NUMERICAL ANALYSIS AND PREDICTION
OF CLOUD AND PRECIPITATION PATTERNS

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ABSTRACT

Present numerical weather products do not include a current analysis or a forecast of large scale occurrence of cloudiness and precipitation. The model used in this paper uses vertical velocities and dew-point depressions at the levels of 850, 700 and 500 mb as inputs to a diagram which determines the accompanying weather parameter. Good results were obtained for a current analysis, but the prognosis scheme used proved to be unsatisfactory at the present time. The model is programmed for the Control Data Corporation 1604 digital computer to utilize the operational output of machine-processed data and analysis produced by the U. S. Navy Fleet Numerical Weather Facility (FNWF), Monterey, California.

The writers wish to express their appreciation to Professor George J. Haltiner of the U. S. Naval Postgraduate School for his assistance, guidance and encouragement in this work. Appreciation is also expressed to the personnel of FNWF for their cooperation and especially Mr. Harry Farnsworth for his aid and advice in programming and performing the machine computations.

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TABLE OF SYMBOLS

D - height anomaly: \( D = Z - \overline{Z} \)

T - absolute temperature

\( T_p \) - dew-point depression

\( \overline{T_p} \) - average dew-point depression

P - large scale precipitation rate

\( P_s \) - precipitation rate due to showers

C - degrees centigrade

w - component of wind speed in the Z direction

\( \overline{w} \) - average component of wind speed in the Z direction

g - vertical component of the acceleration due to gravity

Z - height of an isobaraic surface

m - map-scaling factor: \( m(\theta) = \frac{1 + \sin 60^\circ}{1 + \sin \theta} \)

d - grid-net spacing distance

f - coriolis parameter: \( f = 2 \circ \sin \theta \)

\( \omega \) - total derivative of pressure with respect to time: \( \omega = \frac{dp}{dt} \)

\( \rho \) - atmospheric density

\( T \) - refers to a specific time

\( \gamma_p \) - lapse rate of dew-point depression

\( \phi \) - latitude

\( \mathbf{V} \) - horizontal projection of the wind velocity vector

\( \nabla(c) \) - horizontal gradient operator: \( \nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} \)

\( \mathcal{J}() \) - finite - difference operator corresponding to the Jacobian

(\(^\wedge\)) - scaled parameter, less than one

(\(\sim\)) - parameter in a standard atmosphere
1. Introduction

This study was undertaken to analyse and forecast areas of large scale occurrence of cloudiness and precipitation, including amounts of each, using numerical methods. "Large scale occurrence" refers to the weather associated with systems of the dimensions of the ordinary cyclones and anticyclones of the atmosphere. The parameters used to make the analyses and forecasts were basically vertical motions and moisture content of the atmosphere at the levels of 850, 700 and 500 mb. Present vertical motions and forecast vertical motions were obtained from U. S. Navy Fleet Numerical Weather Facility (FNWF), Monterey, California. The present moisture content, expressed as dew-point depression, was obtained from FNWF and moisture forecasts are an essential part of the program used and will be discussed later.

Once the vertical motion and moisture content at each level are determined, the results are averaged and the Ladag diagram, figure 1, is entered to determine the accompanying cloudiness or precipitation at each grid point. This diagram is similar to one developed by Lewis [1]. It shows cloudiness, showers and precipitation as a function of average vertical motions and dew-point depression from 850 to 500 mb.

The basic numerical program used evolved from one written by Lieutenant Commander E. M. Carlstead, U. S. N.. This investigation involved checking and modifying Carlstead's original program and then verifying the results using hand analysed cloud and precipitation charts.
2. Background

