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Fatigue Phenomena and Their Interpretation

JULY 1966

Prepared by C. C. S. YEN
Materials Sciences Laboratory
Laboratories Division
Laboratory Operations
AEROSPACE CORPORATION

Prepared for BALLISTIC SYSTEMS AND SPACE SYSTEMS DIVISIONS
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California
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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-669.

This report, which documents research carried out from July 1965 through January 1966, was submitted on 13 September 1966 to Captain William D. Bryden, Jr., SSTRT, for review and approval.

This paper was initiated when the author prepared a lecture in a course entitled "Survey of Metal Fatigue" at the University of Southern California, in which Professor A. F. Madayag was the coordinator.

The author wishes to express his appreciation to many fatigue investigators as listed in the References, whose contributions have made this paper possible. The following figures and table have been adapted as indicated: Fig. 1 from Ref. 1; Fig. 2 from Ref. 2; Fig. 3 from Ref. 3; Fig. 4 from Ref. 4; Fig. 5 from Ref. 5; Fig. 6 from Ref. 8; Fig. 7 from Ref. 11; Fig. 8 from Ref. 14; Fig. 9 from Ref. 14; Fig. 10 from Ref. 16; Fig. 12 from Ref. 15; Fig. 13 from Ref. 17; Fig. 16 from Ref. 23; Fig. 17 from Ref. 18; Fig. 18 from Ref. 24; Fig. 19 from Ref. 9; Fig. 20 from Ref. 27; Fig. 21 from Ref. 26; and Table 3 from W. A. Wood, "Some Basic Studies of Fatigue in Metals," Fracture, MIT, John Wiley and Sons Inc., p. 412 (1959).

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Approved

W. C. Riley, Associate Director
Materials Sciences Laboratory
Laboratories Division
Laboratory Operations

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

William D. Bryden, Jr., Capt., USAF
Space Systems Division
Air Force Systems Command
ABSTRACT

Important fatigue data are reviewed, and typical data patterns are developed and interpreted. The causes and effects of materials fatigue and the effects of various loads or stresses on fatigue properties are described. High-level stress (or low-cycle fatigue), low-level stress, mean stress, combined stress, stress concentration, random loading, and the speed of application of cyclic stresses are included in the discussion. The effect of temperature on fatigue is also described.
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INTRODUCTION

With so many automobiles, aircraft, and other machines operating around us, fatigue phenomena of engineering materials are a constant occurrence. Even though this subject has been studied and discussed in many publications, there is a need of a concise, critical review of the important experimental data useful in engineering design, together with scientific interpretations. This paper is intended to fill this need and based on this general review, interesting comparisons, correlations, and insights are developed.

Fatigue data are a record of the causes and effects of fatigue phenomena. Two basic causes that create fatigue problems are repeated stress (which includes stress history and distribution) and a stressed body (which includes materials and structures). The effects include atomic, microscopic, and macroscopic phenomena such as dislocation movements, slip formation, crack nucleation, crack propagation, and complete failure. Usually, engineers are mainly interested in macroscopic phenomena which is, primarily, crack propagation. A diagram showing the cause and effect in fatigue is shown in Table 1; further analysis of the causes is shown in Table 2.

S-N DATA

Since the beginning of fatigue testing by Wohler in 1858, S-N curves have been the "backbone" of fatigue data. The S denotes stress amplitude or the maximum cyclic stress and the N denotes the number of stress cycles to complete fracture. The linear S vs log N scale is the most common one and is used almost exclusively in engineering. Typical S-N curves are shown in Figure 1 for axial loading\(^1\) and in Figures 2 and 3 for rotating beam loading.\(^2,3\) At the low cycle end, the S-N curve is sometimes above the tensile strength. However, in the case of notched
<table>
<thead>
<tr>
<th>CAUSE</th>
<th>EFFECT</th>
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<tbody>
<tr>
<td>Stressed Body</td>
<td>Atomic</td>
</tr>
<tr>
<td>Repeated Stress</td>
<td>Microscopic</td>
</tr>
<tr>
<td>Environment</td>
<td>Macroscopic</td>
</tr>
</tbody>
</table>

