENTS AND CORRELATION OF HEAT TRANSFER IN A SOLID PROPELLANT ROCKET NOZZLE

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ABSTRACT: The two-dimensional transient heat flow in a conical De Laval nozzle subject to a solid propellant exhaust flow was investigated. Local temperature and heat transfer rates at the internal surface were determined from a combined experimental and analytical method developed at the U. S. Naval Ordnance Laboratory. It was found that an analysis assuming one-dimensional heat flow gave essentially the same results as the two-dimensional analysis.

Local heat transfer coefficients were compared with the predictions from a detailed solution of the turbulent boundary layer flow and with the widely used empirical relation of Bartz. The present results were in better agreement with the boundary layer solution, particularly in the prediction of the peak value at the nozzle throat.

The present rocket nozzle data expressed in Nusselt number form showed very good agreement with the steady state heat transfer data observed with a water-cooled copper nozzle. The copper nozzle data was measured at the U. S. Naval Ordnance Laboratory using heated compressed air.

On the basis of the present data plus the copper nozzle data a simple adjustment is derived to include the effects of compressibility and geometry in the conventional Nusselt-Reynolds number correlation. This adjustment decreases the scatter of experimental data by 43 percent.
Measurements and Correlation of Heat Transfer in a Solid Propellant Rocket Nozzle

This report presents the results of an experimental investigation of the heating of a solid propellant rocket nozzle and a comparison with theories.

The author wishes to express his gratitude to Dr. F. Hill and his staff at the Applied Physics Laboratory of the Johns Hopkins University for the use of the rocket test facility and their assistance in performing the tests. He also gratefully acknowledges the efforts of Mr. J. Iandolo who designed and directed the fabrication and instrumentation of the nozzle; of Dr. W. Parr who developed the two-dimensional numerical method for the IBM 7090; of Mrs. C. Piper who assisted in the programming and supporting calculations on the IBM 7090, and of Mr. J. Bott who assisted in the numerical evaluation of the heat transfer data.

R. E. O'DENING
Captain, USN
Commander

K. R. ENKEMHUS
By direction
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<td>specific heat of gas at constant pressure</td>
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<td>$c_{f\infty}$</td>
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<td>q</td>
<td>time rate of heat transfer per unit area</td>
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<td>r</td>
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<td>St_\infty</td>
<td>Stanton number, $h/\rho_\infty u_\infty c_p$</td>
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<tr>
<td>St_i</td>
<td>incompressible Stanton number, $\frac{1}{2}c_{f_i}Pr_{\infty}^{-2/3}$</td>
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<td>T</td>
<td>temperature</td>
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<tr>
<td>t_j</td>
<td>radial distance from nozzle surface to thermocouple location, see figure 2 and Table I</td>
</tr>
<tr>
<td>u</td>
<td>velocity</td>
</tr>
<tr>
<td>x</td>
<td>axial distance, see figure 2 and Table I</td>
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\( \alpha \) thermal diffusivity

\( \gamma \) ratio of specific heats

\( \mu \) viscosity

\( \rho \) density of gas

\( \tau \) time

\( \tau_w \) shear stress

\( \phi \) mass flow

**Superscripts**

\( n \) time reference

\( \cdot \) evaluated at Eckert's reference enthalpy

\( * \) nozzle throat

**Subscripts**

\( aw \) adiabatic wall condition

\( i \) space reference in the axial direction

\( j \) space reference in the radial direction

\( r \) radial direction

\( s \) nozzle surface condition

\( x \) axial direction

\( o \) stagnation condition

\( \infty \) local free-stream condition
INTRODUCTION

It is generally accepted that the exhaust jet from a solid propellant contains nongaseous states even for propellants without solid additives. The effect of these nongaseous states on heat transfer to motor components is not yet well defined (see, for example, ref. (1)). Because of the lack of a good understanding of rocket nozzle heat transfer the designer often relies on empirical relations which have been developed for liquid or gaseous flows to obtain the necessary heat transfer estimates. It follows that the extension of these empirical equations to determine the performance of rocket motors should be verified by experimental results. It can be seen in references (2), (3), and (4) that the agreement among empirical relations themselves and between the empirical relations and the limited experimental data available can vary considerably depending upon the choice of correlation parameters. These empirical relations can be improved upon or at least the limits of application can be determined by comparing them with the results from carefully controlled experiments. This is the objective of the present report.