Considerable work has been done in the past to determine what parameters are best related to the occurrence of precipitation and cloud amounts. Smagorinsky and Collins [2] considered the problem of the mechanics of the formation of precipitation based on the laws of fluid mechanics and used a computer to solve these laws in the form of certain mathematical equations. Williams [3] showed a good relationship existed between positive vorticity advection and precipitation. Panofsky [4], using computed U. S. Weather Bureau Joint Numerical Weather Prediction (JNWP) vertical velocities and the dew-point depression at 850 mb, demonstrated that the probability of precipitation increased with increasing vertical velocities and decreasing dew-point depression. Lewis [1], making use of observed values of 700 mb dew-point depression and JNWP vertical velocities, constructed a nomogram and divided it into four categories: precipitation, cloudy, partly cloudy and clear. Another approach to the problem that showed favorable results was developed by Estoque [5]. He used a graphical technique for predicting precipitation amounts from a two-level model with an assumed vertical velocity profile. Smebye [6] constructed a model using the vorticity equation and the first law of thermodynamics to account for precipitation due to large-scale motions. His computed patterns of precipitation amounts agreed well with those observed. In his model for predicting precipitation amounts in Seattle and western Washington, Wilson [7] used vertical velocities, together with the water content and degree of saturation of the air. Wilson's results gave reasonable estimates of precipitation amounts. Sassman and Allen [8] observed a good differentiation between the occurrence and nonoccurrence of precipitation using vertical velocities and dew-point
depression, averaged over the surface, 850 mb and 700 mb levels. Carlstead [9] has made numerical forecasts of precipitation and cloudiness using the Lewis nomogram to determine the weather category. His method shows promise in predicting large-scale precipitation.
3. The Model

Previous studies by other authors have shown useful correlations between the occurrence of precipitation and cloudiness with vertical motions and moisture content of the air. The vertical velocities were obtained from Fleet Numerical Weather Facility (FNWF), Monterey, California, using a numerical solution of a diagnostic \( \omega \)-equation developed by Haltiner, Clarke, and Lawniczak [10]. The \( \omega \)-forecasts available were in six hour increments. The actual numerical program uses \( w \)-values which are obtained by the hydrostatic relationship

\[
\omega = -\frac{w}{\rho g} \tag{1}
\]

The moisture content of the atmosphere is computed at three levels, 850, 700 and 500 mb. A reasonable measure of moisture content is dew-point depression and this quantity will be defined as \( T_p \). The equation used to forecast \( T_p \) is

\[
T_p(t+1) = T_p(t) - [\nabla \cdot \nabla T_p + w \frac{\partial T_p}{\partial z} + \gamma_p w] \Delta t. \tag{2}
\]

On the right hand side of equation (2) the second term is geostrophic wind advection, the third term vertical advection, and the fourth term the pressure effect. The lapse rate of dew-point depression, \( \gamma_p \), is assumed to be dry adiabatic in nature, and is approximately equal to 8 degrees centigrade per kilometer, depending on temperature and pressure. In finite-difference form, equation (2) then becomes

\[
T_p(t+1) = T_p(t) + \left[ \mathcal{J}(T_p(t), D(t)) \right] \frac{\Delta T_p}{\Delta z} + \gamma_p \frac{\Delta T_p}{\Delta z} \Delta t. \tag{3}
\]

In order to enter the diagram with averages of \( T_p \) and \( w \) the following equations are used:

\[
\overline{T_p} = 0.333(a_1 T_{p850} + b_1 T_{p700} + c_1 T_{p500}) \tag{4}
\]

and

\[
\overline{w} = 0.333(a_2 w_{850} + b_2 w_{700} + c_2 w_{500}) \tag{5}
\]
To give greater weight to low level moisture effects the coefficients used in equation (4) were $a_1 = 1.4$, $b_1 = 1$ and $c_1 = 0.6$. The tentative coefficients used in equation (5) were $a_2 = b_2 = c_2 = 1$. The Ladam diagram, figure 1, is then entered with the two values computed from equations (4) and (5) and a value from 0 to 600 is obtained. The values from 0 to 600 were selected arbitrarily to assist in computer computations. The diagram is reduced to a series of tables and internal values are obtained by linear interpolation.

If the value from the diagram is a number greater than 550, rain is the forecast weather parameter. To determine the amount of precipitation per hour the following equation [11] is used:

$$ P = 3600 \left( a_3 w_{850} - 850 + b_3 w_{700} - 700 + c_3 w_{500} - 500 \right). \quad (6) $$

