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CONTINUOUS DETERMINATION OF THE AVERAGE SOUND VELOCITY OVER AN ARBITRARY PATH

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ABSTRACT

A reasonably accurate determination of the velocity of sound in the open air along a path of arbitrary length by some inexpensive means is the crux of sonic atmospheric sensing. At present, this is accomplished by measuring either time delays or phase differences of an acoustic signal. This paper explores the technique of phase measurement and proposes a direct approach to the problem of determining the sound speed within an arbitrary layer. By means of the phase technique, the initial conditions of the sound velocity and the vector wind may be determined. The temporal distribution of the sound speed is found from an equation for the average sound velocity along the paths between the sound source and two opposing transducers.
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INTRODUCTION

The fundamental problem in sonic atmospheric sensing lies in the temporal determination of an accurate speed of sound in the open air and the separation of the wind effects from the scalar thermodynamic sound speed. It is apparent that the accuracy of the latter is highly dependent upon that of the former. The usual technique (Barrett and Suomi, 1949; and John, 1960) is to measure the difference and sum of transit times of the sound pulses from a source travelling in opposite directions to two transducers, each at a specified distance from the source. At high frequencies and short distances, the difference in transit times is generally of the order of microseconds. However, the same result may be accomplished by measuring the relative phase angles (Coffman and Price, 1962), thereby magnifying the sum and difference by a factor of \(2f\), where \(f\) is the frequency used. The phase technique was utilized over a long path, approximately 60 meters (Low, 1964), and was found to give satisfactory results when compared to the results derived from an array of thermocouples along the same path. Kaimal and Businger's system (1963), constructed on the same principle, measures the vertical wind and temperature variations approximately at a point by artificially presetting certain mean values of the temperature and hence of the sound velocity under windless conditions.

In the present paper, the technique of phase measurement in sonic atmospheric sensing is extended. Instead of attempting to obtain solutions for the wind components from the basic equations, an effort is made to determine continuously a reasonably accurate value of the sound velocity in the open air over an arbitrary path. Then it is a simple matter to extract both the wind and temperature data along the same path.

THEORY

Consider the isosceles triangle ABC in Figure 1 and assume that the wind vector can be resolved along the path of sound propagation, bearing in mind Schotland's (1955) excellent analysis of wind measurement by sonic means. A sound source is placed at the vertex A and a transducer at each of points B and C. In addition to those labeled in the figure, other notations are listed below:

Figure 1
\[ D = AB = AC \]

\[ C_0, C = \] the initial and subsequent velocities of sound;

\[ f, f' = \] the frequencies used at the sound source;

\[ m, n = \] the numbers of wavelengths in the paths AB and AC respectively at the given frequencies;

\[ t_0, t = \] the initial and subsequent transit times from A to B;

\[ t_0', t' = \] the initial and subsequent transit times from A to C;

\[ W_0, W = \] the initial and subsequent wind components;

\[ \lambda, \lambda' = \] the initial wavelengths along the paths AB and AC respectively;

\[ \Delta \phi, \Delta \phi' = \] the phase differences along the paths AB and AC respectively.

Symmetry has been assumed (see Figure 1 and notations).

The initial conditions require that for the frequencies \( f \) and \( f' \) used, the numbers \( m \) and \( n \) must be accurately determined such that:

for path AB

\[ D = m \lambda \quad \text{and} \quad f_\lambda = C_0 - W_0 \cos \alpha \]  

for path AC

\[ D = n \lambda' \quad \text{and} \quad f'\lambda' = C_0 + W_0 \cos \alpha. \]  

Under these conditions, it is preferable, though not necessary, to make \( m \) and \( n \) integers.

The travel time of a wave front emitted from the sound source A to point B, by utilizing equations (1) and (2), is given by

\[ t_0 = \frac{D}{(C_0 - W_0 \cos \alpha)}. \]  

The sound velocity and the wind component are now denoted by \( C \) and \( W \), respectively, and since they no longer represent initial conditions, a new travel time is obtained

\[ t = \frac{D}{(C - W \cos \alpha)}. \]  

The difference, \( \Delta t = t_0 - t \), is the time delay or time difference. From equations (5) and (6) the time difference may be determined as

\[ \Delta t = \frac{D}{C_0 - W_0 \cos \alpha} - \frac{D}{C - W \cos \alpha}. \]  

Solving for the velocity \( C \) by a series of algebraic manipulations, it is found that
\[ C = \frac{C_0 - W_0 \cos \alpha}{1 - \frac{(C_0 - W_0 \cos \alpha) \Delta t}{\Delta \phi}} + W \cos \alpha. \]  

Equation (8) may be simplified by means of equations (1) and (2), noting that \( f/m = (C_0 - W_0 \cos \alpha)/D \).

\[ C = \frac{C_0 - W_0 \cos \alpha + W \cos \alpha}{1 - \frac{\Delta \phi}{2\pi}} \]  

(9)

The difference in time \( \Delta t \) can now be expressed as the difference in phase angle \( \Delta \phi \) by the formula \( \Delta \phi = 2\pi f \Delta t \). Substitution of this formula into equation (9) gives the desired expression for the average sound velocity \( C \) in terms of phase difference with respect to the initial conditions.

\[ C = \frac{C_0 - W_0 \cos \alpha + W \cos \alpha}{1 - \frac{\Delta \phi}{2\pi}} \]  

(10)

By the same series of manipulations, using \( \Delta t' = t_0' - t' \) for the path AC, the following expression for the average sound velocity along AC is found to be:

\[ C = \frac{C_0 + W_0 \cos \alpha}{1 - \frac{\Delta \phi'}{2\pi}} - W \cos \alpha. \]  

(11)

Expressions (10) and (11) form a pair of simultaneous equations and can be solved exactly, if so desired; however, the result may not sufficiently justify the effort. Solving for the average sound velocity \( C \) by combining the above two equations, we obtain

\[ 2C = \frac{(C_0 - W_0 \cos \alpha)/(1 - \Delta \phi/2\pi)}{1 + \Delta \phi/2\pi} \]  

+ \( (C_0 + W_0 \cos \alpha)/(1 - \Delta \phi'/2\pi) \).  

