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APRIL

1965

ROYAL AIRCRAFT ESTABLISHMENT

TECHNICAL REPORT No. 65092

A PEGASUS COMPUTER PROGRAMME FOR THE ASSESSMENT OF THE ACCURACY OF SATELLITE ORBIT DETERMINATION

by

R. H. Gooding

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A PEGASUS COMPUTER PROGRAMME FOR THE ASSESSMENT OF THE ACCURACY OF SATELLITE ORBIT DETERMINATION

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SUMMARY

A computer programme is described by means of which one may estimate the potential accuracy of the elements of a given satellite orbit, if determined from observational data of specified type and assumed accuracy. An application of the programme is made to an orbit of six hours period determined from radar observations at a single station.

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1 INTRODUCTION

There is a recurrent need, in assessment work for possible future projects, for a computer programme which estimates the accuracy with which the parameters of a hypothetical satellite orbit could be determined by use of ground station observations of one or more specified types. Such a programme has now been written for the Pegasus computer and is described in this Report.

The function of the new programme is most easily indicated by comparing the programme with the standard R.A.E. programme for orbit determination[1]. The two main differences of the new programme are the following:-

(i) it is not confined to angular observations, but can deal equally well with observations of range, range rate, direction cosines, etc.,

(ii) as an assessment programme, it does not process actual observations of an actual satellite.

The significance of feature (ii) is that the programme is much simpler to use than a full orbit determination programme would be, and it requires less computer time.

The type of problem to which the new programme refers may be summarised as follows. Nominal values of K unknown parameters of a hypothetical satellite orbit are known, as are also the locations of possible tracking stations. The particular quantities which each station can measure are specified, together with the nominal accuracy associated with each quantity. It is required to estimate the accuracy with which the K orbital parameters could be computed from hypothetical observations over a given number of days.

The difference between the above requirement and that satisfied by a full orbit determination programme may be stated, statistically, rather simply. All observations being subject to random errors, assumed independent and normally distributed, we may consider the K-dimensional variate of errors in orbital parameters computed from a given set of observations. An orbit determination programme estimates, by maximum likelihood, the first and second moments of this variate. An assessment programme, on the other hand, estimates second moments only.

The programme is described in detail in Section 2 of this Report. An application to an assessment problem of recent interest, involving six-hour orbits, is described in Section 3.

One important point must be made. Although the programme does not refer to actual observations, the computer must be told the times at which it

is supposed that observations could be made. Problems, such as (a) finding the periods during which the satellite would be above the horizon at each station and (b) deciding how many independent observations to assume in such periods, are not catered for.

2 THE COMPUTER PROGRAMME

2.1 A brief description

The programme in question is entitled "00.13.29 Error Covariance Estimation for Hypothetical Orbit Determination". The quantities for which covariance estimates are made are certain of the parameters in the Merson model used at R.A.E. as the basis for satellite orbit determination[1]. As in Ref.1 the model parameters are, using standard notation, a_o, e_o, i_o, Ω_o, ω_o, t_o and such additional parameters e_j, i_j, Ω_j, ω_j and n_j as are required; the j suffixed parameters allow polynomial contributions to orbital elements at time t, e.g. $\Sigma\, e_j(t - t_o)^j$ to eccentricity e and $\Sigma\, n_j(t - t_o)^j$ to mean motion n (n_o is given from a_o by Kepler's third law). Some of the parameters are regarded as known quantities and are such that in a genuine orbit determination they would be held fixed at pre-assigned values. The remainder are the 'unknown' parameters which, starting from nominal values, it would be the business of an orbit determination programme to improve by use of observations. Programme 00.13.29 assesses the accuracy with which the unknown parameters, K in number, could be computed in such determination. The distinction between the two types of parameter in the orbital model is one that has been made in the working version of the R.A.E. programme for orbital determination using angular observations only, but it was not present in the original version of that programme described in Ref.1.

Input for the programme consists of the assumed values of <u>all</u> parameters of the hypothetical orbit (including the fixed ones) and of information relating to a number, b, of possible bursts of observational data. A 'burst' is a sequence of observations from the same station, uniformly separated in time; it consists of (a) station position data, (b) type number, indicating the nature of the quantities which could be measured at the station, (c) information as to the number of observations in the burst and the set of uniformly separated times, and (d) the standard deviations of errors in the quantities measured. As remarked in Section 1 (and as explained in connection with another programme on p.14 of Ref.2) no values of hypothetical observed quantities themselves have to be given.

