Reflection and Scattering of Sound
by the Sea Bottom

by

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Presented at the 69th Meeting
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REFLECTION AND SCATTERING OF SOUND
BY THE SEA BOTTOM

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Session K at the 68th meeting of the Acoustical Society of America, Austin, Texas (October, 1964) was devoted to reflection and scattering of sound from the sea bottom. Three invited papers were read, and are presented here. The session was conceived and organized by Aubrey Pryce, Director of Acoustics Programs, Office of Naval Research. Judging from the spirited discussion which followed the papers, a debt of gratitude is owed to Mr. Pryce for his interest in this important and timely subject.

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Reflection and Scattering of Sound by the Sea Bottom

Part I  Theory

H. W. Marsh
AVCO Marine Electronics Office, New London, Connecticut

Abstract

This paper is Part I of a trilogy, the others being Part II - Field Data and Part III - Model Studies. Theoretical topics which are relevant to the most important effects observed in the field are discussed. These are grouped under Intensity of Reflection, Distortion of Reflected Wave and Sound Scattering, and include models predicting effects of absorbing bottoms, layered media and irregular boundaries.

Prepared for the 68th meeting of the Acoustical Society of America, under the sponsorship of The Acoustics Programs, Office of Naval Research.
Introduction

The subject has been of interest at least since the introduction of the echo sounder. It is significant to the design and operation of modern sonars, and will undoubtedly become increasingly important as deep submergence activity develops.

As in most cases, the role of theory is two-fold. On the one hand, theory can provide a guide or prediction under conditions which have not been directly observed. On the other, it can aid in the interpretation and collation of experimental findings. In the present case, theories are elementary, but often are complex and difficult to interpret meaningfully without resort to extensive numerical investigation. Their utility for prediction is seriously limited by a lack of knowledge of the physical character and structure of the sea bottoms. Their application to the interpretation of experiments is hampered by the quality and scope of available data. It happens that most of our knowledge of the sea bottom is acquired through geophysical studies employing sound as a primary tool, either directly in the field, with reflection/refraction shooting and continuous profiling, or in the laboratory with acoustic measurements on core samples. With this in mind, it appears that the actual use of theory is in the development of a self consistent model of the sea bottom, as "seen" through acoustic experiments, for application to acoustic effects.

In this paper, the character of bottom reflected sound will be discussed qualitatively, and theories dealing with various features of the reflection will be outlined. It will be seen that existing theory is not adequate in several situations. This discussion will be developed under three headings: Intensity of Reflection, Distortion of Reflected Wave and Sound Scattering. These have been selected because of their direct significance to sonar problems, and because available data disclose the following facts:
a) Appreciable energy is transmitted into the bottom and not returned to the field in the water

b) There is considerable backscatter, or bottom reverberation

c) There is great variability of effects, sometimes correlated with geographical location, composition and structure of the bottom, acoustic frequency, beam width, angle of incidence and reflection, wave form, etc.

It is clear that any analytical model which can claim generality must be capable of predicting results of these types.

Intensity of Reflection

Figure 1 is a composite of many measurements of "Bottom Loss", showing its dependence upon acoustic frequency and grazing angle. This loss is conceptually related to the reflection coefficient, as shown in Figure 2. This shows the waves reflected and transmitted by a smooth boundary between homogeneous fluids of given densities and sound velocities for an incident wave at grazing angle $\theta$. The bottom loss $N_B$ (Figure 1) is intended to be

$$N_B = 20 \log \left| \frac{A_r}{A_i} \right|$$

or in other words, the bottom loss is the reflection coefficient in dB. It must be emphasized that the results shown here are "average" and not necessarily representative of any particular situation or geographical location. Nevertheless, the apparent dependence upon frequency and grazing angle must be accounted for in our model.

There is available data on the density and sound velocity of sea water, and of a number of deep sea sediments. Mackenzie has shown reflection coefficients measured in the field are well accounted for by correlated
FIGURE I

BOTTOM LOSS vs. GRAZING ANGLE
FIGURE 2  REFLECTION FROM SMOOTH BOUNDARY BETWEEN HOMOGENEOUS FLUIDS
measurements on the density and complex velocity of a number of sediments. He employed Morse's formulae

$$\frac{A_r}{A_i} = \frac{(h - \sigma \sin \phi)^2 + g^2}{(h + \sigma \sin \phi)^2 + g^2}$$

$$h + ig = \frac{k_2 \sin \theta_t}{\beta}$$

$$k_2 = \frac{\omega}{c_2} = \beta + i \alpha$$

$$\sigma = \rho_2 k_1 / \rho_1 \beta$$

In these formulae $\omega$ is the angular frequency and the wave numbers are $k_2 = \omega/c_2$ and $k_1 = \omega/c_1$. Attenuation is admitted in the bottom material through the presence of the term $\alpha$ in $k_2$, implying a complex velocity. To apply the equations, Snell's law is required in the form

$$\frac{\cos \phi}{\cos \theta_t} = (\cos \phi + \frac{n^2 \beta^2}{k_1^2})^{1/2}$$

In his calculations, Mackenzie employed measurements of density and complex velocity made by Hamilton and Shumway, and showed the importance of absorption in the sediments. He extrapolated the values measured at 30 kc/s to frequencies between 0.2 and 5.0 kc/s and concluded that linear extrapolation was best. Additional measurements on acoustical properties of sediments have been reported by Nafe, Nolle, Wood and Hampton. Wood in particular pointed out the extreme importance of care in handling sediment samples, and made in situ measurements in mud. He found a linear dependence of absorption upon frequency and concluded that available theory (Biot, Urick) cannot account for the dependence. Nolle compared values of flow resistance and absorption according to an equation of Morse, and concluded that although the two were in fair agreement, absorption was a better measure.
of flow resistance than the reverse case. On the other hand, sedimentary velocities\(^7\) are in reasonable agreement with those calculated from the Wood\(^1\) equation

$$\frac{1}{c_z^2} = (\Sigma f_i \rho_i) \left(\Sigma f_i C_i\right)$$

with \(f_i\) representing the volume fraction of the \(i^{th}\) constituent and \(C_i\) the compressibilities.