The constants $a$, $b$ and $c$ were obtained through the use of a numerical program called BIMED9 [12] using actual values of precipitation and the vertical velocity and temperature at each of the levels. This program utilizes the six variables and using a best fit procedure determines the values of $a$, $b$ and $c$ in order to solve for the variable $P$. The value of the constants acquired through the use of BIMED9 were $a_3 = 0.01032$, $b_3 = 0.00097$ and $c_3 = -0.00325$. Because the program does not forecast temperatures directly the following hydrostatic approximation is used:

$$ T = T_{std} \left( \frac{Z}{Z_{std}} \right). \quad (7) $$

From this approximation the temperature at each level becomes

$$ T_{850} = 278.5 \left( \frac{D_{700} - D_{1000} + 290,000}{290,000} \right), \quad (8) $$

$$ T_{700} = 268.4 \left( \frac{D_{500} - D_{850} + 411,540}{411,540} \right), \quad (9) $$

and
Next, if the value from the diagram is between 500 and 550, showers is the probable forecast weather parameter. To determine if showers will occur the layer thickness, moisture content and the underlying surface is investigated. Curtis and Panofsky [13] found a good predictor of convective activity by considering the moisture present in the 900 - 700 mb layer and the presence of a heated surface under a tropical air mass. Assuming these relationships apply elsewhere over the hemisphere, it is first determined if a tropical air mass is present by testing to see if the thickness between 1000 and 500 mb is 5,550 meters or greater. If the air mass is tropical in nature it is then determined if sufficient moisture is present in the atmosphere to produce showers. The criterion for this is $T_p$ must be less than 6 degrees centigrade. If the above criteria are satisfied, it is then determined if the point in question is over land or sea. If over sea the shower equation is used directly. If over land the shower equation will be used only if the local time is between 1200 and 2000. The shower equation [11],

$$P_s = .5 \left( 1 - \frac{T}{16} \right),$$

(11)

roughly estimates the rainfall amount to be 50% of the precipitable water in inches per hour.

Finally the cloud amount is determined from the diagram. If the value falls between 300 and 500 cloudy skies are predicted, if between 100 and 300, partly cloudy skies and for values less than 100, clear skies are forecast.

The printed northern hemispheric maps available from this program include the $T_p$ at 850, 700 and 500 mb at any forecast time desired. A
cloudiness and precipitation value is also printed for each grid point. This field is contoured with the following scheme:

(No symbol) . . . . . Less than .3 cloudiness.
• . . . . . . . . . . . . . . . . 0.3 to 0.8 cloudiness.
* . . . . . . . . . . . . . . . . 0.8 to 1.0 cloudiness.
0 . . . . . . . . . . . . . . . . Area of active tropical shower activity and/or area of intermittent large-scale precipitation.
X . . . . . . . . . . . . . . . . Area of large-scale precipitation.

A typical cloudiness and precipitation map is shown in figure 2. Contours have been drawn to separate the various weather categories. Figure 3 depicts the geographical area of the numerical grid used. It is noted that all large-scale precipitation areas are surrounded by "0" characters. Due to an effect of the contouring program the numerical value of showers must be chosen between cloudy and precipitation. This is rationalized to be acceptable because precipitation is often intermittent or showery at the edges of a large-scale precipitation area. The last two printed maps are in units of average precipitation per hour for the previous time step and the total precipitation from the beginning of the forecast period to the present time.

The computations carried out during this investigation were made with a Control Data Corporation 1604 (CDC 1604) digital computer, with a core storage capacity of 32,768 words of 48 bits each. The input data were obtained from the analysis of $T_p$ and $D$ and the forecast of $\omega$ at the levels 850, 700 and 500 mb surfaces by the U. S. Navy Fleet Numerical Weather Facility (FNWF), Monterey, California. A simplified flow diagram of computer operations is shown in figure 4.

In order to utilize available data and computer subroutines from FNWF, the equations were converted to finite-difference form and scaled for fixed-point fractional arithmetic.
A 1977-point, square-net, octagonal grid, circumscribed by latitude 10N on a polar stereographic projection of the Northern Hemisphere was employed to represent the data fields. The net spacing is 381 km at 60N where the projection is true.

