SOME RESULTS AND PROBLEMS IN STOCHASTIC LINEAR PROGRAMMING

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SUMMARY

Results and problems in the ordinary "here-and-now" and "wait-and-see" stochastic linear programming problems are described. A general formulation of the "here-and-now" problem is presented, and an approach for solving a special kind of "here-and-now" problem is suggested.
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1. INTRODUCTION

As evidenced by the number of papers presented recently (e.g., [3], [9], [11], [13], [16], and [20]) whose subjects fall under the general rubric of "stochastic linear programming," there seems to be a current flourishing of interest in this area. It appears to me that work is being done on various sub-problems in this area, but that the area in its entirety has never been given suitable attention, or its problems even formulated concisely. It is our hope to discuss the broad area of stochastic linear programming, establish some consistent terminology, point out where current work fits into this broad picture, and present some basic problems which need solution and suggested avenues of solution.

2. STOCHASTIC LINEAR PROGRAMMING SITUATIONS

The usual linear programming problem is to find the activity vector $x^*$ which minimizes $c^*x^*$ subject to the conditions $A^*x^* = b^*$ and $x^* \geq 0$. Let us rewrite the problem as: find the decision vector $x$ and the slack vector $y$ which minimizes $c'x + f'y$ subject to the conditions $Ax + By = b$, $x \geq 0$, $y \geq 0$. In stochastic linear programming, some or all of the matrices $A$ and $B$ and vectors $b$, $c$, and $f$ are random.
Let us distinguish two stochastic linear programming situations, the "wait-and-see" and "here-and-now" situations. In the "wait-and-see" situation, one first observes the random elements and then solves the (nonstochastic) linear programming problem of finding $x \geq 0$ and $y \geq 0$ which minimizes $c^T x + f^T y$ subject to $Ax + By = b$.

In the "here-and-now" situation, one must (1) find the slack vector $y$ as a function of $x$ and of the random elements which optimizes some criterion and (2) determine a value of the decision vector $x$ which is independent of $y$ and the to-be-observed random elements and which optimizes some criterion which is independent of the observed value of the random elements, from among the vectors $x$ and $y$ such that the probability of feasibility is at least some specified $P$, $0 \leq P \leq 1$. After these vectors are determined, the random elements are observed and the value of $y$ is determined. Following are some examples of "here-and-now" problems.

**Example I:** Among all $x$ and $y$ whose probability of feasibility is at least $P$, find the $y$ which minimizes $c^T x + f^T y$, and determine the value of $x$ which minimizes $\mathbb{E} \min_y (c^T x + f^T y)$.\(^1\)

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\(^1\)Unless otherwise specified, the expectation or probability is with respect to all the random elements.
Example II: Among all $x$ and $y$ whose probability of feasibility is at least $P$, find the $y$ which minimizes $c'x + f'y$, and determine the value of $x$ which maximizes $\Pr \left\{ \min_x (c'x + f'y) \leq k \right\}$ for some fixed preassigned $k$.

Example III: Among all $x$ and $y$ whose probability of feasibility is at least $P$, find the $y$ which minimizes $c'x + f'y$, and determine the value of $x$ which maximizes $\Pr \left\{ \min_y (c'x + f'y) \leq k \right\}$ and $x, y$ feasible for some fixed preassigned $k$.

In the following we shall refer as a practical example to this simple case: A factory has 100 items on hand which may be shipped to an outlet at the cost of $1$ apiece to meet an uncertain demand $d$. In the event that the demand should exceed the supply, it is necessary to meet the unsatisfied demand by purchases on the local market at $2$ apiece. The equations that the system must satisfy are

\[ 100 = x_{11} + x_{12} \]
\[ d = x_{11} + x_{21} - x_{22} \quad (x_{1j} \geq 0) \]
\[ c = x_{11} + 2x_{21} \]

where

$x_{11}$ = number shipped from the factory,
$x_{12}$ = number stored at factory;
$x_{21}$ = number purchased on open market,
$x_{22}$ = excess of supply over demand;
d = unknown demand uniformly distributed between 70 and 120;  
C = total costs.

It is clear that whatever be the amount shipped and whatever be the demand d, it is possible to choose x_{21} and x_{22} consistent with the second equation. The unused stocks x_{12} + x_{22} are assumed to have no value or are written off at some reduced value (like last year's model automobiles when the new production comes in). In this example, x' = (x_{11}, x_{12}), y' = (x_{21}, x_{22}), A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}, c' = (1, 0), f' = (2, 0), and b' = (100, d).