The very high exhaust temperatures of present-day rocket motors, greater than 2000°C, increase the difficulty of measuring heat transfer by conventional experimental techniques. The author had previously developed (ref. (4)) a combined experimental and analytical method for obtaining the transient surface temperature and heat transfer rate at the throat of a solid propellant rocket nozzle where heating is most severe. The previous method assumed one-dimensional heat flow in the radial direction. The present report is an extension of the method to heat flow in both the axial and radial directions to determine the surface temperatures and heat transfer rates over all the nozzle surface. In an attempt to correlate the data, the free stream Nusselt number has been modified for compressibility and geometry.

ANALYSIS

A combined experimental-analytical method which employs a finite difference approximation together with thermocouple data was developed and successfully applied in reference (4) to describe the one-dimensional heat flow at the throat region of a solid propellant rocket nozzle. In the present work the
method is extended to describe the two-dimensional heat flow throughout the nozzle. The resulting temperatures and heat transfer rates from the two-dimensional solution are then compared with the original one-dimensional solution which assumes heat flow only in the radial direction.

Figure 1 illustrates the mesh used in applying the finite difference approximation. The region $R$ is defined by the location of thermocouples on its boundary. The grid lines are orthogonal, with each line crossing one or more thermocouple locations. Therefore, the grid spacings are not necessarily equal but are determined by the thermocouple locations.

The thermocouples, as can be seen in figure 2 and Table 1, have been mounted no closer than 0.1 inch from the inner nozzle surface. This was done to prevent the high surface temperatures from destroying the thermocouples.

The heat flow within region $R$ is described by the two-dimensional transient equation in cylindrical coordinates assuming negligible heat flow in the circumferential direction:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = c_p \rho \frac{\partial T}{\partial t} \quad (1) \]

Equation (1) is solved numerically by the implicit method of finite differences (ref. (5)) which has the important feature that the solution is stable regardless of the choice of time and space increments. This is in contrast to the explicit method which is widely used in heat flow problems and which requires a stability criterion dependent on the time step and mesh spacing.

Several numerical methods are available for the solution of two-dimensional problems in the implicit form. Some of these are discussed in references (6) and (7). The method selected is an alternating direction procedure similar to that presented in reference (8) for steady-state problems described by elliptical equations. This alternating direction procedure may be illustrated as follows. (The actual grid used in the data reduction is rather complicated, but the main ideas may be illustrated with the simple rectangular mesh shown on the following page.)
Assuming that all temperatures at time interval n are known, and for the first half time-step the heat conduction in the r direction is neglected, the following finite difference equation in explicit form can be written for each of the grid points indicated by the crosses: (This neglect of the r heat conduction is equivalent to dropping the r derivatives in equation (1).

\[
\begin{align*}
\frac{1}{x(i+\frac{1}{2}) - x(i-\frac{1}{2})} \left[ n \frac{k(i+\frac{1}{2})}{x(i+1) - x(i)} (T(i+1) - T_i) - n \frac{k(i-\frac{1}{2})}{x(i) - x(i-1)} (T_i - T(i-1)) \right]^{n+\frac{1}{2}} \\
+ n \frac{(\rho c_p)_{i,j}}{\Delta t} (T_{i,j} - T_i) & = (\rho c_p)_{i,j} \frac{T_{i,j}}{\Delta t}
\end{align*}
\]

This results in 12 equations in terms of 20 temperatures. The eight temperatures indicated by circles are known from the thermocouple readings. Hence, the 12 unknown temperatures at the time interval \((n+\frac{1}{2})\) and at the position indicated by the crosses may be found from the 12 equations.