From the foregoing discussion, it may be concluded that an adequate general account of bottom reflectivity can be given in terms of the simple model so far presented. However, for a detailed accounting, the model is inadequate, as may be seen from Figure 3, which is taken from Barnard\(^{14}\). The large variations in reflectivity suggest interference effects, and one is led to consider structured media, or media in which the physical properties are a function of depth of penetration. In fact, the summaries of Nafe\(^7\) show that even in the relatively homogeneous sediments, both density and compressional velocity increase with depth below the ocean floor, with gradients in the case of velocity amounting to 0.5 - 2.0 sec\(^{-1}\). It is an interesting result that in the case of an exponential density variation of the form

$$\rho_2 = \rho_{20} e^{v z}$$

then for normal incidence the reflection coefficient is

$$\frac{A_r}{A_i} = \frac{\left[c_2 \rho_2 - c_1 \rho_1 \sqrt{1 - \frac{\nu^2 c_z^2}{\omega^2}}\right] - i \rho_1 c_1 \nu c_z / \omega}{\left[c_2 \rho_2 + c_1 \rho_1 \sqrt{1 - \frac{\nu^2 c_z^2}{\omega^2}}\right] + i \rho_1 c_1 \nu c_z / \omega}$$

This result, which was pointed out by Berman\(^15\), shows that a frequency dependent reflection coefficient can result from non absorbing material, if it is not homogeneous. By combining effects, a multi-layered bottom of absorbing materials with various constants can be developed, and these may be further generalized to admit rigidity, and hence allow for transverse waves. The general equations accommodating these effects may be found, for example, in Brekhovskikh\(^16\) or Thomson\(^17\). Detailed calculations on specific examples
of this general model have been made at a number of laboratories. A major problem is the lack of values for the several constants involved, especially in the case of shear waves. Some information on shear velocities and absorption is summarized by Nafe and more recent results are given by Ducker.

It would be well to recapitulate at this point. A composite picture of bottom loss was displayed, showing dependence upon grazing angle and frequency. The theory subsequently outlined would appear adequate to account for such behaviour, if sufficiently detailed knowledge of the physical constants of bottom materials were available. The theory, however, demands a plane, monochromatic reflected wave, neither of which hold in the field. For one thing, finite beam widths and pulse lengths are employed in practice, and it is found at times that the bottom loss measured is a function of the technical parameters of the experimental equipments. In short, sound is distorted upon interaction with the sea bottom, and we shall turn our attention next to this distortion.

**Distortion of Reflected Wave**

Since the reflection coefficient of the models considered so far are frequency dependent, some distortion has already been admitted. An important aspect of this is phase distortion. Returning to the Morse equation, there is a phase shift \( \psi \) between reflected and incident waves, given by

\[
\tan \psi = \frac{2 \sigma g \sin \phi}{\sigma^2 \sin^2 \phi - (h^2 + g^2)}
\]

Even in the absence of structure or absorption, a significant phase shift can occur. If there is a critical angle, then for angles more grazing than critical, it is known that the reflection coefficient has unit magnitude, and for the phase
shift
\[
\tan \frac{\psi}{2} = \frac{(1 - n^2 \sec^2 \phi)^{1/2}}{n \sigma \tan \phi}
\]

This phase shift, which is independent of frequency, increases from 0 at the critical angle to \( \pi \) at grazing incidence and will produce distortion in a wide band signal. The effect has been investigated both theoretically and experimentally by Arons\textsuperscript{21} for an exponential pulse. Abramowitz\textsuperscript{22} computed the effect for one cycle of a sine wave, and a number of results have been obtained by Cron\textsuperscript{23}. Figure 4, which was kindly furnished by Cron, illustrates the distortion of a Gaussian pulse with phase shift \( \pi/2 \).

Other examples of phase distortion with various types of frequency dependence could be computed from the general equations identified above. Mackenzie\textsuperscript{3} has computed phase shift for a few absorbing materials, but the subject has not received particular attention, nor (excepting the cited work of Arons) is there reported field data.

Another form of distortion can be introduced by a slight generalization of the model to allow motion relative to the bottom. Frequency distortion will thus be produced by the familiar Doppler components. In Figure 5, suppose the source of sound and point of observation move with velocity \( v \) parallel to an "average" bottom plane, and that \( \phi_i, \phi_r \) are the grazing angles relative to this plane. If the true bottom is inclined at angle \( \Delta \), the frequency of the reflected wave will be

\[
\omega_r = \omega + \frac{\omega v}{c} \left[ \cos \phi_i - \cos \phi_r \right]
\]

and

\[
\phi_i = \phi - \Delta \\
\phi_r = \phi + \Delta
\]

where \( \phi \) is the grazing angle with respect to the true bottom. This frequency distortion could have implications for wide band signal processing, but is perhaps more significant to scattered sound\textsuperscript{24}, which will be considered subsequently.
FIGURE 4. GAUSSIAN PULSE AND HILBERT TRANSFORM

\[ \exp(-t^2) \]

HILBERT TRANSFORM
(90° phase shift)
Another aspect of distortion may be identified under the term "delay distortion". Analytically, the reflected wave is conveniently represented in time domain rather than the frequency domain, and is regarded as a convolution of waves arriving with different epochs. Thus, for an incident plane wave of arbitrary form \( f_i(t) \), the reflected wave will be

\[ f_r(t) = \int_{-\infty}^{\infty} A(\tau) f_i(t - \tau) \, d\tau \]

The "impulse response" \( A(\tau) \) will be characteristic of the material and structure of the bottom, and of the angle of incidence. Of course, the impulse response is determined in principle by the Fourier transform of the reflection coefficient. As a practical matter, however, an approximate calculation of the one may be practical where the other is not. This is especially true when the reflected wave consists of a sequence of more or less overlapping arrivals, as is normally the case in seismic experiments. In this instance, the problem is one of sorting out individual reflections from reasonably well-defined layers, and the impulse response is the sum of a few delta functions. A more interesting situation for theoretical purposes is, however, that in which the properties of the impulse response are known only in some statistical sense, which will be considered below as a scattering phenomenon.

**Sound Scattering**

Another composite of field measurements is shown in Figure 6, which displays "scattering strength" as a function of acoustic frequency and grazing angle. This quantity is conceptually related to the idealized experiment shown in Figure 7. Here, in contradistinction to the reflection diagram, a wave exists proceeding at an angle other than the grazing angle of the incident wave, and is called a scattered wave. The amplitude of the scattered wave is thus a function of the two angles \( \phi \) and \( \theta \). When this amplitude is measured directly
Figure 6. Composite bottom scattering strength