**Table 1. Fatigue Causes and Effects**

1. Dislocation movements 1. Slip formation  Crack Propagation
2. Dislocation multiplications 2. Extrusion & intrusion
3. Other defect movements & interactions 3. Slip saturation 1. Stable stage
5. Plastic strain 3. Critical length
7. Crack nucleation & growth
Table 2. Factors Affecting Fatigue Properties

<table>
<thead>
<tr>
<th>Stress Field or Distribution</th>
<th>Magnitude</th>
<th>Stress Field or Distribution</th>
<th>Axial load</th>
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<tr>
<td></td>
<td></td>
<td>Axiality</td>
<td>Biaxial</td>
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<td>Triaxial</td>
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<td>Combinations</td>
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<td>of the above types</td>
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<td></td>
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<td>Vibratory bending</td>
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<td>Rotary bending</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Axial load</td>
</tr>
<tr>
<td>Spectrum or History or Stress</td>
<td>Stress ratio (or mean stress or static stress)</td>
<td>Prestress or residual stress</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stress wave form: Regular: sinusoidal, square, etc.</td>
<td>Irregular: random</td>
<td></td>
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<tr>
<td></td>
<td>Frequency (cycles per unit time) or strain rate.</td>
<td>Rest period</td>
<td></td>
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<tr>
<td></td>
<td>Varying amplitude</td>
<td>Spectrum</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Sequence</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Random amplitude</td>
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</table>

| Chemistry or Materials | Ferrous and nonferrous alloys, nonmetalics, impurities, etc. |
| Substructure | Point defects, dislocations, sub-boundaries, etc. |

| Microstructure or Material Condition | Raw stock |
| Grain | Shape |
| Inclusion | Size |
| Segregation | Location |
| Residual stress |

| Geometry of Structural Component | Size |
| Shape |
| Notches |
| Surface properties |
| Simple joints - weld, riveted, bolted |
| Other components, from stiffened flat sheet, shell, to others |

| Temperature |
| Vacuum |
| Radiation |
| Air |
| Corrosion |
| Fret corrosion |
Figure 1. Results of fatigue tests on 6061T6 aluminum-alloy specimens under axial load at R = 0.
Figure 2. S-N curves for smooth and notched rotating-beam fatigue specimens of 4340
Figure 3. Relationship between stages of progressive changes observed on 0.97 Mg aluminum alloy

(Redrawn from Ref. 3)
bending specimen, it is close to the notch strength, and for rotating beam specimens, close to the modulus of rupture. This is because the elementary flexural formulas commonly used to calculate the stresses do not represent realistically the distribution of stress. In general, S-N curves represent the progressive structural deterioration and the gradual breaking of cohesive bonds in materials. This process may be analyzed statistically. The S-N curves for nonmetallic materials also indicate structural deterioration\(^4\) (Figure 4).

A collection of S-N data for small, polished, steel-bar, rotating-beam specimens\(^5\) (Figure 5) shows that the endurance limit is approximately 50 per cent of the tensile strength, and the strength at \(10^4\) cycles is approximately 75 per cent of the tensile strength. This curve may be used for preliminary design purposes when fatigue data are not available.

Several attempts have been made to find general mathematical relationships between stress and life\(^6,7\) and several different equations have been proposed to express the S-N relations empirically. Use of these equations reduces the data to a suitable form for data analysis and standardizes curve fitting methods. It also provides some understanding of the S-N relationships.

Weibull\(^6\) proposed an equation

\[
(S - S_e)(N + B)^a = b
\]

where \(S_e\) is the endurance limit, and \(B, a,\) and \(b\) are constants.