Output consists basically of the standard deviations associated with hypothetical determination of all the unknown parameters. Printing is in the same standard format[2] as is used for input of assumed values, zeros being allocated to the fixed parameters. Additional, but optional, output consists of the scaled covariance matrix for the unknown parameters. This is a $K \times K$ matrix and is such that the square roots of the diagonal elements, when de-scaled, are precisely the non-zero standard deviations already referred to. The scaling is irrelevant since the only likely application of the covariance matrix is for re-input into the Pegasus programme[2] for estimating errors in the ephemeris of a satellite (see, in particular, Section 2.5 of Ref.2): the scaling is then automatically allowed for.

2.2 General theory

Let the 'unknown' parameters of the satellite orbit be E_k, for $k = 1,2,\ldots,K$. The notation E_k will also be used for nominal values or initial estimates of these parameters.

Each of the b bursts of hypothetical observations is associated with a particular station. Let S_i designate the station associated with the ith burst, it being understood that a station receives more than one S_i if it is responsible for more than one burst. Let N_i denote the number of observations in the ith burst and take the times of these observations to be t_i, $t_i + \tau_i$, $t_i + 2\tau_i, \ldots, t_i + (N_i - 1)\tau_i$, where t_i and τ_i are known.

If there were a set of actual observations at the given times, they could be used to improve the orbital parameters, say from E_k to E'_k. To give the theory of the assessment problem it is convenient to refer to the hypothetical observations as if they were a real set. This set will include observational errors which lead directly to errors in the improved parameters E'_k. However, the accuracy assessed for the E'_k and given by standard deviations and covariance matrix, is independent of the actual observations (and errors) and depends only on the a priori accuracy estimates associated with them. It is assumed here that the E_k and the E'_k are fairly close, so that the orbital improvement is complete after one iteration; this is equivalent to a linearity assumption as will be seen in due course.

We suppose then that we have a set of observed quantities. Throughout the ith burst let the s_i quantities measured be denoted by θ_q, where $q = 1,2,\ldots, s_i$; for the jth observation of the burst we suppose that θ_q takes the value θ_{qij}; this occurs at time t_{ij}, where $t_{ij} = t_i + (j - 1)\tau_i$. With $s_i = 2$, for example, θ_1 might be right ascension and θ_2 declination; with

$s_i = 6$, θ_1 might be range, θ_2 and θ_3 direction cosines and θ_4, θ_5 and θ_6 the time rates of change of these three quantities. The possible interpretations of the θ_q are listed in Section 2.3, which gives formulae connected with observations of every type covered by the programme.

Although the three suffices q, i and j have been introduced for precision, it will be convenient in most of the following analysis to omit them. We concentrate on a single typical observation θ $(= \theta_{qij})$ made by a station S. We suppose that θ contains an error which derives from a normal distribution having zero mean and the particular a priori standard deviation associated with S.

The actual observation θ may be compared with a theoretical or 'computed' value, θ_c, which is a function of orbital parameters, station position and time, say

$$\theta_c = \theta_c(E_k, S, t).$$

If limitations of the orbital dynamic model are neglected, θ_c is a known function of K + 4 parameters, since E_k, S and t account for K, 3 and 1 respectively. However, we make two further assumptions: that the co-ordinates of the given station are known without error; and that, in contrast with the basic R.A.E. orbit determination programme as originally described[1], time also is error-free. Then only the K orbital parameters are in question and we now write

$$\theta_c = \theta_c(E_k).$$

The comparison of θ with θ_c provides the residual R, where

$$R = \theta - \theta_c(E_k).$$

We are postulating the existence of a complete set of observations from which an orbit improvement, of E_k to E'_k, is to be carried out. Writing

$$\Delta E_k = E'_k - E_k$$

it follows from Taylor's theorem that, to first order,

$$R' = R - \sum_k \frac{\partial\theta}{\partial E_k} \Delta E_k. \qquad (1)$$

Second order terms may be nelgected since the ΔE_k are small; this is the linearity assumption referred to earlier.

Equation (1) can be set up for each θ_{qij} with its associated σ_{qi} (σ is independent of suffix j). The ΔE_k are then found, by the method of least squares, from:-

$$\sum_{q,i,j} (R'_{qij}/\sigma_{qi})^2 = \text{minimum.}$$

This equation is solved by differentiating with respect to each ΔE_k, using equation (1). The resulting 'equations of condition' are best expressed in matrix form, following the notation of Ref.1 as far as possible.

