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Analytical Approximation

Offset Circle Probability Function: We consider the function

\[ q(r, x) = \int_R^\infty e^{-\frac{1}{4}(r^2+x^2)}I_0(rx)\rho d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than \( 0.00037 \) over \((0, \infty)\),

\[ q(0.5, 0.5+y) \approx 1 - \frac{.1045}{\left[1 + .129y + .079y^2 + .056y^3\right]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R \geq 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(x^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0007 over \((0, \infty)\),

\[ q(1, 1+y) \cong 1 - \frac{.267}{\left[1 + .203y + .079y^2 + .062y^3\right]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R > 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0011 over \((0, \infty)\),

\[ q(\lambda + \delta, \lambda + \delta + y) \approx 1 - \frac{.45}{[1 + .227y + .064y^2 + .065y^3]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R > 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R,x) = \int_{\infty}^{R} e^{-r^2} I_0(r x) \, dr \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0013 over \((0,\infty)\),

\[ \lim_{R \to \infty} q(R,R+y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \, dt \]

\[ = 1 - \frac{0.5}{\left[1 + 0.209y + 0.061y^2 + 0.062y^3\right]^4} \]

The parametric form used is convenient for approximating fixed-\(R\) semi-cross-sections of the \(q(R,R+y)\) surface for any \(R \geq 0\) and for \(y\) ranging over \((0,\infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^{2}+x^{2})} I_{0}(\rho x) \rho \, d\rho \]

in which \( I_{0}(z) \) is the usual Bessel function.

To better than .006 over \((0, \infty)\),

\[ \lim_{R \to 0} \frac{1 - q(R, R+y)}{1 - q(R, R)} = e^{-0.25y^{2}} \]

The above gives information concerning a degenerate limiting case in the approximating of fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for \( y \) ranging over \((0, \infty)\).

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