For the next half time-step at time interval \((n+1)\), the procedure is repeated but a somewhat different set of mesh points is used in formulating the equations. The mesh points are indicated by crosses, and the heat conduction in the x direction is neglected.
The finite difference equation for the second half time interval for each of the grid points is then:

\[
\left\{ \frac{1}{r_j} \left( \frac{1}{r(j+\frac{1}{2})} - \frac{1}{r(j-\frac{1}{2})} \right) \right\}
\]

(3)

\[
\begin{bmatrix}
\left( \frac{k}{r(j-\frac{1}{2})} \right) \frac{T(j-1) - T_j}{r_j - r(j-1)} \bigg|_{n+\frac{1}{2}} & \left( \frac{k}{r(j+\frac{1}{2})} \right) \frac{T_j - T(j+1)}{r(j+1) - r_j} \bigg|_{n+\frac{1}{2}}
\end{bmatrix}
\]

Similar to before, the temperature at each of the ten stations indicated by a cross at time interval \((n+1)\) may be computed from equation (3). At time interval \((n+\frac{1}{2})\), the procedure is similar to time interval \((n+\frac{1}{2})\). The alternating procedure is repeated at all succeeding times.

This alternating procedure which is equivalent to computing the one-dimensional heat flow in alternate directions simplifies the machine calculation on the IBM 7090. The numerical problem is reduced to the solution of a tri-diagonal matrix which can be readily solved by Gaussian elimination.

The temperature at the inner nozzle surface is extrapolated from the known results in region R by an equation similar to equation (3). The use of equation (3) for extrapolation will introduce errors which can be kept to a minimum by maintaining the extrapolation distance small. The machine program selects a grid at each extrapolation location. The surface heat transfer rates are determined from the temperature gradient at the surface.

**EXPERIMENTAL PROCEDURE AND INSTRUMENTATION**

Rocket nozzle firings were conducted at the Rocket Tunnel Facility of the Applied Physics Laboratory, Johns Hopkins University. The propellant used was of the standard double base,
end burning, 10-second grain supplied by the Allegany Ballistics Laboratory. This grain produces a nominal operating supply pressure and temperature of 1150 psia and 2500°K, respectively, in the combustion chamber. A detailed discussion of the transport and thermodynamic properties of this propellant is given in references (9) and (10).

A diagram of the heat transfer model consisting of a solid, heavy-wall molybdenum heat sink nozzle is shown in figure 2. The internal nozzle geometry is the standard conical nozzle configuration used at the APL facility. The pertinent dimensions are given in Table 1 and shown in figure 2.

The nozzle was instrumented with a total of forty-five thermocouples placed in nine axial locations as shown. The peripheral thermocouples were used to define the boundary for the region R described previously and also the mesh spacing for the two-dimensional numerical solution. The internal thermocouples were used for checking the results of the one and two-dimensional numerical solution.

All thermocouples were made from platinum and platinum-rhodium 30 gage wires. The junctions were spot-welded onto one-degree tapered molybdenum plugs, with one plug for each axial location. Each plug was inserted into a hand-fitted mating hole which bottomed at approximately .01 inch from the internal surface. All the thermocouples were located in one plane that passed through the nozzle axis. The installation of the wires into the tapered plugs was similar to that reported in reference (4). The emf developed by the thermocouples were recorded on two 50-Channel Midwestern Direct Recording Oscillographs.

The supply pressure was measured with a pressure transducer connected to a static orifice located at the downstream end of the combustion chamber. The supply temperature was measured with an unshielded .020-inch diameter tungsten-iridium thermocouple located in the flow directly ahead of the nozzle inlet.

The nozzle was attached to the combustion chamber by a slotted-thread connector which needed only a quarter turn of the nozzle to seal the passage. This quick connector simplified the installation of the nozzle and prevented damage to the delicate thermocouple wires.

DISCUSSION OF DATA

The characteristic pressure and temperature histories in the combustion chamber are shown in figures 3 and 4, respectively.
The various "dips" in the temperature time curve of figure 4, as concluded from previous tests (see ref. (4)), are caused by the temporary insulation effect of nongaseous products deposited on the bare thermocouple. For the heat transfer analysis, the dashed curve through the maximum values represents the supply temperature of the flow.

Continuous temperature-versus-time recordings of each of the forty-five thermocouples were taken during the course of the 9.5-second run. For the numerical solution the data at time increments of 0.1 and 0.25 seconds were used. The difference in temperature distribution as calculated with these two time increments is negligible.