Let Y, Z and M be the two column vectors and matrix defined by

$$Y = (R_{qij}/\sigma_{qi}), \quad Z = (\Delta E_k) \text{ and } M = \left(\sigma_{qi}^{-1} \frac{\partial \theta_{qij}}{\partial E_k}\right)$$

where we regard q i j as a single (row) suffix. The matrix equation of condition is then (T denoting transposition)

$$M^T(Y - M\,Z) = 0$$

and the solution for Z is given by

$$Z = (M^T M)^{-1} M^T Y.$$

To obtain the required covariance matrix of the computed orbital parameters - and this is just cov Z - we introduce the notation $\bar{Y}$ for the column vector which would be derived from error-free observations and $\bar{Z}$ for the corresponding ΔE_k, the E'_k in this case being the true value of the parameters. Then $\bar{Y}$ and $\bar{Z}$ are the means of populations of all possible Y and Z based on the distribution of errors in observations. So, if $\mathcal{M}$ denotes expectation,

$$\text{cov } Z = \mathcal{M}\{(Z - \bar{Z})(Z^T - \bar{Z}^T)\}$$

$$= (M^T M)^{-1} M^T \mathcal{M}\{(Y - \bar{Y})(Y^T - \bar{Y}^T)\} M(M^T M)^{-1}$$

(since $M^T M$ is a symmetric matrix)

$$= (M^T M)^{-1} M^T \text{ cov } Y\, M(M^T M)^{-1}.$$

But the elements of Y are so weighted that the standard deviation of each is 1, and we assume that these elements are uncorrelated. Thus cov Y is the unit matrix. Hence

$$\text{cov } Z = (M^T M)^{-1}. \qquad (2)$$

Formula (2) is the basis of the Pegasus computer programme. As desired, it is independent of any actual observations (it does not contain Y). The only quantities required are partial derivatives and a priori standard deviations of error. The difference between this assessment formula and the formula for cov Z which would be used for the analysis of actual satellite observations is that the latter contains an additional factor (the ε^2 of Ref.1). This (scalar) factor would be derived from final residuals and would enable a priori error estimates for the observations to be converted into a posteriori estimates.

2.3 Formulae for observation types

The various interpretations for the observed quantities θ must now be listed. Formulae involved in programming the derivatives $\frac{\partial\theta}{\partial E_k}$ will be given.

Depending on the type of observing equipment used, observed quantities fall into six basic categories: range, angles, direction cosines, range rate, angle rates and direction cosine rates. Although angles can be obtained at once from direction cosines, these two categories must be treated separately because of the weighting question. Thus if a station observes direction cosines a fixed standard deviation is assumed for all such observations; the standard deviations of the two angles, to which a given pair of direction cosines could be converted, would depend on the particular observed direction and, in addition, the angular errors would in general be correlated.

Range measurement gives a single quantity, denoted here by ρ. Angle measurement yields two quantities for specifying a line of sight. These may be right ascension and declination, α and δ, or azimuth and elevation. However, no distinction is necessary as will now be demonstrated. Suppose α and δ are the fundamental (independent) quantities; then δ is measurable to the given fixed accuracy, σ_δ, associated with the station, but α only to the variable accuracy, σ_α, given by

$$\sigma_\alpha = \sigma_\delta \sec \delta .$$

This is equivalent to a circular normal distribution for errors in any plane perpendicular to the line of sight and so equivalent, again, to the following

for azimuth and elevation: $\sigma(el)$ fixed, $\sigma(az) = \sigma(el) \times \sec(el)$. Thus azimuth and elevation are effectively fundamental quantities also.

With directiom cosines there is a similar situation, but only so long as we assume that these cosines are always measured with respect to axes in the horizon plane. This is normally the case in practice, for example with Minitrack interferometer observations. Hence we may conveniently suppose the fundamental quantities to be ℓ and m, the direction cosines of a line of sight with respect to ground plane north and east axes respectively. The case of axes in some plane other than the horizon (ground) plane is not covered by the programme.

The remaining quantities which may be observed fall into the three rate categories. They are denoted here by $\dot{\rho}$, $\dot{\alpha}$ and $\dot{\delta}$, and $\dot{\ell}$ and $\dot{m}$.

The 'type' of an observation is determined by the categories of the quantities which are included in the observation. If it is assumed that angles and direction cosines are never included in the same observation, there remain 27 possible types. Of these, 15 have been specifically catered for by the programme. The type no. of each of these is listed in the table below, together with the quantities covered - i.e. interpretations of θ - for each type. It may occasionally be necessary to use the programme with observations of a type not covered by the table. Suppose, for example, that $\dot{\rho}$ ℓ and m are measured (by a combination of Doppler and interferometer). Then the observation may be regarded as of type 15 with ρ $\dot{\ell}$ and $\dot{m}$ measured to very low accuracy (say $\sigma_\rho = 10^9$ metres and $\sigma_{\dot{\ell}} = \sigma_{\dot{m}} = 1000\ (\text{sec})^{-1}$). All the 12 types not specifically covered in the programme may be dealt with in this way.