Valluri\(^7\) derived the following more complex equation which has been applied to only one aluminum alloy:

\[
N = \frac{2}{C} \left[ \frac{\ln(\sigma_u/\sigma) \ln \left[ (\sigma - \sigma_i)/K \right]}{\left[ (\sigma - \sigma_i)/E \right]^2 \left[ (\sigma - \sigma_i)/\sigma_i \right]^2} \right] \]
Figure 4. Fatigue data for phenolic laminates at room temperature

(Redrawn from Ref. 4)
Figure 5. Ratio of fatigue strength to static strength for various steels

Figure 6. Effect of internal defects on fatigue life of 7079-T6 aluminum alloy forged plate stressed in short transverse grain direction
where

\[
\begin{align*}
\sigma &= \text{maximum cyclic stress} \\
\sigma' &= \text{minimum cyclic stress} \\
\sigma_u &= \text{ultimate tensile strength} \\
\sigma_i &= \text{internal stress (approximately the endurance limit)} \\
K, C &= \text{material constants} \\
E &= \text{Young's modulus}
\end{align*}
\]

When identical specimens are fatigue tested at the same stress level, their fatigue lives are generally not the same but vary or scatter a great deal. When many specimens are tested at several stress levels, the test points are scattered as shown in Figure 6. Based on the scatter of data points, a family of curves called S-N-P curves may be drawn as shown in Figure 7, where \( P \) is the probability of survival. It is anticipated that the actual performance for \( P\% \) of all specimens will be above the S-N value indicated by the corresponding \( P \) curve.

LOW-CYCLE AND HIGH-CYCLE FATIGUE

An S-N curve may be divided into the low-cycle range and the high-cycle range; there is no sharp demarcation between these ranges. From 0 to about \( 10^3 \) or \( 10^4 \) cycles is generally considered the low-cycle range and from about \( 10^3 \) or \( 10^4 \) cycles to \( 10^7 \), or higher, the high-cycle range.

Until recently, little attention has been paid to the low-cycle range, and as a result, much of the existing fatigue data are for high cycles only. It soon became apparent that for some pressure vessels, pressurized fuselages, landing gears and wing flap mechanisms, space-ship launching equipment, missiles, etc., only a short fatigue life was required. Consequently, the low-cycle fatigue phenomena began to gain attention.
Figure 7. S-N-P diagram for aluminum alloy 7075 T6
The initial portion of an S-N curve is usually horizontal and flat; for notched specimens, the flat portion is shorter than for plain specimens as shown in Figure 8.\textsuperscript{14, 15} The difference in the initial behavior between notched and plain specimens appears related to their stress-strain curves (Figures 8 and 9).\textsuperscript{14} When the cycles are low, the material fatigue strength is close to the static strength. Consequently, when the notched static strength is higher than the plain tensile strength, the notched fatigue strength at the upper end of S-N curve is also higher than the plain tensile strength. When the cycles are increased, notched fatigue strength drops rapidly to a level that is lower than that for the unnotched fatigue strength.

Another point is noted in the comparison of Figures 8 and 9. Near the top of both the static and the fatigue curves the slope change for the notched specimen is often greater than that for the unnotched. Since the notched specimens show reduced plastic strain in static test and shorter life than the unnotched in fatigue test, the plastic strain may be related to fatigue life. Thus, the plastic strain appears to be a better measure of life than the nominal stress. Indeed, in constant deformation fatigue tests, a straight-line relationship exists between the logarithmic values of either the maximum plastic strain or the range of plastic strain and the lives of the members (Figure 10).\textsuperscript{16} This is expressed by

\[ \Delta \varepsilon_p N^{1/2} = C \]  

where the constant C may be related to the reduction of area R in static tension tests as

\[ C = 0.5 \ln \left( \frac{100}{100-R} \right) \]  

Many reports have considered the static tensile strength as the upper limit of the S-N curve, located arbitrarily at 1/4, 1/2, 3/4, or 1 cycle. It should be noted that the tensile strength test data are usually obtained from a static tension testing machine operating at a low strain rate;
Figure 8. Typical S-N curves for constant-load tests

Figure 9. Typical stress-strain curves
Figure 10. Plastic strain range vs cycles to failure; material, 2S aluminum annealed
fatigue test data are obtained in fatigue testing machines at higher strain rates. The failure mechanism in the low-cycle range is similar to the failure mechanism in static tension, but the failure mechanism in the high-cycle range is different and may be termed "true fatigue." The two mechanisms are compared in Table 3.