3. THE HERE-AND-NOW PROBLEM

3a. Example I

Work has been done on Example I by Beale [2], Dantzig [7], and Madansky [13], in the case where F = 1 and only b is random. The basic consideration of Beale was to consider how one might utilize the solution of the nonstochastic problem where b is replaced by Eb in order to iterate to a solution of the stochastic problem. He suggests that due to certain convexities in the problem, this solution be utilized as a first approximation in an application of the Newton-Raphson method to minimizing E \min_y (c'x + f'y) with respect to x.

Dantzig considered a multistage problem where a decision x_1 has to be made at stage one, then the (random) right-hand side of the first stage of the problem, b_1, is observed, after
which a decision \( x_2 \) has to be made at stage two, etc., for \( m \) stages. He found that \( \text{E}_{b_1, \ldots, b_m} \min (c'x_1 + f'y) \) is convex in \( x_1 \) (where \( y \) includes \( x_2, \ldots, x_m \), as well as slack), and so suggests using appropriate techniques ([5], [6]) to determine \( x_1 \), and hence \( x_2 \) after \( b_1 \) is observed, etc.²

Madansky, in connection with work on inequalities for \( \min \text{E} \min (c'x + f'y) \), the value of the objective function in the "here-and-now" problem of Example I, and \( \text{E} \min \min (c'x + f'y) \), the expected value of the objective function of the "wait-and-see" problem, found the following sufficient (though not necessary) condition for the solution of the stochastic problem to be the solution of the nonstochastic problem where \( b \) is replaced by \( \text{E}b \).

**Theorem.** If \( \min (c'x + f'y) \) can be written as
\[
C_1(b, x) + C_2(b), \quad \text{where} \quad C_1(b, x) \text{ is linear in } b,
\]
then the \( x \) which solves the nonstochastic problem where \( b \) is replaced by \( \text{E}b \) also solves the "here-and-now" problem of Example I where \( P = 1 \) and only \( b \) is random.

¹ Cf. also Theorem 2 of [2].

² Cf. also [4], where by imposing the condition that for each \( i \) \( x_i \) be linear in the \( b_j, j = 1, \ldots, i-1 \), the quantity \( \text{E}_{b_1, \ldots, b_m} \min (c'x_1 + f'y) \), as interpreted in the context of [7], is found to be convex in \( x_1 \).
This theorem generalizes work of Simon [13] and Theil [18] on what is known in the literature as the use of "certainty equivalents" in stochastic problems.¹

The only other results on the version of Example I of the "here-and-now" problem are the inequalities of Madansky [13] on the value of the objective function when only b is random. Let \( C(b, x) = \min_y (c'x + f'y) \). Then in general

\[
E_b \min_x C(b, x) \leq \min_y E_b C(b, x) \leq E_b C(b, \bar{x}(Eb)),
\]

where \( \bar{x}(Eb) \) is the vector which solves the nonstochastic linear programming problem where \( b = Eb \). Also, in the special case considered by Dantzig discussed above, there is the additional inequality

\[
\min_x E_b C(b, x) \geq a_0 + \sum_{j=1}^{r} a_j \bar{x}_j = G(\bar{x})
\]

where \( \bar{x} \) is the \( r \)-dimensional vector which solves the "here-and-now" problem.

¹Reiter [14] studies the general problem of finding sufficient conditions for the solution of a stochastic problem to be that of a "surrogate" problem where the probability distribution is replaced by something simpler. His condition, in our case, is that \( C(b, x) = \sum_{i=1}^{n} A_1(x)B_1(b) \), where \( A_1(x) > 0 \), \( B_1(b) > 0 \), in which case the \( x \) which minimizes \( \sum_{i=1}^{n} A_1(x)EB_1(b) \) solves the stochastic problem. If we let \( n = 3 \), \( B_1(b) = 1 \), \( B_2(b) = b \), \( B_3(b) = C_2(b) \), and \( A_3(x) = 1 \), then \( C(b, x) = C_1(b, x) + C_2(b) = A_1(x) + A_2(x)b + B_3(b) \), so that our theorem is a special case of Reiter's where the "surrogate" is \( Eb \).
\[
\begin{align*}
\alpha_j &= \frac{\partial E_b C(b, x)}{\partial x_j} \bigg|_{x_j = \bar{x}_j(Eb)} , \quad j = 1, \ldots, n, \\
\alpha_0 &= E_b C(b, \bar{x}(Eb)) - \sum_{j=1}^{r} \alpha_j \bar{x}_j(Eb),
\end{align*}
\]

and the derivative of \( E_b C(b, x) \) exists in the neighborhood of \( \bar{x}(Eb) \). Hence, one can determine the coefficients \( \alpha_0, \alpha_1, \ldots, \alpha_r \) without solving Dantzig's "here-and-now" problem and, if one has bounds on \( \bar{x} \), can utilize this information to get a lower bound on \( G(\bar{x}) \). This then yields a lower bound on the value of the objective function of Dantzig's "here-and-now" problem which is independent of the solution of the problem.