The scaled equations corresponding to equations (3), (6) and (11) are equations (12), (13) and (14) respectively.

\[
\hat{T}_{p}(\tau+1) = \hat{T}_{p}(\tau) + \left[ \mathbf{F} \left( \hat{T}_{p}(\tau), \hat{D}_{p} \right) \frac{\hat{w}_{p}^{2}}{d_{f}^{2}} \Delta T \left( \frac{\Delta T}{\Delta Z} + \hat{\gamma}_{p} \right)^{2} \right] \Delta \tau \quad (12)
\]

\[
\hat{P} = 3600 \left( a \hat{w}^{850} \hat{T}^{850} + b \hat{w}^{700} \hat{T}^{700} + c \hat{w}^{500} \hat{T}^{500} \right) 2^{-5} \quad (13)
\]

\[
\hat{P}_{s} = 0.5 \left( 1 - \frac{\hat{T}}{16} \right) 2^{2} \quad (14)
\]

The scaling conventions used in these equations are:

- \(D = \hat{D} \cdot 2^{17}\) (cm)
- \(T_{p} = \hat{T}_{p} \cdot 2^{9}\) (degrees centigrade)
- \(w = \hat{w} \cdot 2^{7}\) (cm/sec)
- \(P = \hat{P} \cdot 2^{11}\) (inches/hr)
- \(P_{s} = \hat{P}_{s} \cdot 2^{11}\) (inches/hr)
- \(T = \hat{T} \cdot 2^{9}\) (degrees centigrade)
- \(m = \hat{m} \cdot 2\) (non-dimensional)
- \(\gamma_{p} = \hat{\gamma} \cdot 2^{9}\) (degrees centigrade - 100 m/sec)
4. Verification of Results and Suggestions for Future Study

Verifications were done by subjective comparison over the continental United States only. Such verification of cloudiness is often difficult due to obscuration by fog and darkness. Using the Lewis nomogram as a guide, the lines delineating the separation of the different weather parameters were shifted in the Ladag diagram until the best fit with actual conditions resulted.

In the verification of current analyses, two precipitation and cloud depiction maps and one precipitation amount map will be discussed. The shower areas are included within the cloudy category on the verifications. Figure 5 shows the 00Z 5 March 1964 map. By inspection it can be seen that the cloud and precipitation patterns agree quite well. If the shower area were included on the numerical map, the precipitation areas in northwestern United States would be in much better agreement. The actual precipitation map showed that in this area the rainfall amounts ranged from a trace to .02 inches during the previous six hours, therefore the maps in actuality are in good agreement. Figure 6, the 12Z 6 March 1964 verification, shows a complex cloud-precipitation pattern. It is felt that programming a complex weather pattern such as this for the computer would be difficult indeed, and that additional parameters besides vertical velocity and dew-point depression are perhaps necessary. Upon inspection of the precipitation area over western New York state, it was noted that the dew-point depression at all three levels, 850, 700 and 500 mb, was large, therefore indicating a partly cloudy sky condition. The low level air trajectory at this time was from the WNW over the Great Lakes and it is felt that sufficient moisture was picked up to account for the light snow showers in this area.
Figure 7 shows the 00Z 05 March 1964 verification of precipitation amounts. The manual analysis was done using total precipitation amounts in the 6-hour period from 18Z 04 March to 00Z 05 March 1964. The numerical analysis involves the computed hourly precipitation rate at 00Z 05 March 1964 and multiplying it by six to get an amount for a 6-hour period. As was previously mentioned the precipitation amounts in the northwestern United States ranged from a trace to .02 inches during the previous six hour period, therefore the verification is not as poor as it first appears to be. In the precipitation area south of the Great Lakes, actual precipitation values ran as high as .68 inches during the previous six hours, however the precipitation amount equation computed a high value of only .38 inches. Further data may permit the determination of better values for the constants $a_3$, $b_3$ and $c_3$.

Results of the verifications of the 24-hour numerical forecast of cloudiness and precipitation patterns, using the prognosis scheme described herein, proved to be rather poor. Two 24-hour prognostic maps are shown in figures 8 and 9. An initial attempt was made to forecast $T_p$ values using only the geostrophic advection term but results were unsatisfactory in regions of developing high and low pressure centers. The forecast $\omega$ values obtained from FNWF for the days in question were far from what actually occurred and it is felt that this is the largest contributing factor to the poor quality of the forecasts. It must be remembered that forecast $w$ values are an essential ingredient to forecast $T_p$ values due both to the vertical advection and pressure effect terms in equation (2). Also the forecast $w$ values are used directly to enter the Ladag diagram to determine the cloudiness and precipitation patterns.