EFFECT OF MEAN STRESS AND COMBINED STRESS

The stresses created in the laboratory for fatigue study are usually sinusoidal or vibrational stresses, and the vibrational stresses are usually expressed in terms of a pair of variables, such as the mean stress $S_m$ and the amplitude $S_a$ (Figure 11). The fatigue S-N data are then determined using a pair of stress variables. In Figure 12, the data were determined by the maximum stress $S_{\text{max}}$ and the ratio of minimum stress to maximum stress $R$. Figure 13 clearly shows the relationships between different pairs of stress variables and between the stress pair and the fatigue life. The mean stress is sometimes also called the static stress or superimposed stress.

When two- or three-dimensional stresses act at a point, they are referred to as combined or complex stresses. The amount of fatigue data for different combinations or states of stresses is overwhelming. Fortunately, there are some general rules on fatigue effect of combined stress that can be used as a guide. One rule proposed by Sines explains not only the effect of combined stresses but also the effect of mean stresses and residual stresses. Sines proposed that the permissible alternation of the octahedral shear stress is a linear function of the sum of the orthogonal normal static stresses, as long as the maximum stress is below the yield strength, or

$$1/3[(P_1 - P_2)^2 + (P_2 - P_3)^2 + (P_3 - P_1)^2]^{1/2} = A - \alpha(S_x + S_y + S_z + R_x + R_y + R_z)$$

(5)
Table 3. Comparison of Low-Cycle and High-Cycle Fatigue

<table>
<thead>
<tr>
<th></th>
<th>Low-Cycle</th>
<th>High-Cycle</th>
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<tbody>
<tr>
<td>Internal stresses and strain hardening</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Net sum of plastic flow</td>
<td>Macro size</td>
<td>Micro size</td>
</tr>
<tr>
<td>Gross sum of plastic flow</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>X-ray disorientation</td>
<td>Large</td>
<td>Small</td>
</tr>
<tr>
<td>Slip</td>
<td>Coarse $\left(10^{-3} - 10^{-4} \text{A}\right)$</td>
<td>Fine (10A)</td>
</tr>
<tr>
<td>Slip plane distortion</td>
<td>Normal</td>
<td>Persistent</td>
</tr>
<tr>
<td>Crack origin in pull-pull load</td>
<td>Interior</td>
<td>Surface</td>
</tr>
<tr>
<td>Crack path</td>
<td>Along max. shear</td>
<td>Cross max. tensile stress</td>
</tr>
<tr>
<td>Fracture</td>
<td>Delayed static</td>
<td>Structure deteriora-tion</td>
</tr>
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</table>

Figure 11. Fatigue stress components
Figure 12. S-N curves for various stress ratios, unnotched specimens 18Cr-9Ni stainless steel sheet, as rolled, 0.039-in.t

(Redrawn from Ref. 15)
Figure 13. Modified Goodman diagram for 2014-T6.
where $P_1$, $P_2$, and $P_3$ are the amplitudes of the alternating principal stresses; $A$ is a material constant that is the octahedral shearing fatigue strength (alternating amplitude) with no superimposed static and residual stresses; $\alpha$ is a material constant that is the proportional factor between static and residual stresses and the permissible alternating octahedral shearing stress; $S_x$, $S_y$, and $S_z$ are the orthogonal static stresses; and $R_x'$, $R_y'$, and $R_z$ are the orthogonal residual stresses.

In addition to Sines' linear equation on the effect of mean stress, there are Goodman's linear law and Gerber's parabolic law proposed for the uniaxial stress system.

Sines' equation

\[ S_a = S_e - CS_m \]  \hspace{1cm} (6)

Goodman's equation

\[ S_a = S_e - S_e \frac{S_m}{T} \]  \hspace{1cm} (7)

Gerber's equation

\[ S_a = S_e - S_e \left( \frac{S_m}{T} \right)^2 \]  \hspace{1cm} (8)

where

- $S_a$ = fatigue strength in terms of the stress amplitude
- $S_e$ = endurance limit when $S_m = 0$
- $C$ = material constant
- $S_m$ = superimposed mean stress
- $T$ = ultimate tensile strength

These equations are compared in Figure 14. In Equation (6), $S_e$ corresponds to $A$ in Equation (5) and $C$ corresponds to $\alpha$. 