As shown in [13], a sufficient condition for \( \min_E E_b C(b, x) = E_b \min_x C(b, x) \) is that \( C(b, x) \) be a linear function of \( b \). A sufficient condition for \( \min_E E_b C(b, x) = E_b C(b, \bar{x}(Eb)) \) is that \( E_b C(b, x) \) be of the form \( x_j^2 - 2\bar{x}_j(Eb)x_j + k \) in each \( x_j \), for then \( \alpha_j = 0 \) for \( j = 1, \ldots, r \), so that \( G(\bar{x}) = E_b C(b, \bar{x}(Eb)) \) and \( E_b C(b, \bar{x}(Eb)) \leq \min_x E_b C(b, x) \leq E_b C(b, \bar{x}(Eb)) \). Examples can be constructed such that \( G(\bar{x}) > E_b \min_x C(b, x) \) and \( G(\bar{x}) < E_b \min_x C(b, x) \), so that one cannot compare these bounds in general.

One can generalize the inequalities further, to the case where all the coefficients in the problem (i.e., the elements
of $A$, $B$, $b$, $c$, and $f$) are random, for even then

$$
\mathbb{E} \min \min_{x,y} (c'x+f'y) \leq \min_{x,y} \mathbb{E} \min (c'x+f'y) \leq \mathbb{E} \left[ \min_{x,y} (c'x+f'y) \middle| x = \bar{x}(\xi) \right],
$$

where $\xi$ denotes $(EA, EB, Eb, Ec, Ef)$. However, the conditions for equality do not generalize.

The major problem area in Example I is that of finding a technique which solves the general problem. The only techniques known are those of Dantzig for the special problem he considered and the use of "certainty equivalents" in the case covered by the above Theorem. It would be of interest to know for what real situations $C(b, x)$ is of the form required by the above Theorem. Also, a possible area of research is on iterative techniques such as that suggested by Beale. The inequalities of Madansky may be used to provide benchmarks on the number of iterations needed to arrive at a solution.

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1Charnes and Cooper [3] suggest that one break the problem into two parts, that of determining the distribution of the optimizing $x_1$'s (since they are functions of the $b_j$'s, $j = 1, \ldots, 1-l$) and that of approximating the $x_1$'s by linear functions, say, of the $b_j$'s, the coefficients being those for which the distribution of this linear function best approximates the distribution of the optimizing $x_1$'s. It is unclear from [3], though, how the distribution of the optimizing $x_1$'s can be determined without actually solving the "here-and-now" problem. Elmaghraby [9] studies the use of the gradient method to solve the problem of [8].
It would be of interest to know for what situations other than that studied by Dantzig is $E C(b, x)$ a convex function of $x$, so that the techniques of [5] and [6] may be used. Also, what is the probability that the "certainty equivalent" solution is feasible in various situations? The answer to this question may be useful in helping someone decide whether he will be willing to suboptimize and solve a solvable problem.

To illustrate the use of the known results for Example I, let us study the simple case given in Section 2. (This case is a single-stage problem of the multistage type which Dantzig studied.) It is clear that if supply exceeds demand ($x_{11} > d$), that $x_{21} = 0$ gives minimum costs and, if $x_{11} \leq d$, that $x_{21} = d - x_{11}$ gives minimum costs. Hence

$$C(b, x) = \begin{cases} x_{11} & \text{if } x_{11} > d \\ x_{11} + 2(d - x_{11}) & \text{if } x_{11} \leq d \end{cases}$$

Since $d$ is assumed to be uniformly distributed between 70 and 120,

$$E_b C(b, x) = \begin{cases} -x_{11} + 190 & \text{if } x_{11} \leq 70 \\ 107.5 + (95 - x_{11})^2/50 & \text{if } 70 < x_{11} \leq 120 \\ x_{11} & \text{if } 120 < x_{11} \end{cases}$$

