SIMULATION OF HELICOPTER AND V/STOL AIRCRAFT

VOLUME II

V/STOL ANALYSIS REPORT

Study, Equations of Motion of Vertical/Short Take-Off and Landing
Operational Flight/Weapon System Trainers

Walter McIntyre

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The report promotes an understanding of V/STOL analysis for simulation purposes and develops equations of motion compatible to either analogue or real time digital solution.

A general set of equations of motion are developed in which axis systems and aerodynamic coefficients are minimized. Equations of motion are then developed for five different V/STOL aircraft wherein the need for additional axis systems and aerodynamic coefficients for a particular V/STOL configuration is developed.

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Foreword

With increasing interest in Vertical/Short Take Off and Landing (V/STOL) aircraft by the military services it was determined that it would be to the advantage of the U.S. Naval Training Device Center to gather together in one report a study of axis systems used for representation of such aircraft, and to develop general aerodynamic equations of motion which may be used in simulation of these aircraft. This effort has been accomplished in this report for five experimental categories of V/STOL aircraft, each of which, as of this date, appear to give promise of becoming operational.

Carmine C. Castellano
Aerospace Staff Consultant
U.S. Naval Training Device Center
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SECTION I

INTRODUCTION

The purpose of this report is to promote an understanding of V/STOL analysis for simulation purposes and to develop equations of motion in a form most compatible to either analog or real-time digital solution. Only V/STOL aircraft configurations are considered in this analysis.

In Section II of the report a method of attack for the general V/STOL problem is developed and various axis systems are selected. The method developed is then applied to a number of different types of V/STOL aircraft. A model and its conceptual representation are developed in Section III. This model may be used as a guideline in developing equations of motion for various types of V/STOL aircraft but cannot be further simplified in general. The various terms in the final equations for a particular aircraft configuration must be analyzed for the magnitude of their contributions before final simplification is possible.

The X-12A tilt-wing and the VZ-1DA tilt-duct aircraft are used to illustrate equation development in Section IV. Equations are presented though not painstakingly developed for the X-19 tilt-prop, the XV-5A fan-in-wing and the P.1127 rotating thrust. A great deal of the symbology used for these different V/STOL aircraft is used just as presented by the respective manufacturer in order that verifying data might be available in the proper form at the earliest possible time. At present it appears that the equations developed will comprise a minimum requirement. Terms in these equations which prove in flight test to be sufficiently small, will be discarded. Analysis of flight test data may enable simulation with less rigorous equations but no such simplification can now safely be made.

A bibliography and Appendix A dealing with basic concepts of axis systems and equations of motion have been provided for the convenience of the reader.

(1) Refer to NAVTRADEVcen 1205-1 for an analysis of single and tandem rotor helicopter configurations.
We will initially consider that the aircraft is operating out of the V/STOL region. This aircraft whether it be a transport, observation plane, subsonic interceptor or any similar type of vehicle can be considered to be operating in a conventional flight regime. A description of both the equations of motion and necessary axis systems for this mode of operation has been done, notably by Connelly and others in References 2 and 7. Connelly, in Reference 2, discusses simulation of fixed-wing aircraft and presents optimum representations of the equations of motion which may be used for construction of a computer. Following the recommendations of Reference 2, we will use wind axes for the description of forces and aircraft body axes for the description of moments in the conventional flight regime.

Appendix A is provided as an aid to the reader. In Appendix A the definitions of inertial and body axes are developed. Transformations between inertial and body axes as well as diagrams illustrating their relationship to one another are shown.

The minimum set of terms in the equations of motion of a V/STOL aircraft that enable a description of its conventional mode of flight will be designated as a necessary description of the particular V/STOL aircraft. When the aircraft is in such a normal flight attitude, fewer axis systems and aerodynamic coefficients may be needed for description than during hover or transition flight. For some V/STOL configurations the axis systems available for fixed-wing aircraft will be sufficient; however for other V/STOL configurations additional axes must be defined. The capability of a V/STOL aircraft to change its physical configuration during vertical and transitional flight may indicate the need of additional axis systems. In addition, the consideration of available aerodynamic data for a particular vehicle may indicate the inclusion or exclusion of certain aerodynamic coefficients while traversing the V/STOL region. The additional terms generated by consideration of the V/STOL region, together with the minimum set of terms developed by consideration of the conventional flight regime, will constitute a necessary and sufficient description of the equations of motion for the V/STOL aircraft.

It is to be expected that any physical change in the actual configuration of the aircraft will probably necessitate the definition of additional axes, but the existence of a change in aircraft configuration or the use of additional thrust producers in the V/STOL region of flight does not necessarily force the development of new axis systems in addition to those already specified for the fixed-wing aircraft. Each type of aircraft must be treated individually and new axes developed only when the effects of the special features of the aircraft configuration cannot be accurately, economically and adequately handled with the axis systems used to describe the conventional mode of flight.
The most simple case of configuration change would be that causing physical rotation of the thrust vector. This can be accomplished by tilting one or more wings at the same or different rates or by tilting two or more ducted fans, propellers or rotors. Thrust rotation can also be accomplished by deflecting the flow from a thrust source such as a ducted fan. Yet another method of achieving V/STOL capability which may require special axis system treatment is the use of special thrust producers in the V/STOL region of flight that are completely separate from the thrust producers utilized in the conventional mode of flight. These separate sources of thrust take the form of jet engines, fans, or completely discrete paths for jet exhaust.

Possible axis systems for aircraft configurations involving such devices are presented below.

PARTICULAR V/STOL CONFIGURATIONS

We will now consider some examples of the use of additional axes to describe particular V/STOL aircraft. In the final consideration of a given V/STOL axis system the form of coefficient and performance data available for the given aircraft will contribute, to a certain extent, to the choice of axis systems for simulation of that aircraft.

TILT-WING. A current example of tilt-wing aircraft is the Tri-Service XC-142A Transport under development by Ling-Temco-Vought, Hiller Aircraft and Ryan Aeronautical Company. Figure 1 is a line drawing of this aircraft. The XC-142A is powered by four G.E. T-64 turbo-prop engines. These engines are mounted on a wing which can be tilted to an angle of 100 degrees from the x-y plane of the aircraft body axes such that hover in a tail wind is possible. The tail assembly is composed of a vertical fin and rudder, a movable horizontal stabilizer (no elevator) and a tail rotor for pitch stabilization.

In order to reduce the problem of keeping track of the relative orientation of the wing chord and the wind as well as the orientation of resulting force vectors, a wing axis system is defined. This system has its origin at the wing pivot point and has its x-z plane coincident with the body axis x-z plane. The system rotates with the wing such that the wing x axis is always parallel to the wing chord. Further, a set of wing stability axes is defined analogous to stability axes of fixed-wing aircraft. This system is rotated about the wing y axis by the angle of attack of the wing; i.e., the angle formed by the projection of the aircraft velocity vector on the x-z plane of the wing axis system and the wing x axis.

Also, in the transition and hover regions the forces developed at each propeller become the primary components of the stability of the aircraft. Therefore, an axis system (the propeller axis system) is specified for each main propeller. It may also be necessary to develop a propeller axis system for the tail rotor of some similar configurations, but this is not necessary for the XC-142A as shown in subsequent portions of this report.
These axis systems enable the development of force and moment expressions for the wing, main propellers and the tail rotor. The resulting forces and moments are transformed to aircraft body axes and then incorporated into the total force and moment equations. These axes in conjunction with inertial, body and wind axes would comprise a sufficient description of axes for simulation of a tilt-wing transport.

TILT-DUCT. An example of the tilt-duct is the VZ-4DA of DOAK Aircraft Company. This is a small aircraft with tiltable ducted fans, one at each wing tip. The tail assembly is composed of a vertical fin and rudder, a horizontal stabilizer and reaction controls. As for the tilt-wing it is desirable to compute the direction and magnitude of the thrust vectors from each ducted fan in order to simulate a trim aircraft in the V/STOL region. In order to accomplish this each rotatable ducted fan should contain an axis system. The resulting force and moments are transformed to the aircraft body axes by considering the angle of tilt of the ducts with respect to the wing. Figure 2 is a line drawing of this aircraft.

Another example of the tilt-duct is the proposed ducted fan VTOL transport, the Bell D2127 of Bell Aerosystem Company. Here again axes for duct velocities and duct thrust vectors are desirable. Attached to the aft side of each duct are wing areas in line with the air flow. These wing areas rotate with the ducts. Forces and moments created by these surfaces can be computed in the axis system defined for the ducts, and then transformed to fuselage axes through the duct tilt angle, thus enabling summation of total forces and moments. In general, the thrust output of a ducted fan varies with its direction of flight with respect to axes located in the duct. The thrust output is not necessarily collinear with the axis of the fan. The components of force perpendicular to this axis become significant in the total moment equations and in general, these components of force have varying lever arms on which to act and become particularly significant terms in vertical or near vertical flight.

TILT-PROP. An example of this type of aircraft is the Curtiss-Wright X-19. Figure 3 is a line drawing of this aircraft which has two engines driving four propellers. Four tilting propellers are mounted in nacelles at the tips of tandem high wings. Stabilization results from variance in the control of pitch at each propeller. Pitch angle control comes from simultaneous change in the pitch of the forward propellers and the aft propellers respectively. Roll angle control results from simultaneous change in the pitch of starboard propellers and the port propellers respectively. Yaw angle control is developed from simultaneous change in the propeller pitch of diagonally opposite propellers. Since the propeller nacelles rotate, a nacelle incidence angle is defined with respect to the aircraft body axes. In order to account for the variance in magnitude and direction of the forces developed at each propeller individual axis systems will be defined. The propeller axis systems in addition to the axis systems of the conventional aircraft enable a sufficient description of the equations of motion of the X-19. Propeller axes are required because of the importance of nonaxial components of thrust in each nacelle.
Figure 2. Three-View of VE-110A.
Figure 3. Three-View of X-19.
"TAIL-WIND. An example of this type of aircraft is the General Electric/ Ryan Aeronautical XV-5A V/STOL research aircraft. Figure 4 is a line drawing of such an aircraft. For this aircraft exhaust gases are emitted straight out the tail pines in normal flight. For transition to, and operation in, vertical flight the tail pines are blocked off and the exhaust gases are diverted to three free wheeling fans—one in the nose and a pair of large lift fans embedded in the wings. The thrust developed out of each fan is controlled by fan exit louvers. The thrust is proportional to fan rpm, and its direction is controlled by the setting of the louvers. If thrust versus fan rpm data and louver position versus thrust data were available such that the direction and magnitude of the thrust at each fan could be described, then forces and moments could be developed in existing wind and body axis systems so that the aircraft could be simulated by methods already developed for subsonic jet aircraft. However, if necessary, an axis system describing louver position could be developed in order to keep track of fan thrust during transition and hover conditions.

ROTATING THRUST AND DEFLected THRUST. An example of rotating thrust is the Hawker P.1127 transonic V/STOL strike fighter. Figure 5 is a line drawing of such an aircraft. A deflected thrust aircraft example is the VTOL trainer support aircraft, X-14A, by Bell Aerosystems Company. The P.1127 is powered by the Bristol Siddeley BS 53 engine which has four simultaneous rotating nozzles. The thrust being ejected from each nozzle is capable of being rotated through an angle of 100 degrees from a positive X-aircraft body axis. Nozzle deflection angles over 90 degrees allow for braking flight during a STOL maneuver. For this system we keep track of the thrust vectors from the exhaust nozzles of the powerplant. This can be done by tracking the rotation of the main exhaust nozzles with respect to body axes. No extra axis systems appear to be necessary.

In the case of the X-14A, thrust diverter position and power output of the engines may give enough information to determine thrust magnitude and orientation.

For both the P.1127 and X-14-A, no additional axis systems are necessary for the development of the equations of motion for the purpose of simulation. Aerodynamic data should enable thrust calculations in wind or body axes, and the consequent direct computation of aerodynamic forces and moments.
Figure 4. Three-View of XV-A.
In this section we will construct a mathematical model which can be applied to the simulation of any V/STOL aircraft. With this model, equations of motion for various V/STOL aircraft configurations are evolved in Section IV. The approach used in deriving these equations of motion will enable the reader to develop simulation equations—as needed—in a systematic and simple manner for V/STOL configurations that are evolved in the future.

The method of attack is to start with the equations of motion representing the conventional mode of operation of the V/STOL aircraft; to choose appropriate axis systems based upon the particular aircraft which are in addition to the inertial, body, stability and wind axes; to determine the V/STOL aerodynamic characteristics not simulated by the aircraft in its conventional flight mode; and finally to write the equations of motion for the V/STOL aircraft. The first objective is to develop the idea of a conventional flight mode.

CONVENTIONAL FLIGHT MODE OF V/STOL AIRCRAFT

The V/STOL aircraft (in the conventional mode of flight) is described by a set of necessary equations. There is no attempt to define whether they apply to single or multi-engine propeller or jet aircraft. The configuration of the aircraft is not defined other than there are control inputs for rudder, elevator, flap and aileron. There is no indication as to low or high performance or mission. Modifications of the basic simulation equations describing this model will depend upon the particular V/STOL aircraft. Consequently, specifying the particular aircraft enables the addition, if warranted, of the factors which allow the necessary equations of the aircraft operating in the conventional flight mode to become a necessary and sufficient set of simulation equations. In conjunction with the following, the development in Appendix A, and Reference 2, we are able to write the equations for a V/STOL aircraft in the conventional flight mode.

MATHEMATICAL MODEL OF CONVENTIONAL FLIGHT MODE OF V/STOL AIRCRAFT. The conventional flight mode is represented by a set of three force and three torque equations. Linear and angular rates are obtained from these equations.

Linear Motion. The following are the force expressions (1)(2).

\[ F_{Ni}^e = F_{Ni}^I \]

\( i = 1, 2, 3 \)

- \( e = \text{external} \)
- \( I = \text{internal} \)
- \( N = \text{inertial axes} \)

(1) See Appendix B of NAVTRADEVCEM 1205-1 and Appendix A herein.
(2) Connelly, reference 2, develops similar equations of motion.
The $\sum F_{e}^{N1}$ are the total external forces which are composed of aero-
dynamic and gravity forces and the $\sum F_{I}^{N1}$ are the total internal forces.

We can view the aircraft as a system of mass points such that the dis-
tance between pairs of mass points are constant throughout the aircraft's
motion. This is a rigid body, and consequently work done by internal
forces between pairs of mass points vanishes. However, since we have a
rigid body the center of mass of the mass points ($m_I$) is concentrated at
a single point and the radius vectors ($r_I$) from the inertial frame to
each arbitrary mass point are replaced by a radius vector $r$ to the center
of mass. We then have the following:

$$\sum F_{e}^{N1} = \sum m_I r_{i} = M r^{N1}$$  \hspace{1cm} (3.1)

$$M = \sum m_I \text{ mass of the aircraft}$$

Equation (3.1) is expanded to give:

$$\sum F_{e}^{N1} = M r^{N1}$$  \hspace{1cm} (3.2)

$$\sum F_{e}^{N2} = M r^{N2}$$

$$\sum F_{e}^{N3} = M r^{N3}$$

To facilitate the description of forces aircraft body axes are
located at the center of mass of the aircraft. Forces described in the
$N_i$ axes are transformed to the $X_i$ body axes as shown in Appendix A. We
have then the following equations:

$$\sum F_{e}^{X1} = M(\dot{V} + Wq_1 - Vr)$$

$$\sum F_{e}^{X2} = M(V + Ur - Wp)$$  \hspace{1cm} (3.3)

$$\sum F_{e}^{X3} = M(\dot{W} + Vp - Ud)$$

In equations (3.3) the following identifications are made:

$$\sum F_{e}^{X1} = \text{Aerodynamic Forces + Gravity Force + Engine Thrust}$$

where $X1 = x$, $X2 = y$, $X3 = z$, $T = \text{engine thrust}$, and $\alpha_T$ is the angle
of the thrust line with respect to the $x$-body axis.
So that there is

\[ \sum F_x = X_a - M g \sin \theta + T \cos \alpha_T \]

\[ \sum F_y = Y_a + M g \cos \theta \sin \phi \]  

\[ \sum F_z = Z_a + M r \cos \theta \cos \phi - T \sin \alpha_T \]  

From (3.3) and (3.4) the force equations in aircraft body axes are:

\[ X_a = M (\dot{V} + Wq_1 - V r) + M g \sin \theta - T \cos \alpha_T \]

\[ Y_a = M (\dot{V} + U r - W p) - M g \cos \theta \sin \phi \]  

\[ Z_a = M (\dot{W} + V p - Uq_1) - M g \cos \phi \cos \phi + T \sin \alpha_T \]  

Angular Motion. Following are the torque expressions for the aircraft in conventional flight. It is practically a universal practice to develop aircraft angular motion in body axes both in the aircraft manufacturing and simulator industry. For simulation purposes this practice avoids the added problem of transformations of the products and moments of inertia which would be required if other axis systems were used. From Appendix A we write for the torque components the usual dynamic terms involving products and moments of inertia, angular velocities and accelerations, and the coupling of angular velocities.

\[ L_a = I_{11} \dot{p} - I_{13} (\dot{r} + p q_1) + (I_{33} - I_{22}) q_1 r \]

\[ M_a = I_{22} \dot{q}_1 - I_{13} (r^2 - p^2) + (I_{11} - I_{33}) p r \]  

\[ N_a = I_{33} \dot{r} - I_{13} (\dot{p} - q_1 r) + (I_{22} - I_{11}) p q_1 \]  

In the equations of 3.6, the x-z plane of the aircraft body axes is a plane of symmetry. Consequently products of inertia \( I_{12}, I_{21}, I_{32} \) are zero. \( L_a, M_a \) and \( N_a \) are the aerodynamic coefficients necessary to describe any V/STOL aircraft in conventional flight.
<table>
<thead>
<tr>
<th>COEFFICIENT</th>
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<td>$X_a$</td>
<td>$C_x (\alpha, Ma)$</td>
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<tr>
<td></td>
<td>$C_x (\beta)$</td>
</tr>
<tr>
<td></td>
<td>$C_x_{6F}$</td>
</tr>
<tr>
<td>$Y_a$</td>
<td>$C_y \beta$</td>
</tr>
<tr>
<td>$Z_a$</td>
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<td>$C_z_{6F}$</td>
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<td>$C_z_{6E}$</td>
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<td>$C_1_p$</td>
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<td>$C_1_r$</td>
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Table 1. Dimensionless Coefficients Used for Basic Aircraft

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<table>
<thead>
<tr>
<th>( M_a )</th>
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<th>FORCE OR TORQUE EXPRESSION</th>
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<tr>
<td>( C_m^6F )</td>
<td>( \frac{1}{2} \rho V^2 Sc C_m^6F )</td>
<td>( \frac{1}{2} \rho V^2 Sc C_m^6F \cdot 6F )</td>
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<tr>
<td>( C_m^6E )</td>
<td>( \frac{1}{2} \rho V^2 Sc C_m^6E )</td>
<td>( \frac{1}{2} \rho V^2 Sc C_m^6E \cdot 6E )</td>
</tr>
<tr>
<td>( C_m q_1 )</td>
<td>( \frac{1}{4} \rho V^2 Sc C_m q_1 )</td>
<td>( \frac{1}{4} \rho V^2 Sc C_m q_1 \cdot q_1 )</td>
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<tr>
<td>( C_m ^{\alpha} )</td>
<td>( \frac{1}{4} \rho V^2 Sc C_m ^{\alpha} )</td>
<td>( \frac{1}{4} \rho V^2 Sc C_m ^{\alpha} \cdot \alpha )</td>
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<th>FORCE OR TORQUE EXPRESSION</th>
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<td>( C_n^\beta )</td>
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<td>( \frac{1}{2} \rho V^2 Sb C_n^\beta \cdot \beta )</td>
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<td>( \frac{1}{2} \rho V^2 Sb C_n^\delta A \cdot \delta A )</td>
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<td>( C_n^\delta R )</td>
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<td>( \frac{1}{2} \rho V^2 Sb C_n^\delta R \cdot \delta R )</td>
</tr>
<tr>
<td>( C_n^p )</td>
<td>( \frac{1}{4} \rho V Sb^2 C_n^p )</td>
<td>( \frac{1}{4} \rho V Sb^2 C_n^p \cdot p )</td>
</tr>
<tr>
<td>( C_n^r )</td>
<td>( \frac{1}{4} \rho V Sb^2 C_n^r )</td>
<td>( \frac{1}{4} \rho V Sb^2 C_n^r \cdot r )</td>
</tr>
</tbody>
</table>

\( V \) = Aircraft velocity  
\( \rho \) = Air density  
\( S \) = Wing area  
\( b \) = Wing span  
\( c \) = Mean aerodynamic chord

Table 1. Dimensionless Coefficients Used for Basic Aircraft (Cont'd.)

15
EXAMINATION OF COEFFICIENTS FOR V/STOL AIRCRAFT IN CONVENTIONAL FLIGHT MODE. Following the work presented by Connelly in Reference 2, we will state that the coefficients listed in Table 1 constitute a necessary number of aerodynamic coefficients in order to develop the aerodynamic forces and moments for a V/STOL aircraft in conventional flight. In many cases actual manufacturers' data may not be available for just these specific coefficients. For example, data are usually available in terms of coefficient of lift \( C_L \) or drag \( C_D \) which in turn can be transformed to give \( C_x \) and \( C_z \). Table 1 is then a list of what is considered to be a minimum number of aerodynamic coefficients to describe the aircraft in conventional flight. Modification of this list is possible when the flight of the aircraft is described in the V/STOL region.

For an actual V/STOL aircraft the equations for \( X, Y, Z, L_a, M_a \) and \( N_a \) may contain terms due to the specific effect of various aircraft components—the wing, propeller(s) or ducted fan, the fuselage, vertical tail and/rudder, horizontal tail, and devices to insure trim flight such as a small variable direction thrust propeller or ducted fan. Each of these forces and moments due to the various aircraft components will have associated aerodynamic coefficients—some common to the aircraft in conventional flight and other aerodynamic coefficients applicable only to the particular V/STOL aircraft.

STATEMENT OF EQUATIONS FOR V/STOL AIRCRAFT IN CONVENTIONAL FLIGHT MODE. From the force equations (3.5) and the torque equations (3.6) and the data of Table 1 we can write a set of equations for the aircraft in conventional flight. It must be remembered that these equations are only guidelines. They give us an insight into what will be contained in the simulation of a V/STOL aircraft. The equations which follow (3.7 to 3.12) are written in aircraft body axes and do not include effects due to landing gear or other variations in external stores or the aircraft configuration.

\( X \) Force Equation

\[
\frac{1}{2} \rho \frac{V^2}{2m} \left[ C_x(a, M_a) + C_x(\gamma) + C_x(\delta) \cdot 6F \right] = m(U + W_1 - V_r) + mg \sin \theta - T \cos \alpha_T \\
\text{or} \quad (U + W_1 - V_r) = \rho \frac{V^2}{2m} \left[ C_x(a, M_a) + C_x(\gamma) + C_x(\delta) \cdot 6F \right] - g \sin \theta \\
+ \frac{T}{m} \cos \alpha_T
\]

\( Y \) Force Equation

\[
\frac{1}{2} \rho \frac{V^2}{2m} y_b \delta = m(V + U_r - W_p) - mg \cos \theta \sin \phi \\
\text{or} \quad (V + U_r - W_p) = \rho \frac{V^2}{2m} y_b \cdot \delta + g \cos \theta \sin \phi
\]
NAVTRADEVCEI 1205-2

Z Force Equation

\[ \frac{1}{2} \rho \bar{V}^2 \left[ C_\alpha (\alpha, M) + C_{\delta F} \cdot 5F + C_{\delta E} \cdot 5E \right] \]

\[ = m(W + \bar{V}_p - Uq) - mg \cos \theta \cos \phi + T \sin \alpha_T \]

or

\[ \dot{(W + \bar{V}_p - Uq)} = \rho \frac{\bar{V}^2}{2m} \left[ C_\alpha (\alpha, M) + C_{\delta F} \cdot 5F + C_{\delta E} \cdot 5E \right] \]

\[ + g \cos \theta \cos \phi = \frac{T}{m} \sin \alpha_T \]

Roll Equation

\[ \frac{1}{4} \rho \bar{V}_{Sb} (2VC_{\beta} \cdot \beta + 2VC_{\delta A} \cdot 5A + 2VC_{\delta R} \cdot 5R + bC_{p} \cdot p) \]

\[ + bC_{r} \cdot r \]

\[ = I_{11} \dot{\beta} - I_{13} (\dot{r} + pq_1) + (I_{33} - I_{22}) q_1 \dot{r} \]

or

\[ \dot{p} = \frac{I_{13}}{I_{11}} (\dot{r} + pq_1) - \frac{(I_{33} - I_{22})}{I_{11}} q_1 \dot{r} \]

\[ + \sqrt{\bar{V}_{Sb}} \frac{I_{11}}{4} \left( 2VC_{\beta} \cdot \beta + 2VC_{\delta A} \cdot 5A + 2VC_{\delta R} \cdot 5R + bC_{p} \cdot P + bC_{r} \cdot r \right) \]

Pitch Equation

\[ \frac{1}{4} \rho \bar{V}_{Sc} [2VC_m (\alpha, M) + 2VC_{\delta F} \cdot 5F + 2VC_{\delta E} \cdot 5E + cC_{mq_1} \cdot q_1] \]

\[ = I_{22} \dot{q} - I_{13} (r^2 - p^2) + (I_{11} - I_{33}) pq \]

or

\[ \dot{q} = \frac{I_{13}}{I_{22}} (r^2 - p^2) - \frac{(I_{11} - I_{33})}{I_{22}} pq \]

\[ + \frac{\bar{V}_{Sc}}{4I_{22}} [2VC_m (\alpha, M) + 2VC_{\delta F} \cdot 5F + 2VC_{\delta E} \cdot 5E + cC_{mq_1} \cdot q_1 - cC_{mq_2} \cdot \dot{\alpha}] \]
Yaw Equation

\[
\frac{1}{I_1} \cdot \frac{\dot{V} - V b (2Vc \cdot \beta + 2Vc \cdot \delta A + 2Vc \cdot \delta R + bC \cdot p)}{n_\beta} 
+ \frac{\dot{T}_r}{n_r} + \frac{\dot{r}}{n_r} \cdot r = I_{33} \dot{r} - I_{13} (\dot{p} - q_1 r) + (I_{22} - I_{11}) p q_1
\]  

or

\[
\dot{r} = \frac{I_{13}}{I_{33}} (\dot{p} - q_1 r) - \frac{(I_{22} - I_{11})}{I_{33}} p q_1
\]

\[
+ \frac{\dot{V} - V b (2Vc \cdot \beta + 2Vc \cdot \delta A + 2Vc \cdot \delta R + bC \cdot p + bC \cdot r)}{n_\beta} \cdot \frac{1}{n_\beta}
\]

ADDITIONAL AXIS SYSTEM SELECTION

The next step in the mathematical model is to choose the necessary additional axis systems in order to describe the particular flight performance of a given V/STOL aircraft. In Section II axis systems were discussed in conjunction with particular V/STOL configurations.