The correlation between the experimental results and the one and two-dimensional numerical methods were made at three selected time stations. These were at three seconds; when free-stream flow conditions should have stabilized; at an intermediate station of six seconds; and just before burnout at 9.5 seconds. The temperature distributions at each of the nine axial locations at these three time stations are shown in figures 5 through 13. The computed two-dimensional solution showed good agreement with measured data at the 9.5-second time station. At the early times of three and six seconds, the computed two-dimensional solution appeared to be lower than experimental data; the lag was more severe at three seconds, particularly at the thicker wall stations. Since there is no reason to doubt the experimental data, the explanation for this discrepancy is that the alternating procedure selected for solving equation (2) introduces a time lag in the computed temperatures at the early highly transient conditions.

Figure 14 is a graph of the computed surface temperature at times 3 and 9.5 seconds. In addition to the anticipated high temperature at the nozzle throat area, a second hot spot occurred at slightly less than one inch downstream from the nozzle throat (station G). This second temperature maximum can also be seen in the map of the isotherms at 6 and 9.5 seconds shown in figures 15 and 16, respectively. The exact cause of this second maximum is not known; however, it may be associated with the flow separation and reattachment observed with the same nozzle geometry in other tests.

It is noted in figures 15 and 16 that the calculated isotherms are wavy near the inner surface. As one goes away from this surface, the waviness is retained in the one-dimensional curves but is diminished in the two-dimensional ones. One expects the waviness to diminish as one gets further from the hot surface—hence, it appears that the two-dimensional calculation is a more accurate description of the heat conduction process than the one-dimensional.
Local nozzle surface heat transfer coefficients are shown in figure 17. The rocket nozzle data show fair agreement with the turbulent boundary layer solutions of Persh and Lee, (ref. (11)) which assumed ideal gas flow and constant wall temperatures of 555°K and 2222°K. Also plotted is the widely used solution of Bartz (ref. (12)) which gives a higher heat transfer coefficient at the throat and downstream. Unlike the ideal gas flow calculation of reference (11) where maximum heat transfer is predicted to occur slightly upstream of the nozzle throat, the rocket nozzle results show the peak to occur slightly downstream. The upstream heat transfer coefficients are lower than the boundary layer prediction possibly due to the observed large combustion deposits in this region. The high data point at approximately 0.9 inch is due to the probable reattachment of separated flow discussed earlier.

For engineering purposes, the maximum temperature and the maximum stress are of prime importance. Both of these occur at the nozzle surface. The computed results show that the difference between the one-dimensional solution and the two-dimensional solution in computing the surface temperature and heat transfer are within experimental accuracy for the test configuration used.

CORRELATION OF DATA

The conventional correlation of nozzle heat transfer data in terms of Nusselt number versus Reynolds number is shown in figure 18. The steady state heat transfer data of reference (13), which covered an extensive range of Reynolds number, is also plotted and appeared to be in very good agreement with the present rocket nozzle data. The scatter of the data may be represented by the distance between the two dashed lines drawn. The upper line is drawn through the higher data points and the lower line is drawn through the lowest data point parallel to the upper line. The two dashed lines deviate from their arithmetic mean by ± 49 percent.

It has been noted in reference (13) for air flow and also reported in reference (2) for a simulated liquid propellant flow that the heat transfer rates are higher in the subsonic nozzle inlet region than at the supersonic expansion region for the same Reynolds number. This is also true for the present rocket nozzle data and is illustrated in figure 18. The subsonic data are shown by the hollow symbols for both the air flow data of reference (13) and for the present rocket nozzle flow data. These appear to be in good agreement with the subsonic turbulent pipe flow relation of Dittus and Boelter (ref. (14)) which is represented by the line of the following expression:
Nu = 0.0265 Re^{0.8} Pr^{0.3} \tag{4}

From the observations described in the preceding paragraph, it appears that a compressibility correction may reduce the scatter of the data. This compressibility correction is intended as an attempt to remove the Mach number and wall temperature effects but not the experimental scatter or the difference between the one- and two-dimensional solutions or between solutions for different times.