Note that $E_b C(b, x)$ is convex in $x$. This function attains its minimum, 107.5, when $x_{11} = 95$. Hence, $\min \limits_{x} E_b C(b, x) = 107.5$. 
One also sees that \( \min C(b, x) = d \), so that \( \mathbb{E}_b \min C(b, x) = \mathbb{E}d = 95 \). Also, since \( \mathbb{E}(\mathbb{E}b) = 95 \),

\[
C(b, \mathbb{E}(\mathbb{E}b)) = \begin{cases} 
95 & \text{if } 95 > d \\
-95 + 2d & \text{if } 95 \leq d 
\end{cases}
\]

and \( \mathbb{E} C(b, \mathbb{E}(\mathbb{E}b)) = 107.5 \). Our inequalities yield

\[
95 \leq \min \mathbb{E}_b C(b, x) \leq 107.5,
\]

and we note that the upper bound is sharp.

Since this example is a degenerate multistage problem of the type studied by Dantzig, let us determine \( G(\mathbb{E}) \). Here

\[
\frac{\partial \mathbb{E}_b C(b, x)}{\partial x_{11}} = \begin{cases} 
-1 & \text{if } x_{11} \leq 70 \\
(x_{11} - 95)/25 & \text{if } 70 < x_{11} \leq 120 \\
1 & \text{if } 120 < x_{11}
\end{cases}
\]

and \( \mathbb{E}_{11}(\mathbb{E}b) = 95 \), so that \( \alpha_1 = 0, \alpha_0 = \mathbb{E}_b C(b, \mathbb{E}(\mathbb{E}b)) = 107.5 \) and \( G(\mathbb{E}) = 107.5 \). Hence this lower bound is sharp, as well, in this case.

Note that the sufficient condition for \( \min \mathbb{E}_b C(b, x) = \mathbb{E}_b C(b, \mathbb{E}(\mathbb{E}b)) \) given above is satisfied in this case, so that we expect both \( G(\mathbb{E}) \) and the upper bound to be sharp. Note also that both the optimum \( x_{11} \) and \( \mathbb{E}(\mathbb{E}b) \) are 95. This is to be expected, because \( C(b, x) \) is linear in \( b \) for each \( x \), and so satisfies the requirement of the Theorem stated above.

Finally, note that \( C(b, x) \) is not linear in \( b \) for all \( x \), so the sufficient condition for \( \min \mathbb{E}_b C(b, x) = \mathbb{E}_b \min C(b, x) \) is not satisfied, and in fact in this example equality of these values does not hold.
3b. Other Examples

I have seen no work on either of the other examples given in Section 2. A natural approach which might be taken in solving Example II where only $b$ is random is to replace $b$ by a vector $b_\gamma$ where $\Pr \{ b \leq b_\gamma \} = \gamma$, and solve the nonstochastic linear programming problem for $x_\gamma$, say. One might then search for the largest $\gamma$ and concomitant $x_\gamma$ such that $x_\gamma$ and $y(x_\gamma)$ are feasible with probability $P$ or more and such that $C(b_\gamma, x_\gamma) = k$. This decision solves the problem of Example II.

The problems involved in implementing this approach, aside from the "search" problem, are the following. In multidimensions, $b_\gamma$ is not unique, and so one must first study each $b_\gamma$, for a given $\gamma$, such that $\Pr \{ b \leq b_\gamma \} = \gamma$. Also, as $\gamma$ increases, at least one component of $b_\gamma$ increases. However, though $x_\gamma$ is a continuous function of $b_\gamma$, it may not be the case that by increasing $\gamma$, $\Pr \{ C(b_\gamma, x_\gamma) \leq k \}$ will increase.

Let us illustrate how, in a one-dimensional uncertainty problem, where these difficulties do not exist, use of the $\gamma$-th quantile of the distribution of $b$ will yield the solution of the problem of Example II where only one component of $b$ is random and where $P = 1$. We consider the illustration studied above.

Let us write $d_\gamma = 70 + 50\gamma$, $0 \leq \gamma \leq 1$. We note that, as a function of $d$, 
\[
\min_{x} C(b, x) = \begin{cases} 
0 & d \leq 0 \\
\begin{align*}
 d & 0 < d \leq 100 \\
2d - 100 & 100 < d
\end{align*}
\end{cases}
\]

or rewriting in terms of \( \gamma \),

\[
\min_{x} C(b, x) = \begin{cases} 
0 & \gamma \leq -1.4 \\
70 + 50\gamma & -1.4 < \gamma \leq .6 \\
40 + 100\gamma & .6 < \gamma
\end{cases}
\]

Note also that \( \min_{x} C(b, x) = 70 + 50\gamma \) when only \( x_{11} \) and \( x_{12} \) are the only (possible) non-zero components of the solution vector, and \( \min_{x} C(b, x) = 40 + 100\gamma \) when only \( x_{11} \) and \( x_{21} \) are the only (possible) non-zero components of the solution vector.