V/STOL AERODYNAMIC COEFFICIENTS

Once a system of axes is selected, additional coefficients may be specified. These could relate to thrust for jet, propeller of ducted fan. They could be functions for speed brakes, slats, external stores, landing gear, drop tanks, bomb bay doors or louvers for control of air flow. The inclusion of any such coefficients is best illustrated by the tilt-wing example in Section IV.

V/STOL EQUATIONS OF MOTION

With the equations of the V/STOL aircraft in conventional flight, axis systems, and a study of the particular V/STOL aerodynamics discussed up to this point, we are now ready to write the equations of motion for the V/STOL aircraft. No small angle, or small term simplifications have been made in the force or torque equations of the aircraft in conventional flight. Simplifications may be made during the development of the equations for a particular aircraft.
In this section equations of motion will be developed for five V/STOL configurations. These configurations—tilt-wing, tilt-duct, tilt-prop, fan-in-wing, and rotating thrust—incorporate individually and collectively the designs of V/STOL aircraft currently being developed. A particular aircraft, either one that is proposed or a prototype will be used as an example of each of these configurations. The first type to be considered is the tilt-wing.

**TILT-WING**

The Vought-Hiller-Ryan XC-142A Tri-Service Transport will be used in the example in the development of tilt-wing equations of motion. The equations from Section III will be modified so that they are applicable to an aircraft such as the XC-142A.

In order to obtain some idea of the XC-142A let us consider Figure 1, Table 2 and Table 3. In Figure 1, a three view arrangement of the XC-142A is shown. Table 2 contains some of the physical characteristics of the aircraft and Table 3 is a list of definitions of symbols contained in the XC-142A equations.

With this general idea of the XC-142A, we may now develop the mathematical model for the XC-142A. First, define, if necessary, applicable axis systems. Second, develop additional aerodynamic coefficients. Finally, write the equations of motion for the XC-142A.
### CHARACTERISTIC

<table>
<thead>
<tr>
<th></th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Gross Weight</td>
<td>37,474 pounds</td>
</tr>
<tr>
<td>Center of Gravity (aft of the leading edge of the mean aerodynamic chord (MAC))</td>
<td>10% MAC</td>
</tr>
<tr>
<td></td>
<td>28% MAC</td>
</tr>
<tr>
<td>Wing</td>
<td></td>
</tr>
<tr>
<td>Total Area</td>
<td>534.37 ft²</td>
</tr>
<tr>
<td>Span</td>
<td>67.5 ft</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>8.53</td>
</tr>
<tr>
<td>Dihedral Angle</td>
<td>-2.12°</td>
</tr>
<tr>
<td>Airfoil Section</td>
<td>NACA 63-318 (Mod)</td>
</tr>
<tr>
<td>Trail Edge Flaps - Double slotted</td>
<td></td>
</tr>
<tr>
<td>Maximum Deflection</td>
<td>60°</td>
</tr>
<tr>
<td>Deflection for take off (STOL)</td>
<td>40°</td>
</tr>
<tr>
<td>Deflection for landing (STOL)</td>
<td>60°</td>
</tr>
<tr>
<td>Leading Edge Flaps</td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>87°</td>
</tr>
<tr>
<td>Ailerons - Plain</td>
<td></td>
</tr>
<tr>
<td>Maximum Deflection (wing up)</td>
<td>+ 50°</td>
</tr>
<tr>
<td>Maximum Deflection (wing down)</td>
<td>+ 15°</td>
</tr>
<tr>
<td>Horizontal Stabilizer - All moving</td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td>163.5 ft²</td>
</tr>
<tr>
<td>Span</td>
<td>31.14 ft</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>5.08</td>
</tr>
<tr>
<td>Dihedral Angle</td>
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</tr>
<tr>
<td>Airfoil Section (root)</td>
<td>NACA 0015</td>
</tr>
<tr>
<td>Airfoil Section (tip)</td>
<td>NACA 0012</td>
</tr>
<tr>
<td>Maximum Deflection (leading edge up)</td>
<td>60°</td>
</tr>
<tr>
<td>Maximum Deflection (leading edge down)</td>
<td>15°</td>
</tr>
<tr>
<td>Hinge Line, % of tail mean geometric chord</td>
<td>13%</td>
</tr>
<tr>
<td>Vertical Tail</td>
<td></td>
</tr>
<tr>
<td>Area: Fin to rudder hinge</td>
<td>95 ft²</td>
</tr>
<tr>
<td>Area: Rudder aft of hinge</td>
<td>21.6 ft²</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>1.87</td>
</tr>
<tr>
<td>Airfoil Section (root)</td>
<td>NACA 0018</td>
</tr>
<tr>
<td>Airfoil Section (tip)</td>
<td>NACA 0012</td>
</tr>
<tr>
<td>Fuselage</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>50 ft</td>
</tr>
<tr>
<td>Length (including tail rotor)</td>
<td>58.12 ft</td>
</tr>
<tr>
<td>Outside Height</td>
<td>10.72 ft</td>
</tr>
<tr>
<td>Outside Width</td>
<td>9.25 ft</td>
</tr>
<tr>
<td>Maximum cross-sectional area</td>
<td>90 ft²</td>
</tr>
<tr>
<td>Propellers</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>15.5 ft</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>4</td>
</tr>
<tr>
<td>Tail Rotor</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>8 ft</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>3</td>
</tr>
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</table>

Table 2. Selected Physical Characteristics IC-142A
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Fuselage</td>
</tr>
<tr>
<td>W</td>
<td>Wing</td>
</tr>
<tr>
<td>n</td>
<td>Main propellers (n = 1, 2, 3, h) - top view left to right</td>
</tr>
<tr>
<td>VT</td>
<td>Vertical Tail</td>
</tr>
<tr>
<td>HS</td>
<td>Horizontal Stabilizer</td>
</tr>
<tr>
<td>TR</td>
<td>Tail Rotor</td>
</tr>
<tr>
<td>(\alpha_F)</td>
<td>Angle of attack - fuselage</td>
</tr>
<tr>
<td>(\alpha_w)</td>
<td>Angle of attack - wing</td>
</tr>
<tr>
<td>(\alpha_{hs})</td>
<td>Angle of attack - horizontal stabilizer</td>
</tr>
<tr>
<td>(i_t)</td>
<td>Incidence angle - horizontal stabilizer</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Downwash angle</td>
</tr>
<tr>
<td>(\beta_F)</td>
<td>Sideslip angle - fuselage</td>
</tr>
<tr>
<td>(\beta_w)</td>
<td>Sideslip angle - wing</td>
</tr>
<tr>
<td>(i_w)</td>
<td>Incidence angle - wing</td>
</tr>
<tr>
<td>(\xi)</td>
<td>(\xi = i_w - \alpha_w)</td>
</tr>
<tr>
<td>(V_B)</td>
<td>Total velocity in aircraft body axes</td>
</tr>
<tr>
<td>(V_W)</td>
<td>Total velocity in wind stability axes</td>
</tr>
<tr>
<td>(c)</td>
<td>Mean aerodynamic chord</td>
</tr>
<tr>
<td>(b)</td>
<td>Wing span</td>
</tr>
<tr>
<td>(S)</td>
<td>Total area of wing</td>
</tr>
<tr>
<td>(S_p)</td>
<td>Total area of main propeller disks</td>
</tr>
<tr>
<td>(q_w)</td>
<td>Dynamic pressure due to power on wing effects</td>
</tr>
<tr>
<td>(q_l)</td>
<td>Dynamic pressure - free stream</td>
</tr>
<tr>
<td>(q_{hs})</td>
<td>Dynamic pressure at horizontal stabilizer</td>
</tr>
</tbody>
</table>

Table 3. Definition of Symbols Used in XG-MIIA Equations
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{T,S}$</td>
<td>Coefficient of thrust at wing due to inflow velocity</td>
</tr>
<tr>
<td>$T$</td>
<td>Total thrust of main propellers</td>
</tr>
<tr>
<td>$J_n$</td>
<td>Advance ratio of main propeller</td>
</tr>
<tr>
<td>$J_{TR}$</td>
<td>Advance ratio of tail rotor</td>
</tr>
<tr>
<td>$N_n$</td>
<td>RPM - main propeller</td>
</tr>
<tr>
<td>$N_{TR}$</td>
<td>RPM - tail rotor</td>
</tr>
<tr>
<td>$N_o$</td>
<td>Nominal RPM - main propeller</td>
</tr>
<tr>
<td>$B_n$</td>
<td>Blade pitch angle - main propeller</td>
</tr>
<tr>
<td>$T_n$</td>
<td>Main propeller thrust</td>
</tr>
<tr>
<td>$N_n$</td>
<td>Main propeller thrust component normal to $T_n$</td>
</tr>
<tr>
<td>$Y_n$</td>
<td>Main propeller moment (initially turning)</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Main propeller moment (initially pitching)</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>Main propeller torque</td>
</tr>
<tr>
<td>$D_n$</td>
<td>Diameter main propeller</td>
</tr>
<tr>
<td>$D_{TR}$</td>
<td>Diameter tail rotor</td>
</tr>
<tr>
<td>$B_{TR}$</td>
<td>Blade pitch angle - tail rotor</td>
</tr>
<tr>
<td>$T_{TR}$</td>
<td>Thrust - tail rotor</td>
</tr>
<tr>
<td>$I_E$</td>
<td>Inertia of main propeller blades and shaft</td>
</tr>
<tr>
<td>$\omega_E$</td>
<td>Angular velocity main propeller</td>
</tr>
<tr>
<td>$\dot{\omega}_E$</td>
<td>Angular acceleration main propeller</td>
</tr>
<tr>
<td>$I_{TR}$</td>
<td>Inertia of tail rotor blades and shaft</td>
</tr>
<tr>
<td>$\omega_{TR}$</td>
<td>Angular velocity tail rotor</td>
</tr>
<tr>
<td>$\dot{\omega}_{TR}$</td>
<td>Angular acceleration tail rotor</td>
</tr>
</tbody>
</table>

Table 3. Definition of Symbols Used in YC-1424 Equations (Cont'd.)
AXIS SYSTEMS FOR XC-142A. In order to describe the dynamics and aerodynamics of the tilt-wing aircraft, one axis system—the propeller axis system—in addition to the inertial, body, stability and wind axes is required. The wind axis system is complicated by the tilt of the wing. Figure 6 shows the wing tilt angle ($i_w$) and the variable distances ($x_{ac}$ and $z_{ac}$) that track the aerodynamic center (a.c.) of the wing as the wing is tilted through the angle $i_w$. The aerodynamic center of the wing as located in the x-z body axes plane is the origin of the wing stability axes. The wing stability axis system is parallel to the aircraft stability axis system and has its origin translated by $x_{ac}$ and $z_{ac}$ from the aircraft stability axis origin (which is located at aircraft c.g. nominally) as in Figure 6.

The additional axis system, the propeller axis system, is shown in Figure 7. The axis system is repeated for each main propeller, so that there are actually four main propeller axis systems. The propeller axis system will enable the development of the propeller thrust ($T_n$) propeller torque ($Q_n$) in terms of propeller power, a normal force ($N_n$) perpendicular to $T_n$ along the propeller blade, and propeller moments ($M_n$) and ($Y_n$).

Further development of these propeller forces and moments will be considered in the discussion of aerodynamic effects.
cos \psi_n = \cos \beta_n \cos \alpha_y \\
\sin \psi_n = \cos \beta_n \sin \alpha_y \\
\cos \beta_n

Moments: \( M_n \) and \( I_n \)

(1) \( \beta_n = 0, \ i_w = 0 \)
- \( M_n \) is in the \( x-s \) plane
- \( I_n \) is in the \( x-y \) plane

(2) \( \beta_n \neq 0, \ i_w = 0 \)
- \( M_n \) tilted from \( x-s \) plane by \( \pm \beta_n \).
- \( I_n \) tilted from \( x-y \) plane by \( \pm (90-\beta_n) \).
- Obtain components of \( M_n \) and \( N_n \) by multiplying by \( \sin \beta_n \) and \( \cos \beta_n \).

(3) \( \beta_n \neq 0, \ i_w \neq 0 \)
- In addition to (2) \( M_n \) rotated in \( x-s \) plane by \( \pm i_w \).
- \( N_n \) rotated from \( x-y \) plane by \( \pm i_w \).

Figure 7. Main Propeller Axis System

Revision No. 1
Revised 28 May 1961
At this point in the discussion we will recall the equations of motion of the basic aircraft as discussed in Section III. The internal moments (the right side of the moment equations) now include gyroscopic effects due to the rotating mass of an engine. The gyroscopic effects are due to the main engines and the tail rotor. For the main engines we have the following terms representing gyroscopic effects:

- \( (I_E \Omega) \cos i_w - q_1 (I_E \Omega) \sin i_w \) for \( L \) term.
- \( p(I_E \Omega) \sin i_w + r(I_E \Omega) \cos i_w \) for \( M \) term.
- \( - (I_E \Omega) \sin i_w - q_1 (I_E \Omega) \cos i_w \) for \( N \) term.

For the tail rotor we have:

- \( q_1 I_{TR} \Omega_{TR} \) for \( L \) term.
- \( 0 \) for \( M \) term.
- \( - pI_{TR} \Omega_{TR} \) for \( N \) term.

In these terms \( p \), \( q_1 \) and \( r \) are respectively the angular velocities around the \( x \), \( y \) and \( z \) body axes. The wing tilt angle is \( i_w \). \( I_E \) and \( I_{TR} \) are respectively, main engine and tail rotor inertias and \( \Omega \) are respectively, engine and tail rotor rotational speed.

The equations of motion without expansion of the aerodynamic terms and incorporating the engine gyroscopic and tail rotor terms become:

- \( X = m(U + Wq_1 - Vr) + mg \sin \theta \)
- \( Y = m(\dot{V} + Ur - Vp) - mg \cos \theta \sin \phi \)
- \( Z = m(\dot{W} + Vp - Uq_1) - mg \cos \theta \cos \theta \)
- \( L = I_{11} \dot{\phi} - I_{13}(r^2 + p^2) + (I_{22} - I_{23})q_1 r + (I_E \Omega) \cos i_w - q_1 (I_E \Omega) \)
- \( M = I_{22} \dot{q}_1 - I_{13}(r^2 - p^2) + (I_{11} - I_{23})p \dot{r} + p(I_E \Omega) \sin i_w \)
- \( N = I_{33} \dot{r} - I_{13}(\dot{\phi} - q_1 r) + (I_{22} - I_{11})pq_1 - (I_E \Omega) \sin i_w - q_1 (I_E \Omega) \)

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The aerodynamic forces and moments—$X_a$, $Y_a$, $Z_a$, $L_a$, $M_a$ and $N_a$ will be developed for the tilt-wing aircraft so that a complete set of equations of motion is presented.

**AERODYNAMIC FORCES AND MOMENTS - XC-142A.** In the equations of motion developed for the basic aircraft in Section III a set of aerodynamic force and moment terms were developed as a function of specific dimensionless and aerodynamic coefficients. We will develop aerodynamic force and moment expressions based upon the particular configurations of the XC-142A tilt-wing aircraft.

In order to develop expressions for $X_a$, $Y_a$, $Z_a$, $L_a$, $M_a$ and $N_a$, contributions from the major airframe components of the XC-142A aircraft will be considered separately. The major components to be considered are the wing, the main propellers, the vertical tail and rudder, the horizontal stabilizer, the tail rotor and the fuselage. After the aerodynamic force and moment expressions for each of these major components are developed, they will be summed to get the total aerodynamic force and moment expressions. These total force and moment expressions can then be visually checked with the aerodynamic force and moment expressions developed for the basic aircraft in Section III. The expressions in Section III will be contained in the aerodynamic force and moment expressions developed for the tilt-wing XC-142A.

Before continuing we will develop the aircraft body axes to inertial axes transformation. The matrices are:

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos \psi \cos \theta$</td>
<td>$\sin \psi \cos \theta$</td>
<td>$- \sin \theta$</td>
</tr>
<tr>
<td>$\cos \psi \sin \theta \sin \phi$</td>
<td>$\sin \psi \sin \theta \sin \phi$</td>
<td>$\cos \theta \sin \phi$</td>
</tr>
<tr>
<td>$- \sin \psi \cos \phi$</td>
<td>$\sin \psi \cos \phi$</td>
<td>$\cos \theta \cos \phi$</td>
</tr>
</tbody>
</table>

In these matrices $N_1$ is north, $N_2$ is east and $N_3$ down for the inertial axes, and $X_1$ is $x$, $X_2$ is $y$ and $X_3$ is $z$ for the aircraft body axes.

$\psi$, $\theta$ and $\phi$ are conventional Euler angles. Their definitions in developing the above matrices are shown in Appendix A.

The inertial to body axes rates are:

\[
\begin{align*}
    r &= -\dot{\psi} \sin \theta + \dot{\phi} \\
    q_1 &= \dot{\psi} \cos \theta \sin \phi + \dot{\theta} \cos \phi \\
    r &= \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi
\end{align*}
\]
And conversely:
\[ \phi = p + q_1 \tan \theta \sin \phi + r \tan \theta \cos \phi \]
\[ \theta = q_1 \cos \phi - r \sin \phi \]
\[ \psi = r \frac{\cos \phi + q_1 \sin \phi}{\cos \theta} \]

We are now ready to develop the aerodynamic forces and moments for each major component of the aircraft. The first to be considered will be the effect of the main propellers. It should be noted that the expressions developed are for hovering, transition and conventional flight.

Main Propeller. There are four mutually similar systems of axes used to describe forces and moments generated by the main propellers during hover and low aircraft velocities. An analysis which was developed by Ling-Temco-Vought (Refererence 6) for the XC-142A is adopted herein for the description of main propeller forces and moments. The subscript \( n \) \((n = 1, 2, 3, 4)\), denotes the particular propeller. They are numbered left to right looking from the top—1 and 2 are port propellers; 3 and 4 are starboard propellers. The approach used to develop the required propeller equations is to first consider the propeller geometry, next state the aerodynamic coefficients and finally write expressions for the force and moment contributions of the main propellers.

1. Main Propeller Geometry. From Figure 7 the wind vector with respect to each propeller is formed as \( V_n = u_n + v_n + w_n \).

For each propeller we may define an angle \( \psi_0 \) which is the angle between the component of the total aircraft velocity \( (V_B) \) in the x-z plane of the body axes and the propeller axis line. If \( i_w = 0 \), then

\[ \psi_0 = \alpha_p \]

\[ \psi_0 = (i_w + \alpha_p) \]

From geometric considerations in Figure 7 we may form the expression for \( \psi_n \) which is the angle between the \( u_n \) and \( V_n \) velocities.

\[ \cos \psi_n = \frac{u_n}{v_n} = \cos \beta_p \cos \psi_0 \]

\[ \beta_p = \sin^{-1}(\frac{v}{V_B}) \]

\[ \sin \psi_n = \frac{(w^2 + v^2)^{1/2}}{v_n} \]

\[ \frac{\cos \beta_p \sin \psi_0}{\cos \beta_n} \]

(h.1)
In a like manner expressions are formed for $\beta_n$ which is defined as the angle between the velocities $V_n \sin \psi_n$ and $w_n$.

\[
\sin \beta_n = \frac{V_n \sin \psi_n}{V_n \sin \psi_n} = \frac{\sin \beta_F}{\sin \psi_n}
\]

\[
\cos \beta_n = \frac{w_n}{(w_n^2 + v_n^2)^{1/2}} = \frac{\cos \beta_F \sin \psi_0}{\sin \psi_n}
\]

The velocity expressions $u_n$, $v_n$ and $w_n$ will now be developed. The origins of each propeller axis are located along a line parallel to the $y$-body axis at approximately the center of mass of each nacelle. In Figure 7 the origins of the propeller axis systems are rotated from $V_R \cos \beta$ by angle $\psi_0$. Each propeller axis system origin is located by $x_n$, $y_n$ and $z_n$ body axis components which are multiplied by the appropriate body axis angular velocity in order to give tangential velocity components of $u_n$, $v_n$ and $w_n$.

\[
u_n = V_R \cos \beta_F \cos \psi_0 - y_n (p \sin i_w + r \cos i_w) + x_n q_1 \sin i_w
\]

\[
u_n = V_R \sin \beta_F
\]

\[
u_n = V_R \cos \beta_F \sin \psi_0 + y_n (p \cos i_w - r \sin i_w) - x_n q_1 \cos i_w
\]

In equations (4.3) and (4.5) the term $y_n$ is a constant. In fact $y_1 = y_4$ and $y_2 = y_3$ since the inboard and the outboard propellers are each the same distance from the $x$-$z$ body axis plane. The component $x_n$ and $z_n$ vary as a function of wing tilt.

2. Main Propeller Aerodynamic Coefficients. The aerodynamic coefficients presented for the main propellers follow those presented by Reference 6. First let us define the advance ratio ($J_n$) for each main propeller and the advance ratio normal to the propeller disk ($J'_n$).

\[
J_n = \frac{60V_n}{\nu_n D} \text{ and } J'_n = J_n \cos \psi_n
\]
The symbol $N_n$ is the particular propeller RPM, the number 60 changes RPM to RPS, $D$ is the diameter of the propeller and $\beta_n$ the blade pitch angle of the particular propeller. The aerodynamic coefficients are then developed in terms of advance ratio and blade pitch.

\[ C_T = C_{T0} + \frac{3C_T}{2J_n} (J_n')^2 + \frac{3}{2J_n} \frac{\partial C_T}{\partial \beta_n} \frac{\partial \beta_n}{\partial J_n} B_n J_n' \]  
\[ C_P = C_{P0} + \frac{3C_P}{2J_n} (J_n')^2 + \frac{3}{2J_n} \frac{\partial C_P}{\partial \beta_n} \frac{\partial \beta_n}{\partial J_n} B_n J_n' \]  
\[ C_N = \frac{3}{2J_n} \left[ \frac{\partial C_N}{\partial J_n} \right] B_n J_n \sin \psi_n \]  
\[ C_Y = \frac{3}{2J_n} \left[ \frac{\partial C_Y}{\partial J_n} \right] B_n J_n \sin \psi_n + \frac{3}{2J_n} \left[ \frac{\partial C_Y}{\partial J_n} \right] J_n' J_n \sin \psi_n \]  
\[ C_M = \frac{3}{2J_n} J_n' \left[ \frac{\partial C_M}{\partial \psi_n} \right] \psi_n J_n' \]  

$C_T$ is the coefficient of thrust ($T_n$), $C_P$ is the coefficient of power used to express torque ($Q_n$), $C_N$ is the coefficient of normal thrust ($N_n$) -- the thrust component perpendicular to $T_n$. $C_Y$ and $C_M$ are the lateral and longitudinal hub moment coefficients that appear during wing tilt.