-20-
Figure 14. Comparison of Sines', Gerber's, and Goodman's relations
Sines' equation covers the maximum stress level only up to the yield strength, but Goodman's and Gerber's laws cover the maximum stress levels up to the ultimate tensile strength. Sines' is more general in that it covers three dimensional stress states, and by adjusting the value of C, it can be made to coincide with Goodman's. It also can be used as a linear approximation for Gerber's parabola under yield strength.

**Effect of Stress Concentration**

Fatigue is a major cause of aircraft and machine component failure and practically all fatigue failures are stress concentration failures. The fatigue crack usually originates at a point where the stress is highest and the stress peaks often occur at the surface. Therefore, fatigue failures are surface failures, and the fatigue damage before cracking is only skin deep. Fatigue cracks initiated at the surface are shown in Figure 15. The depth of fatigue damage beyond some crack tips has been found to be less than 0.020 in. or 0.030 in. 19, 20

There are several reasons why stress peaks, fatigue damages, and crack origins occur on the surface.

1. The applied load is rarely truly axial; some bending or twisting practically always exists. Further, the part often has stress raisers on the surface, and as a result, the stress distribution is never uniform, with the maximum stress usually located at the surface. Even if the applied stress is uniform, the residual or assembly stress often reaches a maximum at the surface. 21, 22

2. The corrosion and erosion damage always roughens the smooth surface and introduces pits and notches, which, in turn, introduce stress concentrations when under load.

3. Almost all types of fractures (fatigue, creep, ductile or brittle) start from a microscopic local region with shear stress concentration. In fatigue fractures, microscopic plastic deformation or slip occurs more readily on the surface than in the interior. It roughens the surface and introduces extrusions and intrusions. The intrusion is a sharp notch (a stress raiser) that starts the crack.
Figure 15. A portion of the cross section of a stainless steel tube, 1/2-in o. d. and 0.028-in. wall thickness. (This photograph illustrates a typical fatigue crack starting from the tube internal surface; magnification is 100X.)
4. Basically, a surface may be considered as a crystal defect. Further, an atomically perfect surface is not practical to prepare. Because of the vulnerable free surface, a fatigue weakening or strengthening process may be only a surface treatment, especially near the stress concentration area. New types of surface treatment, should be checked for their fatigue effect. Sometimes internal defects and large second-phase particles can also introduce stress concentration at the interface.

To study the stress concentration effect on fatigue, test specimens are usually designed with notches or holes. The ratio of the peak stress in the notched specimen to that of a corresponding unnotched specimen is called the theoretical stress concentration factor $K_t$. The peak stress at the root of the notch may be determined mathematically, photoelastically, or by X-ray measurement. The peak stress in the unnotched specimen is always calculated from an elementary stress formula such as $S = P/A$, $S = Mc/I$, or $S = Tc/J$.

Since the peak stress is raised by a factor $K_t$ due to the notch, it is expected that the strength of the notched specimen would be reduced by $K_t$. In fact, the strength reduction of notched specimens tends to increase with $K_t$ but usually is not as large as $K_t$. The ratio of the nominal fatigue strength of an unnotched specimen to that of a notched specimen is called the strength reduction factor, the fatigue notched factor, or the effective stress concentration factor, and is denoted by $K_f$ or $K_e$. Usually $K_f$ is smaller than $K_t$.

To explain why $K_f$ tends to be smaller than $K_t$, continuum mechanics postulates that the theoretical peak stress $K_t S_n$ is lowered to $K_t S_{n+}$ by plastic flow (Figure 16), where $S_n$ is the nominal stress in the corresponding unnotched specimen as found by an elementary formula. The increment of the plastic stress $K_t S_{n+}$ over the nominal stress $S_n$ divided by the increment of the theoretical or elastic stress $K_t S_n$ over the nominal $S_n$ is defined as an index for the notch sensitivity $q$. Thus
Figure 16. Lowering of peak stress by plastic action

(Redrawn from Ref. 23)
\[
q = \frac{K_f S_n - S_n}{K_t S_n - S_n} = \frac{(K_f - 1) S_n}{(K_t - 1) S_n} = \frac{K_f - 1}{K_t - 1}
\]

For engineering in steel, Petersen relates notch sensitivity to notch radius and tensile strength (Figure 17). This subject has been reviewed thoroughly by Yen and Dolan.