Two relatively simple compressibility correction methods which have been successfully applied to flat plate flow were tried here for nozzle flow. The first is by evaluating the data at a reference enthalpy (see ref. (15)) which is defined as:

\[ \frac{H'}{H_\infty} = 1 + 0.032 M^2 + 0.58 \left( \frac{H_S}{H_\infty} - 1 \right) \tag{5} \]

The results of applying the reference enthalpy method, shown in figure 19, showed no improvement in the scatter of the rocket nozzle data. However, the cold-flow data of reference (13) was reduced from the original scatter of ± 44.6 percent when correlated in terms of free-stream properties to a scatter of ± 35.7 percent when correlated by the reference enthalpy method. A possible reason for the lack of favorable results for the rocket nozzle data is that the constants in equation (5) were empirically determined for a particular set of conditions. These original constants, which were adjusted to fit low temperature flat plate data as demonstrated in reference (16), are not necessarily correct for high temperature nozzle flows in which a pressure gradient exists. However, as shown, the method does reduce somewhat the scatter of cold flow data with pressure gradient.

The second method is by the empirical Winkler-Cha skin friction formula (see ref. (16)) where the local skin friction coefficient can be approximated by:

\[ \frac{c_{f_i}}{c_{f_\infty}} = \left( \frac{T_O}{T_\infty} \right)^{\frac{1}{6}} \left( \frac{T_{aw}}{T_s} \right)^{\frac{1}{4}} \tag{6} \]

where \(c_{f_i}\) is the incompressible skin friction coefficient, which is a known function of the momentum thickness Reynolds number. The right hand side of equation (6) may be looked upon as a function of Mach number and wall temperature which when multiplied by \(c_{f_\infty}\) removes the Mach number and wall temperature dependencies. If one further assumes that Colburn's version of Reynolds analogy holds, that is
then it may be shown that

\[ \frac{c_{f_{\infty}}}{2} = St_{\infty} \Pr_{\infty}^{2/3} \]

(7)

then it may be shown that

\[ Nu_{i} = \left( \frac{T_{w}}{T_{s}} \right)^{\frac{1}{3}} \left( \frac{T_{aw}}{T_{s}} \right)^{\frac{1}{4}} Nu_{\infty} \]

(8)

where

\[ Nu_{i} = St_{i} Re_{\infty} \Pr_{\infty} \]

(9)

In other words, the same factors which remove these variations from the skin friction coefficient for a flat plate should also remove them from the Nusselt number.

The results of using the Winkler-Cha method are shown in figure 20. Similar to the result from the reference enthalpy method, the rocket nozzle data showed negligible change while the low temperature data was reduced from + 44.6 percent to + 34.1 percent. It should be noted that both the reference enthalpy and the Winkler-Cha methods give nearly the same improvement to the low temperature data and as shown in figures 19 and 20 put the low temperature data in line with the higher data points of the rocket nozzle results.

Another method of correlation was carried out as follows. In analogy with figure 22 of reference (17), the term \((T_{aw}/T_{s})^{\frac{1}{4}}\) was omitted from equation (6). We define

\[ Nu_{m} = \left( \frac{T_{w}}{T_{s}} \right)^{\frac{1}{3}} Nu_{\infty} \]

(10)

Fig. 21 shows the results. The overall scatter of both the low temperature and rocket nozzle data in figure 21 (excluding the data at the second maximum temperature region which appeared outside the band drawn) was reduced from the original + 49 percent to 36 percent. Comparison of figures 18 through 21 shows that of the four methods tried, this modified version of the Winkler-Cha formula, equation (10), does the best job in reducing the scatter in the heat transfer data.