3. Main Propeller Force and Moment Expressions. Before expressing the force and moment contribution in body axes due to the propellers, the individual propeller forces and moments developed in propeller axes are stated in terms of equations (4.7), (4.8), (4.9), (4.10) and (4.11).

\[ T_n = D^4 \left( \frac{N_n}{N_o} \right)^2 \left( \frac{\rho}{\rho_o} \right) C_{Tn} \]  
\[ N_n = D^4 \left( \frac{N_n}{N_o} \right)^2 \left( \frac{\rho}{\rho_o} \right) C_{Nn} \]  
\[ Y_n = D^5 \left( \frac{N_n}{N_o} \right)^2 \left( \frac{\rho}{\rho_o} \right) C_{Yn} \]
(\dot{v}_n = \frac{1}{\rho_o} \frac{1}{n^2} \left( \frac{\rho_o}{\rho_i} \right)^2 c_{\text{M}_n} \right) \tag{4.15}

(\dot{\gamma}_n = \frac{1}{\rho_o} \frac{1}{n^2} \left( \frac{\rho_o}{\rho_i} \right)^2 \gamma_{\text{F}_n} \right) \tag{4.16}

In equations (4.12) through (4.16) \( N_o \) is the maximum RPM of the propellers and \( \rho_o \) is the air density at sea level on a standard day. These equations then enable us to write the propeller force and moment contributions in aircraft body axes. Observe that each equation is subscripted by \( n \) so that each propeller individually influences the forces and moments.

\[
(\Delta \alpha_p) = \frac{1}{n} (T_n \cos i - N_n \cos \theta_n \sin i_w) \tag{4.17}
\]

\[
(\Delta \gamma_p) = \frac{1}{n} (-N_n \sin \theta_n) \tag{4.18}
\]

\[
(\Delta \beta_p) = \frac{1}{n} (-T_n \sin i_w - N_n \cos \theta_n \cos i_w) \tag{4.19}
\]

\[
(\Delta L_a) = -(\Delta \alpha p_1 - (\Delta \alpha_p)_2 - (\Delta \alpha_p)_3) y_1 -(\Delta \alpha p_2 - (\Delta \alpha_p)_3) y_2
\]

\[
- [(\Delta \beta_p)_1 + (\Delta \beta_p)_2] z_1 - [(\Delta \beta_p)_2 + (\Delta \beta_p)_3] z_2
\]

\[
\frac{1}{n} (Y_n \cos \theta_n \sin i_w) - \frac{1}{n} (M_n \sin \theta_n \sin i_w) \tag{4.20}
\]

\[
(\Delta M_p) = M_{\text{FIVOT}} + \frac{1}{n} T_n (\cos i_w) z_{\text{FIVOT}} + \frac{1}{n} T_n (\sin i_w) x_{\text{FIVOT}}
\]

\[
- (\gamma_1 \cos \theta_1 \sin i_w + N_4 \cos \theta_4 \sin i_w) z_1
\]

\[
- (N_2 \cos \theta_2 \sin i_w + N_3 \cos \theta_3 \sin i_w) z_2
\]

\[
+ (\gamma_1 \cos \theta_1 \cos i_w + N_4 \cos \theta_4 \cos i_w) x_1
\]

\[
+ (N_2 \cos \theta_2 \cos i_w + N_3 \cos \theta_3 \cos i_w) x_2
\]

\[
- \frac{1}{n} (Y_n \cos \theta_n) + \frac{1}{n} (M_n \cos \theta_n) \tag{4.21}
\]
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Where \( w_{T_{PIVOT}} \) = \( 1.625(T_1 + T_4) + 1.092(T_2 + T_3) \)

\[
(\Delta_X a)_p = -[(\Delta X a) _{p1} - (\Delta X a) _{p4}] y_1 - [(\Delta X a) _{p2} - (\Delta X a) _{p3}] y_2
\]

\[
+[(\Delta Y a) _{p1} + (\Delta Y a) _{p4}] x_1 + [(\Delta Y a) _{p2} + (\Delta Y a) _{p3}] x_2 \tag{4.22}
\]

\[
\frac{1}{2} \sum_{n=1}^{4} (Y_n \cos \theta_n) \cos i_w \frac{1}{2} \sum_{n=1}^{4} (Y_n \sin \theta_n) \cos i_w
\]

In equations (4.17) through (4.22) the letter \( p \) denotes the effects of the main propellers and \( p \) subscripted \( p_n \) where \( n = 1, 2, 3, 4 \) is a particular propeller. For example, in \((\Delta L a)_p\) the term \((\Delta Z a) _{p1}\) equals \((-T_1 \sin i_w - N_1 \cos \theta_1 \cos i_w\)). In order to better appreciate these equations let us consider the aircraft in normal forward flight where the propeller wind vector is parallel to the x-z plane \((\theta_n = 0)\) and there is no tilt of the wing \((i_w = 0)\). The equations (4.17) through (4.22) then become:

\[
(\Delta X a)_p = \sum_{n=1}^{4} T_n \tag{4.23}
\]

\[
(\Delta Y a)_p = 0 \tag{4.24}
\]

\[
(\Delta Z a)_p = \frac{1}{2} \sum_{n=1}^{4} N_n \tag{4.25}
\]

\[
(\Delta L a)_p = -(-N_1 + N_4) y_1 - (-N_2 + N_3) y_2 \tag{4.26}
\]

\[
(\Delta M a)_p = M_{T_{PIVOT}} + \sum_{n=1}^{4} T_n Z_{PIVOT} \tag{4.27}
\]

\[
+ (N_1 + N_4) x_1 + (N_2 + N_3) x_2 + \sum_{n=1}^{4} M_n \tag{4.28}
\]

Equation (4.23) is the total thrust and (4.25) is the total normal force due to the propellers. Equation (4.26) is the rolling moment contribution which will be zero if outboard \((n=1 \text{ and } 4)\) and inboard \((n=2 \text{ and } 3)\) normal propeller forces are balanced; (4.27) is the pitching moment contribution; and (4.28) is the turning moment contribution which will be negligible if
the outboard \((n=1\) and \(l_i)\) and inboard \((n=2\) and \(3)\) thrusts are balanced and \(EY_n = 0\). The incremental propeller forces and moments equations \((4.17)\) through \((4.22)\) will be included in the total aerodynamic forces and moments.

Wing. The calculation of wing aerodynamics forces and moments is complicated by the wing tilt during vertical and transition flight. These forces and moments are developed in wing stability axes by first considering the wing geometry and then defining wing aerodynamic coefficients in accord with Reference 6.

1. Wing Geometry. In wing stability axes there occurs an induced velocity \((AV)\) due to the propeller wash across the wing. This gives the effect of increased lift. In order to describe the effect, a coefficient of thrust of the wing \((C_{T,S})\) is defined as a function to total aircraft velocity \((V_R)\).

\[
C_{T,S} = \frac{T}{q_w S_p} 
\]

where \(S_p\) is the total disk area of the four propellers and \(q_w\) is the wing dynamic pressure.

\[
q_w = (q + \frac{T}{S_p}), \text{ where } q = \frac{1}{2} \rho V_B^2
\]

At low forward speed during transition and in hover the effect of \(C_{T,S}\) is at a maximum. The particular function of \(C_{T,S}\) is dependent upon wing tilt and wing flap angle which should be determined from manufacturer's data.

From the geometry in Figure 8 the \(u_w\) component of wing velocity is \(u_p + AV\). The velocity \(u_p\) is the average of the \(u_n\) velocities developed in propeller axes and is the predominant velocity effect in wing axes. \(u_p\) is then the velocity of the aircraft \((V_B \cos \theta_p)\) and as such defines the body axes velocities \(J\) and \(W\). Any \(V\) in body axes is equal to \(v_w\) since we are concerned with wing stability axes and \(u_p\) is contained in the \(x-z\) body axes and \(x-z\) wing axes planes. The total velocity in wing axes \((v_w)\) is as follows:

\[
v_w = (u_p^2 + (u_p + AV)^2 + v_w^2)^{1/2}, \text{ where } v_w = V 
\]

From Figure 8, \(w_p = w_w = -U \sin i_w + W \cos i_w\)

and \(u_p = U \cos i_w + W \sin i_w\)
Figure 8. Wing Axes
From the foregoing the wing angle of attack ($\alpha_w$), and the wing sideslip angle ($\beta_w$) can be stated as:

$$\alpha_w = \tan^{-1} \left( \frac{W_p}{u_p + AV} \right), \quad (4.32)$$

with the sign of $\alpha_w$ as positive down from plus $x_w$.

$$\beta_w = \tan^{-1} \left( \frac{V}{\sqrt{(u_p + AV)^2 + W_p^2}} \right), \quad (4.33)$$

The expression for induced velocity ($AV$) is derived from momentum theory.\(^1\) $AV$ is similar to the mean inflow velocity ($W_i$) developed for helicopter inflow analysis.\(^2\)

$$AV = -u_p + (u_p^2 + \frac{2T}{\rho S_p})^{1/2} \quad (4.34)$$

Rearranging we have:

$$(u_p + AV) = (u_p^2 + \frac{2T}{\rho S_p})^{1/2}$$

but $V_w = u_p + AV$ \quad (4.35)

and $u_p = V_R$, so that

substituting for $(u_p + AV)$ and $u_p$, and multiplying by $\rho$ we have:

$$\frac{1}{2} \rho V_w^2 = \frac{1}{2} \rho V_R^2 + \frac{T}{S_p}$$

$q = 1/2 \rho V_w^2$, where $q$ is the dynamic pressure

but $q_w = 1/2 \rho V_w^2$

so that we have:

$$q_w = (q + \frac{T}{S_p}) \quad (4.36)$$

---

1. For example, see *Airplane Aerodynamics*, Reference 5.
2. NAVTRADEVCEN 1205-1, Section 3.
In this manner $q_w$ develops naturally from the momentum theory definition of $\Delta V$. The wing dynamic pressure is the pressure used in equation (4.29) for $C_{T_s}$. Consequently, we have now related the coefficient $C_{T_s}$ with the induced velocity, $\Delta V$, through the wing dynamic pressure, $q_w$. We are now ready to develop the wing aerodynamic coefficients.

2. Wing Aerodynamic Coefficients. In order to develop the wing forces and moments five aerodynamic coefficients will be defined as in Reference 6. Since the development is in wing axes, rolling ($p$) and turning ($r$) rates necessary to define these coefficients are transformed from body axes. The pitching rate ($q_1$) is the same in wing axes since $q_1$ lies in the $x$-$z$ plane and the $x_w$-$z_w$ plane. The transformation of the angular rates ($p$ and $r$) is accomplished by a rotation, $\xi$. From Figure 8 $\xi$ is equal to $(i_w - \alpha_w)$. We can then write for the wing rolling rate ($p_w$) and the wing turning rate ($r_w$) the following equations.

\[
p_w = p \cos \xi - r \sin \xi \quad (4.37)
\]
\[
r_w = p \sin \xi + r \cos \xi \quad (4.38)
\]

The aerodynamic coefficients for the wing are $C_D$, $C_L$, $(C_{I_w})$, $(C_{m_w})$ and $(C_{n_w})$. They are defined as follows in accordance with their development in Reference 6.

\[
C_D = C_{D_o} + KC_L + \frac{3C_D}{\delta F} \cdot \delta F + \frac{2C_D}{\delta F} \cdot \delta F \quad (4.39)
\]

The strong flap dependence ($\delta^2 F$) in the $C_D$ expression above, is due to the importance of flap during transition.

\[
C_L = C_{L_o} + C_{L_{\delta F}} \cdot \delta F + C_{L_{\delta A}} \cdot \delta A + \frac{1}{2V_w} C_L p_w + \frac{1}{2V_w} C_{L_r} \cdot r_w \quad (4.40)
\]

\[
(C_{I_w}) = C_{I_o} + C_{I_{\delta F}} \cdot \delta F + C_{I_{\delta A}} \cdot \delta A + \frac{b}{2V_w} C_{I_p} \cdot p_w + \frac{b}{2V_w} C_{I_{r}} \cdot r_w \quad (4.41)
\]

\[
(C_{m_w}) = C_{m_o} + \frac{3C_m}{\delta F} \cdot \delta F + \frac{c}{2V_w} C_{m_{q_1}} \cdot q_1 \quad (4.42)
\]

\[
(C_{n_w}) = C_{n_o} + C_{n_{\delta F}} \cdot \delta F + C_{n_{\delta A}} \cdot \delta A + \frac{b}{2V_w} C_{n_p} \cdot p_w + \frac{b}{2V_w} C_{n_r} \cdot r_w \quad (4.43)
\]
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3. Wing Force and Moment Expressions. Before writing the force and moment expressions for the wing, equations (4.39) through (4.43) will be transformed to the aircraft body axes through the angle $\xi$.

The body axes wing coefficients are then:

\[ (C_x) = -CD \cos \xi - CL \sin \xi \]  \hspace{1cm} (4.44)

\[ (C_z) = CD \sin \xi - CL \cos \xi \]  \hspace{1cm} (4.45)

\[ (C_l) = (C_l) \cos \xi + (C_n) \sin \xi \]  \hspace{1cm} (4.46)

\[ (C_m) = (C_m) - \frac{x_{ac}}{c} (C_x) + \frac{x_{ac}}{c} (C_z) \]  \hspace{1cm} (4.47)

\[ (C_n) = -(C_z) \sin \xi + (C_n) \cos \xi \]  \hspace{1cm} (4.48)

\( x_{ac} \) and \( z_{ac} \) are the respective distances from the c.g. of the aircraft in body axes to the aerodynamic center (ac) of the wing in the \( x-z \) plane. \( c \) is the mean aerodynamic chord.

The wing force and moment contributions to the total force and moment equations, (4.44) through (4.48) can be expressed as forces and moments. For example, if we have a coefficient \( C_x \), a force, \( X_a \), is immediately defined as \( C_x S q \). \( S \) is the wing area of the aircraft and \( q \) the dynamic pressure. The coefficients in equations (4.44) through (4.48) are immediately expressible in terms of their respective force and moment contributions.

\[ (\Delta X_a) = (C_x) S [f(C_T,s) q_w] \]  \hspace{1cm} (4.49)

\[ (\Delta Z_a) = (C_z) S [f(C_T,s) q_w] \]  \hspace{1cm} (4.50)

\[ (\Delta L_a) = (C_l) bS [f(C_T,s) q_w] \]  \hspace{1cm} (4.51)

\[ (\Delta M_a) = (C_m) cS [f(C_T,s) q_w] \]  \hspace{1cm} (4.52)

\[ (\Delta N_a) = (C_n) bS [C_T,s] q_w \]  \hspace{1cm} (4.53)

3. Use of coefficients to express forces and moments can be reviewed in References 2 and 7.
Equations (4.49) through (4.53) are the wing force and moment contributions. \([f(CT, S) q_w]\) is the dynamic pressure term incorporating the effects of having thrust produced by the propellers and having wash across the wing. Consider what happens if the aircraft is flying and the engines are turned off \((T = 0)\). Then \(q_w = q\) from equation (4.36) and \(f(T, S) \approx 1\) from data available in Reference 6. Thus \([f(CT, S) q_w]\) \(\approx q\) with the engines off and equations (4.49) through (4.53) are wing force and moment equations that would be expected to occur in a jet aircraft.

Vertical Stabilizer and Rudder. Forces and moments for vertical tail \((vt)\) and rudder arise from the relative wind pushing against the vertical tail surfaces thereby causing a turning moment due to control input to the rudder. This gives side force, as well as rolling and turning moments.

Wind pushing against the vertical tail and rudder yields a side force \((Y_a)_{vt}\) and rolling moment \((\Delta L_a)_{vt}\) respectively which are non-dimensionalized in terms of aerodynamic coefficients as \(C_y\) and \(C_l\). The rolling moment is coupled in rolling velocity \(p\) and turning velocity \(r\). We can then write for \(C_y\) and \(C_l\) the expressions, (4.54) and (4.55).

\[
C_y = C_y \beta_F + C_{6R} \theta_F \quad (4.54)
\]

\(C_y\) is the change in side force coefficient with changing sideslip angle. \(\beta_F\) acts as a damping term. \(C_{6R}\) is the change in side force coefficient with rudder deflection and represents the controllable term in the expression. This is important in the use of automatic stabilization systems for aircraft.

\[
C_l = C_l \beta_F + C_{6R} \theta_F + \frac{b}{2V_B} [C_{1p} \cdot p + C_{1r} \cdot r] \quad (4.55)
\]

\(C_l\) is the change in rolling moment with variation in sideslip angle. \(\beta_F\) is caused by the position of the vertical tail normally above the X axis and other factors as wing sweepback and dihedral. \(C_{6R}\) is the change in rolling moment coefficient due to rudder deflection. \(C_{1p}\) is the roll damping derivative. \(C_{1r}\) is the change in rolling moment coefficient with change in yawing velocity.

Rudder deflection will give a turning moment \((\Delta N_a)_{vt}\) which can be expressed by the aerodynamic coefficient \(C_{n}\), and the expression for \(C_n\) is:
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\[ C_n = C_{nB} + b_R + C_{nSR} + \frac{b}{2V_R} [C_p + C_{nR} + r] \]  (4.56)

\( C_{nB} \) is the weathercock or static directional derivative, \( C_{nSR} \) is rudder effectiveness, the change in yawing moment coefficient with rudder deflection, \( C_{nR} \) is the change in yawing moment coefficient with varying rolling velocity. \( C_{nR} \) is the yaw damping derivative. The tail is the main contribution to \( C_{nR} \).

The forces and moments for the vertical tail can then be expressed in the following equations:

\[ (\Delta V_a)_{vt} = C_y \frac{q_{vt}}{q} \]  (4.57)

\[ (\Delta L_a)_{vt} = C_t \frac{q_{vt}}{q} \]  (4.58)

\[ (\Delta N_a)_{vt} = C_n \frac{bSq}{q} \]  (4.59)

Here \( q \) is the free stream dynamic pressure and \( q_{vt} \) is the vertical tail dynamic pressure. This force and these moments will be included in the total aerodynamic forces and moments.

Horizontal Stabilizer. The equations for forces and moments for the horizontal stabilizer (hs) will be developed in the standard manner.

We define \( \alpha_t = \alpha_F + \gamma_t - \epsilon \)

Here \( \alpha_t \) is the angle of attack of the tail, \( \alpha_F \) is the angle of attack of the fuselage, \( \gamma_t \) is the angle of incidence of the tail and \( \epsilon \) is the downwash angle.

In accord with Reference 6 we have

\[ \epsilon = [h - 0.5h_21 (i_w + \alpha_F)] [L_w f(C_{tT})] + i_w - a_w + a_F \]

for the YC-1h2A.

The lift \( (C_{tL}) \) and \( (C_{tT}) \) coefficients of the tail can then be expressed in the following relations...
The horizontal stabilizer (hs) can contribute forces in the x and z directions, and a pitching moment. The equations are as follows:

\[
(\Delta X_a)_{hs} = -[-C_{l_t} \cos (i_t - a_t) + C_{l_t} \sin (i_t - a_t)] \frac{q_{hs}}{q} \quad (h.60)
\]

\[
(\Delta Z_a)_{hs} = [-C_{l_t} \sin (i_t - a_t) + C_{l_t} \cos (i_t - a_t)] \frac{q_{hs}}{q} \quad (h.61)
\]

\[
(\Delta M_a)_{hs} = - (\Delta X_a)_{hs} \cdot h_{hs} + (\Delta Z_a)_{hs} \cdot l_{hs}
\]

\[
+ \frac{c^2 S \rho}{4} \left[ C_{m_{q\perp}} \cdot q_1 V_S + C_{n_d} \cdot \dot{w} \right] \quad (h.62)
\]

\(l_{hs}\) is distance from aircraft c.g. to the aerodynamic center (a.c.) of the horizontal stabilizer and \(h_{hs}\) is the height of a.c. above the c.g. Both \(l_{hs}\) and \(h_{hs}\) are measured in the x-z plane of the aircraft body axes. \(S\) is the wing area, \(\rho\) is the air density, \(c\) is the mean aerodynamic chord and the angle \((i_t - a_t)\) is used to transform \(C_{l_t}\) and \(C_{D_t}\) to body axes.

\(C_{m_{q\perp}}\) is the pitch derivative-the change in pitching moment coefficient as pitch velocity is changed. \(C_{n_d}\) is due to the fact that when the wing undergoes a change in angle of attack, there is a time lag before the change in downwash is felt at the tail surfaces.

\((\Delta X_a)_{hs}, (\Delta Z_a)_{hs}\) and \((\Delta M_a)_{hs}\) will be included in the total aerodynamic forces and moments.

Tail Rotor. The z force \((\Delta Z_a)_{TR}\) and the pitching moment \((\Delta M_a)_{TR}\) developed at the tail will be obtained by finding a tail rotor (TR) advance ratio \((J_{TR})\). From \(J_{TR}\) the tail rotor thrust coefficient \((C_{T_{TR}})\) and tail rotor power coefficient \((C_{p_{TR}})\) will be found. In turn, the tail rotor thrust \((T_{TR})\) and torque \((C_{T_{TR}})\) is obtained and consequently \((\Delta Z_a)_{TR}\) and \((\Delta M_a)_{TR}\). \(T_{TR}\) is positive in the -z direction.
We define the total tail rotor velocity \( V_{TR} \) as

\[
V_{TP} = \left[ (u_{TP})^2 + (w_{TP})^2 \right]^{1/2}
\]

Here \( u_{TR} \) and \( w_{TR} \) are defined as:

\[
u_{TR} = V_n \sin \left( \psi_{TR} \right)
\]

\[
w_{TR} = V_n \cos \left( \psi_{TR} \right) - l_{TR} q_1
\]

\( l_{TR} \) is the distance from the center of the tail rotor hub to the aircraft c.g., and \( \psi_{TR} \) locates the tail rotor with respect to the aircraft body axes.

\[
\psi_{TR} = 90^\circ + (\alpha_F - \zeta)
\]

Here \( \zeta \) is again the downwash angle and is defined as for the horizontal stabilizer.

The advance ratio for the tail rotor is

\[
J_{TR} = \frac{60 V_{TR}}{N_{TR} D_{TR}} \quad \text{or} \quad J_{TR}^* = \frac{60(-w_{TR})}{N_{TR} D_{TR}}
\]

where \( N_{TR} \) is the RPM and \( D_{TR} \) the diameter of the tail rotor.

We now define \( (C_{TR}) \) and \( (C_{pTR}) \) as is done in Reference 6.

\[
C_{T_{TR}} = C_{T_{TR}} (B_{TR}) + \frac{2C_{T_{TR}}}{\beta J_{TR}^*} (J_{TR}^*)
\]

\[
C_{p_{TR}} = \frac{\beta^2 C_{p_{TR}}}{\beta B_{TR}^2} (B_{TR})^2
\]

Here \( B_{TR} \) is the collective pitch of the tail rotor blades and \( C_{T_{TR}} (B_{TR}) \) is developed from a curve of \( C_{T_{TR}} \) plotted versus \( B_{TR} \).

The thrust \( (T_{TR}) \) and torque \( (Q_{TR}) \) of the tail rotor is then written as:

\[
T_{TR} = \frac{N_{TR}}{\frac{D_{TR}}{\beta}} \left( \frac{N_{TR}}{\beta} \right)^2 C_{T_{TR}}
\]

\[
Q_{TR} = \frac{D_{TR}}{\frac{\beta}{\beta}} \left( \frac{N_{TR}}{\beta} \right)^2 C_{p_{TR}}
\]
Consequently the force and moment terms can be written directly.

\[(\Delta Z_a) = T_{TR} \]  
\[(\Delta M_a) = T_{TR} l_{TR} \]  
\[(\Delta N_a) = Q_{TR} \]  

Fuselage. In a very direct manner we can write the effects of the fuselage \((F)\) on the total aerodynamic forces and moments.

We have for the forces

\[ (\Delta X_a) = -\frac{1}{2} \rho V^2 S C_{D_o} \]  
\[ C_{D_o} \] is the equilibrium drag coefficient.

\[ (\Delta Y_a) = +\frac{1}{2} \rho V^2 S C_{y\beta_F} \cdot \beta_F \]  
\[ C_{y\beta_F} \] is the change in side force with respect to a changing sideslip angle.

\[ (\Delta Z_a) = -\frac{1}{2} \rho V^2 S C_{L\alpha_F} \cdot \alpha_F \]  
\[ C_{L\alpha_F} \] is the change in lift coefficient with varying angle of attack. This is also known as the lift curve slope.

We have for the moments:

\[ (\Delta L_a) = 0 \]  
\[ (\Delta M_a) = \frac{1}{2} \rho V^2 S c C_{m_o} + \frac{1}{2} \rho V^2 Sc C_{m_{\alpha_F}} \cdot \alpha_F \]  
\[ C_{m_o} \] is the aerodynamic pitching moment coefficient in equilibrium flight and \( C_{m_{\alpha_F}} \) is the longitudinal static stability derivative.

\[ (\Delta N_a) = \frac{1}{2} \rho V^2 S b C_{n\beta_F} \cdot \beta_F \]  
\[ C_{n\beta_F} \]
is the static directional or "weathercock" derivative.

In the above expressions \( q = \frac{1}{2} \rho V^2 \) which is the dynamic pressure. 

\( b \) is the wing span, \( c \) is the mean aerodynamic chord and \( S \) is the wing area.

For hovering and in transition regions \( \beta_F \) is assumed small so that 
\[ \cos \beta_F = 1. \]

EQUATIONS OF MOTION - XC-1h2A. The aerodynamic force and moment terms developed for each of the major aircraft components will be combined and finally expressed in the equations of motion for the total aircraft.

Total Aerodynamic Forces and Moments. The total aerodynamic forces and moments are as follows:

\[ X_a = (\Delta X_a)_P + (\Delta X_a)_w + (\Delta X_a)_h + (\Delta X_a)_F \]
\[ Y_a = (\Delta Y_a)_P + (\Delta Y_a)_v + (\Delta Y_a)_F \]
\[ Z_a = (\Delta Z_a)_P + (\Delta Z_a)_w + (\Delta Z_a)_h + (\Delta Z_a)_T + (\Delta Z_a)_F \]
\[ L_a = (\Delta L_a)_P + (\Delta L_a)_w + (\Delta L_a)_v \]
\[ M_a = (\Delta M_a)_P + (\Delta M_a)_w + (\Delta M_a)_h + (\Delta M_a)_T + (\Delta M_a)_F \]
\[ N_a = (\Delta N_a)_P + (\Delta N_a)_w + (\Delta N_a)_v + (\Delta N_a)_T + (\Delta N_a)_F \]

Equations of Motion Expanded. The forces and moments are presented in body axes. These equations are subject to simplification in representation of aerodynamic propeller forces and moments, when flight test data are available from which to determine typical magnitudes of terms.