- Mackenzie, 530 cps to 60 kcs, Ref. 27
- Urick; 500 cps to 1000 cps, Ref. 40
- "; 4 kcs to 8 kcs, Ref. 40
- Patterson; 2.5 kcs, Ref. 31
- Marsh; 268 cps to 1.3 kcs, Ref. 41
FIGURE 7. SCATTERING FROM IRREGULAR BOTTOM
it is found to depend in a systematic way upon such factors as the duration of the incident wave and the directional properties of the receiving instrument. For this reason it is customary to measure the so-called scattering coefficient, which is illustrated in Figure 8. Here is shown a sinusoidal pulse of duration $T$, intensity $I_1$ at the "scattering region" $dM$. The received intensity is $I_s$ at time $t = t_1 + t_2$, with $t_1$ being the travel time from source to $dM$ and $t_2$ the travel time from $dM$ to receiver. The scattered intensity is evidently a function of the scattering coefficient $A$ and the particular geometry which related $dM$ to the travel time $t_1$, $t_2$. This geometrical relation may, in more complicated cases, depend upon the directional properties of source and receiver. In the field, the scattering intensity $I_s$ is measured, but the scattering coefficient $A$ is desired, and some model is necessary for data reduction. The scattering may be thought to originate from inhomogeneities within the bottom, in which case $dM$ involves a volume element and $A$ is the "scattering coefficient per unit volume". Nolte²⁵ considered this point of view in studying backscatter from sand layers in a laboratory tank, but no theory has been developed for application in the field, and we shall not consider volume scattering further.²⁶

As a second possibility, the scattering region may be regarded as lying upon the surface of the bottom. In this case, providing $T$ is small compared with $t_1$ and $t_2$, we have

$$J(\theta, \phi, \psi) = \frac{c^2 t_1 t_2 I_s}{dA I_0}$$

In this equation, $I_0$ is the intensity of the source at unit distance and $dA$ the area of the surface scattering region. The angle $\psi$ measures the inclination of the plane containing source, receiver and $dA$ from the vertical to the surface.

The quantity thus represented is the surface scattering coefficient for unit area, although it is a numeric and independent of the units employed.
\[ I_s(t) = \int \frac{I_\varepsilon(t_1) A(t_1, t_2)}{t_2} dM \]

\[ t < t_1 + t_2 < t + T \]

**FIGURE 8. GEOMETRY FOR DEFINITION OF SCATTERING COEFFICIENT**
In the special case $\theta = \pi - \phi$, $\psi = 0$ we have the back scattering coefficient, and the back scattering strength is

$$N_b = 10 \log J$$

We have already seen a summary of reported values of $N_b$ in Figure 6. There are two general features to the reported data, the roughly s shaped dependence upon grazing angle, and substantial independence upon frequency. Our model must now account for these features. There are two types of models in use, which we may designate as phenomenological, and analytic. In the former category, Lambert's law has been employed, which states that

$$J = \delta \sin \theta \sin \phi$$

$\delta$ being a constant of the bottom material. Although this law provides an approximate empirical representation for many circumstances, there is no theory which can account for either the law or the dependence of $\delta$ upon other properties (However, see Kuo). For backscatter, Lambert's law gives

$$J = \delta \sin^2 \phi$$

Generalizations of this result have been made in the form

$$J = \delta \sin^n \phi$$

Some measurements are better represented by $n = 1$ than $n = 2$, but again, there is no theoretical account for such behavior.

A second phenomenological model has been proposed by Patterson, in which scattering is regarded as reflection from facets in the bottom interface, which are suitably inclined. He postulates that the backscattered intensity is:

$$I_b = -\infty \sum_{m=\infty}^{\infty} \sum_{L=0}^{D(m) f(L) B(\theta, L) d(L)}$$

18
In this expression, $D(m)$ is the distribution of facet slopes $m$ (assumed to be Gaussian), $f(L)$ is the "strength" of the facets (assumed to be proportional to their length $L$), $B(0, L)$ is the beam pattern of the facets as a function of size and grazing angle (assumed to be that of a line reflector) and $d(L)$ is the distribution of length which is assumed to be Rayleigh. As with an earlier model, he also multiplies the expression above by the reflection coefficient to allow for loss into the bottom. By suitable choice of constants in this model, Patterson obtains a good fit to some reported data excepting at the lowest grazing angles. He comments further on the possibility of using Schooley's concept of "facet tolerance" to allow for some curvature in the facets. Patterson's results are of value at least for empirical purposes, but similar results can be obtained by analytic methods, as will be seen. Further, Eaglesfield has objected to the use of the reflection coefficient in this manner.

All analytic methods are approximate either in principle or in application, but start with an attempt to solve the boundary value problem for an irregular surface. A good survey of methods as of 1958 has been presented by Lysanov. We shall confine our attention to two more recent developments, which are particularly applicable to the sea bottom. These consider the interface to be described by a function $z(x, y)$ representing the elevation $z$ at a point $(x, y)$ from the mean bottom plane. The particular properties of interest are such statistics of $z$ as the joint probability distribution

$$p\left[z(x_1, y_1); z(x_2, y_2)\right],$$

which is assumed to depend only upon the separation between the two points $(x_1, y_1)$ and $(x_2, y_2)$. In that case, there is the autocovariance

$$\rho(x, y) = \langle z(x_1, y_1) z(x_1 + x, y_1 + y) \rangle$$
and the (power) spectrum

\[ A(k_x, k_y) = \iint e^{i(k_x x + k_y y)} \rho(x, y) \, dx \, dy \]

For isotropic surfaces, if \( r^2 = x^2 + y^2 \), the auto covariance is a function only of \( r \), and there is the wave number spectrum

\[ A^2(K) = \int_0^\infty J_0(Kr) \rho(r) \, dr \]

In considering radar backscatter for a totally reflecting earth, Hayre obtained the following expression for the scattering coefficient:

\[ J = \frac{u}{\pi \sqrt{2}} \left[ \frac{\pi - 2\phi}{\cos \phi} \right] e^{-\nu \sum_{n=1}^{\infty} \frac{\nu^n}{(n-1)! (2u+n^2)^{3/2}}} \]

\[ u = (k \, B \sin \phi)^2 ; \quad \nu = (2 \, k \sigma \cos \phi)^2 \]

\[ \rho(r) = a^2 e^{-r/B} \]

He employed a slight extension of the basic work of Eckhart, using a modified Kirchhoff-Huygens method, and assumed the surface roughness to be normally distributed with variance \( \sigma^2 \) and the indicated exponential autocovariance. He found good agreement with various reported measurements, and also obtained estimates of \( \sigma \) and \( B \) from army contour maps of various continental US areas. These results could be applied to the sea bottom if the bottom were to be regarded as totally reflecting.

If the first order perturbation results which Marsh developed for the sea surface are applied to a totally reflective bottom, there is obtained the expression

\[ J = \frac{\sin^4 \phi \, k^3 \, A^2(K)}{\pi \cos \phi} \]

\[ K = 2k \cos \phi \]
We note that if $A^2(K)$ is proportional to $K^{-3}$, then the backscatter will be independent of the acoustic wave number $k$. This expression has been employed to estimate the spectrum of several naturally occurring surfaces, where measurements of backscatter were available. Results are shown in Figure 9, along with values computed from the expressions of Hayre. The curves for the "Throcher Area" were developed from a fine grained survey map made available by WHOI, using Hayre's technique, and the curve for Lake Travis was taken from Horton, who employed a similar mapping method. The solid parts of the curves cover ranges of wave number where either acoustic measurements are available, or where the contour method is believed applicable; dashes are extrapolation. On the whole, an asymptotic form of $K^{-3}$ is fairly typical (or $K^{-2}$, in the case of an exponential autocovariance.