It is felt that definite improvements could be made in the forecast model used. It was noted that on the forecast $T_p$ fields, certain areas
became either very wet or very dry. The normal five point Jacobian, used in the geostrophic advection term may tend to over-emphasize extreme values of \( T_p \) and certainly can quickly distort small-scale features. These errors could be reduced by using upstream advection or possibly a nine-point Jacobian with a smaller truncation error. Similarly, improvement could also be made in the vertical advection term. The only \( T_p \) values available from FNWF were the 850, 700 and 500 mb levels. In computing the vertical derivatives at 850 and 500 mb it was necessary to use a forward difference derivative if \( w \) was positive at 850 mb or negative at 500 mb. In taking this forward difference derivative the assumption is made that the \( T_p \) gradient is constant above and below 700 mb, which certainly does not always hold. If \( T_p \) values were available at 1000 and 400 mb a better finite-difference approximation could be used.
5. **Conclusions**

It is the authors' opinion, in view of the results obtained, that a numerical analysis of cloud and precipitation chart, patterned after this study be made available on an operational basis. It is felt that this information would be of significant value to the numerical product user.

The limitations of an analysis and forecasting scheme such as described herein are several. It is felt that one of the larger errors in the forecasts stems from incorrect placement and magnitude of the forecast vertical motion centers. This, in turn, is a direct function of the quality of the sea-level and 500 mb predictions. Second, the precipitation model depends upon empirical relationships that determine precipitation amounts, and these relationships are not fully understood. Third, there is no allowance for the obtaining of moisture from below the 850 mb level and in particular any increase of water vapor resulting from surface evaporation. Fourth, the analysis of the dew-point depression field by FNWF, Monterey, has been shown by Lieutenant Commander D. Howard [14] to be in error by as much as 6 to 8 degrees centigrade, even over dense data areas. This error is, of course, magnified over sparse data areas. Fifth, the use of a one-hour forecast time step rather than the six-hour step now used should definitely improve results. Last, the effects of latent heat of condensation where precipitation is forming have not been included in any way. In this connection the effect on vertical velocity would perhaps be most important.

To show the factor of machine speed over the same job done manually, the following figures are presented: Lewis' technique took about one-half hour to compute the forecast dew-point depression for one station at the single level of 700 mb. The parcel trajectory was computed using charts spaced 12 hours apart. The present machine technique will produce a 24-hour
dew-point depression forecast for 1977 points using time steps of 6 hours at three separate levels. This 24-hour forecast takes only 10 minutes, a gain factor of 35,586. In addition, the machine produces the forecast in chart form along with auxiliary charts. Although improvements are indeed necessary for this forecast model, the time is near at hand when electronic computers will actually forecast moisture parameters on an operational basis.
Enter diagram with average vertical motion, $\bar{w}$, and average dew-point depression, $\bar{T}_p$, to determine accompanying weather parameter where

$$\bar{w} = .333 (w_{850} + w_{700} + w_{500})$$

and

$$\bar{T}_p = .333 (1.4T_{850} + T_{700} + .6T_{500}).$$
Figure 2 - Numerical analysis of cloud and precipitation patterns at 00Z 03 March 1964 with manually drawn contours.
Figure 3 - Polar stereographic projection of the Northern Hemisphere to the scale of figure 2.
Figure 4 - Flow diagram of computer computations.

Start with
\( \tau = 0 \)

Read in fields changing \( \omega \) to \( w \).

Is \( \tau = 0 \)?

yes

Forecast \( T_p \) at \( \tau + 6 \) hrs.

Find \( T + w \)

Go into diagram

Is it raining?

yes

Calculate precip. & \( \Sigma \) precip.

no

Are showers possible?

yes

Showers are possible

yes

Calculate Showers & \( \Sigma \) precip.

no

Print Maps

no

Increase \( \tau \) by 6 hrs.

yes

Stop

no
Figure 5 - Manual versus numerical analysis of cloud and precipitation patterns at 00Z 05 March 1964.
Figure 6 - Manual versus numerical analysis of cloud and precipitation patterns at 12Z 06 March 1964.
Figure 7 - Manual versus numerical analysis of 6 hour precipitation amounts at 00Z 05 March 1964.
Figure 8 - Manual analysis of cloud and precipitation patterns at 00Z 04 March 1964 versus 24-hour numerical forecast from 00Z 03 March 1964.
Figure 9 - Manual analysis of cloud and precipitation patterns at 12Z 07 March 1964 versus 24-hour numerical forecast at 0Z 08 March 1964.

- **Clear**
- **Partly Cloudy**
- **Cloudy**
- **Precipitation**