Let us say we wish \( C(b, x) \leq k \). Hence

\[
x = \begin{cases} 
\begin{align*}
x_{11}, x_{12} & \text{if } \gamma \leq (k-70)/50 \text{ and } 0 \leq \gamma \leq 0.6 \\
x_{11}, x_{21} & \text{if } \gamma \leq (k-40)/100 \text{ and } 0.6 < \gamma \leq 1
\end{align*}
\end{cases}
\]

and finding \( x \) which maximizes \( \Pr \{ C(b, x) \leq k \mid P = 1 \} \) is the same as finding \( x \) which maximizes \( \gamma \) subject to the above restraints on \( \gamma \). For example, if \( k = 110 \),

\[
x = \begin{cases} 
\begin{align*}
x_{11}, x_{12} & \text{if } \gamma \leq 0.8 \text{ and } 0 \leq \gamma \leq 0.6 \\
x_{11}, x_{21} & \text{if } \gamma \leq 0.7 \text{ and } 0.6 \leq \gamma \leq 1
\end{align*}
\end{cases}
\]

i.e.,

\[
x = \begin{cases} 
\begin{align*}
x_{11}, x_{12} & \text{if } \gamma \leq 0.6 \\
x_{11}, x_{21} & \text{if } \gamma \leq 0.7
\end{align*}
\end{cases}
\]
Hence $y$ is maximized if $x = (x_{11}, x_{21})$, in which case $y = .7$, $d_y = 105, x_{11} = 100$, and $x_{21} = 105 - 100 = 5$. For the decision $x_{11} = 100, x_{21} = 5$, the probability of feasibility is 1 and the probability of the objective function being less than or equal to 110 is maximized.

4. The Wait-and-see Problem

Tintner [19] has considered the problem where one wishes to optimize, over a set of possible distributions of the random elements of the problem, a "preference functional" of the (random) value of the objective function of the "wait-and-see" problem. Examples of such a preference functional are $E(\min \min (c'x + f'y))$ and $\Pr \left\{ \min \min (c'x + f'y) \leq k \right\}$. He suggests an approximate procedure for obtaining the distribution of the value of the objective function in case the random elements are normally distributed.

The only other work on the "wait-and-see" linear programming problem are the inequalities of Madansky [13] when only $b$ is random, and unpublished work of Talacko and Rockafellar (cf. [17]).

It has been shown in [13] that

$$\min_x C(Eb, x) \leq E_b \min_x C(b, x) \leq \min_x E_b C(b, x),$$

and also, that if $b$ is an $n$-dimensional vector with finite

$\text{\textsuperscript{1}}$See [10] for a special type of "wait-and-see" problem which reduces to a "wait-and-see" quadratic programming problem.
range, \([\beta_1, \beta_2]\), then

\[ E_b \min_x C(b, x) \leq \sum_{j=1}^{n} \frac{(-1)^{j} (\beta_{\phi_j} - Eb_j)}{(\beta_{2j} - \beta_{1j})} \min_x C(\beta_{\phi_1}, \ldots, \beta_{\phi_n}) \]

\[ = H^*(Eb) \]

where \(\phi_j (j = 1, \ldots, n)\) takes on the values 1 and 2,
\(\bar{\phi}_j = 3 - \phi_j\), \(\phi\) is the set of \(2^n\) \(n\)-vectors of 1's and 2's, and \(\beta_{\phi_j}^k\) is the \(k\)-th element of the vector \(\beta_{\phi_j}\). The inequality

\[ \min_x C(Eb, x) \leq E_b \min_x C(b, x) \]

was first derived, in a different manner from that of [13], by Vajda [20].

A necessary and sufficient condition is given in [13] for

\[ H^*(Eb) = E_b \min_x C(b, x) = \min_x C(Eb, x) \]

namely that \(\min_x C(b, x)\) be linear in \(b\). Also, it is shown that a sufficient condition for

\[ \min_x E_b C(b, x) = E_b \min_x C(b, x) = \min_x C(Eb, x) \]

is that \(C(b, x)\) be linear in \(b\). Examples have also been constructed where \(H^*(Eb)\) is greater than \(\min_x E_b C(b, x)\) and where it is less than \(\min_x E_b C(b, x)\).