Marsh's results are defective at grazing incidence, where they vanish too rapidly. To remedy this, and also allow for non-totally reflecting material, Kuo has extended his results by solving the boundary value problem for two fluids, and obtained to first order

$$J^1 = |\mathcal{R}(\gamma)|^2 J$$

$$\mathcal{R}(\gamma) = \frac{p_2 c_2 \gamma - p_1 c_1 \eta}{p_2 c_2 \gamma + p_1 c_1 \eta} + 2 \frac{(1 - \gamma^2) p_2 c_2^2 (p_2 - p_1)}{(p_2 c_2 \gamma + p_1 c_1 \eta)^2}$$

$$\eta = \sqrt{1 - \frac{c_2^2 (1 - \gamma^2)}{c_1^2}}, \quad \gamma = \sin \phi$$

$J$ is the coefficient for total reflection. The quantity $\mathcal{R}$ may be called the modified reflectivity. It is seen to bear a family resemblance to the reflection coefficient, to which it is equal at normal incidence. However, the two differ significantly near grazing incidence, and Kuo's results provide a greatly
FIGURE 9.  GEOPHYSICAL ELEVATION SPECTRA

RMS ROUGHNESS IN METERS

<table>
<thead>
<tr>
<th>Location</th>
<th>RMS Roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Surface</td>
<td>0.8, 4.3</td>
</tr>
<tr>
<td>Arctic Ice</td>
<td>3.2</td>
</tr>
<tr>
<td>Lawrence, Kansas</td>
<td>1.0</td>
</tr>
<tr>
<td>Toca Nas, Arizona</td>
<td>10.0</td>
</tr>
<tr>
<td>Mountain Park, N.M.</td>
<td>300</td>
</tr>
<tr>
<td>Norwegian Sea Bottom</td>
<td></td>
</tr>
<tr>
<td>Abyssal Plain</td>
<td></td>
</tr>
<tr>
<td>Treshar Area</td>
<td>12, 20</td>
</tr>
<tr>
<td>Lake Travis</td>
<td>10.6</td>
</tr>
</tbody>
</table>

\[10^3 \log A^2 (K), \text{ meter}^3\]
improved fit to a number of experimental results, including bottom reverberation, as well as backscatter from ice and from sand surfaces.

The Marsh-Kuo theory may also be applied to scattering in any direction, and could be tested, for example, against the "side scatter" reported by Urick. In the forward direction, it is shown that there is a specular component of magnitude given by the reflection coefficient for the smooth interface. At directions close to the specular, there will be scattered energy of amounts depending upon the elevation spectrum and material properties. Accordingly, there is a scattered contribution to the apparent reflectivity or bottom loss measured with practical beamwidths and pulse lengths. The calculation of this contribution is a tedious exercise in numerical integration, and has been carried out only for totally reflecting surfaces. Related calculations apply to the delay distortion in wave form, because of the dispersion of arrival times of the scattered waves.

If the same equations are applied to the joint statistics of the scattered field at points separated in time and/or space, it turns out that, at least to the first perturbation order, the scattered field is totally incoherent. It is thus suggested that the total return from the bottom can be separated into reflected and scattered components by measurement of the coherence or related properties of the return relative to the incident wave. Figure 10 gives some results obtained with very wide band explosives. The direct arrival and bottom return were low pass filtered and the normalized cross correlation of the filtered signals computed. The maximum value of this correlation is called the "average coherence", and is shown as a function of the low pass cut off frequency. For this particular bottom, the scattered return represents about one-half of the total, varying between 20 and 80%.
FIGURE 10. AVERAGE COHERENCE OF BOTTOM ARRIVALS
Summary

In conclusion, we may summarize the outstanding deficiencies in presently available theory. There is no adequate theory for the attenuation which is so important in real materials. Scattering effects are large, but the theories are either oversimplified or too cumbersome. Although we have not covered several important aspects of the subject, including shallow water sound propagation and various classes of seismic waves, these two deficiencies are probably at the top of any list which might be compiled.
1. Much of the data, and related documentation is not available in the open literature. Some general information may be found in the following:

   Physics of Sound In The Sea, Part I: Transmission; Office of Technical Services, U. S. Dept. of Commerce


10. Hampton, L. D. , Acoustic Properties of Sediments, University of Texas, Austin, Texas (1964)


References
(cont'd)


15. Berman, A., Scattering and Reflection Theory, Hudson Laboratories, Dobbs Ferry, New York (unpublished manuscript)


18. These include The Defense Research Laboratory, Austin; The Naval Electronics Laboratory, San Diego and The Underwater Sound Laboratory, New London, Conn.

19. That shear is important is emphasized, for example by F. R. Spitznogle and E. G. McElroy, J. Acoust. Soc. Am. 35, 1808 (1963)


24. This has been discussed for backscatter by B. G. Hurdle and R. H. Ferris, J. Acoust. Soc. Am. 34, 742 (A) (1962)


References
(cont'd)


33. Eaglesfield, C. C., J. Acoust. Soc. Am. 36, 1217 (June 1964)


37. Sound Reflection and Scattering by the Ocean Boundaries, AVCO Corporation, New London, Conn. (December 1963)


REFLECTION AND SCATTERING OF SOUND
BY THE SEA BOTTOM

PART II. FIELD DATA

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ABSTRACT

A review is made of bottom reflection experiments in both shallow-and deep-water ocean areas, illustrating frequency- and angle-dependent reflection effects. Interpretations of these measurements are made in the light of knowledge of the geological character of the regions involved. It is concluded that existing field data on bottom reflection in the specular direction is consistent with a model which considers the bottom to be flat and stratified into absorbing layers.

This paper is Part II of a trilogy to be presented at Session K of the 68th Meeting of the Acoustical Society of America at Austin, Texas, 21-24 October 1964. (Part I: H. W. Marsh, "Theory," Part III: C. W. Horton, "Model Studies."

INTRODUCTION

I intend to limit the scope of my discussion of field data to the reflection of sound in the specular direction, rather than in back or side directions.
GEOMETRY OF REFLECTION IN THE SPECULAR DIRECTION

Slide 1

Idealized Geometry for Reflection of Sound from the Sea Bottom In the Specular Direction

The first slide illustrates the idealized geometry of the situation we shall consider. In practice, positioning the receiver and source in this manner does not mean that all the energy arriving via the bottom reflection path actually arrives at the receiver by a specular, or mirror-like, reflection mechanism. A very irregular sea bottom would tend to produce many paths from the source to the receiver. However, the interpretation of the reception of forward reflected sound as largely a specular phenomenon is common. Thus the spreading along the path is considered to be inverse square except as modified by refraction. After sea water attenuation is accounted for, the remainder of the loss is assumed to arise from the penetration of energy into the bottom.