RANDOM LOADING AND CUMULATIVE DAMAGE

As pointed out, the fatigue data of many specimens are always scattered over a wide band (Figures 6 and 7). Similarly, the service load magnitudes often have several different values as shown in Figure 18.

The aircraft service loads that contribute to fatigue include many different situations: gust load in clear air and storm; maneuver loadings; landing impacts; taxiing and ground handling; ground-air cycles; buffeting, stalling, and supersonic shock wave instability; and acoustical noise due to propeller tip or jet and aerodynamic boundary layer noise. In all of these loadings, the sequence of the load magnitudes varies, more or less, in a random manner (e.g., the random sequence of the high and low flight-gust loads as represented in Figure 19).

The standard practice in the laboratory is to test each specimen at a constant load amplitude and obtain an S-N curve, but in service, the load on each critical part varies a great deal. The problem is how to use the constant amplitude laboratory data or the S-N curve in designing parts that will resist variable amplitude service loads. One approach is to use the equation proposed independently by Palmgren and by Miner

\[
\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots + \frac{n_m}{N_m} = 1
\]
Figure 17. $q$ vs $r$ curves for steels, bending or axial loading
Figure 18. Statistical frequency distribution diagrams of (1) fatigue loads and (2) fatigue strengths of samples. (Abscissas indicate load or strength magnitudes.)

Figure 19. Random sequence of flight loads
or simply

\[ \sum \frac{n}{N} = 1 \]

where \( n_1, n_2, n_3 \ldots n_m \) are the number of cycles applied at different stress levels of \( S_1, S_2, S_3 \ldots S_m \) respectively. Each \( n \) causes partial damage and the total \( \sum n \) causes complete failure. \( N_1, N_2, N_3 \ldots N_m \) are the number of cycles at one of the stress levels of \( S_1, S_2, S_3 \ldots S_m \) respectively, but each \( N \) causes complete failure. Thus, according to this equation, the damage to a part at a given stress \( (S_1, S_2, \text{ or } S_m) \) increases directly with the increase of the number of cycles at that stress.

This equation has been used by designers for many years, but at the same time, it has been criticized by researchers. It has been noticed in many test results that \( \Sigma n/N \) is far from 1 and that the fatigue damage is not linearly related to the number of cycles or the cycle ratio \( n_1/N_1 \). Also, interaction between the fatigue damages at various stress levels has been neglected. In this interaction, there is a sequence effect. The fatigue damage that results from the application of the high load followed by a low load is not the same as the damage that results from application of a low load followed by a high load.

Although aware of its limitations, designers still use Miner's equation as a preliminary guide. Cummings stated that for the design engineer, there appears to be, for the present, no alternative to using the linear cumulative damage hypothesis (Miner's equation) as a starting point. In another report, twenty different methods of assessing cumulative damage were reviewed, the conclusion reached was that the linear cumulative damage procedure (Miner's equation) warranted recommendation for its simplicity, versatility, and accuracy (commensurate with the data currently available) when satisfactory constant amplitude S-N type data can be provided for a specific structure.
Random-load fatigue testing is time consuming and costly, so the data are limited and do not permit a full evaluation of the use of Miner's equation for random loading conditions. According to Freudenthal, $25\varepsilon_n/N$ is always less than 1. In some cases, it is as low as 0.13, but it generally falls between 0.20 and 0.60. Head also reported that Miner's equation overestimated the fatigue life in random loading by factors of from 2 to 3.5. The primary reason for this discrepancy, according to Freudenthal, is the interaction of fatigue damage between various load amplitudes.