1. X Force Equation

\[
\left( C_x \frac{E f (C_T, S)}{q} q_w + \sum_{n=1}^{\frac{L}{2}} (T_n \cos i_w - N_n \cos \beta_n \sin i_w) \right) - \left( C_D \cos (i_t - \alpha_t) + C_L \sin (i_t - \alpha_t) \right) \frac{q_h s}{q} - q C_{L_0} \]

\[ = n(\frac{H}{2} + \frac{\omega q}{2} - r) + \omega \cdot \sin \theta \]
2. Y Force Equation.

\[ \sum_{n=1}^{\infty} \left( -N_n \sin \theta_n \right) + C z \ b \ S q \left( \frac{q_{T1}}{q} \right) + S q \ C y_{\beta_p} \cdot \varepsilon_F \n \]

\[ = m(\dot{V} + U_r - W_p) - mg \cos \theta \sin \phi \]


\[ (C_z) b \ S f(C_T, S) q_w + \sum_{n=1}^{\infty} \left( -T_n \sin i_w - N_n \cos \theta_n \cos i_w \right) \n \]

\[ + [C_{D_t} \sin (i_t - a_t) + C_{L_t} \cos (i_t - a_t)] \ S q \left( \frac{q_{T1}}{q} \right) + T_{TR} - S q_{\beta_p} \cdot \varepsilon_F \n \]

\[ = m(W + V_p - U_q) - mg \cos \theta \cos \phi \]

4. Roll Equation.

\[ (C_1) b \ S f(C_T, S) q_w \n \]

\[ + \left[ (+T_1 \sin i_w + N_1 \cos \theta_1 \cos i_w) - (+T_4 \sin i_w + N_4 \cos \theta_4 \cos i_w) \right] \n \]

\[ + \left[ (+T_2 \sin i_w + N_2 \cos \theta_2 \cos i_w) - (+T_3 \sin i_w + N_3 \cos \theta_3 \cos i_w) \right] \n \]

\[ + \left[ (+N_1 \sin \theta_1 + N_4 \sin \theta_4) x_1 + (+N_2 \sin \theta_2 + N_3 \sin \theta_3) z_2 \right] \n \]

\[ = I_{11} \dot{\phi} + I_{13} (\dot{r} + p q_1) + (I_{33} - I_{22}) q_1 r + (I_{12} \dot{\phi}) \cos i_w \n \]

\[ = q_1 (I_E) \sin i_w + q_1 I_{TR} \dot{O}_{TR} \]

5. Pitch Equation.

\[ (C_m) b \ S f(C_T, S) q_w \n \]

\[ + M_{TP} + \sum_{n=1}^{\infty} T_n (\cos i_w) x_{\text{pivot}} + \sum_{n=1}^{\infty} T_n (\sin i_w) x_{\text{pivot}} \n \]
NAVTRADECEN 1205-2

\[- (N_1 \cos \beta_1 \sin \dot{w} + N_4 \cos \beta_4 \sin \dot{w}) z_1 \]
\[- (M_2 \cos \beta_2 \sin \dot{w} + N_3 \cos \beta_3 \sin \dot{w}) z_2 \]
\[+ (N_1 \cos \beta_1 \cos \dot{w} + N_4 \cos \beta_4 \cos \dot{w}) x_1 \]
\[+ (N_2 \cos \beta_2 \cos \dot{w} + N_3 \cos \beta_3 \cos \dot{w}) x_2 - \sum_{n=1}^{\infty} \frac{N_n \sin \beta_n}{q} \]
\[\sum_{n=1}^{\infty} (M_n \cos \beta_n) \]
\[+ [C_D \sin (i_t - \alpha_t) + C_L \sin (i_t - \alpha_t)] S q(h_s) \cdot h_s \]
\[+ [C_D \sin (i_t - \alpha_t) - C_L \cos (i_t - \alpha_t)] S q(h_s) \cdot l_s \]
\[+ \frac{c^2 S_p}{h} [C_m \cdot q V_B + C_{m_a} \cdot \dot{w}] + T_{TR} \cdot l_{TR} + q S c(m_a + m_F \cdot a_F) \]
\[= I_{22} q + I_{33}(r^2 - p^2) + (I_{11} - I_{33}) pq \]
\[+ p(I_{E_{\alpha}}) \sin i_{w} + r(I_{E_{\alpha}}) \cos i_{w} \]


\[(C_m) \cdot b S q(C_T, S) q_w \]
\[- [(T_1 \cos \dot{w} - N_1 \cos \beta_1 \sin \dot{w}) - (T_4 \cos \dot{w} - N_4 \cos \beta_4 \sin \dot{w}) y_1 \]
\[- [(T_2 \cos \dot{w} - N_2 \cos \beta_2 \sin \dot{w}) - (T_3 \cos \dot{w} - N_3 \cos \beta_3 \sin \dot{w}) y_2 \]
\[+ (N_1 \sin \beta_1 + N_4 \sin \beta_4) x_1 - (N_2 \sin \beta_2 + N_3 \sin \beta_3) x_2 \]
\[+ \sum_{n=1}^{\infty} \frac{N_n \cos \beta_n}{q} \cdot \frac{C_{\alpha F}}{C_{\theta F}} \cdot \dot{w} \cdot \frac{C_{\alpha F}}{C_{\theta F}} \cdot \dot{w} \cdot q \]
\[= I_{33} \dot{\phi} + I_{13}(p - q_1) + (I_{22} - I_{11}) pq_1 \]
\[- (I_{E_{\alpha}}) \sin i_{w} - q_1(I_{E_{\alpha}}) \cos i_{w} - p \]

No simplifications have been made on the right side of these six equations. External stores, rough air, or landing gear conditions have not been developed in these equations.
TILT-DUCT

The VZ-1DA VTOL aircraft of DOAK Aircraft will be used in the exam-
nel of simulation equations of motion for a tilt-duct aircraft. Much
of the development which follows has been based upon information gleaned
from References 1 and 8. Soule and Baumgarten in Reference 8 present a
method for calculating the aerodynamic forces and moments for the VZ-1DA
and Peul in Reference 1 presents a study of handling qualities for a
tilt-duct aircraft. From this information and the presentation set
forth for the tilt-wing example, a well-defined set of engineering simula-
tion equations for the tilt-duct will be presented realizing that an
actual simulation will be dependent upon the type and usefulness of the
aerodynamic data available from the manufacturer of the aircraft.

In order that we may have some idea of the specific details of the
VA-1DA, Figure 2 is a three-view drawing of the VZ-1DA, Table 4 is a
selected list of physical characteristics of the aircraft and Table 5
is a list of symbols used in developing the equations of motion. As
much as possible, repetition of symbols will be used in the various
examples of V/STOL aircraft in this section of the report.

From this general idea of the VA-1DA, we may now develop the mathe-
matical model for the VA-1DA. First, define, if necessary, applicable
axis systems. Second, develop the aerodynamic coefficients. Finally,
write the equations of motion for the VA-1DA. As needed we will borrow
from the results obtained in the tilt-wing equations of motion and from
the results obtained in References 1 and 8.

AXIS SYSTEMS FOR THE VA-1DA. In order to describe the dynamics and aero-
dynamics of the tilt-duct aircraft, one axis system, the duct-axis system,
in addition to the inertial, body, stability and wind axes is required.
The wind axis system is complicated by the tilt of the ducts. Both of
the ducts tilt through the same angle simultaneously. Figure 9 shows the
wing tilt angle (\(i_w\)) and the duct incidence angle (\(i_D\)). Duct incidence
is described in the aircraft stability axes. The force and moment coef-
ficients for the ducted fans are described in the duct axis system. There
are two duct-axis systems—one located at each duct. This is similar to
the repeated propeller axis systems developed for the XG-142A. We define
the duct axis system by the following three mutually orthogonal velocity
vectors. First, there is a side duct velocity (\(V_s\)) parallel to the y
body axis and in a minus y direction. Second, there is an axial velocity
(\(V_A\)) into the center of the duct. Finally, there is a normal velocity
(\(V_R\)) parallel to the x-z plane of the aircraft body axis system and
perpendicular to \(V_s\) and \(V_A\). If there is no duct incidence, the duct-axis
systems as shown in Figure 10 are similar to the x, y, z body axes. The
duct-axes (\(x_M\), \(y_M\), \(z_M\), \(x_D\), \(y_D\), \(z_D\)) are right-handed systems with
y parallel to \(y_M\) and \(y_D\) and x in the same direction as \(x_M\) and \(x_D\) and
z in the same direction as \(z_M\) and \(z_D\) when \(i_w\) is zero.
Airplane:

Over-all length 31.2 ft.
Over-all span (including ducts) 25.5 ft.
Over-all height 10.3 ft.

Propulsive Device:

Fan
- Type fixed pitch
- Number of blades 8
- Diameter 4 ft.
- Blade root chord 0.312 ft.
- Blade tip chord 0.184 ft.
- Solidity 0.117
- Blade angle (.75R) 0.360 radians
- Hub to tip diameter ratio 0.333
- Fan location, position of hub from duct lip 0.806 ft.

Duct
- Centerbody Diameter (maximum) 1.333 ft.
- Over-all length 5.79 ft.

Trim Vane
- Area 5.66 ft.$^2$
- Span 4.525 ft.
- Airfoil section NACA 0009
- Flap chord, 25 percent 0.313 ft.
- Movement 0 to + 0.349 radians

Inlet Guide Vanes
- Number of vanes 14
- Area (per vane) 0.333 ft.$^2$
- Span 1.333 ft.
- Chord (constant) 0.25 ft.
- Airfoil section NACA 652 - 015
- Movement 0.1745 radians

Wing Group:

Wing
- Area (excluding ducts) 96.0 ft.$^2$
- Span (excluding ducts) 16.0 ft.
- Mean aerodynamic chord 6.08 ft.
- Aspect ratio 2.67
- Airfoil section NACA 2418 (mod.)
- Dihedral 0 radians

Table 4. Selected Aircraft Characteristics - W-14
Aileron
Area (per aileron) 6.0 ft.$^2$
Span 3.0 ft.$^2$
Movement ± .262 radians

Empenage Group:

Horizontal Tail
Area (projected) 28.5 ft.$^2$
Span (projected) 11.8 ft.$^2$
Mean aerodynamic chord 2.5 ft.$^2$
Aspect ratio 4.9
Airfoil section NACA 0012 (Mod.)
Incidence variable, 0 to + .1745
Dihedral 0.1745 radians
Elevator movement
(stabilizer incidence, 0 radians) +0.472 radians (T.E. down)
-0.472 radians (T.E. up)
(stabilizer incidence, +1.745 radians) +0.297 radians (T.E. down)
-0.576 radians (T.E. up)

Vertical Tail
Area 13.9 ft.$^2$
Span 5.2 ft.$^2$
Mean aerodynamic chord 2.8 ft.$^2$
Aspect ratio 1.95 (geometric); 3.02 (effective)
Airfoil section NACA 0012 (Mod.)

Fuselage:
Length (excluding reaction control vanes) 29.3 ft.$^2$
Maximum height (approx.) 4.12 ft.$^2$
Maximum width 3.0 ft.$^2$

Reaction Control Vanes
Area (within 15 inch tail pipe diameter) 1.042 ft.$^2$
Span (within 15 inch tail pipe diameter) 1.25 ft.$^2$
Aspect ratio (effective) 1.32
Airfoil section NACA 0009
Movement ± 0.612 radians

Table 4. Selected Aircraft Characteristics - VZ-4DA (Cont'd.)
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Aspect ratio</td>
</tr>
<tr>
<td>$c_T$</td>
<td>Thrust coefficient (for velocity = $v_D$)</td>
</tr>
<tr>
<td>$\bar{c}_T$</td>
<td>Thrust coefficient (for velocity = $v_D$)</td>
</tr>
<tr>
<td>$c_N$</td>
<td>Normal force coefficient = normal force$/\rho n^2 D^4$</td>
</tr>
<tr>
<td>$c_S$</td>
<td>Side force coefficient = side force$/\rho n^2 D^4$</td>
</tr>
<tr>
<td>$c_M$</td>
<td>Pitching moment coefficient = Pitching Moment$/\rho n^2 D^5$</td>
</tr>
<tr>
<td>$c_L$</td>
<td>Total rolling moment coefficient for ducted-fan</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Total pitching moment coefficient for ducted-fan</td>
</tr>
<tr>
<td>$c_n$</td>
<td>Total yawing moment coefficient for ducted-fan</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift Coefficient</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag Coefficient</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Duct chord</td>
</tr>
<tr>
<td>$c$</td>
<td>Wing chord</td>
</tr>
<tr>
<td>$D$</td>
<td>Fan diameter</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$i_D$</td>
<td>Ducted fan incidence angle</td>
</tr>
<tr>
<td>$i_W$</td>
<td>Wing incidence angle</td>
</tr>
<tr>
<td>$i_{HT}$</td>
<td>Horizontal tail incidence angle</td>
</tr>
<tr>
<td>$i_{VT}$</td>
<td>Vertical tail incidence angle</td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>Axial advance ratio ($V_D/D$)</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate (slugs/sec.)</td>
</tr>
<tr>
<td>$N$</td>
<td>Fan speed (R.P.S.)</td>
</tr>
<tr>
<td>$\dot{p}$</td>
<td>Roll rate about $X$-Axis (Rad./Sec.)</td>
</tr>
<tr>
<td>$\dot{p}_W$</td>
<td>Roll rate about $v_W$ (Rad./Sec.)</td>
</tr>
<tr>
<td>$b_v$</td>
<td>Duct span</td>
</tr>
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</table>

Table 1. Definition of Symbols - NAVTRADEVGEN 1205-2
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
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</thead>
<tbody>
<tr>
<td>h</td>
<td>Wing span</td>
</tr>
<tr>
<td>$q_l$</td>
<td>Pitch rate about Y-Axis (Rad./Sec.)</td>
</tr>
<tr>
<td>r</td>
<td>Yaw rate about Z-Axis (Rad./Sec.)</td>
</tr>
<tr>
<td>S</td>
<td>Area - Wing</td>
</tr>
<tr>
<td>$T_R$</td>
<td>Residual Thrust</td>
</tr>
<tr>
<td>$V_{A-}$</td>
<td>Axial velocity into duct</td>
</tr>
<tr>
<td>$V_{R-}$</td>
<td>Side velocity at duct entrance parallel to X-Z plane</td>
</tr>
<tr>
<td>$V_{S-}$</td>
<td>Side velocity at duct entrance parallel to Y-Axis</td>
</tr>
<tr>
<td>$V_E$</td>
<td>Jet exhaust velocity (see Appendix G)</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>Moment arm from center of gravity along X-Axis</td>
</tr>
<tr>
<td>$\ddot{y}$</td>
<td>Moment arm from center of gravity along Y-Axis</td>
</tr>
<tr>
<td>$\ddot{z}$</td>
<td>Moment arm from center of gravity along Z-Axis</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>$\alpha_D$</td>
<td>Arc tan ($V_{R-}/V_{A-}$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Sideslip angle</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>Arc tan ($V_{S-}/V_{A-}$)</td>
</tr>
<tr>
<td>$\delta_D$</td>
<td>Angular displacement</td>
</tr>
<tr>
<td>$\lambda \varepsilon$</td>
<td>Downwash (+) or upwash (-)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass density of air</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>Mass density of jet efflux</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Sidewash angle</td>
</tr>
<tr>
<td>D</td>
<td>Duct (Subscript)</td>
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Table 5. Definition of Symbols (Cont'd.)
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<tbody>
<tr>
<td>F</td>
<td>Elevator</td>
</tr>
<tr>
<td>F</td>
<td>Fan, fuel or fuselage</td>
</tr>
<tr>
<td>HT</td>
<td>Horizontal tail</td>
</tr>
<tr>
<td>I</td>
<td>Interference</td>
</tr>
<tr>
<td>IV</td>
<td>Inlet vanes</td>
</tr>
<tr>
<td>j</td>
<td>Denotes port (j = 1) or starboard (j = 2)</td>
</tr>
<tr>
<td>M.G.C.</td>
<td>Mean geometric chord</td>
</tr>
<tr>
<td>N</td>
<td>Centerbody nose, nose or normal</td>
</tr>
<tr>
<td>OL</td>
<td>Zero lift</td>
</tr>
<tr>
<td>R</td>
<td>Radial or rudder</td>
</tr>
<tr>
<td>RC</td>
<td>Reaction control</td>
</tr>
<tr>
<td>T</td>
<td>Test or thrust</td>
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<tr>
<td>TV</td>
<td>Trim vane</td>
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<td>VH</td>
<td>Horizontal vane</td>
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<td>VT</td>
<td>Vertical tail</td>
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<tr>
<td>VV</td>
<td>Vertical vane</td>
</tr>
<tr>
<td>W</td>
<td>Wing</td>
</tr>
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</table>

Table 5. Definition of Symbols (Cont'd.)
\[ i_D = i_w + \delta_D \]

- \( i_w \) = wing incidence angle
- \( \delta_D \) = change in duct angle
- \( i_D \) = duct incidence angle

**Figure 9. Duct Tilt Axis**
VA Axial velocity into duct
VR parallel to x-z plane
VS parallel to y-axis

\[ \alpha_D = \tan^{-1}\left(\frac{V_R}{V_A}\right) \]
\[ \rho_D = \tan^{-1}\left(\frac{V_S}{V_A}\right) \]

x, y, z are body axes.

Figure 10. Duct Fan Axis
Before developing the aerodynamic forces and moments it should be noted that the VA-141D has gyroscopic effects due to the rotating masses in the duct fan. In the following expressions \( i_w \) is the fixed wing incidence angle and \( \delta_D \) is the variable duct angle. For the ducted fans we have:

\[
\begin{align*}
&+ (I_{E-E} \cos(i_w + \delta_D) - qI(I_{E-E}) \sin (i_w + \delta_D) ) \\
&+ p(I_{E-E} \sin(i_w + \delta_D) + r(I_{E-E}) \cos (i_w + \delta_D) \\
&- (I_{E-E} \sin(i_w + \delta_D) + qI(I_{E-E}) \cos (i_w + \delta_D)
\end{align*}
\]

for \( I_a \) term.

These expressions will be added to the internal moments in the final equations for the VZ-141D. Next the forces and moments-- \( X_a, Y_a, Z_a, L_a, M_a, \) and \( N_a \) will be developed for the tilt-duct so that a complete set of equations of motion can be written.

AERODYNAMIC FORCES AND MOMENTS VZ-141D. In order to develop expressions for the aerodynamic external forces and moments, contributions from the major airframe components will be considered separately. The major components to be considered are the wing, the ducted fans, the vertical stabilizer and rudder, the horizontal stabilizer and elevator, the fuselage and the reaction controls. As with the XC-142A after the aerodynamic force and moment expressions are developed for each of these major components, they will be summed to get the total aerodynamic force and moment expressions.

Wing. The calculation of wing aerodynamic forces and moments is relatively straightforward. In Reference 8 a definition of the wing aerodynamic coefficients is developed. These coefficients contain interference and damping effects. The aerodynamic coefficients for the wing in accordance with Reference 8 are as follows:

\[
\begin{align*}
(C_D)_w &= (C_D)_o + \frac{3C_D}{2(C_L)} \cdot C^2_{Lg} \\
(C_L)_w &= (C_L)_o + (C_L) \cdot q_w \\
(C_y)_w &= (C_y)_o \cdot q_w + (C_y) \cdot \frac{b}{2V_w} \cdot p_w + (C_y) \cdot \frac{b}{2V_w} \cdot r_w \\
(C_z)_w &= (C_z)_o \cdot q_w + \frac{b}{2V_w} (C_z) \cdot p_w + \frac{b}{2V_w} (C_z) \cdot r_w + (C_z) \cdot 8A \cdot 5A \\
(C_m)_w &= (C_m)_o \cdot \frac{c}{2V_w} (C_m)_o \cdot q_1 + \frac{3C_m}{3\delta F} \cdot \delta F
\end{align*}
\]

(4.72)  
(4.73)  
(4.74)  
(4.75)  
(4.76)
In Figure 8 we see how $a_w$ is defined. For purposes of clarity the wing incidence angle, $i_w$, has been exaggerated. $\Delta V$ is the induced velocity previously defined in equation (4.34) which is derived from wash effects across the wing. In Figure 8, we see that:

$$V_w^2 = (u_w + \Delta V)^2 + v_w^2 + w_w^2.$$  

So that:

$$a_w = \tan^{-1}\left(\frac{w_w}{u_w + \Delta V}\right) = \sin^{-1}\frac{w_w}{V_w \cos \beta_w}$$  

and:

$$\beta_w = \tan^{-1}\left[\frac{V_w}{\sqrt{(u_w + \Delta V)^2 + w_w^2}}\right]$$  

In equation (4.78) and (4.79) $V_w = V$ and at low velocities where $\Delta V \approx 0$, then $V_w = V$. 

In order to transform wing data to body axis we rotate through an angle where $\xi = (i_w - a_w)$.

Before writing force and moment expressions for the wing, equations (4.72) through (4.77) are transformed to body axes by rotating through the angle $\xi$ as in the similar analysis for the XC-142A (Section IV).

$$(C_x)_w = (C_L)_w \sin \xi - (C_D)_w \cos \xi$$  

$$(C_z)_n = -(C_L)_w \cos \xi - (C_D)_w \sin \xi$$  

$$(C_x)_w = (C_L)_w \cos \xi - (C_n)_w \sin \xi - (C_y)_w \left(\frac{x_{a,c}}{c}\right)$$  

$$(C_m)_w = (C_m)_w + \frac{z_{a,c}}{c} (C_x)_w - \frac{x_{a,c}}{c} (C_z)_w$$  

$$(C_n)_w = (C_n)_w \cos \xi + (C_z)_w \sin \xi + (C_y)_w \left(\frac{x_{a,c}}{c}\right)$$  

$x_{a,c}$ and $z_{a,c}$ are the respective distances from the c.g. of the aircraft in aircraft body axes to the aerodynamic center (a.c.) of the wing in the x-z plane. $c$ is the mean aerodynamic chord.

As in the tilt-wing example, (Section IV) Equations (4.80) through (4.84) are readily expressed as force and moment contributions of the wing.
In equations (4.85) through (4.90) $S$ is the wing area, $c$ is the mean aerodynamic chord, $b$ is the wing span and $q_w$ is the wing dynamic pressure ($q_w = 1/2 \rho V_w^2$). These equations will be incorporated in the total force and moment equations.

**Ducted Fan.** In the calculation of ducted fan aerodynamic forces and moments wake, interference and damping are included in the definition of the aerodynamic coefficients. Six coefficients $C_{Tj}$, $C_{Nj}$, $\tilde{C}_{Sj}$, $\tilde{C}_{m_j}$, $\tilde{C}_{Lj}$, and $\tilde{C}_{Nj}$ are defined in Reference 8 to describe the behavior of the ducted fan. These coefficients are as follows:

$$
\bar{C}_T = \bar{C}_{Tj} + \bar{C}_{Tj} + \Delta \bar{C}_{TIVj} \quad (4.91)
$$

$\bar{C}_T$ is the total thrust coefficient. $\bar{C}_{Tj}$ is the change in thrust due to varying thrust, $\bar{C}_{Tj}$ is the change in thrust due to varying interference effects and $\Delta \bar{C}_{TIVj}$ is the change in thrust due to the inlet vanes to the duct. $j=1$ for the port duct and $j=2$ for the starboard duct.

$$
\bar{C}_{Nj} = \bar{C}_{NFj} + \bar{C}_{NDj} + \bar{C}_{NIj} + \bar{C}_{Nj} + \bar{C}_{NIVj} + \bar{C}_{NTVj} \quad (4.92)
$$

$\bar{C}_{Nj}$ is the total normal force coefficient. $\bar{C}_{NFj}$ results from fan
effects, \( \tilde{C}_{N_{D_j}} \) results from duct effects, \( \tilde{C}_{N_{I_j}} \) comes from interference effects, \( \tilde{C}_{N_{N_j}} \) is the change in normal force due to varying normal force, \( \tilde{C}_{N_{IV_j}} \) is the change in normal force due to the inlet vanes, and finally \( \tilde{C}_{N_{TV_j}} \) is the change in normal force due to the trim vanes.

\[
\tilde{C}_{S_j} = \tilde{C}_{S_{F_j}} + \tilde{C}_{S_{D_j}} + \tilde{C}_{S_{N_j}} + \tilde{C}_{S_{IV_j}} \tag{4.93}
\]

\( \tilde{C}_{S_j} \) is the side force coefficient. \( \tilde{C}_{S_{F_j}} \) results from fan effects, \( \tilde{C}_{S_{D_j}} \) results from duct effects, \( \tilde{C}_{S_{N_j}} \) is the change in side force due to varying normal force and \( \tilde{C}_{S_{IV_j}} \) is the change in side force due to inlet vanes.

\[
\tilde{C}_{M_j} = \tilde{C}_{M_{F_j}} + \tilde{C}_{M_{D_j}} + \tilde{C}_{M_{I_j}} + \tilde{C}_{M_{N_j}} + \tilde{C}_{M_{IV_j}} + \tilde{C}_{M_{TV_j}} \tag{4.94}
\]

\( \tilde{C}_{M_j} \) is the total pitching moment coefficient for the ducted fan.

\( \tilde{C}_{M_{F_j}} \), \( \tilde{C}_{M_{D_j}} \), \( \tilde{C}_{M_{I_j}} \), result respectively from fan, duct and interference effects.

\( \tilde{C}_{M_{N_j}} \) is the change in pitching moment due to varying normal force.