The definition of the "bottom reflection coefficient" as the ratio of the reflected to incident sound pressure at the sea bottom interface is unambiguous for the idealized situation. Minus twenty log to the base ten of this ratio is called the "bottom loss" in db. In practice, however, a pulse of sound from the source will often produce a cluster of arrivals at the receiver at a time close to that corresponding to the specular path. For most bottoms, multiple arrivals are caused by reflection from the sea surface and from sub-bottom strata. Therefore the answer one gets for reflection coefficient of the bottom depends to some degree upon the exact time of measurement, the averaging period, the pulse
length and the directivity of the receiver. It is conventional to correct
for the extra path, or paths, due to the sea surface reflection but the
sub-bottoms are properly considered a part of the bottom impedance.

DEPENDENCE OF BOTTOM LOSS ON
FREQUENCY AND GRAZING ANGLE

Prior to 1950 no published data can be found on the dependence of
bottom loss on frequency and grazing angle. During the early 1950's
the Underwater Sound Laboratory collected a sizable amount of data on
bottom loss in the deep ocean and summarized these data, and some
from other sources, in the frequency- and angle-dependent curves
presented by Dr. Marsh in Part I. A few years ago, these curves were
revised at the Underwater Sound Laboratory to take into account the
surface reflection path contribution to the measurements of the reflected
sound. This path tends to provide a 6 db boost to long pulse signals
taken with nondirectional sources and receivers and produces an
apparent bottom loss that is lower than the true value by some 6 db.

The marked frequency dependence of the bottom loss data presented
in Part I was initially a mystery and not accounted for by either
Rayleigh or modified Rayleigh two-fluid models of the sea bottom
interface, even though such models have upon occasion given an adequate
explanation of the angular dependence of reflectivity at a single
frequency.¹ ² Schulkin suggested that a bottom consisting of an absorb-
ing fluid overlying a rigid sub-bottom would be a physically credible
model for explaining the observed frequency dependence.³

Cole later refined Schulkin's model to a two-fluid absorbing bottom
and noted that this model not only accounted for frequency dependence
but also explained interference effects which were evident in the field data taken in specific locations, as shown in the next slide.

INTERFERENCE EFFECTS WITH CHANGE IN ANGLE

<table>
<thead>
<tr>
<th>substance</th>
<th>ρ (g/cm³)</th>
<th>c (ft/sec)</th>
<th>α (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WATER</td>
<td>1.03</td>
<td>49.0</td>
<td>0</td>
</tr>
<tr>
<td>Silty Sand</td>
<td>1.60</td>
<td>5341</td>
<td>0.65</td>
</tr>
<tr>
<td>SAND</td>
<td>1.90</td>
<td>5616</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Layer thickness = 4.2 ft.

This is an example of an interference effect produced by the phase of a sub-layer reflection acting successively in concert and in opposition with the phase of the reflection from the water-bottom interface as the inclination angle is changed. The data points are statistically significant, with a standard error of each point being less than 1 db. The theoretical computation was based on bottom properties arrived at from the data as follows. The critical angle effect at 19 degrees permitted solving for the velocity of the upper layer. The spacing between the peaks tells us the upper layer must have been 4.2 feet thick. The properties of the sub-layer were assumed equal to those of a typical sand layer. The number of remaining independent variables is not as great as it might seem since the high correlation between density, velocity, and attenuation in sediments is well established. The attenuation at 4.5 kc was extrapolated downward in frequency according
to the accepted first power relation from the high frequency data of Shumway’s. The agreement between the theoretical computation and the data points is so close that a rather convincing case seems to be presented not only for the nature of the reflecting mechanism but for the acoustic constants of the sediments themselves.

The closest existing core samples were three cores taken by Lamont some 150 miles distant from this deep water location. They showed sand layers varying in depth from 1 to 10 feet. While multiple sand layers are often found, a two-layered bottom model is not as naive as it might at first appear to be, since both the high reflectivity and the high attenuation of the uppermost sand layer tend to minimize contributions from deeper layers. However, the model can be extended to any arbitrary number of solid or fluid layers by methods presented by Brekhovskikh.

Similar deductions can be made from field data previously presented by Mackenzie. The experimental points are not densely enough distributed in angle to attempt a complete inference of a cyclical behavior,
but the curve computed from the deduced core properties at least seems to be consistent with the actual behavior of the experimental data. Note the agreement between the amplitude excursion of the computed curve and the experimental points, as well as the tendency for both to smooth out at the smaller angles. This time the sand sub-layer was inferred to be at a depth of 80 feet.

Dr. Marsh in Part I has already discussed Barnard's tank experiments in which similar interference effects were produced by a two-layer sediment structure and correlated with computations based on the acoustic properties of the sediments.

### THE PREDICTION OF REFLECTIVITY FROM CORE MEASUREMENTS

![Comparison of Computations from In Situ Core Data with Reflectivity Observations](slide4)

This slide represents our first attempt to compare computations from measured in situ core properties with acoustic reflectivity measurements. A single core, taken at the 2750-fathom location of
the measurement on the Sohm Abyssal Plain, showed a sand layer at 2.3 feet. The Hamilton and Shumway semi-empirical relations between sediment properties and velocity or attenuation were used to provide the estimated acoustic properties. The densities were measured directly. Two sets of data were obtained, of which the means are shown here, one at 20° and one at 30°. The standard error of these points is less than a db. While this can hardly be put forth as a complete verification of the model, it is at least a start and offers some encouragement that the calculations from such a model are useful.

Some further unpublished comparisons of this type in other areas suggest that such computations will tend to indicate too high a loss where deep reflecting strata exist underneath comparatively transparent surface sediments. That is, a core may show no reflecting strata simply because it does not go deep enough. The pattern that seems to be developing concerning the usefulness of reflectivity predictions from cores is that the presence of sand layers within a core will be a good indicator of a comparatively strong reflecting bottom. The absence of reflecting layers in a core, however, does not necessarily indicate that the bottom reflectivity will be poor. It may simply mean that the core is not deep enough to show highly reflective substrata.

LATERAL VARIABILITY

One objection to predicting bottom reflectivity from core measurements relates to the popular conception of the ocean bottom as a rather chaotic medium, varying rapidly as geographic position is changed and defying anything but a statistical description. Geological observations
do not bear this out. While confused bottoms may be found in some locations, sub-bottom profiler tracks show other areas to be stratified over many miles. Ewing has shown layer by layer core analysis to reveal detailed similarity in cores separated by as much as 80 miles.