**EFFECT OF TEMPERATURE AND FREQUENCY**

The temperature of the materials and the structures and the frequency (defined as time rate of the load cycle repetition) influence fatigue life. Generally speaking, the higher the temperature, or the lower the frequency, the lower will be the fatigue life or strength (Figures 20 and 21). 15, 26-28

From the viewpoint of thermal activation on materials properties, the temperature and the frequency effect are closely interrelated. This relationship was brought out by Daniels and Dorn in their fatigue study of annealed pure aluminum. 26 Within the scope of their tests, the cycles to failure $N$ is related to the temperature $T$, frequency $\nu$, and stress $\sigma$ through thermal activation energy $\Delta H$ and the gas constant $R$ by

$$N = f(\nu e^{\Delta H/RT}, \sigma)$$

(11)

where $f$ is an experimentally determinable function (Figure 21). Thus for a given stress, the same number of cycles can be endured at the same value of $\nu e^{\Delta H/RT}$, and the effect of higher testing frequencies are comparable to the effects of lower temperatures.

From a metallurgical point of view, higher temperatures may cause instability in the microstructure of a metal, depending on the alloy, the temperature, the time of exposure, and the situation. Some of the phenomena are: recovery, phase change, grain coarsening, recrystallization, reprecipitation, reversion, diffusion, aging, overaging, sigma-phase formation, graphitization, decarburization, annealing, oxidation, and other
Figure 20. Fatigue strength of wrought aluminum alloys at various temperatures

(Redrawn from Ref. 27)
Figure 21. Composite N-temperature diagram, annealed pure aluminum (2450 psi)

(Reprinted from Ref. 26)
corrosion and chemical changes. Many of these phenomena are recognized or are suspected of being factors that affect fatigue properties.

From a macroscopic point of view, the higher temperatures often hasten crack initiation, but they may retard crack propagation and increase critical crack length. The lower temperatures often retard crack initiation, hasten crack propagation, and reduce critical crack length.

In high-temperature fatigue, failure or the initiation of failure is sometimes excessive deformation or creep instead of the pure progressive fracture often encountered in low-temperature fatigue testing. Hence, there is a close relationship between high-temperature fatigue and creep that should be considered when the thermal activation process is important. This relationship is shown in Equation (11) and Figure 21, where the fatigue parameter \( ve^{\Delta H/RT} \) or \( \log e v + \frac{\Delta H}{RT} \) is essentially the same as the creep parameter \( t e^{-\Delta H/RT} \), where \( t \) is the time under stress. 29

**SUMMARY**

It has been shown that the entire S-N curve may be divided into two ranges: near the top, the low-cycle fatigue; near the bottom, the high-cycle fatigue. Low-cycle fatigue and high-cycle fatigue operate in the metal microstructure with different basic mechanisms. In the S-N curves, a third quantity that describes the scatter, the probability of survival or the minimum curve is also useful, but it necessitates extensive experimentation.

The stress that causes fatigue damage always has two components (e.g., the mean stress and the amplitude or the maximum stress and the ratio R). We have reviewed Sines', Goodman's, and Gerber's equations to express the effect of mean stress. Stresses can combine in two- or three-dimensional directions in different states, but the most important stress is the maximum shear or the octahedral shearing stress.
Based upon the experiences of practical engineers, it can be said that fatigue failures are tension failures, stress concentration failures, and surface failures, and that fatigue damage is only skin deep.

Since the service-load amplitude is not constant, a method of assessing cumulative damage is necessary. For this purpose, industry has been using Miner's equation extensively despite criticism by researchers.

The effects of temperature and load frequency on mechanical properties are sometimes related (e.g., high frequency effects may be equivalent to low temperature effects). The temperature effect is one of the most critical fatigue problems in the design of supersonic and hypersonic vehicles.

While many are still concerned with the atomic and microscopic processes by which a fatigue crack develops in a solid material, engineers are always faced with a new spectrum of fatigue problems in connection with new vehicles (supersonic and hypersonic transports), new materials (composites), or new environment (outer space).
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FATIGUE PHENOMENA AND THEIR INTERPRETATION

Important fatigue data are reviewed, and typical data patterns are developed and interpreted. The causes and effects of materials fatigue and the effects of various loads or stresses on fatigue properties are described. High-level stress (or low-cycle fatigue), low-level stress, mean stress, combined stress, stress concentration, random loading, and the speed of application of cyclic stresses are included in the discussion. The effect of temperature on fatigue is also described.
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Abstract (Continued)