A closer look at the nozzle throat data of figure 21, which is replotted in figure 22, shows that a straight line can be drawn through these data points. Further analysis of the subsonic data showed a systematically increasing deviation from this straight line with increasing distance (decreasing Reynolds number) from the nozzle throat. This is illustrated by the crosses on figure 22 which represent the data of reference (13).
for one test condition. The dashed line indicates the trend of the subsonic data. Since the free stream temperature changed very little in the subsonic region, the modified Winkler-Cha compressibility correction is negligible at the nozzle inlet. An inspection of the parameters used in the correlation suggest that the nozzle geometry may not be fully accounted for in the applied correction. Therefore, an added geometry correction factor in the form of \((d/d^*)^2\) was tried and appeared to work satisfactorily. The exponent \(l\) was determined by calculating \(d/d^*\) and adjusting \(l\) by trial until the data points fell nearest to the line drawn through the nozzle throat data points. This resulted in a value of \(l\) equal to 0.6. Combining the above geometry factor together with the modified compressibility factor of Winkler-Cha results in the following adjustment in the Nusselt number correlation:

\[
\frac{N_u_e}{N_u_o} = \left(\frac{T_o}{T_{\infty}}\right)^{0.5} \left(\frac{d^*}{d}\right)^{0.6}
\]  

(11)

For isentropic one-dimensional flows where the diameter is related to the Mach number, equation (11) can be written in a different form:

\[
N_u_e = N_u_o \left(\frac{3}{10}\right) \left[\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{3}{2}}\left(\frac{T_o}{T_{\infty}}\right)^{\frac{3}{2}}\left(\frac{T_o}{T_{\infty}}\right)^{\frac{1}{2}}\right]^{\frac{7}{10} \gamma-1.3}
\]

(12)

A constant effective value of \(\gamma = 1.25\) may be assumed. This value is computed at the mean temperature between stagnation and nozzle throat surface temperatures (see refs. (9) and (10)).

The graph of the effective Nusselt number versus Reynolds number is shown in figure 23. The deviation of the data from the mean value is reduced from the original ± 49 percent to ± 28 percent. A line drawn through the data would result in the following expression:

\[
N_u_e = 0.001 \text{Re}_{\infty}
\]

(13)

The foregoing adjustment of conventional correlation parameters is an attempt to reduce the wide scatter of rocket nozzle heat transfer data. The extent of applicability of this correlation will need further experimental support. Equations (11), (12), and (13) were derived from data which extended over the following ranges:

\[
4.0 \times 10^5 \leq \text{Re}_{\infty} \leq 4.1 \times 10^6
\]

\[
1.0 \leq \left(\frac{T_o}{T_{\infty}}\right) \leq 2.1
\]
From equations (11) and (13) and the definition of the Stanton number, the following equation can be derived to compute the local heat transfer coefficient:

\[ h = 0.001 \left( \frac{T_0}{T_\infty} \right)^{-0.5} \left( \frac{d^*}{d} \right)^{-0.5} \left( \frac{c_p}{A} \right)^{0.5} \]  

(14)

In many instances the specific heat and Prandtl number change very little throughout the flow temperature range. Equation (14) can then be reduced further to a constant plus three terms which involve only the mass flow, local diameter, and the local Mach number. The local heat transfer coefficient computed by equation (14) is compared with the present rocket nozzle data in figure 17.

SUMMARY AND CONCLUSIONS

Detailed investigations of the two-dimensional transient heat flow have been made for a heat sink nozzle subject to a solid propellant flow. A finite difference implicit numerical method was employed to describe the two-dimensional heat conduction and to obtain the internal surface heat transfer rates. The surface temperature and heat transfer were compared and were found to agree with the results from the one-dimensional heat flow analysis to within the measuring accuracy.

Peak nozzle throat heat transfer coefficients agree with the predictions from a detailed turbulent boundary layer solution. Propellant deposits and flow separation and reattachment affected the heat transfer upstream and downstream of the nozzle throat respectively. Nusselt number correlation showed very good agreement with steady state heat transfer data observed with a water-cooled copper nozzle.