In a preliminary attempt to explore lateral variability in reflectivity, a 2200-fathom area was selected. Curve 1 in this slide shows the average measured reflectivity at each of three angles, 39°, 27.5°, and 16°. In curve 2 the reflection point for each angle has been laterally displaced 1 mile by shifting the source-receiver geometry. The mean change in reflectivity over the 1 mile distance is slightly greater than a db, which is comparable to the standard error of each observation. Curve 3 shows an attempt to repeat the first curve upon returning to the general area 4 days later. It is unfortunately not possible to say how close we were to the original area 4 days later because of limitations in navigational accuracy which could have given us a positioning error of several miles. In any event, except for the 39-degree point, there is no significant difference in the results in curve 3 from the other 2 curves. This suggests we were operating in an acoustically homogeneous patch having an extent of at least several miles.
This slide shows the result of a second experiment in an area 12 miles removed from the first, which we call here "Area II." The results from the first area are shown by the shaded region. Those from the second area are separated into initial measurements and those obtained after a revisit to the station 4 days later. Again navigational errors imply that it is likely there was a separation of several miles in the reflection points over the 4-day time span. As in Area I, we see in the Area II data that good repeatability is obtained over lateral distances of a few miles. However, the difference in the Area I and II curves shows that there is a significant change in reflectivity properties over a separation of 12 miles.

Cores have been taken in Areas I and II but have not yet been analyzed. Mackenzie plans to look at these two areas in detail with the bathyscaph TRIESTE during the next year. It will obviously be of great interest to be able to obtain a physical description of these bottoms. In any event we see that the reflectivity properties of the bottom are not chaotic but show evidence of an orderly behavior with change in point of reflection.
EFFECT OF SCATTERING

Another popular conception about bottom reflectivity in the specular direction is that the ocean bottom must be rough enough in detail to make the reflection process more of a scattering process than a specular reflection. However, as Urick has pointed out, the received levels observed in the specular direction so greatly exceed those in side and back directions, that the reflectivity must at least show a great concentration of energy in the forward direction.\(^1\)

![Diagram of test setup](image)

Slide 7
Test Setup for Determining Importance of Non-Specular Reflection in the Forward Direction

With this setup we decided to investigate in a particular location whether or not energy received in directions close to specular is comparable in magnitude with that received in the specular direction. A four wavelength high 3.5 kc transmitting array with a vertically steerable beam was used as a source. The receiver was 19,000 yards away on a specular angle of 30 degrees. If the specular path were far stronger than any non-specular path, the beam pattern should be traced out as the beam is vertically steered. Strong non-specular arrivals would tend to widen or perhaps completely destroy the beam pattern.
Here we have plotted in our relative levels received from the hydrophone at 19,000 yards as we change the depression angle of our transmission beam. Shown as the dashed curve is the short-range, direct path beam pattern determined with a hydrophone at a 30-degree depression angle as the depression angle of the transmission beam is changed. The evident close similarity between the direct-path and bottom-reflection patterns indicates that any non-specular transmission path via the bottom was far weaker than the specular path in this location.

SUMMARY AND CONCLUSIONS

To summarize, our observations presented in this paper, of bottom reflection in the specular direction, have indicated that in both deep and shallow water, existing field data are consistent with a model which considers the bottom to be flat and stratified into absorbing layers. This model is capable of explaining the dependence of bottom loss on frequency, location, and incident angle. Geological and
acoustical reflectivity properties can be related by considering the bottom impedance as derived from the established correlation between the physical properties of the sediments and their fundamental velocity and absorption characteristics. Behavior of the field data discussed in this paper has shown no evidence that scattering processes play a significant role in either adding to or subtracting from the sound reflected in the specular direction.

Our conclusions are, however, limited to locations which represent only a few specks on world-wide charts of the ocean bottom. More field data must obviously be collected over a greater sample of bottom locations before we can lay claim to a broad understanding of bottom reflectivity. Future acoustical investigations should be backed up with all possible means of observing in situ bottom characteristics in order to facilitate the testing and improvement of reflectivity models.
LIST OF REFERENCES


REFLECTION AND SCATTERING OF SOUND BY THE SEA BOTTOM

Part III. Model Studies

C. W. Horton

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Abstract

This paper is Part III of a trilogy, the others being Part I - Theory and Part II - Field Data. The advantages offered by measurements of reflection and scattering of sound from models of the sea bottom are discussed. Model studies provide a useful supplement both to theoretical analyses and to field studies. Selected experiments performed in the laboratory are reported which are valuable in interpreting effects observed in the field. The models illustrate the effects of fluid and rigid layered media and of rough surfaces.
I believe that the two preceding papers have convinced everyone in the audience that the study of the reflection and scattering of sound from the sea bottom presents many significant problems both of a theoretical and of a practical nature. In fact, the problems are so extensive that one should use every possible method of attacking them.

Scale models have been used frequently as a laboratory tool for the study of complex problems in the earth sciences, and a wide range of problems have been studied in this manner. The following list of scale model experiments is illustrative only and is by no means complete: ocean currents such as the Gulf Stream (von Arx\textsuperscript{1}), propagation of seismic waves, generation of seismic waves by simulated earthquakes, electromagnetic waves in geophysical prospecting (Yost\textsuperscript{2}), reflection of acoustic waves from artificially produced water waves (Wood\textsuperscript{3}), and propagation of acoustic waves in turbulent media (Stone\textsuperscript{4}).

The advantages offered by scale models are many. First, they provide a significant guide to the validity of the mathematical approximations introduced in the theoretical work. In all but the most simple of problems, even though the physical situation is idealized, one still obtains often a solution which is a power series in some parameter associated with the problem. When the expansion is formidable, one obtains only the first few terms, and frequently no assessment can be made of the remainder term. Sometimes, one cannot even be certain if the series is convergent or, if not, if it is an asymptotic solution. A carefully designed program of model studies can offer a significant guide to the range of validity of the theoretical analysis and show with respect to which parameter the expansion should be made.
Sometimes one introduces into the theoretical problem approximations which are physical rather than mathematical in nature. An example of this type of simplification arises when one analyzes the reflection of sound waves from a rough sea bottom. The problems are greatly simplified if one assumes either that the bottom is a pressure release surface or that it is perfectly rigid. These assumptions are commonly made in the study of this problem (Eckart, Marsh, Uretsky). In the first paper of the series Dr. Marsh has presented arguments to the effect that one can modify the solutions for these ideal bottoms by multiplication by a suitable factor so that they will apply to other types of bottoms. The validity of this modification can be tested extensively with scale models.