\( \tilde{C}_{M_{IV_j}} \) and \( \tilde{C}_{M_{TV_j}} \) are respectively the changes in pitching moment due to inlet and trim vanes.

\[
\tilde{C}_{L_j} = - \left( -1 \right)^j \left( \frac{Y_D}{D} \right) \left[ \tilde{C}_{T_j} \sin i_D + \tilde{C}_{N_j} \cos i_D \right] \left( \frac{1}{2} \sin \theta_{D_j} \right) \left( \tilde{C}_{T_{D_j}} \sin i_D \right)
\]

\[
- \left[ \left( \frac{-F}{D} \right) \tilde{C}_{S_{F_j}} + \left( \frac{-X}{D} \right) \tilde{C}_{S_{D_j}} + \left( \frac{X_{IV_j}}{D} \right) \tilde{C}_{S_{IV_j}} \right] \sin i_D + K_1 \sin i_d \tag{4.95}
\]

\( \tilde{C}_{L_j} \) is the total rolling moment coefficient for the ducted fan.
$k_1$ is a function of axial advance ratio of the ducted fan.

\[
\tilde{\eta}_j = -(-1)^j \left( \frac{y_D}{D} \frac{c}{T_j} \cos i_D - \tilde{C}_{N_j} \sin i_D \right) \left( \frac{1}{2} \sin \beta_{D_j} \right) \left( \tilde{C}_{T_{D_j}} \cos i_D \right) \\
- \left( \frac{x_F}{D} \tilde{C}_{S_{F_j}} + \left( \frac{x_D}{D} \tilde{C}_{S_{D_j}} \right) + \left( \frac{x_{IV}}{D} \tilde{C}_{S_{IV_j}} \right) \cos i_D \right) + k_1 \cos i_D
\]  

$\tilde{\eta}_j$ is the total yawing moment coefficient for the ducted fan.

Before writing the force and moment equations these coefficients are re-expressed as coefficients representing both ducted fans.

\[
\left( \tilde{C}_x \right)_D = k_2 \sum_{j=1}^{2} \left( \tilde{C}_{T_{j}} \cos i_D - \tilde{C}_{N_{j}} \sin i_D \right) 
\]

\[
\left( \tilde{C}_z \right)_D = -k_2 \sum_{j=1}^{2} \left( \tilde{C}_{T_{j}} \sin i_D + \tilde{C}_{N_{j}} \cos i_D \right) 
\]

\[
\left( \tilde{C}_y \right)_D = k_2 \sum_{j=1}^{2} \tilde{C}_{S_{j}} 
\]

\[
\left( \tilde{C}_t \right)_D = -k_2 \frac{D}{b} \sum_{j=1}^{2} \tilde{C}_{L_{j}} 
\]

\[
\left( \tilde{C}_m \right)_D = k_2 \frac{D}{c} \sum_{j=1}^{2} \tilde{C}_{N_{j}} 
\]

\[
\left( \tilde{C}_n \right)_D = k_2 \frac{D}{b} \sum_{j=1}^{2} \tilde{C}_{N_{j}} 
\]

The equations for duct forces and moments are then as follows:

\[
(\Delta X_a) = k_3 \left( \tilde{C}_x \right)_D 
\]

\[
(\Delta Y_a) = k_3 \left( \tilde{C}_y \right)_D 
\]
(\Delta Z_a) = K_3 (\bar{c}_z) \quad K_3 = \frac{\rho}{2} (\frac{b}{n} b^2) S \quad (4.105)

(\Delta L_a) = K_3 b (\bar{C}_l) \quad (4.106)

(\Delta M_a) = K_3 c (\bar{C}_m) \quad (4.107)

(\Delta N_a) = K_3 t (\bar{C}_n) \quad (4.108)

Equations (4.103) to (4.108) will be included in the total aerodynamic force and moment expressions.

Vertical Stabilizer and Rudder. Forces and moments for the vertical tail and rudder will be defined in the same manner as for the XC-142A and Reference 8. The associated coefficients are as follows:

\[
C_y = C_{y \beta} + \beta + C_y \delta R
\]

\[
C_l = C_{l \beta} + \beta + C_l \delta R + \frac{pb}{2V_B} C_p + \frac{rb}{2V_B} \ C_r
\]

\[
C_n = C_{n \beta} + \beta + C_n \delta R + \frac{pb}{2V_B} C_p + \frac{rb}{2V_B} \ C_r
\]

The forces and moments for the vertical tail can then be expressed in the following equations.

(\Delta Y_a)_{VT} = C_y b \sqrt{T} \left(\frac{q_{VT}}{q}\right) \quad (4.112)

(\Delta L_a)_{VT} = C_l b \sqrt{T} \left(\frac{q_{VT}}{q}\right) \quad (4.113)

(\Delta N_a)_{VT} = C_n b \sqrt{T} \left(\frac{q_{VT}}{q}\right) \quad (4.114)

This force and these moments will be included in the total aerodynamic forces and moments.

Horizontal Stabilizer and Elevator. The aerodynamic forces and moments for the horizontal stabilizer will be defined in a similar manner as for the XC-142A. Elevator movement is considered in the z-force,
and the pitching moment. The angle of attack of the tail $\alpha_t$ will be a
function of interference damping terms, and considerations of downwash.
The lift and drag coefficients of the tail can be expressed in the following relations:

$$C_{L_t} = C_{L_{\alpha_t}} \cdot \alpha_t$$  \hspace{1cm} (4.115)

$$C_{D_t} = C_{L_t} + K$$ \hspace{1cm} (4.116)

The horizontal stabilizer (HS) can contribute forces in the x and z directions and a pitching moment. The equations are as follows:

$$(AX_a)_{HS} = [-C_{D_{\alpha_t}} \cos (i_t - \alpha_t) - C_{L_{\alpha_t}} \sin (i_t - \alpha_t)] \frac{q_{HS}}{q}$$ \hspace{1cm} (4.117)

$$(AZ_a)_{HS} = [C_{D_{\alpha_t}} \sin (i_t - \alpha_t) - C_{L_{\alpha_t}} \cos (i_t - \alpha_t)] \frac{q_{HS}}{q}$$ \hspace{1cm} (4.118)

$$\cdot qScC_{m_{\delta E}}$$

$$(A\alpha_a)_{HS} = -qScC_{m_{\delta E}} \cdot \delta E$$ \hspace{1cm} (4.119)

$\delta E$ is the distance from the aircraft c.g. to the aerodynamic center of the horizontal stabilizer and $h_{HS}$ is the height of the a.c. above the c.g. $i_t$ is the incidence angle of the tail. Equations (4.117) to (4.119) will be included in the total aerodynamic force and moment expressions.

Fuselage. In a very direct way we can write the effects of the fuselage (F) on the total aerodynamic forces and moments. They are as follows:

$$(AX_a)_F = -qS \cdot C_{D_0}$$ \hspace{1cm} (4.120)

$$(AY_a)_F = qS \cdot C_{Y_B} \cdot \beta$$ \hspace{1cm} (4.121)

$$(AZ_a)_F = -qS \cdot C_{L\alpha} \cdot \alpha$$ \hspace{1cm} (4.122)
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\[(\Delta m_a)^F = q \frac{Sc}{C_{m o}} + C_{m a} \cdot \theta \]  \hspace{1cm} (4.121)

\[(\Delta m_a)^F = q \frac{Sc}{C_{n \theta}} \cdot \theta \]  \hspace{1cm} (4.124)

Equations (4.120) to (4.124) will be included in the total aerodynamic forces and moments.

Reaction Controls. The control vanes (horizontal (VH) and vertical (VV)) are located in the exhaust flow of the jet engine. The reaction control (RC) vanes provide longitudinal and lateral control in the hover mode. The following coefficients are defined as in Reference 1.

\[C_{L_{VH}} = k_{VH} \]

\[C_{L_{VV}} = k_{VV} \]

\[C_{D_{VH}} = k \ C_{L_{VH}} \]

\[C_{D_{VV}} = k \ C_{L_{VV}} \]

\[T_r = \dot{m} (V_E - U) \text{ -- residual thrust (T_r)} \]

\[C_{m_{RC}} = \frac{\ddot{x}_{VH}}{c_v} C_{L_{VH}} - \frac{\ddot{x}_{VV}}{c_v} C_{D_{VV}} \]  \hspace{1cm} (4.125)

\[C_{n_{RC}} = \frac{\ddot{x}_{VV}}{r_v} C_{L_{VV}} - \frac{\ddot{r}_{VV}}{r_v} C_{n_{VV}} \]  \hspace{1cm} (4.126)

\[\ddot{x}_{VH}, \ddot{x}_{VV}, \text{ and } \ddot{r}_{VV} \text{ are moment arms to control vanes.} \]

\[C_x = - \left[ C_{D_{VH}} + C_{D_{VV}} \right] \]  \hspace{1cm} (4.127)

\[C_y_{RC} = - C_{L_{VV}} \]  \hspace{1cm} (4.128)

\[C_z_{RC} = - C_{L_{VH}} \]  \hspace{1cm} (4.129)

The force and moment equations then follow from equations (4.125) to (4.129) as:
Equations (14.130) to (14.134) will be included in the total aerodynamic forces and moments.

EQUATIONS OF MOTION VZ-HDA. The aerodynamic force and moment terms developed for the VZ-HDA for each of the major aircraft components will be combined and set equal to their corresponding internal force and moment expressions developed in axis system section for the VZ-HDA.

Total Aerodynamic Forces and Moments. These expressions are as follows:

\[ X_a = (\Delta X_a) + (\Delta X_{aw}) + (\Delta X_{awD}) + (\Delta X_{awFS}) + (\Delta X_{awRC}) \]

\[ Y_a = (\Delta Y_a) + (\Delta Y_{aw}) + (\Delta Y_{awD}) + (\Delta Y_{awVT}) + (\Delta Y_{awF}) + (\Delta Y_{awRC}) \]

\[ Z_a = (\Delta Z_a) + (\Delta Z_{aw}) + (\Delta Z_{awD}) + (\Delta Z_{awHS}) + (\Delta Z_{awF}) + (\Delta Z_{awRC}) \]

\[ L_a = (\Delta L_a) + (\Delta L_{aw}) + (\Delta L_{awF}) + (\Delta L_{awRC}) \]

\[ M_a = (\Delta M_a) + (\Delta M_{aw}) + (\Delta M_{awD}) + (\Delta M_{awHS}) + (\Delta M_{awF}) + (\Delta M_{awRC}) \]

\[ N_a = (\Delta N_a) + (\Delta N_{aw}) + (\Delta N_{awD}) + (\Delta N_{awVT}) + (\Delta N_{awF}) + (\Delta N_{awRC}) \]

Equations of Motion.- These equations are subject to simplification in the interference region and in the representation of duct fan forces and moments when further flight test data is available for the determination of typical magnitudes of terms.
1. X Force Equation
\[
\begin{align*}
(t - a_t) \cos t + (i_t - a_t) \sin t \cdot \frac{q_{HS}}{q} \\
(\cos (i - a) + \sin (i - a) \cdot \frac{q_{HT}}{q}) \\
+ (C_x) S, q_w + K_3(C_x) - q_t S C_{D_0} + q_E S_y(C_x) + T_r \\
= m(\dot{v} + \omega q_T - V_r) + m v \sin \theta
\end{align*}
\]

2. Y Force Equation
\[
\begin{align*}
(C_y) S, q_w + K_3(C_y) - q y \cdot \frac{q_{VT}}{q} + q_t S C_y \cdot \beta + q_E S_y(C_y) \\
= m(\dot{V} + U_T - W_T) - m g \cos \theta \sin \phi
\end{align*}
\]

3. Z Force Equation
\[
\begin{align*}
(C_D \sin (i_t - a_t) - C_{L_t} \cos (i_t - a_t)) \cdot \frac{q_{HS}}{q} \\
+ (C_z) S, q_w + K_3(C_z) \\
+ q S C m_{E} \cdot \delta_E - q S C_{L_a} \cdot \alpha + q_E S_y(C_z) \\
= m(\dot{W} + V_T - U q_T) - m g \cos \theta \cos \phi
\end{align*}
\]

4. Roll Equation
\[
\begin{align*}
(C_t) b S, q_w + K_3(b(C_t) - C_t) b \cdot \frac{q_{VT}}{q} \\
= I_{11} \dot{\delta} - I_{13} (\dot{r} + p q_1) + (I_{33} - I_{22}) q_1 r \\
+ (I_E q_T) \cos (i_T + \delta_D) - q_1 (I_E q_T) \sin (i_T + \delta_D)
\end{align*}
\]

5. Pitch Equation
\[
\begin{align*}
(C_m) c S, q_w + K_3(c(C_m) - q S(c + C_m a) + q_E S y(c(C_m) \\
- (\Delta X_a) \cdot h_{HS} + (\Delta Z_a) \cdot i_{HS} + \frac{c^2 S_p}{L_t} [C_m \cdot q_{11} V + C_m a \cdot \dot{W}] \\
= I_{22} \dot{\delta} - I_{13} (r^2 + p^2) + (I_{11} - I_{33}) \alpha
\end{align*}
\]

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6. Yaw Equation

\[ p(I_p^E_E) \sin (i_w + \delta_D) + r(I_p^E_E) \cos (i_w + \delta_D) \]

No simplifications have been made on the right side of these six equations.

TILT PROPELLER

The Curtiss-Wright X-19 V/STOL aircraft will be used as the example in the development of tilt propeller (tilt prop) V/STOL simulation equations of motion. The development which follows is based upon the type of arguments presented for the tilt-wing and information gleaned from Reference 4.

In order to gain some idea of the X-19 let us consider Figure 3 and Table 6. In Figure 3, a three-view arrangement of the X-19 is shown. Table 6 is a list of definition of symbols contained in the X-19 equations.

With this general idea of the X-19, we may now develop the mathematical model for the X-19. First we define, as necessary applicable axis systems. Second, develop aerodynamic coefficients. Finally, write the equations of motion for the X-19.

AXIS SYSTEMS FOR X-19. It appears that the forward and aft wing chords are parallel to the x-body axis of the aircraft. In this aircraft the aft wing functions as the horizontal stabilizer with elevator. The total velocity of the wing, \( V_w \), can be expressed easily in aircraft body axes. In addition separate axis systems to describe propeller velocities are not necessary since the propeller velocities can be expressed in terms of aircraft body axis velocities (\( U, V, W \)). Consequently, no additional axis systems will be necessary other than the conventional inertial, body, stability or wind axes.

In addition, there will be no gyroscopic effects from the rotating masses of the propellers and their drive shafts. The four propellers are mechanically linked to rotate at the same speed and the starboard propeller turns in the opposite direction of the port propellers. If the propellers failed these could be gyroscopic effects.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>F</td>
<td>Fuselage</td>
</tr>
<tr>
<td>w</td>
<td>Wing</td>
</tr>
<tr>
<td>n</td>
<td>Main propellers n = 1, 2, Front. n = 3, 4 Aft. Top view, left to right</td>
</tr>
<tr>
<td>vt</td>
<td>Vertical Tail</td>
</tr>
<tr>
<td>$\alpha_F$</td>
<td>Angle of attack - fuselage</td>
</tr>
<tr>
<td>$\alpha_{wf}$</td>
<td>Angle of attack - wing forward</td>
</tr>
<tr>
<td>$\alpha_{wa}$</td>
<td>Angle of attack - wing aft</td>
</tr>
<tr>
<td>$\beta_F$</td>
<td>Sideslip angle - fuselage</td>
</tr>
<tr>
<td>$\beta_{wf}$</td>
<td>Sideslip angle - wing forward</td>
</tr>
<tr>
<td>$\beta_{wa}$</td>
<td>Sideslip angle - wing aft</td>
</tr>
<tr>
<td>ip</td>
<td>Incidence angle - propeller nacelle</td>
</tr>
<tr>
<td>$V_B$</td>
<td>Total velocity in aircraft body axes</td>
</tr>
<tr>
<td>$V_w$</td>
<td>Total velocity in wind stability axes</td>
</tr>
<tr>
<td>c</td>
<td>Mean aerodynamic chord</td>
</tr>
<tr>
<td>b</td>
<td>Wing span</td>
</tr>
<tr>
<td>S</td>
<td>Total area of wing</td>
</tr>
<tr>
<td>$q_{wf}$</td>
<td>Dynamic pressure at forward wing due to power-on effects</td>
</tr>
<tr>
<td>$q_{wa}$</td>
<td>Dynamic pressure at aft wing due to power-on effects</td>
</tr>
<tr>
<td>q</td>
<td>Dynamic pressure - free stream</td>
</tr>
<tr>
<td>$q_{vt}$</td>
<td>Dynamic pressure at vertical tail</td>
</tr>
<tr>
<td>T</td>
<td>Total thrust of main propellers</td>
</tr>
<tr>
<td>$J_n$</td>
<td>Advance ratio main propeller</td>
</tr>
<tr>
<td>$J_n^1$</td>
<td>Advance ratio normal to propeller disk</td>
</tr>
<tr>
<td>$N_n = N$</td>
<td>RPM - main propeller</td>
</tr>
</tbody>
</table>

Table 6. Definition of Symbols Used in X-19 Equations
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td>Nominal RPM - main propeller</td>
</tr>
<tr>
<td>$\beta_n$</td>
<td>Blade pitch angle - main propeller</td>
</tr>
<tr>
<td>$T_n$</td>
<td>Main propeller thrust</td>
</tr>
<tr>
<td>$N_n$</td>
<td>Main propeller thrust component normal to $T_n$.</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Main propeller moment (initially pitching)</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Main propeller torque coefficient</td>
</tr>
<tr>
<td>$D_n$</td>
<td>Diameter main propeller</td>
</tr>
</tbody>
</table>

Table 6. Definition of Symbols Used in X-19 Equations (Cont'd.)
AERODYNAMIC FORCES AND MOMENTS - X-19. In order to develop expressions for $X_a$, $Y_a$, $Z_a$, $L_a$, $M_a$, and $N_a$, contributions from the major airframe components of the X-19 will be considered separately as was done for the tilt-wing aircraft. The major components to be considered are the propellers, the forward wing and aft wings, the vertical tail and rudder, and the fuselage.

Propeller. The inclination of the propeller nacelles from the $x$-body axis is defined by propeller shafts, angle of tilt, $i_p$. In Figure 3 we see the angle $i_p$ and the geometry defining the total propeller velocity, $V_n$. For the velocity, $V_n$, $n$ denotes the particular propeller ($n = 1, 2, 3, 4$). The front left propeller is number 1, the front right propeller is number 2, the rear left propeller is number 3 and the rear right propeller is number 4. Each propeller nacelle is located by $x_n$, $y_n$ and $z_n$ body axis components. Due to the symmetric placement of the propellers: $x_1 = -x_2, x_3 = -x_4, y_1 = -y_2, y_3 = -y_4, z_1 = -z_2$ and $z_3 = -z_4$. These body axes moment arms ($s_n$, $y_n$ and $z_n$) are multiplied by the appropriate body axis angular velocity ($p$, $q_1$ and $r$) in order to give tangential velocity components of the propeller velocity components.

$$u_n = U - y_n(p \sin i_p + r \cos i_p) - q_1(x_n \sin i_p - z_n \cos i_p)$$ \hspace{1cm} (4.135)

$$v_n = V + x_n r$$ \hspace{1cm} (4.136)

$$w_n = W + y_n(p \cos i_p - r \sin i_p) - q_1(x_n \cos i_p + z_n \sin i_p)$$ \hspace{1cm} (4.137)

We will now define the propeller aerodynamic coefficients $C_{T_n}$, $C_p$, $C_N$ and $C_M$. These coefficients are the same as those for the main propellers of the XC-142A, however there is no $C_y$ for the X-19, since the port propellers turn in an opposite direction to the starboard propellers. First let us define the advance ratio ($J_n$) for each propeller and the advance ratio normal to the propeller disk ($J'_n$)

$$J_n = \frac{60 V_n}{N_n D},$$

$$J'_n = J_n \cos \psi_n$$ \hspace{1cm} (4.138)
The symbol $n_n$ (the particular propeller RPM and since all propellers are mechanically linked to turn at the same rate $n = n$). The number of changes RPM to RPS, $D$ is the diameter of the propeller disk and $R_n$ is defined as the blade pitch angle of the particular propeller. The aerodynamic coefficients can then be developed in terms of advance ratio and blade pitch. The angle $\psi_n$ is the angle between the propeller thrust vector and $v_n$ as shown in Figure 7. $\psi_n$ is defined the same as for the tilt wing as is $n_n$—equations (4.1) and (4.2).

\[
C_{T_n} = C_{T_0} + \frac{\partial C_T}{\partial J_n} J_n' + \frac{\partial^2 C_T}{\partial R_n \partial J_n} R_n J_n' + \frac{\partial^2 C_T}{\partial \beta_n \partial J_n} \beta_n J_n' \tag{4.139}
\]

\[
C_{P_n} = C_{P_0} + \frac{\partial C_P}{\partial J_n} J_n' + \frac{\partial^2 C_P}{\partial B_n \partial J_n} B_n J_n' + \frac{\partial^2 C_P}{\partial \beta_n \partial J_n} \beta_n J_n' \tag{4.140}
\]

\[
C_{N_n} = \frac{3}{\partial R_n} \left[ \frac{\partial (C_n \cot \psi)}{\partial J_n} \right] R_n J_n' \sin \psi_n \tag{4.141}
\]

\[
C_{M_n} = \frac{3}{\partial J_n} \left[ \frac{\partial C_M}{\partial \psi_n} \right] \psi_n J_n' \tag{4.142}
\]

$C_{T_n}$, $C_{P_n}$, $C_{M_n}$ are defined similar to equations (4.7), (4.8), (4.9) and (4.11). Since blade pitch is the prime control parameter in the X-19, the second partial derivative $\frac{\partial^2}{\partial \beta_n \partial J_n}$ is retained in equations (4.139) and (4.140) whereas $\frac{\partial^2}{\partial J_n^2}$ is neglected. $C_{T_n}$ is the coefficient of thrust ($T_n$), $C_{P_n}$ is the coefficient of power used to express torque ($C_n$), $C_{N_n}$ is the coefficient of normal thrust ($N_n$) and $C_{M_n}$ is the longitudinal propeller hub moment coefficient ($M_n$).

Before expressing the force and moment contributions in body axes due to the propellers, we will define $T_n$, $N_n$, $M_n$ and $Q_n$ (Reference 6).

\[
T_n = \beta_n \left( \frac{N}{N_o} \right) \left( \frac{\rho}{\rho_o} \right) C_{T_n} \tag{4.143}
\]

\[
N_n = \beta_n \left( \frac{N}{N_o} \right) \left( \frac{\rho}{\rho_o} \right) C_{N_n} \tag{4.144}
\]
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\[ M_n = \frac{D^5}{N_o} \left( \frac{N}{N_o} \right) \frac{\rho_n}{\rho_o} C_{M_n} \quad (4.145) \]

\[ Q_n = \frac{D^5}{2} \left( \frac{N}{N_o} \right) \frac{\rho_n}{\rho_o} C_{p_n} \quad (4.146) \]

\( N_o \) is the maximum RPM of the propellers and \( \rho_o \) is the air density at sea level on a standard day.

From equations (4.143) through (4.146) we can then write the propeller force and moment expressions in a manner similar to equations (4.17) through (4.22) of the XC-142A.

\[ (\Delta X_a)_p = \sum_{n=1}^{\text{L}} \left( T_n \cos \theta_p - N_n \cos \theta_n \sin \phi_p \right) \quad (4.147) \]

\[ (\Delta Y_a)_p = \sum_{n=1}^{\text{L}} (-N_n \sin \theta_n) \quad (4.148) \]

\[ (\Delta Z_a)_p = \sum_{n=1}^{\text{L}} (-T_n \sin \theta_p - N_n \cos \theta_n \cos \phi_p) \quad (4.149) \]

\[ (\Delta L_a)_p = -\left[ (\Delta Z_a)_p \right] y_1 - \left[ (\Delta Z_a)_p \right] y_2 - \left[ (\Delta Z_a)_p \right] y_3 + \left[ (\Delta Y_a)_p \right] z_1 + \left[ (\Delta Y_a)_p \right] z_2 + \left[ (\Delta Y_a)_p \right] z_3 \]

\[ \sum_{n=1}^{\text{L}} \frac{M_n \sin \theta_n \sin \phi_p}{n} \quad (4.150) \]

\[ (\Delta M_a)_p = x_1(T_1 + T_2) \sin \phi_p - x_3(T_3 + T_4) \sin \phi_p \]

\[ - z_1(T_1 + T_2) \cos \phi_p - z_3(T_3 + T_4) \cos \phi_p \]

\[ + z_1(N_1 \cos \theta_1 + N_2 \cos \theta_2) \sin \phi_p \]

\[ + z_3(N_3 \cos \theta_3 + N_4 \cos \theta_4) \sin \phi_p \]

\[ + x_1(N_1 \cos \theta_1 + N_2 \cos \theta_2) \cos \phi_p \quad (4.151) \]
- $x_3(N_3 \cos \beta_3 + N_4 \cos \beta_4) \cos \gamma_p$

$$+ \frac{1}{h} \sum_{n=1}^{N_n} M_n \cos \beta_n$$

$$(\Delta N_p) = [(x_{a1} - (x_{a2}) y_1 + [(x_{a3} - (x_{a4}) y_3$$

$$+ [(y_{a1} + (y_{a2}) x_1 - [(y_{a3} + (y_{a4}) x_3 \frac{1}{h} \sum_{n=1}^{N_n} M_n \sin \beta_n \cos \gamma_p$$

Note that $x_1, x_3, y_1, y_3, z_1$ and $z_3$ are positive values. The propeller force and moment equations (4.147) through (4.152) will be included in the total aerodynamic forces and moments.