On the basis of the present data plus the copper nozzle data a simple adjustment is derived to include the effects of compressibility and geometry in the conventional Nusselt-Reynolds number correlation. This adjustment decreases the scatter of experimental data by 43 percent. The data can be approximated by the expression

\[ \text{Nu}_e = 0.001 \text{ Re}_\infty \]  

(13)
from which a simple equation for the local heat transfer coefficient was obtained.

\[
h = 0.001 \left( \frac{T_o}{T_\infty} \right)^{-0.5} \left( \frac{d*}{d} \right)^{-0.6} \left( \frac{d}{A} \right) \left( \frac{C_p \rho_m}{Pr_m} \right)
\]  (14)

The above expression is applicable over the following ranges:

\[4.0 \times 10^5 \leq Re_m < 4.1 \times 10^6\]
\[1.0 < \left( \frac{T_o}{T_\infty} \right) < 2.1\]
\[1.4 < \left( \frac{T_{aw}}{T_S} \right) < 10.3\]
\[0.35 < (d*/d) < 1.0\]
REFERENCES


(7) Varga, Richard S., "Matrix Iterative Analysis," Prentice-Hall, 1962, Chap. 7 and 8


(13) Lieu, B. H., "Air-Film Cooling of a Supersonic Nozzle," NOLTR 64-65, October 1964


TABLE I

Thermocouple Locations
(See figure 2)

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FIG 3  MEASURED CHAMBER PRESSURE AS A FUNCTION OF TIME FROM IGNITION
FIG 5 TEMPERATURE DISTRIBUTIONS AT STATION A
FIG 6 TEMPERATURE DISTRIBUTIONS AT STATION B

TIME

\[ \tau = 9.5 \text{ SEC} \]

\[ \tau = 6.0 \text{ SEC} \]

\[ \tau = 3.0 \text{ SEC} \]
FIG 7 TEMPERATURE DISTRIBUTIONS AT STATION C
FIG 8 TEMPERATURE DISTRIBUTIONS AT STATION D
FIG 9 TEMPERATURE DISTRIBUTIONS AT STATION E
FIG 10 TEMPERATURE DISTRIBUTIONS AT STATION F
FIG 11 TEMPERATURE DISTRIBUTIONS AT STATION G
FIG 12 TEMPERATURE DISTRIBUTIONS AT STATION H
FIG 13 TEMPERATURE DISTRIBUTIONS AT STATIONS 1
FIG 14 NOZZLE SURFACE TEMPERATURE DISTRIBUTIONS
EQN. (14) \( C_{pd}Pr = 0.7 \text{ Cal/gm}^\circ \text{K} \)

BARTZ. (REF. (12))

PERSH AND LEE (REF. (11))

O ONE DIMENSIONAL SOLUTION, \( \tau = 6.0 \text{ SEC.} \)

O ONE DIMENSIONAL SOLUTION, \( \tau = 9.5 \text{ SEC.} \)

O TWO DIMENSIONAL SOLUTION, \( \tau = 6.0 \text{ SEC.} \)

O TWO DIMENSIONAL SOLUTION, \( \tau = 9.5 \text{ SEC.} \)

FIG 17 LOCAL HEAT TRANSFER COEFFICIENT
Fig 18 NuSselt-Reynolds number correlation based on free-stream properties.
FIG 19 NUSSELT-REYNOLDS NUMBER CORRELATION BASED ON ECKERT'S REFERENCE ENTHALPY
FIG 20 NUSSELT—REYNOLDS NUMBER CORRELATION BASED ON FREE-STREAM PROPERTIES AND USING THE WINKLER-CHA COMPRESSIBILITY CORRECTION
ONE DIMENSIONAL SOLUTION, $\tau = 6.0 \text{ sec.}$

ONE DIMENSIONAL SOLUTION, $\tau = 9.5 \text{ sec.}$

TWO DIMENSIONAL SOLUTION, $\tau = 6.0 \text{ sec.}$

TWO DIMENSIONAL SOLUTION, $\tau = 9.5 \text{ sec.}$

LIEU, (REF. (13))

$\pm 36\% \text{ DEVIATION}$

FIG 21 NUSSLEIT—REYNOLDS NUMBER CORRELATION BASED ON FREE-STREAM PROPERTIES AND USING THE MODIFIED WINKLER–CHA COMPRESSIBILITY CORRECTION
FIG. 22 NUSSELT - REYNOLDS NUMBER CORRELATION FOR NOZZLE THROAT DATA
FIG 23 NUSSELT–REYNOLDS NUMBER CORRELATION USING THE MODIFIED WINKLER–CHA COMPRESSIBILITY CORRECTION AND THE SHAPE FACTOR \((a/b)^{0.6}\)
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