Another assumption of a physical nature that is frequently made in these problems is the neglect of the shear modulus of the materials in the ocean bottom. The argument usually offered is either that the layers below the ocean bottom are unconsolidated sediments which have negligible shear modulus, or that since the incident and reflected waves are longitudinal, one can obtain an adequate description of reflection and scattering coefficients by neglecting the shear modulus of the bottom. Evidence from scale models will be offered below to show that the shear modulus cannot be neglected in these problems.

Studies with scale models offer equally significant guides to the design of field programs for the measurement of data. There are so many physical parameters which might affect the reflection of sound from the sea bottom that it is easy to fail to record some of the significant parameters. Thus in the elaborate studies on transmission that were made near San Diego during World War II the only information known and recorded about the bottom
was the simple descriptions, mud, silt, sand,... Now it is known that
the layering of the upper ten to twenty feet has a significant effect on
the reflection coefficient and hence on the transmission ranges obtained
in some of these experiments.

A second example of an important physical parameter is the bottom
topography. It may be very difficult to determine the contours of the
sea bottom with sufficient accuracy to predict fully the scattering phen-
omena. Results on scale models will be presented below which indicate
that bottom features of quite moderate relief have significant effect on
the behavior of scattered sound. This suggests that topographic maps of
the sea bottom must be made with much greater detail than hitherto. In
fact, it may not be possible to measure bottom relief with the accuracy
necessary to predict all of the details of the scattering phenomena. In
this case one must devote more effort to acquiring an understanding of the
factors that govern the bottom relief, factors such as grain size distrib-
tion, silt content, ocean currents and stratification.

One significant feature of scale model studies is their economy.
This is most strikingly illustrated when one compares the cost of labora-
tory studies with measurements made at sea. Laboratory studies are not
as economical as theoretical studies, although the difference in these
costs has become much less now that large digital computers are used ex-
tensively. It is easy to be misled in thinking about the cost and diffi-
culty of scale model studies. Good scale model studies cannot be carried
out without carefully constructed, high quality equipment. The physical
and acoustic properties of water make it an excellent medium for the scale
models. Unfortunately, however, this means that when one increases the
frequency in order to compress the physical size of the model, the wave-
length and the period are both decreased. Consequently, the accuracy with
which one must position the transducers and the scattering bodies and the precision of the electronic timing circuits increases. One is required to use heavy, rigid support stands and the positioning mechanisms must be carefully designed and constructed. However, once a good model has been constructed, it is possible to carry out a large suite of measurements rapidly and thereby to make extensive studies of the various parameters such as grazing angle.

It is not necessary to dwell upon the difficulty of good field measurements since most of the members of the audience have had some experience in this area.

SCALE MODEL STUDIES OF REFLECTION FROM SAND SURFACES

The scientists at DRL have used model experiments of various kinds in their work in underwater acoustics. One of the earliest of these programs was carried out during 1952-54 by Professor A. W. Nolle and his students under a contract with the Bureau of Ships. They studied the acoustic properties of water-filled sands and also studied back scattering from the surfaces of these sands. Figure 1 shows the experimental arrangement used in these back scattering measurements.

These workers found that even though they sieved the sand, removed the air by boiling, packed the sand by mechanical agitation, and smoothed the surface carefully, the amplitude of the back scattered signal fluctuated by ±3 dB when the reflection point was moved over the surface of the water-sand interface. This fluctuation was attributed to small ripples in the sand, ripples so small that they could be seen only when an intense beam of light was shone on the surface at a low grazing angle.
FIGURE 1

THE MODEL AND RELATED EQUIPMENT USED BY PROFESSOR NOLLE AND HIS CO-WORKERS
A specific example of this phenomenon will be described. Back scattering measurements were carried out with acoustic pulses of length 30 μsec and center frequency 500 kcps. The grazing angle was 20° and the effective beam diameter was 5 cm. The transducer was moved horizontally so that the point of reflection changed uniformly with time. Thus an oscillogram was obtained which showed the amplitude of the back scattered sound versus distance along the scattering surface. Figure 2 contains a reproduction of an oscillogram obtained in this manner. The record corresponds to a distance of 20 cm on the bottom. Figure 3 is an autocorrelation function based on 50 samples with a Δx = 0.4 cm. The sand used in these experiments was from the sea bottom near Panama City. The sand was sieved and all particle sizes below 20 mesh were retained.

These results illustrate a point mentioned earlier that surprisingly small departures from flatness of the reflecting surface will produce significant changes in the scattering from the surface. The substantial negative value for the autocorrelation function at Δx = 2.5 cm cannot be attributed to the granular nature of the sand but it is caused most probably by small ripples on the surface.

After these experiments were terminated the scale model studies at DRL were directed toward measurements of echo structure for discrete objects. Although many interesting results were obtained, such as the first demonstration of the creeping waves predicted by Franz, these results are not germane to the present discussion and must be passed over.
FIGURE 2

THE AMPLITUDE OF THE BACKSCATTERED WAVE AS A FUNCTION OF THE POSITION ON THE BOTTOM. THE LENGTH OF THE SAMPLE CORRESPONDS TO 20 CMS ON THE BOTTOM.
FIGURE 3
AUTOCORRELATION FUNCTION OF THE BACKSCATTERED SOUND INTENSITY AS A FUNCTION OF THE DISPLACEMENT OF THE REFLECTION POINT
REFLECTION FROM FLAT AND SINUSOIDAL SURFACES

In 1960 a new program of model studies on reflection from the sea bottom was initiated at DRL under the sponsorship of the Bureau of Ships. This work has been carried out under the supervision of Mr. Garland Barnard, to whom I am indebted for the use of some of the following material.

Figure 4 shows the new model tank which is eight feet in diameter and has a water depth of 7.2 feet. The horizontal and vertical tracks shown in the slide enable one to position the transducers. A 5-in. diameter shaded, circular piston operating at 100 kc/sec was used to insonify layers of various fluid and solid media supported near the bottom of the tank by a 4 ft by 4 ft tray.

Dr. Marsh has shown one experimental curve obtained with this equipment and the corresponding theoretical curve for a fluid sediment. It is of interest to show the behavior of reflection from a layer with rigidity.

A model was constructed with the layers shown in Fig. 5. The experimental and theoretical curves for the reflection coefficient are shown in Fig. 6. The agreement is quite good except over the angular range $52^\circ$ to $62^\circ$. It turns out that the theoretical response in this range is highly sensitive to the value used for the attenuation of shear waves in the limestone. Since the critical grazing angle for the sediment-limestone interface is $52^\circ$, the theoretical response for smaller grazing angles is not sensitive to the value of this attenuation. This model study suggests that the rigidity of the layers in the sea bottom is of significance in bottom reflections.