(12)

Generally, \( \Delta \phi, \Delta \phi' \ll 2\pi, 2\pi \) respectively; it is reasonable to take the following approximations:

\[ \frac{1}{1 - \Delta \phi/2\pi} \approx 1 + \Delta \phi/2\pi, \text{ and} \]  

\[ \frac{1}{1 - \Delta \phi'/2\pi} \approx 1 + \Delta \phi'/2\pi. \]  

(13)

Upon expansion of equation (12) using the above approximations and factoring, we obtain two terms.
which are not much different from zero. Thus, a reasonably accurate expression
for the continuous average sound velocity along an arbitrary path is given by

\[ C = C_0 \left[ 1 + \frac{1}{2n} \left( \Delta \alpha / m + \Delta \alpha' / n \right) \right]. \]  

(15)

The average acoustic virtual temperature (Suomi, 1949) in degrees absolute
along the path, which will not be appreciably different from the average free
air temperature along the same path with low humidity conditions, is readily
obtainable from the average sound velocity \( C \) given above by means of the Laplace
formula.

To obtain the average horizontal wind component across the paths, equations
(10) and (11) are solved simultaneously and, with approximations, yield

\[ 2W \cos \alpha = 2 W_0 \cos \alpha - (C_0 - W_0 \cos \alpha) \Delta \alpha / 2n \]

\[ + (C_0 + W_0 \cos \alpha \Delta \alpha') / 2n. \]  

(16)

Equations (1), (2), (3), and (4), the initial conditions, and the fact that
\( \cos \alpha = d/D \) are used to simplify this expression. The average horizontal wind
component \( W \) is then given by

\[ W = W_0 - \frac{1}{2n\alpha} \left( f \Delta \alpha - f' \Delta \alpha' \right). \]  

(17)

DISCUSSION AND CONCLUSIONS

The initial conditions may conceivably cause some concern as to their mean-
ing. Actually, the setting of the initial conditions could be termed calibra-
tion which is a required step for any instrument. Under normal weather condi-
tions, this setting need not be at all altered if there is no drift in the elec-
tronic equipment.

In the course of developing the equations in the preceding section, no re-
strictions have been imposed thereon, except for approximations which are con-
sidered reasonable and should in no manner jeopardize the precision of measure-
ment within the limitations of the electronic equipment. Equations (15) and
(17) are generalized forms; however, the frequencies chosen do not have to be
different, in which case either \( m \) or \( n \) may take on the value of a noninteger.
The use of a single frequency does not greatly simplify the computations, but it
has the advantage of presenting a visual picture of the wind effect when the
temperature remains unchanged during a short interval of observation, and it may
bring about some economy in instrumentation. Furthermore, as may be obvious
from equations (2) and (4), the wind effect will cause the phase angles \( \Delta \alpha \) and
\( \Delta \alpha' \) to assume opposite signs, thereby indicating the sense of the wind direc-
tion, while the temperature effect will cause them to have the same sign, either
negative or positive, depending on the initial calibration values.
The system is highly sensitive to wind and temperature changes; therefore, the frequency or frequencies to be used should be selected with care. Unless a higher resolution is required, the lower audible frequencies should be used for reasons noted below. As shown before, $\Delta f = 2f \pi t$. Normally a change of 1°C in temperature will cause a change of about 0.6 meter per second in sound speed. Now assume that the time difference $\Delta t$, as a result of a change of 1°C, is about 1 millisecond. It is easily seen that at $f = 100$ cps, $\Delta f = 0.36\degree$; at $f = 1000$ cps, $\Delta f = 3.6\degree$, etc. The same reasoning holds for a change in the wind field. Moreover, the longer the path $D$, the larger the time difference $\Delta t$ and the greater the desirability to use lower frequencies.

As the angle $\alpha$ in Figure 1 becomes larger and larger, the value of $W \cos \alpha$ approaches the vertical component of the vector wind. As the angle $\alpha$ approaches 90 degrees, the system will have become a device for temperature measurement, the vertical components being negligible under normal conditions. The average free air temperature in degrees absolute along the path may then be obtained directly from the observed phase angle

$$T = T_0 (1 + 0.00556 \Delta \phi / m).$$

(18)

At the other extreme, as the value of $\cos \alpha$ approaches one, the system assumes the familiar geometry employed in sonic anemometry-thermometry. As the distance between the transducers and sound source is reduced, approximate point measurement is attained.

It may be recalled that in the text the word "wind component" has been employed to emphasize the fact that the wind so obtained is not the total wind vector but a component thereof, depending on the orientation of the system. If two more transducers are available so that each is oriented orthogonally to the next, it is then possible to obtain the average total wind field across the array.
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Abstract

A reasonably accurate determination of the velocity of sound in the open air along a path of arbitrary length by some inexpensive means is the crux of sonic atmospheric sensing. At present, this is accomplished by measuring either time delays or phase differences of an acoustic signal. This paper explores the technique of phase measurement and proposes a direct approach to the problem of determining the sound speed within an arbitrary layer. By means of the phase technique, the initial conditions of the sound velocity and the vector wind may be determined. The temporal distribution of the sound speed is found from an equation for the average sound velocity along the paths between the sound source and two opposing transducers.
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