Talacko and Rockafellar [17] have studied the problem of obtaining confidence limits on \(\min_x C(b, x)\) by using quantiles of the distribution of \(b\). They also have generalized these confidence limits to the case where \(A, B, c,\) and \(f\) are also random. As yet, this work is unpublished.

A variant of the use of "certainty equivalents" was studied in Babbar in [1]. He considered the modified "wait-and-see" problem (where the criteria are those of Example I)
where the activity vector $x$ but not its value must be determined before the random elements are observed, and after the random elements are observed the value of $x$ must be determined so as to "best" satisfy the resulting restraints of the problem. More precisely, he studied the approximate distribution of the value of $x$, where the vector $x$ is that vector which solves the nonstochastic problem where the random elements are replaced by their expected values. Unfortunately, as Wagner [21] noted, this vector may not be feasible after the random elements are observed, and so the distribution studied is not necessarily the distribution of a vector which even satisfies the problem.

Also, we note from [13] that the expected value of the objective function of the modified "wait—and—see" problem is not only at least as great as the expected value of the objective function of the "wait—and—see" problem, which is to be expected, but also is at least as great as the value of the objective function of the "here—and—now" problem. Hence, one is better off in the modified "wait—and—see" situation of Babbar to recast the problem as a "here—and—now" problem (if, of course, the "here—and—now" problem can be solved). Other comments on Babbar's procedure are made in [21].

The outstanding problems in the "wait—and—see" area are those of determining the distribution, or at least the mean and variance, of the objective function and of the value of the optimizing vector under various distributional assumptions.
about the random variables. It may also be fruitful to look
further at Tintner's approximations in the case he considered
and derive approximate means and variances of these quantities.

One should also note from Theorem 3 of [2] that if both
A and b are random, then \( \min_y (c'x + f'y) \) is a convex function
of both A and b. Let \( C(A, b, x) = \min_y (c'x + f'y) \), in this
case. Then, by the same argument as in [12], one can show
that \( \min_x C(EA, Eb, x) \leq E_{A,b} \min_x C(A, b, x) \leq \min_x E_{A,b} C(A, b, x) \)
and, if the elements of both A and b are distributed over a
finite range, then one can determine an upper bound of the
form of \( H^*(Eb) \), say \( H^*(EA, Eb) \), by the method of [12].

To return once again to the simple case studied earlier,
we record once again that \( E_b \min_x C(b, x) = 95 \), which was less
than \( \min_x E_b C(b, x) = 107.5 \). When \( d = Ed = 95 \),

\[
C(Eb, x) = \begin{cases} 
  x_{11} & \text{if } x_{11} > 95 \\
  -x_{11} + 190 & \text{if } x_{11} \leq 95 
\end{cases}
\]

and \( \min_x C(Eb, x) = 95 \). Finally, in this case

\[
H^*(Eb) = \frac{(-1)(70 - Ed)}{(120 - 70)} \min_x C(120, x) + \frac{(-1)^2(120 - Ed)}{(120 - 70)} \min_x C(70, x)
\]

\[
= Ed = 95.
\]

Hence \( 95 \leq E_b \min_x C(b, x) \leq 95 \). But this was to be expected,
since \( \min_x C(b, x) = d \) and hence satisfies the necessary and
sufficient condition for equality of these three quantities.
REFERENCES


Inequalities for Stochastic Linear Programming Problems,
The RAND Corporation, Research Memorandum RM-2287,
November 13, 1958.

14. Reiter, S., "Surrogates for Uncertain Decision Problems:
Minimal Information for Decision Making," Econometrica,

15. Simon, H. A., "Dynamic Programming under Uncertainty with

Abstract No. 552–15, Notices, American Mathematical

17. Talacko, J. V., and R. T. Rockafellar, "Estimation of the
Confidence Interval for the Optimum Stochastic Linear
Functional," Marquette University, unpublished.

18. Theil, H., "A Note on Certainty Equivalence in Dynamic

19. Tintner, G., "Stochastic Linear Programming with
Applications to Agricultural Economics," Proceedings of
the Second Symposium on Linear Programming, National
pp. 197–227.

20. Vajda, S., "On Stochastic Linear Programming," presented
at the Thirtieth Session of the International Statistical

21. Wagner, H. M., "On the Distribution of Solutions in Linear
Programming Problems," Journal of the American Statistical