**Forward Wing.** The forward wing forces and moments are developed as for the XC-142A in Section IV. It is assumed that the wing incidence angle, $\alpha_W$, is zero and that the induced velocity, $\Delta V$, is defined as equation (4.34) when $\gamma_p$ is zero. $\Delta V$ will change as a function of $\cos \gamma_p$ and flap position when the aircraft is in hover and transition regions. Consequently, as a preliminary estimate, the wing velocity $u_{wf}$ is as follows:

$$u_{wf} = u_p + (\Delta V)_{TP} \quad (4.153)$$

where $(\Delta V)_{TP} = K \Delta V \cos \gamma_p$

For forward flight where $\gamma_p$ equals zero then $u_p$ equals $V_B$ and $K = 1$. $K$ is a function of flaps and interference effects as the propeller wash hits the wing at high $\gamma_p$ angles. $K$ and $\cos \gamma_p$ will tend to decrease $\Delta V$ as $\gamma_p$ increases. As in equation (4.36) $q_{wf}$ is defined similarly as:

$$q_{wf} = \left( q + \frac{2(T_1 + T_2)}{2\pi D} \right) \quad (4.154)$$

The forward wing aerodynamic coefficients are $C_D$, $C_L$, $(C_x)_{wf}$, $(C_m)_{wf}$ and $(C_n)_{wf}$. $C_D$ and $C_L$ defines $(C_x)_{wf}$ and $(C_z)_{wf}$ respectively.
Since the development of these coefficients is in wing axes, rolling and turning rates necessary to define these coefficients are transformed from body axes. The only difference is in the angle of attack of the wing, \( \alpha_{wf} \), which is from equation (4.32) as follows:

\[
\alpha_{wf} = \tan^{-1} \left( \frac{w_p}{u_p + (\Delta V)_p} \right) \tag{4.155}
\]

where \( w_{wf} = w_p; \) \( w = u_p \sin \theta \)

and \( u_{wf} = u_p + (\Delta V)_p; \) \( U = u_p \cos \theta \)

In a similar manner \( \beta_{wf} \) is defined as:

\[
\beta_{wf} = \tan^{-1} \frac{V}{V_{wf}} \tag{4.156}
\]

where \( v_p = v_{wf} = V \)

and \( V_{wf} = [w^2 + (U + \Delta V)_p^2 + v^2]^{1/2} \]

We write for the wing rolling rate \( (p_{wf}) \) and the wing turning rate \( (r_{wf}) \) the following equations:

\[
p_{wf} = p \cos \alpha_{wf} + r \sin \alpha_{wf} \tag{4.157}
\]

\[
r_{wf} = -p \sin \alpha_{wf} + r \cos \alpha_{wf} \tag{4.158}
\]

These are just equations (4.37) and (4.38) where \( \xi = -\alpha \) since \( i_w = 0 \) in \( \xi = (i_w - \alpha_{wf}). \)

Following the tilt wing the aerodynamic coefficients for the wing are as follows:

\[
C_D = C_{D_o} + C_{D\delta F} \cdot \delta F \tag{4.159}
\]

\[
C_L = C_{L_o} + C_{L\delta F} \cdot \delta F + C_{L_{\alpha F}} \cdot \alpha_{wf} \tag{4.160}
\]

\( C_D \) and \( C_L \) do not reflect as strong a flap dependence as in the XC-112A.

This is a preliminary estimate of \( C_D \) and \( C_L \) which may be modified by
further manufacturer data. \( (C_e)_{\text{nwf}} \), \( (C_m)_{\text{nwf}} \) and \( (C_n)_{\text{nwf}} \) will be defined as for the XC-142A.

\[
(C_e)_{\text{nwf}} = C_e - \beta_{\text{nwf}} + \frac{b}{2V_{\text{w}}} C_{\text{n}} \cdot P_{\text{nwf}} + \frac{b}{2V_{\text{w}}} C_{\text{n}} \cdot r_{\text{nwf}} \quad (4.161)
\]

\[
(C_m)_{\text{nwf}} = C_m + C_{\text{nf6F}} \cdot 5F + \frac{c}{2V_{\text{w}}} C_{\text{q1}} \quad (4.162)
\]

\[
(C_n)_{\text{nwf}} = C_n + \beta_{\text{nwf}} + \frac{b}{2V_{\text{w}}} C_{\text{n}} \cdot P_{\text{nwf}} + \frac{b}{2V_{\text{w}}} C_{\text{n}} \cdot r_{\text{nwf}} \quad (4.163)
\]

Equations (4.159) through (4.163) will now be written in body axes by rotating through an angle \((-\alpha_{\text{w}})\).

\[
(C_x)_{\text{nwf}} = -C_D \cos \alpha_{\text{nwf}} + C_L \sin \alpha_{\text{nwf}} \quad (4.164)
\]

\[
(C_z)_{\text{nwf}} = -C_D \sin \alpha_{\text{nwf}} - C_L \cos \alpha_{\text{nwf}} \quad (4.165)
\]

\[
(C_l)_{\text{nwf}} = (C_e)_{\text{nwf}} \cos \alpha_{\text{nwf}} - (C_n)_{\text{nwf}} \sin \alpha_{\text{nwf}} \quad (4.166)
\]

\[
(C_m)_{\text{nwf}} = (C_m)_{\text{nwf}} - \frac{Z_{\text{ac}}}{c} (C_x)_{\text{nwf}} - \frac{X_{\text{ac}}}{c} (C_z)_{\text{nwf}} \quad (4.167)
\]

\[
(C_n)_{\text{nwf}} = (C_e)_{\text{nwf}} \sin \alpha_{\text{nwf}} + (C_n)_{\text{nwf}} \cos \alpha_{\text{nwf}} \quad (4.168)
\]

\( X_{\text{ac}} \) and \( Z_{\text{ac}} \) are the respective distance from the c.g. of the aircraft in body axes to the aerodynamic center (a.c.) of the forward wing in the x-z plane. \( c \) is the mean aerodynamic chord of the forward wing.

The wing force and moment contributions can then be written from the coefficients expressed in equations (4.161) through (4.168). \( S \) is the forward wing area and \( q_{\text{w}} \) the dynamic pressure at the wing.

\[
(\Delta X_{\text{a}})_{\text{nwf}} = (C_x)_{\text{nwf}} S q_{\text{nwf}} \quad (4.169)
\]

\[
(\Delta Z_{\text{a}})_{\text{nwf}} = (C_z)_{\text{nwf}} S q_{\text{nwf}} \quad (4.170)
\]

\[
(\Delta L_{\text{a}})_{\text{nwf}} = (C_l)_{\text{nwf}} bS q_{\text{nwf}} \quad (4.171)
\]

\[
(\Delta M_{\text{a}})_{\text{nwf}} = (C_m)_{\text{nwf}} cS q_{\text{nwf}} \quad (4.172)
\]

\[
(\Delta N_{\text{a}})_{\text{nwf}} = (C_n)_{\text{nwf}} bS q_{\text{nwf}} \quad (4.173)
\]
Vertical Stabilizer and Rudder. The discussion is the same as for the XC-142A since there is obvious similarity in configuration of the vertical stabilizer for the propeller wash effects on the vertical stabilizer for the XC-142A and the X-19. Forces and moments for vertical tail (vt) and rudder arise from the relative wind pushing against the vertical tail surfaces thereby causing a turning moment due to control input to the rudder. This gives side force, as well as rolling and turning moments.

Wind pushing against the vertical stabilizer and rudder yields a side force \( (\Delta N_a)_{vt} \) and rolling moment \( (\Delta L_a)_{vt} \) respectively which are non-dimensionalized in terms of aerodynamic coefficients as \( C_y \) and \( C_\ell \).

The rolling moment is coupled in rolling velocity \( p \) and turning velocity \( r \). We can then write for \( C_y \) and \( C_\ell \) the following expressions.

\[
C_y = C_{y_b} \cdot \beta + C_{y_6R} \cdot \delta R
\]

\( C_{y_b} \) is the change in side force coefficient with changing sideslip angle. It acts as a damping term. \( C_{y_6R} \) is the change in side force coefficient with rudder deflection and represents the controllable term in the expression. This is important in the use of Automatic Stabilization Equipment (ASE).

\[
C_\ell = C_{\ell_b} \cdot \beta + C_{\ell_6R} \cdot \delta R + \frac{\Delta b}{2V_B} C_{\ell_p} + \frac{rb}{2V_B} C_{\ell_r}
\]

\( C_{\ell_b} \) is the change in rolling moment with variation in sideslip angle. \( C_{\ell_6R} \) is the change in rolling moment coefficient due to rudder deflection. \( C_{\ell_p} \) is the roll damping derivative. \( C_{\ell_r} \) is the change in rolling moment coefficient with change in yawing velocity.

Rudder deflection will give a turning moment \( (\Delta N_a)_{vt} \) which can be expressed by the aerodynamic coefficient \( C_n \).

\[
C_n = C_{n_b} \cdot \beta + C_{n_6R} \cdot \delta R + \frac{\Delta b}{2V_B} C_{n_p} + \frac{rb}{2V_B} C_{n_r}
\]

\( C_{n_b} \) is the weathercock or static directional derivative. \( C_{n_6R} \) rudder effectiveness, the change in yawing moment coefficient with rudder deflection. \( C_{n_p} \) is the change in yawing moment coefficient...
with varying rolling velocity. $C_{n_r}$ is the yaw damping derivative.

The tail is the main contribution to $C_{n_r}$.

The forces and moments for the vertical tail can then be expressed in the following equations:

$$(\Delta Y_a)_{vt} = C_y \frac{q_{vt}}{q}$$ (4.177)

$$(\Delta L_a)_{vt} = C_i bS_q \frac{q_{vt}}{q}$$ (4.178)

$$(\Delta N_a)_{vt} = C_n bS_q \frac{q_{vt}}{q}$$ (4.179)

Here $q$ is the free stream dynamic pressure and $q_{vt}$ is the vertical tail dynamic pressure. These force and moments will be included in the total aerodynamic forces and moments.

Aft Wing. The aft wing (wa) has two propellers placed in a similar configuration as the forward wing and acts as a combination wing and horizontal stabilizer. When the four propellers are simultaneously rotated by the angle $\theta_p$, the aft wing is either in or out of the front propeller wash. If the aft wing is out of the front propeller wash, the aft wing will be practically a duplicate of the forward wing in aerodynamics. If the aft wing is in the front propeller wash, the aft wing has induced velocity effects from its own and the front propellers. The description of downwash, as for the XC-112A Horizontal Stabilizer is not applicable to the X-19. However, since the aft wing has elevator control and no aileron control, the forces and moments developed will lie in the x-z body axes plane as for a horizontal stabilizer.

As for the forward wing, the aft wing incidence angle is assumed to be zero. Induced velocity effects for the aft wing are as follows:

$$u_{wa} = u_p + (\Delta V)_{TP}$$ (4.180)

$$q_{wa} = (q + \frac{T_1 + T_2 + T_3 + T_4}{\pi D^2})$$ (4.181)

Equation (4.180) is the same as (4.153), but is for the aft wing out of front propeller wash. $V_{wa}$ is the same as $V_{wf}$ out of wash, and $V_{wa}$ differs from $V_{wf}$ by the effect of induced velocity in wash effects. For the aft wing in front propeller wash, equation (4.180) becomes:
\[ u_{wa} = u_p + 3/2(\Delta V)_{TP} \] (4.182)

where \((\Delta V)_{TP} = K_2 \Delta V \cos \theta_p\)

Notice that if \(\theta_p = 0\), \((\Delta V)_{TP} = \frac{3K_2}{2} \Delta V = \frac{3}{2} (K_2 \approx 1 \text{ at } \theta_p = 0)\)

\(K_2\) accounts for interference effects. From momentum theory the induced velocity, \(\Delta V\), does not double on the aft wing from the effect of two propellers blowing on the wing in parallel but \(\Delta V\) increases by one-half. The coefficients \((C_D)_{wa}\), \((C_L)_{wa}\) and \((C_m)_{wa}\) are defined similar to equations (4.159), (4.160) and (4.162), but using elevator controls instead of flap controls.

\[(C_D)_{wa} = C_D + C_{D6E} \cdot \alpha_{wa} \] (4.183)

\[(C_L)_{wa} = C_L + C_{L6E} \cdot \alpha_{wa} + C_{L\phi} \cdot \alpha_{wa} \] (4.184)

\[(C_m)_{wa} = C_m + \frac{c}{2V_{wa}} C_{m1} \cdot \alpha_{wa} + C_{m2} \cdot \omega \] (4.185)

The coefficients \((C_D)_{wa}\), \((C_L)_{wa}\) and \((C_m)_{wa}\) are rotated to body axes by the aft wing angle of attack \(\alpha_{wa}\). The coefficients in body axes are similar to equations (4.164), (4.165) and (4.167).

\[(C_x)_{wa} = -(C_D)_{wa} \cos \alpha_{wa} + (C_L)_{wa} \sin \alpha_{wa} \] (4.186)

\[(C_z)_{wa} = -(C_D)_{wa} \sin \alpha_{wa} - (C_L)_{wa} \cos \alpha_{wa} \] (4.187)

From the coefficients defined in equations (4.185), (4.186) and (4.187) the force and moment contributions of the aft wing are as follows:

\[(\Delta X_{wa}) = (C_x)_{wa} S q_{wa} \] (4.188)

\[(\Delta Z_{wa}) = (C_z)_{wa} S q_{wa} \] (4.189)

\[(\Delta M_{wa}) = (C_m)_{wa} - \frac{Z_{ac}}{c} (\Delta X_{wa}) + \frac{X_{ac}}{c} (\Delta Z_{wa}) \] (4.190)

In equation (4.190), \(X_{ac}\) and \(Z_{ac}\) are the respective distances from the aircraft c.g. to the aerodynamic center (ac) of the aft wing in the x-z plane. \(c\) is the mean aerodynamic chord of the aft wing.
Fuselage. In a very direct manner we can write the effects of the fuselage \( F \) on the total aerodynamic forces and moments as for the XC-l.\( \text{h} \)2A.

We have for the forces

\[
(\Delta X_F) = -\frac{1}{2} \rho V^2 R S C_D \tag{4.191}
\]

\( C_D \) is the equilibrium drag coefficient.

\[
(\Delta Y_F) = +\frac{1}{2} \rho V^2 R S C_y \cdot \beta_F \tag{4.192}
\]

\( C_y \) is the change in side force with respect to a changing sideslip angle.

\[
(\Delta Z_F) = -\frac{1}{2} \rho V^2 R S C_L \cdot \alpha_F \tag{4.193}
\]

\( C_L \) is the change in lift coefficient with varying angle of attack.

This is also known as the lift curve slope.

We have for the moments:

\[
(\Delta L_F) = 0 \tag{4.194}
\]

\[
(\Delta M_F) = \frac{1}{2} \rho V^2 R S c C_{m_o} + \frac{1}{2} \rho V^2 R S c C_{m_{\alpha_F}} \cdot \alpha_F \tag{4.194}
\]

\( C_{m_o} \) is the aerodynamic pitching moment coefficient in equilibrium flight and \( C_{m_{\alpha_F}} \) is the longitudinal static stability derivative.

\[
(\Delta N_F) = \frac{1}{2} \rho V^2 R b C_{n_{\beta_F}} \cdot \beta_F \tag{4.195}
\]

\( C_{n_{\beta_F}} \) is the static directional or "weathercock" derivative.

In the above expressions \( q = \frac{1}{2} \rho V^2 \) which is the dynamic pressure, \( b \) is the forward wing span, \( c \) is the mean aerodynamic chord and \( S \) is the forward wing area.
For hovering and in transition regions $\beta_F$ is assumed small so that $\cos \beta_F = 1$.

EQUATIONS OF MOTION - X-19. The aerodynamic force and moment terms developed in this section for each of the major aircraft components of the X-19 will be combined and finally expressed in the equations of motion.

Total Aerodynamic Forces and Moments. The total forces and moments are as follows:

\begin{align*}
X_a &= (AX_a)_p + (AX_a)_w w + (AX_a)_w a + (AX_a)_F \cos \beta_F \\
Y_a &= (AY_a)_p + (AY_a)_w t + (AY_a)_F \\
Z_a &= (AZ_a)_p + (AZ_a)_w w + (AZ_a)_w a + (AZ_a)_F \\
L_a &= (AL_a)_p + (AL_a)_w w + (AL_a)_w t \\
M_a &= (AM_a)_p + (AM_a)_w w + (AM_a)_w a + (AM_a)_F \\
N_a &= (AN_a)_p + (AN_a)_w w + (AN_a)_w a + (AN_a)_F
\end{align*}

Equations of Motion Expanded. The forces and moments are expressed in body axes. These equations are subject to revision when further data are available concerning the X-19.

1. X Force Equation.

\begin{align*}
\dot{L} &= \sum_{n=1}^{N} \left( T_n \cos \beta_n \cos \beta_n \sin \beta_n + (C_x)_n \frac{S}{W} w \right) \\
&\quad + (C_x)_n \frac{S}{W} a - C_{D_0} \frac{S}{W} \\
&= m \left( \ddot{V} + W_q - \dot{V} \right) + mg \sin \theta
\end{align*}

2. Y Force Equation.

\begin{align*}
\dot{L} &= \sum_{n=1}^{N} \left( -T_n \sin \beta_n \right) + (C_y) \frac{S}{W} t + C_{y_{\beta}} \frac{S}{W} \beta_F \\
&= m \left( \ddot{V} + W_q - \dot{V} \right) - mg \cos \theta \sin \phi
\end{align*}

\[ h \sum_{n=1}^{N} (-T_n \sin \theta_n - N_n \cos \theta_n \cos \phi_n) + (c_z)_w f \] Sn wf

+ (c_z)_wa Sn wf - Sq C_{\alpha F} \cdot \alpha F

= m(\omega + \nu - Uq_1) - mg \cos \theta \cos \phi

4. Roll Equation.

\[-(T_1 - T_2) \sin \theta_1 + (N_1 \cos \theta_1 - N_2 \cos \theta_2) \cos \phi_1 \] y_1

\[+(T_3 - T_4) \sin \theta_3 + (N_3 \cos \theta_3 - N_4 \cos \theta_4) \cos \phi_3 \] y_3

\[-[N_1 \sin \beta_1 + N_2 \sin \beta_2] z_1 - N_3 \sin \beta_3 + N_4 \sin \beta_4] z_3 \]

\[- \sum_{n=1}^{N} M_n \sin \beta_n \sin \phi_n + (C)_{\beta F} bS_{\alpha F} + C bS_{\phi F} \]

\[+ I_{13} (\dot{r} + pq_1) - (I_{33} - I_{22}) q_1 \]

\[= I_1 \ddot{p} \]

5. Pitch Equation.

\[ [x_1(T_1 + T_2) - x_3(T_3 + T_4)] \sin \theta_1 - [z_1(T_1 + T_2) + z_3(T_3 + T_4)] \cos \theta_1 \]

\+[z_1(N_1 \cos \beta_1 + N_2 \cos \beta_2) + z_3(N_3 \cos \beta_3 + N_4 \cos \beta_4)] \sin \theta_1 \]

\[+[x_1(N_1 \cos \beta_1 + N_2 \cos \beta_2) - x_3(N_3 \cos \beta_3 + N_4 \cos \beta_4)] \cos \theta_1 \]

\[+ \sum_{n=1}^{N} M_n \cos \phi_n + (C)_w f cS_{\alpha F} + (C)_w f cS_{\phi F} + qcS(C_m + C_{m_a F} \cdot \alpha_F) \]

\[+ I_{13} (r^2 - \dot{p}^2) - (I_{11} - I_{33}) pr \]

\[= I_{22} q_1 \]
c. Yaw Equation.

\[ [(T_1 - T_2) \cos \beta_p + (N_2 \cos \beta_2 - N_1 \cos \beta_1) \sin \beta_p] y_1 \]

\[ [(T_3 - T_4) \cos \beta_p + (N_4 \cos \beta_4 - N_3 \cos \beta_3) \sin \beta_p] y_3 \]

\[ + (N_3 \sin \beta_3 + N_4 \sin \beta_4) x_3 - (N_1 \sin \beta_1 + N_2 \sin \beta_2) x_1 \]

\[ - \sum_{n=1}^{4} M_n \sin \beta_n \cos \beta_p + (C_{1w} b S q \omega f + C_{2w} b S q \omega t + q B S C_n \beta_F \cdot \beta_F) \]

\[ + I_{13}(\beta - q_1) - (I_{22} - I_{11}) q_3 \]

\[ = I_{33} \]

*External stores, rough air or landing gear conditions are not included in these equations.*
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FAN-IN-WING

The General Electric/Ryan Aeronautical XV-5A V/STOL airplane will be used as the example in the development of fan-in-wing V/STOL simulation equations of motion. The equations of Section III will be directly applicable to the simulation equations for this aircraft.

In order to obtain some idea of the XV-5A consider Figure 1 and Table 7. In Figure 1, a three-view arrangement of the XV-5A is shown. Table 7 contains some of the physical characteristics of the aircraft.

With this general idea of the XV-5A, we may now develop the mathematical model for the XV-5A. First, we will define, if necessary, applicable axis systems. Second, we develop additional aerodynamic coefficients. Finally, we write the equations of motion for XV-5A.

AXIS SYSTEMS FOR THE XV-5A. The XV-5A has no change in its physical configuration during the hover, transition and normal flight. The only gross change from hover to normal flight is from thrust generated from the nose and wing fans to thrust from the tail pipes. The thrust developed by the fans during hover and transition should be available as a function of fan revolutions per second (rps) and fan louver position which directs the thrust of the fans. The thrust of the aircraft is described sufficiently in the axis systems used for ordinary jet aircraft simulation (Reference 2). Consequently, no axis systems in addition to the inertial, body, stability or wind axes will be necessary, to develop equations of motion for the XV-5A.

In addition, there will be no gyroscopic effects from the wing fans since they remain in the same geometric plane, are free wheeling in opposite directions and are driven from a common gas source at approximately the same rps. The fan in the nose, also free wheeling, does not give any appreciable lift, but is used as a pitch control. The doors in the fuselage ventral to the nose fan are linked to the control stick in order to control the flow of air through the nose fan thus giving pitch control.

\[ I_{NF} \dot{\alpha}_{NF} \] \( q_1 \), \( L_a \) term.
\[ I_{NF} \dot{\alpha}_{NF} \] \( \dot{p} \), \( M_a \) term.
\[ I_{NF} \dot{\alpha}_{NF} \] \( \dot{q}_1 \), \( N_a \) term.

It may be determined after consideration of data from the manufacturer that these terms will be negligible, but for the present they will be included in the equations of motion.
Weight

Empty 9,200 lbs.
Design Gross 12,000 lbs.
Overload Ferry (Conventional Take off) 13,600 lbs.

Dimensions

Wing Span 29.8 ft.
Fuselage Length (over-all) 44.5 ft.
Height 14.7 ft.

Powerplant

Lift Fan System - G.E. X353-5B
- Fan Diameter, Installed 76 inches
- Fan Thickness 14.5 inches

Turbojet - G.E. J85-5
- Over-all Length with Diverter Valve 67.2 inches
- Maximum Diameter 17.7 inches

Table 7. Selected Physical Characteristics - XV-5A

AERODYNAMIC FORCES AND MOMENTS XV-5A. It is proposed that since the XV-5A has no special problems in development of the equations of motions that equations such as (3.7) through (3.12) or those by Connelly, in Reference 2, be used for simulation purposes. In equations 3.7 and 3.9 the engine thrust will be defined to conform with the fan-in-wing.

The total engine thrust, T, is composed of either fan thrust, T_F, or jet thrust T_J.

\[ T_F = T_{F1} + T_{F2} \]

1 is port fan
2 is starboard fan

Due to cockpit control of the diverter valve, T is equal to T_F or T_J. For example, when the aircraft has enough forward speed to be supported on wing lift the diverter valve is moved so that all the incoming air passes through the jet engine for forward thrust (i.e. T_J) and the "an inlet and exits are closed. There is a three to four second delay before the fan develops 100% lift when the diverter valve is switched from T_J to T_F. This delay is easily simulated and rids the thrust of a discontinuity between T_J and T_F.
Hover Flight XV-5A. The equations for hover flight are equations (3.7) through (3.12) with the aerodynamic coefficients considered negligible. We can then write the following expressions:

\[ \dot{m} \left( v + \dot{w} - \dot{V} \right) = T_j \cos \alpha_T + T_F \sin \theta_L \]

\[ \dot{m} \left( v + \dot{w} - \dot{V} \right) = g \cos \theta \sin \gamma \]

\[ \dot{m} \left( w + \dot{v} - \dot{w} q_1 \right) = T_j \sin \alpha_T - T_F \cos \theta_L + f(T_{NF}) \]

\[ p = \frac{I_{13}}{I_{11}} r - \frac{I_{NF}}{I_{11}} q_1 + \frac{\dot{V}}{I_{11}} \left( T_{F1} \cos \theta_{L1} - T_{F2} \cos \theta_{L2} \right) \]

\[ + \frac{g S_b}{I_{11}} C_{\alpha_{6A}} \cdot 6A \]

\[ q_1 = \frac{I_{NF}}{I_{22}} p + \frac{I_{NF} T_{NF} \cos \theta_{NL}}{I_{22}} + \frac{g S_c}{I_{22}} C_{m_{6E}} \cdot 6E \]

\[ r = \frac{I_{13}}{I_{33}} p + \frac{I_{NF} \dot{\alpha}_{NF}}{I_{33}} + \frac{\dot{V}}{I_{33}} \left( T_{F1} \sin \theta_{L1} - T_{F2} \sin \theta_{L2} \right) \]

\[ + \frac{g S_b}{I_{33}} C_{n_{6R}} \cdot 5R \]

In these equations \( \theta_L \) is the angle the fan thrust is rotated (\( \theta_L \) is a function of louver angle) and \( \dot{y} \) is the moment arm from the \( x \)-axis (parallel to the \( y \)-axis) to the center of either wing fan. \( \theta_{L1} \) and \( \theta_{L2} \) refer to louver angles at the port and starboard fans. Cross-coupling of angular rates is assumed negligible for the hover condition. The gyroscopic effects of the nose fan (NF) are included in equations (4.199), (4.200) and (4.201). These gyroscopic terms will be zero in conventional flight. Reaction moments are included in equations (4.199), (4.200) and (4.201) to account for aileron (6A), elevator (6E) and rudder (6R) control motion during hover. The reaction moments are contained in \( C_{\alpha_{6A}}, C_{m_{6E}} \) and \( C_{n_{6R}} \) so that they phase naturally into the transition and conventional flight equations of motion. Pitch control is represented by nose fan thrust \( (T_{NF}) \). The pitching moment created by the nose fan is \( f(T_{NF}) \), where \( I_T \) is the moment arm from the center of the nose fan to the c.g. of the aircraft.
Transitional Flight. As in the equations for transitional flight for the P.1127 (Section IV), the transitional equations for the XV-5A are composed of the equations (3.7) through (3.12). Consequently, we can write the equations of motion for the fan-in-wing directly.