A reflecting surface that has been analyzed quite extensively is the corrugated surface whose cross-section is a sinusoid. When such a surface
FIGURE 4

THE TANK AND EQUIPMENT USED IN THE NEW SERIES
OF SCALE MODEL MEASUREMENTS AT
FIGURE 5

ONE OF THE MODELS USED IN THE STUDY OF REFLECTION FROM A STRATIFIED BOTTOM
FIGURE 6

A COMPARISON BETWEEN EXPERIMENTAL MEASUREMENTS AND THEORETICAL CALCULATIONS FOR THE REFLECTION OF SOUND FROM THE MODEL OF FIGURE 5
is constructed of a pressure release material, it can be used as a model of the ocean surface or as an aid in the study of reflection from the sea bottom. Further interest in the problem results from the fact that there are a large number of theoretical studies of the problem although most of these analyses contain serious restrictions on the magnitude of the relief and/or the slope.

La Casce and Tamarkin have made some scale model measurements on a sinusoidally corrugated surface and have compared their data with some of the approximate theories. Uretsky has developed an exact solution that contains no explicit restrictions on the parameters of the reflecting surface. However, the computations required to evaluate his solution are rather extensive when the relief or the slope of the surface is large. Uretsky has made numerical comparison of the prediction of his theory with the results obtained by La Casce and Tamarkin.

When one examines these comparisons between experiment and theory for the sinusoidal surface, one concludes that although the measurements of La Casce and Tamarkin are important and interesting, the range of parameters in their experiments was not sufficiently extensive to provide a thorough test of the various theoretical analyses. Consequently, we felt that there was a need for more detailed measurements on a model having significant relief to provide a critical test of the exact and approximate theories in the literature.

A sinusoidal surface of peak-to-peak amplitude 3 cm and wavelength 4.5 cm was constructed out of styrofoam. Extensive measurements of scattering were made using frequencies of 100 to 400 kcps. This range of frequencies corresponds to $\lambda = 1.46$ cm to 0.37 cm which falls in the critical range in which neither the high-frequency nor the low-frequency approximations are
valid. Some results of this work were reported yesterday by Mr. Barnard. Concurrently with these experimental measurements, theoretical calculations are being carried out with the aid of the formulas developed by Uretsky. Mr. Spitznogle, who is currently on leave of absence from Mine Defense Laboratory to do graduate study here at The University of Texas, is working on this aspect of the problem.

Figure 7 shows a theoretical and experimental curve for the corrugated surface. It should be stressed that there has been no adjustment in the vertical positions of either curve.

The experimental measurements on corrugated surfaces will be of great help to workers on the theoretical problem. The theory is sufficiently difficult that one can easily overlook significant features. It is expected that the results of the experimental measurements will suggest new avenues for the theoretician to follow. In particular, probe measurements of pressure in the valleys of the model may help resolve questions about the completeness of the set of functions used in the expansion. That this is a problem of current interest can be seen from the letter to the editor by Murphy and Lord\textsuperscript{11} in the August issue of JASA.

**REFLECTION FROM ROUGH SURFACES**

The sea bottom does not in general have the regularity of a sinusoidal surface, so one must consider models with a more irregular surface. We could not locate any contour maps of the sea bottom which were sufficiently detailed for a model so we were forced to look elsewhere for a random surface. It was discovered that some of the aeromagnetic maps prepared by the Canadian Geological Survey, when interpreted as topographic maps, yielded rough surfaces suitable for a model sea bottom. Figure 8 shows one of these maps from
Figure 7
The scattering of sound by a pressure-release sinusoid
FIGURE 8
A SIMPLIFIED VERSION OF AN AEROMAGNETIC MAP FROM CANADA USED AS A MODEL FOR A RANDOM SURFACE.
a region in the Northwest Canadian Shield. The area represented in this
map is 16 miles x 16 miles and the paper map itself is 16 in. x 16 in.
Four of these maps in the form of a square were used as the pattern for
constructing a relief model.

The model was constructed of low-density styrofoam with the base and
sides of reinforced fiberglass. Enough steel was used in this reinforce-
ment so that the model would remain submerged. The horizontal dimensions
are 32 in. x 32 in., while the vertical relief was constructed in approxi-
mately 70 steps of height 1/32 in. This gave an overall relief of greater
than two inches. This is approximately 3.5\% at a frequency of 100 kcps.
Figure 9 is a photograph of the model. It is obvious that the hills and
valleys are elongated and have a trend in the direction N12E. This value
for the trend is based on a statistical analysis, and one sees from the
figure that the trends visible on the model are distributed about this
value.

Each of the four aeromagnetic maps which served as the pattern for
the relief model was sampled at a network of points forming a square grid
1/2 inch on a side. This yielded 1,069 sample values per map, which were
used to compute autocovariance functions. Figure 1 shows the autocovariance
function for one of the four maps used in the relief model. This plot shows
more strikingly the N12E trend seen in the photograph of the model. This
figure is a contour representation of the fundamental statistical function,
the autocovariance function, that is introduced in theoretical studies of
reflection from rough surfaces. With the aid of the autocovariance function
one can predict the scattered intensities that will be measured experimentally
in the model tank. There are numerous theoretical papers devoted to the
problem of scattering of sound by rough surfaces of which only those by

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FIGURE 9
THE MODEL BUILT TO STUDY THE REFLECTION OF SOUND FROM A ROUGH, PRESSURE RELEASE SURFACE
FIGURE 10
THE AUTOCOVARIANCE FUNCTION
FOR THE MAP SHOWN IN FIGURE 8
Eckart\textsuperscript{5} and Marsh\textsuperscript{6} will be mentioned. The relief of the present model is so large compared with the horizontal that one cannot test the low-frequency limits of the theoretical expressions. This defect will be remedied with another model having less vertical relief.

It might be mentioned that the latest issue of Geophysics has an article by Horton, Hempkins, and Hoffman\textsuperscript{12} which shows the autocovariance functions of each of the individual maps and relates their characteristics to the geology of the region.

At the present time one of my graduate students, Mr. Muir, is developing special cases of Eckart's theory so that comparison can be made between this theory and the experimental measurements.

SUMMARY

It is apparent that a wide variety of problems can be attacked successfully with the aid of scale models. The accuracy with which a scale model can be constructed enables one to control the critical parameters. This means, for example, that one can determine with certainty whether anomalous values of scattering amplitudes that are observed in field measurements are caused by interference effects or are due to random fluctuations of small probability. It seems desirable that any large scale program of acoustic measurements at sea should be preceded by a laboratory scale model study which is a replica of the procedures planned for the field. These laboratory tests would enable one to optimize the field techniques.

Since this paper was completed Barnard, et al.,\textsuperscript{13} have published an account of their work on scale models.
References


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