EQUATIONS OF MOTION XV-5A. The equations of motion are expressed in aircraft body axes.

**X Force Equation**

\[
m (\dot{W} + Wq_1 - \dot{V}_r) = \frac{\rho v^2 S}{2} \left[ C_x (\alpha, Ma) + C_x (\theta) + C_{x_{6F}} \cdot 6F \right]
+ T_1 \cos \alpha_1 - T_F \sin \theta_L - mg \sin \theta
\]

**Y Force Equation**

\[
m (\dot{V} + \dot{U}_r - Wp) = \frac{\rho v^2 S}{2} \left[ C_y (\alpha, Ma) + C_y \cdot \theta + C_{y_{6R}} \cdot 6R \right] + mg \cos \theta \sin \phi
\]

**Z Force Equation**

\[
m (\dot{W} + \dot{V}_p - Uq_1) = \frac{\rho v^2 S}{2} \left[ C_z (\alpha, Ma) + C_{z_{6F}} \cdot 6F + C_{z_{6E}} \cdot 6E \right]
- T_1 \sin \alpha_1 - T_F \cos \theta_L + f(T_{NF}) + mg \cos \theta \cos \phi
\]

**Roll Equation**

\[
p = \frac{I_{13}}{I_{11}} (\dot{r} + p q_1) - \left( \frac{I_{33} - I_{22}}{I_{11}} \right) q_1 r + \frac{I_{NF} q_{NF}}{I_{11}} q_1
\]

\[
+ \frac{\rho v^2 S b}{2 I_{11}} \left[ C_{l_{\beta}} \cdot \theta + C_{l_{6A}} \cdot 6A + C_{l_{5R}} \cdot 5R + \frac{b}{2} \left( C_{l_p} \cdot p + C_{l_r} \cdot r \right) \right]
+ \frac{V}{I_{11}} (T_{F_1} \cos \theta_{L_1} - T_{F_2} \cos \theta_{L_2})
\]
Pitch Equation

\[ q_1 = \frac{I_{13}}{I_{22}} (r^2 - p^2) - \left( \frac{I_{11} - I_{33}}{I_{22}} \right) pr - \frac{I_{NF} \Omega_{NF}}{I_{22}} p \]

\[ + \frac{V_{R}^2}{2I_{22}} \left[ C_m (\alpha, M_a) + C_{m_{6F}} \cdot 5F + C_{m_{6E}} \cdot 5E + \frac{C}{2V_B} (C_{m_{q_1}} \cdot q_1 + C_{m_{\alpha}} \cdot \dot{\alpha}) \right] \]

\[ + \frac{f(T_{NF}) I_{T}}{I_{22}} \]

Yaw Equation

\[ r = \frac{I_{13}}{I_{33}} (p - q_1 r) - \left( \frac{I_{22} - I_{11}}{I_{33}} \right) pq_1 + \frac{I_{NF} \Omega_{NF}}{I_{33}} \]

\[ + \frac{V_{B}^2 S_b}{2I_{33}} \left[ C_{n_{\beta}} + C_{n_{6A}} \cdot 5A + C_{n_{6R}} \cdot 5R + \frac{b}{2V_B} (C_{n_p} \cdot p + C_{n_r} \cdot r) \right] \]

\[ + \frac{Y}{I_{33}} (T_F \sin \theta_1 - T_F \sin \theta_2) \]

External stores, rough air or landing gear conditions are not expressed in the above equations.
ROTATING THRUST

The Hawker P.1127 will be used as an example in the development of rotating thrust V/STOL simulation equations of motion. The equations of Section III as well as information from Hawker Aircraft Ltd. (Reference 9) will be directly applicable to the development of the simulation equations.

In order to obtain some idea of the P.1127 let us again consider Figure 5 and Table 8. In Figure 5, a three-view arrangement of the P.1127 is shown, whereas Table 8 contains some of the physical characteristics of the aircraft.

With this general idea of the P.1127, we may now develop the mathematical model for the P.1127. First, we define, if necessary, applicable axis systems. Second, we develop any additional aerodynamic coefficients. Finally, we write the equations of motion for the P.1127.

AXIS SYSTEMS FOR THE P.1127. The argument presented here is similar to that of the fan-in-wing. The only physical change in the configuration of the P.1127 from hover, through transition to normal flight is the rotation of the thrust nozzles. This rotation can be adequately described in the aircraft body axis system. In accord with Reference 9, body axes can be used during hover flight and stability axes can be used during transition and conventional flight in order to specify stability and control derivatives. Consequently, no axis systems in addition to the inertial, body, stability or wind axes will be necessary to develop equations of motion for the P.1127.

AERODYNAMIC FORCES AND MOMENTS - P.1127. In Reference 9, Hawker Aircraft lists equations of motion for hover and transitional flight. During hover, control is maintained by the reaction controls which obtain control power from the cockpit stick and rudder pedals. There are two roll reaction controls (one near each wing tip) which operate in conjunction with the ailerons from command signals generated by the cockpit stick. Next, there are two pitch reaction controls (one under the nose and one under the tail of the aircraft) which operate in conjunction with the all movable tail plane from command signals generated by the cockpit stick. Finally, there are two yaw reaction controls (one on each side of the tail of the aircraft) which operate in conjunction with the rudder from command signals generated by the rudder pedals.
### Weight

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>10,280 lbs.</td>
</tr>
<tr>
<td>Design Gross</td>
<td>15,749 lbs.</td>
</tr>
</tbody>
</table>

### Wing

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>186.40 ft.²</td>
</tr>
<tr>
<td>Span</td>
<td>22.82 ft.</td>
</tr>
<tr>
<td>Incidence to fuselage datum</td>
<td>1.75°</td>
</tr>
<tr>
<td>Dihedral Angle</td>
<td>-12°</td>
</tr>
<tr>
<td>Aerodynamic Mean Chord</td>
<td>8.667 ft.</td>
</tr>
</tbody>
</table>

### Horizontal Tail (all movable)

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>44.20 ft.²</td>
</tr>
<tr>
<td>Span</td>
<td>12.00 ft.</td>
</tr>
<tr>
<td>Dihedral Angle</td>
<td>-15° 50'</td>
</tr>
</tbody>
</table>

### Fuselage

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Length</td>
<td>42.00 ft.</td>
</tr>
<tr>
<td>Overall Height</td>
<td>10.75 ft.</td>
</tr>
</tbody>
</table>

### Vertical Tail

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>26.10 ft.²</td>
</tr>
<tr>
<td>Span</td>
<td>5.625 ft.</td>
</tr>
<tr>
<td>Height from fuselage datum</td>
<td>8.167 ft.</td>
</tr>
</tbody>
</table>

### Underfin

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>6.0 ft.²</td>
</tr>
<tr>
<td>Distance of tip from fuselage datum</td>
<td>1.67 ft.</td>
</tr>
</tbody>
</table>

### Flaps

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>13.25 ft.²</td>
</tr>
<tr>
<td>Movement</td>
<td>0 to 50°</td>
</tr>
</tbody>
</table>

### Reaction Controls

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of roll reaction nozzles from fuselage $C_L$ centerline</td>
<td>11.20 ft.</td>
</tr>
<tr>
<td>Distance of forward pitch reaction nozzle from moment reference center</td>
<td>17.55</td>
</tr>
<tr>
<td>Distance of rear pitch reaction nozzle from moment reference center</td>
<td>20.55 ft.</td>
</tr>
</tbody>
</table>

Table 8. Selected Physical Characteristics P.1127
**Reaction Controls (Cont'd.)**

Distance of yaw reaction nozzles from moment reference center.  

| Distance of Yaw Reaction Nozzles from Moment Reference Center | 20.15 ft. |

**Aerodynamic Controls**

<table>
<thead>
<tr>
<th>Component</th>
<th>Area</th>
<th>Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aileron Area (Port and Starboard)</td>
<td>8.75 ft$^2$</td>
<td>$\pm 12^\circ$</td>
</tr>
<tr>
<td>Rudder Area</td>
<td>5.25 ft$^2$</td>
<td>$\pm 15^\circ$</td>
</tr>
<tr>
<td>Tailplane Net Area</td>
<td>38.08 ft$^2$</td>
<td>$\pm 12^\circ$, $-10^\circ$</td>
</tr>
<tr>
<td>Trim Range</td>
<td>$+5 1/2^\circ$, $-2 1/2^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

**Intake**

| Area                           | 9.3 ft$^2$ |

**Powerplant**

<table>
<thead>
<tr>
<th>No. and Model</th>
<th>(1) Pegasus B.S. Pg. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Ducted fan lift/thrust engine</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>Bristol Siddeley</td>
</tr>
<tr>
<td>By-Pass Ratio</td>
<td>1.4</td>
</tr>
<tr>
<td>Length</td>
<td>8.25 ft.</td>
</tr>
<tr>
<td>Diameter</td>
<td>4.00 ft.</td>
</tr>
<tr>
<td>Maximum Engine Rating</td>
<td>18,000 lb.</td>
</tr>
<tr>
<td>Maximum Fan RPM (% design)</td>
<td>101.0</td>
</tr>
</tbody>
</table>

Table 8. Selected Physical Characteristics P.1127 (Cont'd.)
From Reference 9 further information concerning thrust is obtained. There are four jet nozzles operating from the engine. These nozzles are rotated simultaneously. The front port and starboard nozzles are cold jets while the aft port and starboard nozzles are hot jets.

The cold jets are oriented 5° out from the x-z body axis plane. The hot jets are a function of \( \theta_j \) and are given by the following expression:

\[
\phi_H = 7.5^\circ + 0.05 \theta_j
\]

The total thrust vector (four nozzles) is:

\[
T = T_{GC} \cos 5^\circ + T_{GH} \cos \phi_H
\]

\[
T = 0.996 T_{GC} + T_{GH} \cos \phi_H
\]

where \( T_{GC} \) is the gross thrust from the two cold jet nozzles and \( T_{GH} \) is the gross thrust from the two hot jet nozzles. We will turn to aerodynamic forces and moments during hover and transitional flight.

Hover Flight. The equations used by Hawker in Reference 9 for hover are next stated. Nomenclature has been changed in these equations to agree with usage in this report.

\[
\dot{r} = \frac{L_A}{I_{11}} \cdot A
\]

\[
\dot{q}_1 = \frac{M_E}{I_{22}} \cdot E
\]

\[
\dot{r} = \frac{N_R}{I_{33}} \cdot R
\]

\[
\dot{v} = \frac{g}{w} T \cos (\theta_j + \alpha_T) - g \sin \theta
\]

\[
\dot{v} = \frac{g}{w} T \sin (\theta_j + \alpha_T) + g \cos \theta \sin \phi
\]

In the above equations \( \dot{r} = \frac{d^2 \phi}{dt^2} \), \( \dot{q}_1 = \frac{d^2 \theta}{dt^2} \) and \( \dot{r} = \frac{d^2 \theta}{dt^2} \). \( L_A \), \( M_E \), and \( N_R \) are aerodynamic moments due to reaction controls where \( A \) is the aileron, \( E \) is the tailplane and \( R \) the rudder. \( \theta_j \) is the angle of
deflection of the engines nozzles from the centerline ($\phi_L$) of the engine, $\alpha_T$ is the angle between $\phi_L$ and the x body axis and $T$ is engine thrust.

Equations (4.204) through (4.209) will be accepted as hover equations and incorporated in the final equations of motion.

Transitional Flight. Again we turn to Reference 9 and quote here the transitional flight equations. These equations will be stated in the nomenclature used in this report. Appropriate comments will be made for each equation so as to compare it with the math model of Section III. There are no explicit thrust terms in the X and Z forces. The thrust terms will be in the final equations -- (4.216) through (4.221).

\[ \dot{m}V = qS \left[ C_U + C_\alpha + C_\deltaF \cdot \deltaF \right] - mg(\theta) \quad (4.210) \]

Comparing equation (4.210) to (3.7), there is no $C_x(\beta)$ coefficient in (4.210). In (4.210), $C_U + C_\alpha$ corresponds to $C_x(\alpha, Ma)$ of (3.7) and $mg \sin \theta \approx mg(\theta)$ where $\theta$ is a small angle.

\[ \dot{m}V = qS \left[ C_y + C_\deltaA + C_\deltaR \cdot \deltaR \right] + mg(\phi) \quad (4.211) \]

Comparing equation (4.211) to (3.8), there is no $C_y$ of $C_\deltaA$ in (3.8). In jet aircraft $C_y$ and $C_\deltaR$ are usually small. They are included in (4.211) because of their importance in the transition region. The term $mg \cos \theta \sin \phi \approx mg(\phi)$ where $\theta$ and $\phi$ are considered small angles.

\[ \dot{m}W = qS \left[ C_z + C_\alpha + C_\deltaF \cdot \deltaF \right] + mUq_1 \quad (4.212) \]

Comparing (4.212) to (3.9), there is no $C_z \cdot \deltaE$ term in (4.218). This term should be in the final equations since $\deltaE$ does affect the Z - Force. The term, $C_U + C_\alpha$ corresponds to $C_z(\alpha, Ma)$.

In the moment expressions Hawker has assumed that cross-coupling of angular rates is negligible so the terms like $pq \approx 0$ and $r^2 \approx 0$. 

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Roll.

\[ \frac{I_{13}}{I_{11}} \dot{r} = \rho \frac{V_{B}^2 S_{b}}{2I_{11}} \left[ C_\delta + C_\delta \alpha + C_\delta A + C_\delta R \right] \]

\[ + \frac{b}{2V_B} C_p \cdot p + \frac{b}{2V_B} C_l \cdot r \]  \hspace{1cm} (4.213)

Equation (4.213) is the same as equation (3.10) if cross-coupling of angular rates is neglected.

Pitch.

\[ \dot{q}_1 = \rho \frac{V_{B}^2 S_c}{2I_{22}} \left[ C_m_a + C_m_\alpha + C_m_\delta \alpha + C_m_\delta E \right]
\]

\[ + \frac{c}{2V_B} C_m_\alpha \cdot q_1 \]  \hspace{1cm} (4.214)

Yaw.

\[ \dot{z} - p \frac{I_{13}}{I_{33}} = \rho \frac{V_{B}^2 S_b}{2I_{33}} \left[ C_n_\beta + C_n_\delta \alpha + C_n_\delta A + C_n_\delta R \right] \]

\[ + \frac{b}{2V_B} C_n_p \cdot p + \frac{b}{2V_B} C_n_r \cdot r \]  \hspace{1cm} (4.215)

Equation (4.215) is the same as equation (3.12) if cross-coupling of angular rates is neglected.

From the hover equations and transitional equations final force and moment expressions will be written.

EQUATIONS OF MOTION - P.1127. The equations of motion herein presented are written by considering the equations (3.7) through (3.12) and the Hawker hovering and translational equations. In equations (3.7), (3.8) and (3.9), the velocity, \( U \), is predominant. The equations are written in body axes.

X Force Equation.

\[ m (\dot{U} + Wq_1 - Vr) = \rho \frac{V_{B}^2 S}{2} \left[ C_x U + C_x \alpha + C_x \delta F + C_x (\beta) \right] \]

\[ + T \cos (\theta_j + \alpha_T) - mg \sin \theta \]  \hspace{1cm} (4.216)

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Y Force Equation

\[
m (V + Ur - Wp) = \frac{V_B^2}{2} \left[ C_Y + \beta + C_{y6A} + C_{y6R} \right] + mg \cos \theta \sin \phi
\]  

(4.217)

Z Force Equation

\[
m (W + Vp - Uq_1) = \frac{V_B^2}{2} \left[ C_z + C_{z6F} + C_{z6E} \right] + T \sin (\theta_j + \alpha_T) + mg \cos \theta \cos \phi
\]  

(4.218)

Roll Equation

\[
p = \frac{I_{13}}{I_{11}} (r + pq_1) - \left( \frac{I_{33} - I_{22}}{I_{22}} \right) q_1 r + \frac{LA}{I_{11}} \cdot \frac{V_B^2}{2} \left[ C_{\phi} + \beta + C_{\phi5A} \right]
\]  

(4.219)

Pitch Equation

\[
q_1 = \frac{I_{13}}{I_{22}} (r^2 - p^2) - \left( \frac{I_{11} - I_{33}}{I_{22}} \right) pr + \frac{M_E}{I_{22}} \cdot \frac{V_B^2}{2} \left[ C_{m_{\theta}} + C_{m_{\alpha}} + C_{m_{6E}} \right]
\]  

(4.220)

Yaw Equation

\[
r = \frac{I_{13}}{I_{33}} (p - qr_1) - \left( \frac{I_{22} - I_{11}}{I_{33}} \right) p q_1 + \frac{N_R}{I_{33}} \cdot R + \frac{V_B^2}{2I_{33}} \left[ C_{n_{\phi}} + \beta \right]
\]  

(4.221)

For \( W \approx 0, V \approx 0 \) and a small value of \( Ur \), equations (4.216) through (4.221) are equivalent to the hover equations (4.204) through (4.209) since all the aerodynamic coefficients in brackets are practically zero.
As $\mu$ becomes greater ($\theta_j$ approaches zero) $L_{A}$, $M_{E}$, and $N_{R}$ phase out and the aerodynamic terms are predominant as in equations (4.210) through (4.215). The transitional equations then carry directly into conventional flight. External stores, rough air, or landing gear conditions are not developed in equations (4.216) through (4.221).
NAVTAEVCEN 1205-2

SECTION V

BIBLIOGRAPHY


9. P.O.N. No. 614, Hawker Aircraft LTD.
The derivation of the equations of motion for an aircraft flying in a conventional manner has been done by means of a notation that is economical of space. Naturally, there is a sophisticated degree of abstractness in manipulating equations developed in this notation; however, no more than the simplest best known expressions in mechanics and the representation of direction cosines are actually being used. The idea being developed in an equation can be received as a whole using this notation instead of seeing bits and pieces of derivation. The main terms of an equation are immediately apparent, and initial assumptions as to importance of various terms in the equation can be applied in order to give an orderly routine of simplification of the general equation for a particular set of assumptions.

NOTATION

The notation employed uses the device of subscripting and superscripting a particular symbol with an algebraic variable which will vary over a prescribed range. It is actually the range and summation techniques developed in Tensor Analysis. More precisely, what is used and manipulated are Cartesian Tensors. These are particularly simple tensors in three dimensions using Cartesian coordinates. In fact, for the purposes of these derivations the only tensor quality is the adoption of the range and summation conventions plus a few manipulative methods, and the advantage gained from Tensor Analysis is a concise notation.

e - SYMBOL

\[ \epsilon_{ijk} \text{ or } \epsilon_{abc} \] is defined in the following manner:

\[ \epsilon_{ijk} = +1 \text{ if } ijk \text{ is an even permutation of 123} \]

\[ \epsilon_{ijk} = -1 \text{ if } ijk \text{ is an odd permutation of 123} \]

\[ \epsilon_{ijk} = 0 \text{ if } i = j \text{ or } j = k \text{ or } i = k \text{ or } i = j = k \]

For example:

\[ +1 = \epsilon_{123} = \epsilon_{231} = \epsilon_{312} \]

\[ -1 = \epsilon_{132} = \epsilon_{321} = \epsilon_{213} \]

\[ 0 = \epsilon_{112} = \epsilon_{122} = \epsilon_{121} = \epsilon_{111} \]
First Order Kronecker Deltas
\[ \delta_{i}^{j} = +1 \text{ if } i = j \]
\[ \delta_{i}^{j} = 0 \text{ otherwise (} i \neq j \) \]

Second Order Kronecker Deltas
\[ \delta_{ij}^{ab} = +1 \text{ if } a + b, i + j, a = i, b = j \]
\[ \delta_{ij}^{ab} = -1 \text{ if } a + b, i + j, a = j, b = i \]
\[ \delta_{ij}^{ab} = 0 \text{ otherwise} \]

This scheme can be generalized to \( n \)th order Kronecker Deltas. However, since the coordinate systems being used in this report are three dimensional, the highest order Kronecker Delta encountered will be three.

Third Order Kronecker Deltas
\[ \delta_{ijk}^{abc} = +1 \text{ if } a + b + c, a + c \text{ and } ijk \text{ is an even permutation of } abc. \]
\[ \delta_{ijk}^{abc} = -1 \text{ if } a + b + c, a + c \text{ and } ijk \text{ is an odd permutation of } abc. \]
\[ \delta_{ijk}^{abc} = 0, \text{ otherwise} \]

Product of Two \( \varepsilon \) Symbols
\[ \varepsilon_{abc} \varepsilon_{ijk} = \delta_{ijk}^{abc} \]

In particular a special case is
\[ \varepsilon_{a}^{abc} \varepsilon_{ijk} = \delta_{ijk}^{a} \delta_{ik}^{ac} \]

Example:
Does \( \varepsilon_{123}^{321} = \delta_{13}^{3} \)?
\[ (+1) (-1) = -1 \]
\[ -1 = -1 \quad \text{Q.E.D.} \]
Expansion of Second Order Kronecker Delta

$$\delta_{ab} = \delta_{a}^{a} \delta^{b}_{b} - \delta_{a}^{b} \delta^{a}_{b}$$

Example:

Does $$\delta_{21}^{12} = \delta_{2}^{1} \delta_{1}^{2} - \delta_{1}^{1} \delta_{2}^{2}$$?

$$(-1) = (0) (0) - (1) (1)$$

$$(-1) = (-1) \quad Q.E.D.$$  

Example:

$$\delta_{23}^{11} = \delta_{2}^{1} \delta_{3}^{1} - \delta_{1}^{1} \delta_{2}^{1}$$

$$0 = (0) (0) - (0) (0)$$

$$0 = 0$$

e - Symbol Times Kronecker Delta

$$\delta_{d}^{e} \delta_{m}^{j} = \delta_{d}^{e} \delta_{j}^{m}$$

Example:

$$\delta_{1}^{1} \delta_{j}^{m} = \delta_{j}^{m}$$

But $$\delta_{1}^{1} = +1$$

$$\delta_{j}^{m} = \delta_{j}^{m}$$

Example:

$$\delta_{2}^{1} \delta_{j}^{m} \delta_{2}^{2} = \delta_{j}^{m} \quad \delta_{2}^{2} = 0$$

Possibilities

$$\delta_{2}^{1} \delta_{3}^{1} = \delta_{3}^{1}$$

$$0 = 0$$

or

$$\delta_{2}^{1} \delta_{1}^{2} = \delta_{1}^{2}$$
RANGE AND SUMMATION CONVENTIONS. The definitions are as follows:

Range Convention. When an uncapitalized letter (superscript or subscript) occurs unrepeated in a term, it is understood to take all the values 1, 2 and 3, where 3 is the number of dimensions of the space; i.e., three dimensional space.

Summation Convention. When an uncapitalized letter (superscript or subscript) occurs repeated in a term, then summation with respect to this letter is understood, where the extent of summation is 1...3.

For example, consider the following expression:

\[ F^X_j = M(v^X_j + v^X_\omega x e_{jck}) \]

In \( F^X_j \), the \( j \) indicates a range: \( j = 1, 2 \) or 3

So there is:

\[ F^X_1 = M(v^X_1 + v^X_\omega x e_{1ck}) \]
\[ F^X_2 = M(v^X_2 + v^X_\omega x e_{2ck}) \]
\[ F^X_3 = M(v^X_3 + v^X_\omega x e_{3ck}) \]

Notice now \( v^X_\omega x e_{jck} \). In \( F^X_1 \), \( F^X_2 \) and \( F^X_3 \) there are two repeated indices--c and k. This would indicate a summation process according to the summation convention:

\[ F^{X1} = \sum_{c,k=1}^3 M (v^X_1 + v^X_\omega x e_{1ck}) \]
\[ F^{X2} = \sum_{c,k=1}^3 M (v^X_2 + v^X_\omega x e_{2ck}) \]
\[ F^{X3} = \sum_{c,k=1}^3 M (v^X_3 + v^X_\omega x e_{3ck}) \]

Expanding \( F^{X1} \), it is written:

\[ F^{X1} = \sum_{c,k=1}^3 \left( v^{X1} + v^{X1} x e_{111} + v^{X2} x e_{112} + v^{X3} x e_{113} + v^{X1} x e_{121} + v^{X2} x e_{122} + v^{X3} x e_{123} + v^{X1} x e_{131} + v^{X2} x e_{132} + v^{X3} x e_{133} \right) \]
Collect non-vanishing terms so that

\[ F^{X1} = (v^{X1} + v^{X3}X2e_{123} + v^{X2}X3e_{132})M \]

\[ F^{X1} = (v^{X1} + v^{X3}X2 - v^{X2}X3)M \]

\[ F^{X2} \text{ and } F^{X3} \text{ are treated in a similar manner.} \]

In the above expansion of \( F^{X1} \), the summation and range conventions were used. It should be noted that there is an easier method to arrive at the expanded \( F^{X1} \).

\[ F^{X1} = M(v^{X1} + v^{Xk}xke_{1ck}) \]

Note: \( e_{1ck} \) to be non zero can only have \( c \) and \( k \) equal to 2 or 3. That is:

\[ e_{1ck} = e_{123} \text{ and } e_{132} \]

Then for \( F^{X1} \) we write the non-vanishing terms immediately:

\[ F^{X1} = M(v^{X1} + v^{X3}X2e_{123} + v^{X2}X3e_{132}) \]

\[ F^{X1} = (v^{X1} + v^{X3}X2 - v^{X2}X3)M \]

Thus, by noting the non zero properties of the \( e \) - symbol, the summation process can be shortened.

CONTENTS OF NOMENCLATURE. This is a description of the more important individual designations that will appear in the terms of the equations.

Position - Particle to Particle. (1) This is designated by a vector \( \vec{r} \). In the indexed notation \( \vec{r} \) becomes \( r_{ti} \), which indicates that \( \vec{r} \) originates at the origin of the \( t \)-axes. There may be other vectors such as \( r_{si} \) or \( r_{ni} \).

(2) \( i = 1, 2, 3 \) gives the particular component of \( r_{ti} \).

(3) To differentiate between various \( r_{ti} \), a subscript is added. There results \( r_{ti0}, r_{t1}, r_{t2}, \ldots \) or for a particle: \( r_{tip} \).

Velocity. This is:

\[ v_{ti}^0 = \vec{f}_{ti} \]

\[ v_{t1}^1 = \vec{f}_{ti} \]

\[ v_{t2}^2 = \vec{f}_{ti} \]

and for a particle \( v_{tip}^p = \vec{f}_{ti} \)
All dots over symbols refer to derivatives with respect to time.

Remember that $t_i$ can be replaced by $y_i$, $z_i$, $s_i$ or some other set of coordinates.

Acceleration. This is:

$$a^s_i = \ddot{s}_i = \dot{y}_i$$

$$a^y_i = \ddot{y}_i = \dot{s}_i$$

and for a particle $a^p_i = \ddot{y}_i = \dot{s}^p_i$.

Mass. A mass point is designated as $m_p$. When deriving equations of motion a summation of mass points is made.

Example:

For fuselage particle: $\int m_p = M_F$ (mass of fuselage)

Force. In general: $F = m\ddot{r}$ (vector form)

Now in indexed notation for example: $f^t_i = m_p a^t_i$

But the equations are developed by knowing what particles are being manipulated. If the expression for the fuselage particle is being sought the final form will be:

$$f^t_i = m_p a^t_i$$

where $a^t_i$ is acceleration and is a function of direction cosines, position vectors, and rates of change of both.

Integrating: $\int f^t_i = F_F$ Subscript $F$ fuselage

$$\int m_p = M_F$$ Mass of fuselage particle

Torque. Vectorial: $\vec{T} = \vec{r} \times \vec{F}$

Now in indexed notation: $t^{sm} = m_p a^{sj}_{ij} r^{sd}_{ij}$

Integrating: $\int t^{sm} = T_F$ if $a^{sj}_{ij}$ is the acceleration associated with the fuselage particle.

Inertia. Vectorial: The angular momentum $\vec{J} = \sum_{i=1}^{n} \vec{m}_i \times (\vec{r}_i \times (\vec{r}_i \times \vec{r}_i))$

or $\vec{J} = \frac{1}{2} \sum_{i=1}^{n} m_i (y_1^2 + z_1^2) - \omega_y \vec{r}_1 \times m_i x_1 y_1 - \omega_z \vec{r}_1 \times m_i x_1 z_1$

+ $\frac{1}{2} \sum_{i=1}^{n} m_i (x_1^2 + z_1^2) - \omega_y \vec{r}_1 \times m_i x_1 y_1 + \omega_z \vec{r}_1 \times m_i x_1 z_1$

+ $\frac{1}{2} \sum_{i=1}^{n} m_i (x_1^2 + y_1^2)$
Moments of inertia are terms like:
\[ \sum m_i (y_i^2 + z_i^2) = I_{xx} \]

Products of inertia are terms like:
\[ \sum m_i x_i y_i = I_{xy} \]

Now in indexed notation:
\[ I_{pq} = \int m_r x_a r_a d \]

or
\[ \int m_r x_a r_a d = -I_{ad} + \frac{1}{2} (I_{11} + I_{22} + I_{33}) \delta_{13} \]

\[ I_{pq} \]
defines both moments and products of inertia. To make this clear consider the following examples:

Example: Moment of inertia
\[ I_{11} = \int m_r x_a r_a \delta_{1a} \]

Now \( \delta_{1a}, \delta_{12}, \text{and} \delta_{13} \) are non zero

Then:
\[ I_{11} = \int m_r x_2 x_2 \delta_{12} + \int m_r x_3 x_3 \delta_{13} \]
\[ \delta_{12} = +1, \delta_{13} = +1 \]
\[ I_{11} = \int m_r [(r_2)^2 + (r_3)^2] \]

Example: Product of inertia
\[ I_{12} = \int m_r x_a r_a \delta_{2d} \]

Now \( \delta_{1a}, \delta_{12}, \text{and} \delta_{13} \) are non zero

Then:
\[ I_{12} = -\int m_r x_2 x_1 \]

Example:
Let $a = 1$ and $d = 1$

\[
\begin{align*}
\mathbf{m}_{pp}^{X_1 X_1} &= - \frac{I_{11}}{2} + \frac{I_{22}}{2} + \frac{I_{33}}{2} \\
\delta_1 &= +1 \\
I_{11} &= \mathbf{m}_{pp}^X \left[ (r_p^{X_2})^2 + (r_p^{X_3})^2 \right] \\
I_{22} &= \mathbf{m}_{pp} \left[ (r_p^{X_3})^2 + (r_p^{X_1})^2 \right] \\
I_{33} &= \mathbf{m}_{pp}^X \left[ (r_p^{X_1})^2 + (r_p^{X_2})^2 \right]
\end{align*}
\]

So that:

\[
\begin{align*}
\mathbf{m}_{pp}^{X_1 X_1} &= - \frac{I_{11}}{2} + \frac{I_{22}}{2} + \frac{I_{33}}{2} \\
\mathbf{m}_{pp}^{X_1 X_1} &= - \mathbf{m}_{pp}^{X_2 X_2} - \mathbf{m}_{pp}^{X_3 X_3} \\
&+ \mathbf{m}_{pp}^{X_3 X_3} + \mathbf{m}_{pp}^{X_1 X_1} \\
&+ \mathbf{m}_{pp}^{X_1 X_1} + \mathbf{m}_{pp}^{X_2 X_2}
\end{align*}
\]

**AXES SYSTEMS**

Two different axis systems are described—the inertial system and the fuselage system in terms of direction cosines. Direction cosines are the angles which define a vector in a particular axes system. The matrices developed in this appendix relate the measurement of a unit length in one direction as measured in a different direction. This is accomplished by the reorientation of one axis system with respect to another axes system. The method used is to consider that the axes systems have common origins and then by rotating the axes of one system with respect to the other achieve coincidence. The direction cosines measure this rotation.

In this report objects in three dimensional space are described by the use of Cartesian coordinates which are orthogonal by definition. Orthogonality allows the transpose of the direction cosine matrix to equal its inverse. It is sufficient to describe the rotation of one axes system with respect to another by the use of three angles (Euler angles).
INERTIAL AXES SYSTEM. The system is identified by \( N \) axes \((a = 1, 2, 3)\).

- \( N_1 \) is in the horizontal plane with positive direction North.
- \( N_2 \) is in the horizontal plane with positive direction East.
- \( N_3 \) is normal to the \( N_1N_2 \) plane with positive direction down.

THE FUSELAGE AXES SYSTEM. The system is identified by \( x \) axes \((a = 1, 2, 3)\).

This axes system corresponds to the body axes system used in normal aircraft parlance. The origin is fixed in the fuselage (usually at the nominal center of mass). The origin of the \( x \)-axes is separated from the origin of the \( N \)-axes by a vector \( r_0 \)—see Figure 11.

- \( x_1 \) is in the horizontal plane with positive direction noseward.
- \( x_1 \) is not necessarily parallel to the water line axis defined by the aircraft manufacturer.
- \( x_2 \) is in the horizontal plane with positive direction right.
- \( x_3 \) is normal to the \( x_1x_2 \) plane with positive direction down.

INERTIAL TO FUSELAGE AXES DIRECTION COSINE MATRICES. Three angles, the Euler angles, are sufficient to locate the fuselage in inertial space. The inertial system is translated such that its origin coincides with the origin of the fuselage (i.e. the nominal c.m. of the aircraft). Specific rotations of the \( N \)-axes are then made so that a unit of measure in the \( N \)-axes can be expressed in the \( x \)-axes. In each case the particular rotation is in a plane so that there is a rotation about the third axis—that is the rotation written as a matrix is \( \sin \alpha \) sines and cosines of the rotating angle with the orthogonal property of the axes systems stating that \( \sin^2 \alpha + \cos^2 \alpha = 1 \). This will become clear.

The three rotations are \( \psi \), \( \theta \) and \( \phi \).

Yaw by Angle \( \psi \) Around \( N_3 \) Axis. Here an intermediate system \( N' \) is defined by the rotation of \( \psi \) around \( N_3 \).

Define \( C \) as direction cosine matrix.
Figure A1. Inertial and Fuselage Axes
Pitch by Angle \( \theta \) Around \( N'2 \) Axis. Here an intermediate system \( N'' \) is defined by the rotation of \( \theta \) around \( N'2 \).

\[
\begin{array}{ccc}
N''1 & N''2 & N''3 \\
N'1 & \cos \theta & 0 & \sin \theta \\
0 & 1 & 0 & \\
N'3 & -\sin \theta & 0 & \cos \theta \\
\end{array}
\]

Roll by Angle \( \phi \) Around \( N''1 \) Axis. This third rotation arrives at the \( x \)-axes.

\[
\begin{array}{ccc}
x1 & x2 & x3 \\
N''1 & 1 & 0 & 0 \\
N''2 & 0 & \cos \phi & -\sin \phi \\
N''3 & 0 & \sin \phi & \cos \phi \\
\end{array}
\]

Combination of Three Rotations \((\psi, \theta, \phi)\). To get the final result—the relation between the inertial coordinate system and the fuselage coordinate system—a matrix multiplication is performed.

This multiplication is

\[
C_N = C_{N'Y} \times C_{N''X} \times C_{X'\psi} = C_{X'\psi}
\]

\[
\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1 \\
\end{array}
\times
\begin{array}{ccc}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta \\
\end{array}
\times
\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi \\
\end{array}
= C_{X'\psi}
\]

\[
\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1 \\
\end{array}
\times
\begin{array}{ccc}
\cos \theta & \sin \theta & \sin \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \\
\end{array}
= C_{X'\psi}
\]
<table>
<thead>
<tr>
<th>x1</th>
<th>x2</th>
<th>x3</th>
</tr>
</thead>
<tbody>
<tr>
<td>cosφ cosθ</td>
<td>cosφ sinθ sinφ</td>
<td>cosφ sinθ cosφ</td>
</tr>
<tr>
<td>-sinφ cosθ</td>
<td>+sinφ sinθ</td>
<td></td>
</tr>
<tr>
<td>sinφ cosθ</td>
<td>sinφ sinθ sinφ</td>
<td>sinφ sinθ cosφ</td>
</tr>
<tr>
<td></td>
<td>+cosφ cosθ</td>
<td></td>
</tr>
<tr>
<td>-sinθ</td>
<td>cosθ sinφ</td>
<td>cosθ cosφ</td>
</tr>
</tbody>
</table>

Note that \( C_N^{x_β} C_{x_β}^{Na} = 1 \)

where \( C_N^{x_β} = (C_{x_β}^{Na})^{-1} = (C_{x_β}^{Na})^T \)

\( (\quad)^{-1} \) Inverse matrix

\( (\quad)^T \) Transpose matrix

**INERTIAL TO FUSELAGE AXES RATES.** \( \dot{\phi}, \dot{\theta}, \dot{\psi} \) and \( \ddot{\psi} \) are measured as \( p, q, \) and \( r \) around the \( x_1, x_2 \) and \( x_3 \) axes respectively. The dot is the total derivative with respect to time.

\( \dot{\psi} \) Occurs Around the \( N_3 \) Axis. To get the direction cosine components refer to \( C_{x_β}^{Na} \).

\( \dot{p} = \dot{\psi} (-\sin \theta) \)

\( \dot{q}_1 = \dot{\psi} \cos \theta \sin \phi \)

\( \dot{r} = \dot{\psi} \cos \theta \cos \phi \)

\( \dot{\theta} \) Occurs around the \( N'2 \) Axis. To get the direction cosine components refer to

\( C_{N'}^{x_β} = C_{x_β}^{N''} x_β \)

\( \dot{p} = \dot{\theta} (0) \)

\( \dot{q}_1 = \dot{\theta} \cos \phi \)

\( \dot{r} = \dot{\theta} (-\sin \phi) \)

\( \dot{\phi} \) Occurs Around the \( N''_1 \) axis. To get the direction cosine components refer to \( C_{x_β}^{N''_1} \).

\( \dot{p} = \dot{\phi} (1) \)

\( \dot{q}_1 = \dot{\phi} (0) \)

\( \dot{r} = \dot{\phi} (0) \)
Combination of Three Rates \((p, q_1, r)\).

\[
p = \dot{\psi} (-\sin \theta) + \theta (0) + \dot{\theta} (1)
\]

\[
q_1 = \dot{\psi} \cos \theta \sin \phi + \dot{\theta} \cos \phi + \dot{\phi} (0)
\]

\[
r = \dot{\psi} \cos \theta \cos \phi + \dot{\theta} (-\sin \phi) + \dot{\phi} (0)
\]

In matrix notation this becomes

\[
\begin{bmatrix}
p \\
q_1 \\
r
\end{bmatrix} =
\begin{bmatrix}
-\sin \theta & 0 & 1 \\
\cos \theta \sin \phi & \cos \phi & 0 \\
\cos \theta \cos \phi & -\sin \phi & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix}
\]

By forming the inverse there is

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix}
= \begin{bmatrix}
p \\
q_1 \\
r
\end{bmatrix} \times
\begin{bmatrix}
0 & \sin \phi & \cos \phi \\
0 & \cos \phi & -\sin \phi \\
1 & \tan \theta \sin \phi & \tan \theta \cos \phi
\end{bmatrix}
\]

DERIVATION OF FORCE AND TORQUE EQUATIONS

The derivation of force and torque equations is done in concise notation. Certain assumptions are present in the use of these aircraft equations of motion. These assumptions are as follows:

(a) The earth is assumed to be a flat plane with gravity acting normal to this plane and it is not rotating or translating through space. In the derivation of the equations of motion, Newton's law, \(F = ma\), is used in its most simple form---there is no mass variation with change in time.

(b) The \(x_1\) and \(x_3\) fuselage axes define a plane of symmetry for the aircraft.

(c) There is a fixed center of gravity.

(d) There is no relative motion between the earth and its atmosphere.

The equations will now be developed using the inertial and fuselage axes, Newton's law, and direction cosines of the Eulerian angles.

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Revised 23 May 1964
FORCE ASSOCIATED WITH THE FUSILEAR PARTICLE. The position of \( m_i \) for the fusilear particle with respect to Ni frame of reference is stated as:
\[
\mathbf{r}_i = \mathbf{r}_{i0} + \mathbf{r}_p \cdot \mathbf{C}_{\mathbf{x}}. \quad \text{See Figure A1.}
\]

The inertial system velocity is given as:
\[
\mathbf{v}_i = \mathbf{v}_{i0} + \mathbf{v}_p \cdot \mathbf{C}_{\mathbf{x}} \quad \text{by differentiation of } \mathbf{r}_i.
\]
\[
\mathbf{v}_i = \mathbf{v}_{i0} + \mathbf{v}_p \cdot \mathbf{r}_a \quad \text{since } \mathbf{r}_p = \text{constant}
\]

But \( \mathbf{C}_{\mathbf{x}} = \mathbf{C}_{\mathbf{x}} \), therefore
\[
\mathbf{v}_i = \mathbf{v}_{i0} + \mathbf{v}_p \cdot \left( \mathbf{C}_{\mathbf{x}} \cdot \mathbf{c}_{\mathbf{abc}} \right) \quad \text{by substitution (A1)}
\]

Note: For vector treatment of this, refer to Reference 3, *Theoretical Physics - Mechanics* by F. W. Constant, pages 69 to 71.

The inertial system acceleration is given by a similar procedure as
\[
\mathbf{a}_i = \mathbf{a}_{i0} + \mathbf{a}_p \cdot \mathbf{C}_{\mathbf{x}}
\]
\[
\mathbf{a}_i = \mathbf{a}_{i0} + \mathbf{a}_p \cdot \left( \mathbf{C}_{\mathbf{x}} \cdot \mathbf{c}_{\mathbf{abc}} \right)
\]
\[
\mathbf{a}_i = \mathbf{a}_{i0} + \mathbf{a}_p \cdot \left( \mathbf{C}_{\mathbf{x}} \cdot \mathbf{c}_{\mathbf{abc}} \right)
\]
\[
\mathbf{a}_i = \mathbf{a}_{i0} + \mathbf{a}_p \cdot \left( \mathbf{C}_{\mathbf{x}} \cdot \mathbf{c}_{\mathbf{abc}} \right)
\]

For a particular mass particle \( m_p \) it can be said that
\[
\mathbf{r}_i = m_p \mathbf{a}_i
\]
\[
\mathbf{r}_i = m_p \mathbf{a}_i
\]

and transforming to the x-axis
\[
\mathbf{r}_j = m_p \mathbf{a}_i \mathbf{C}_{\mathbf{x}} \quad \text{as (A3)}
\]

Substituting (A2) into (A3) yields
\[
\mathbf{r}_j = m_p \mathbf{a}_i \mathbf{C}_{\mathbf{x}}
\]

Interchange b and h where they both appear in the same term and then by changing b to j
\[
\mathbf{r}_j = m_p \mathbf{a}_i \mathbf{C}_{\mathbf{x}}
\]

(A4)
Now is written:
\[ v^{N_i} = 0^{N_i} \]
\[ v^x_j = 0^{N_i} o_x^j \]
\[ v^x_j = a^{N_i} o_x^j + r^{N_i} o_x^j e_{jkc}, \tag{A5} \]
\[ v^x_k = a^{N_i} o_x^k. \tag{A6} \]

Substituting (A6) into (A5) gives:
\[ v^x_j = a^{N_i} o_x^j + v^x_k x c e_{jkc}. \]

Rearranging and letting \( e_{jkc} = -e_{jck} \),
\[ v^{N_i} o_x^j = v^x_j + v^x_k x c e_{jck}. \tag{A7} \]

Substituting (A7) into (A6) offers:
\[ p^x_j = m_p [v^x_j + v^x_k x c e_{jck}] + r^x_p x a x c e_{ajc} + r^x_p o_x^j x c e_{ahc e hig}. \]

Integrating over \( p^x_j \), \( m \) and \( m r^x_a \) finally yields:
\[ p^x_j = m_p [v^x_j + v^x_k x c e_{jck}] + r^x_p x a x c e_{ajc} + r^x_p o_x^j x c e_{ahc e hig}. \]

Since
\[ e_{ahc e hig} = g_{hig} = b_{hig} = a_{hig} e_{ca}, \]
\[ r^x_j = x_p [v^x_j + v^x_k x c e_{jck}] + x_p r^x_a x c e_{ajc} + x_p o_x^j x c e_{ahc e hig}. \tag{A8} \]

FOR JKE DUE TO THE FUSELAGE PARTICLE. The definition of torque offers:
\[ \tau^x_j = r^x_j x d + \tau^x_j x c e_{jmd} \]
\[ = \tau^x_j x d + \tau^x_j x c e_{jmd}. \]

That is, by knowing the force, the torque of a particle can be developed by substituting for \( p^x_j \) the terms \( r^x_j x d \) from (A8), realizing, of course, that this \( \tau^x_j \) is the acceleration expression developed for the fuselage particle. Then it follows from the substitution that:
\[ \tau^x_j = \tau^x_j x d (v^x_j + v^x_k x c e_{jck}) + r^x_p x a x c e_{ajc} + r^x_p o_x^j x c e_{ahc e hig} x e_{jmd} \tag{A9} \]
By substituting (A9) into the expression

\[ T_{xm} = \frac{T_{xm}}{F} \]  
\[(A10)\]

There is obtained

\[ T_{xm} = \int_{p} r_{p} \left( v_{x}^{i} + v_{x_{c}} e_{jck} \right) e_{jmd} \]
\[ + \int_{p} r_{p} \left( e_{ajc} e_{jmd} \right) \]
\[ + \int_{p} r_{p} \left( x_{c} \omega_{c} \delta_{j} g_{e} \right) \]
\[ + \int_{p} r_{p} \left( x_{c} \omega_{c} \delta_{j} g_{e} \right) \]
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\[ + \int_{p} r_{p} \left( x_{c} \omega_{c} \delta_{j} g_{e} \right) \]
\[-\omega \omega x g_I \rho e^\text{cmd} + \omega \omega x c_I \rho e^\text{cmd} \]
\[+ \omega x c x g \frac{1}{2}(I_{11} + I_{22} + I_{33}) e^\text{cmd} - \omega x c x c \frac{1}{2}(I_{11} + I_{22} + I_{33}) e^\text{cmd} \]
\[-\omega x c x g_I \rho e^\text{cmd}. \]

\[ (A18) \]

Then, in conclusion,
\[ T_{F} = I_{F}^{cm} x c x g_I \rho e^\text{cmd} \]
\[ + I_{F}^{cm} \omega x c - \omega x g_I \rho e^\text{cmd} \]
\[ (A19) \]

which is obtained by substitution of \((A15)\) and \((A18)\) into \((A11)\).

**Expansion of Force and Torque Equations.** Equation \(A8\) is the force equation and equation \(A19\) is the torque equation. The terms in these equations are defined as follows:

(a) \(M_F\) is the aircraft mass.
(b) \(V_x\) are body axes velocities.
(c) \(\omega\) is an angular velocity around the appropriate body axis.
(d) \(r_F\) is the displacement of the center of gravity from origin of the \(x\)-axes. Here \(r_F = 0\).
(e) \(F_F\) are body axes forces.
(f) \(T_F\) are body axes torques.

Dropping the subscript \(F\) these equations are then written as:
\[ F = M(\dot{V} + V x \omega \omega x e_j c_k) \]
\[ T = I_{cm} \omega x c - \omega x g I \rho e^\text{cmd} \omega x c \]

Expanding these equations there is:
\[ F_x = M(\dot{V}_x + V x_3 \omega x 2 - V x_2 x 3) \]
\[ F_x = M(\dot{V}_x + V x_1 \omega x 3 - V x_3 \omega x 1) \]
\[ F_x = M(\dot{V}_x + V x_2 \omega x 1 - V x_1 \omega x 2) \]
\[ T_x = I_{11} \dot{\omega} x 1 + I_{13} \dot{\omega} x 3 + \omega x x_1 x 1 + I_{13} + \omega x x_2 x 3 I_{33} - \omega x x_3 x 2 I_{23} \]
\[ T_x = I_{33} \dot{\omega} x 3 + I_{31} \dot{\omega} x 1 + \omega x x_2 x 3 I_{31} - \omega x x_1 x 3 I_{33} + \omega x x_3 x 1 I_{11} + \omega x x_1 x 3 I_{11} \]

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Revision No. 1

Revised 28 May 1964
Since the aircraft is symmetric with the $x_1 - x_3$ plane only the $I_{13}$ products of inertia exist. $I_{13}$ carries a minus sign.

Finally, by using more conventional nomenclature these force and moment equations will be more readily recognized. $F_{x1}$, $F_{x2}$, $F_{x3}$, $T_{x1}$, $T_{x2}$ and $T_{x3}$ are the total external forces and torques, and as such denote aerodynamic effects (also gravity and thrust effects if applicable).

- $F_{x1} = X_a$, total aerodynamic $x$-force.
- $F_{x2} = Y_a$, total aerodynamic $y$-force.
- $F_{x3} = Z_a$, total aerodynamic $z$-force.
- $T_{x1} = L_a$, total aerodynamic rolling moment.
- $T_{x2} = M_a$, total aerodynamic pitching moment.
- $T_{x3} = N_a$, total aerodynamic turning moment.

- $v_{x1} = U$, velocity along $x$-body axis.
- $v_{x2} = V$, velocity along $y$-body axis.
- $v_{x3} = W$, velocity along $z$-body axis.
- $\omega_{x1} = p$, angular velocity around $x$-body axis.
- $\omega_{x2} = q_1$, angular velocity around $y$-body axis.
- $\omega_{x3} = r$, angular velocity around $z$-body axis.

$M = m$, the aircraft mass.

The equations for the basic aircraft then become the following expressions.

- $X_a = m(\dot{U} + Wq_1 - Vr)$
- $Y_a = m(U + Ur - Wp)$
- $Z_a = m(\dot{W} + Vp - Uq_1)$
- $L_a = I_{11}\dot{\phi} - I_{13}(\dot{f} + p q_1) + (I_{33} - I_{22})q_1 r$
- $M_a = I_{22}q_1 - I_{13}(r^2 - p^2) + (I_{11} - I_{33})p r$
- $N_a = I_{33} f - I_{13}(\dot{\beta} - q_1 r) + (I_{22} - I_{11})pq_1$