Observation types

Type no.	Quantities observed	Type no.	Quantities observed
1	ρ	8	ρ, ℓ, m
2	α, δ	9	$\dot{\rho}, \dot{\alpha}, \dot{\delta}$
3	ℓ, m	10	$\dot{\rho}, \dot{\ell}, \dot{m}$
4	$\dot{\rho}$	11	$\rho, \dot{\rho}$
5	$\dot{\alpha}, \dot{\delta}$	12	$\alpha, \delta, \dot{\alpha}, \dot{\delta}$
6	$\dot{\ell}, \dot{m}$	13	$\ell, m, \dot{\ell}, \dot{m}$
7	ρ, α, δ	14	$\rho, \alpha, \delta, \dot{\rho}, \dot{\alpha}, \dot{\delta}$
		15	$\rho, \ell, m, \dot{\rho}, \dot{\ell}, \dot{m}$

Turning our attention to the formulae for partial derivatives, we let r, $\bar{\alpha}$, $\bar{\delta}$ denote geocentric spherical polar co-ordinates, corresponding to the topocentric ρ, α, δ. Then if X, Y, Z be station co-ordinates in the

normal geocentric system of axes,

$$\rho \cos\delta \cos\alpha = r \cos\bar{\delta} \cos\bar{\alpha} - X,$$

$$\rho \cos\delta \sin\alpha = r \cos\bar{\delta} \sin\bar{\alpha} - Y$$

and

$$\rho \sin\delta = r \sin\bar{\delta} - Z.$$

The partial derivatives of r, $\bar{\alpha}$ and $\bar{\delta}$ depend on the Merson model for satellite motion and are given in full elsewhere[1]. Assuming these derivatives to be known, we require formulae for the derivatives of observations θ in the six basic categories. We start with the first two categories, formulae for the derivatives of ρ, α and δ being obtainable from the equations just given. For generality we take differentials of both sides, employing matrix notation. It is interesting to note that time derivatives can most simply be dealt with by use of moving axes, fixed in the earth, which instantaneously coincide with the normal inertial axes (x axis towards the vernal equinox and z axis towards the north pole). In this case $\Delta X = \Delta Y = \Delta Z = 0$ and if ω_E is the angular velocity of the earth we get

$$\begin{pmatrix} -\cos\alpha & 0 & -\sin\alpha \\ -\sin\alpha & 0 & \cos\alpha \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -\cos\delta & 0 & \sin\delta \\ \sin\delta & 0 & \cos\delta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta\rho \\ \rho\cos\delta(\Delta\alpha - \omega_E \Delta t) \\ \rho\Delta\delta \end{pmatrix} =$$

$$\begin{pmatrix} -\cos\bar{\alpha} & 0 & -\sin\bar{\alpha} \\ -\sin\bar{\alpha} & 0 & \cos\bar{\alpha} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -\cos\bar{\delta} & 0 & \sin\bar{\delta} \\ \sin\bar{\delta} & 0 & \cos\bar{\delta} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta r \\ r\cos\bar{\delta}(\Delta\bar{\alpha} - \omega_E \Delta t) \\ r\Delta\bar{\delta} \end{pmatrix}. \quad (3)$$

When the standard R.A.E. orbital determination programmes were written[1] time derivatives were needed for correction of observation times and equation (3) was developed with the Δt terms kept in. For the present programme, despite the fact that observations of time differentiated quantities are allowed for, formulae for time derivatives have not been used. Dropping the Δt terms and inserting the matrices on the left-hand side of equation (3) we get

$$\begin{pmatrix} \Delta\rho \\ \rho\cos\delta\,\Delta\alpha \\ \rho\Delta\delta \end{pmatrix} = R_1 R_2 R_3 \begin{pmatrix} \Delta r \\ r\cos\bar{\delta}\,\Delta\bar{\alpha} \\ r\Delta\bar{\delta} \end{pmatrix}$$

where $R_1 = \begin{pmatrix} -\cos\delta & \sin\delta & 0 \\ 0 & 0 & 1 \\ \sin\delta & \cos\delta & 0 \end{pmatrix}$, $R_2 = \begin{pmatrix} \cos A & 0 & -\sin A \\ 0 & 1 & 0 \\ \sin A & 0 & \cos A \end{pmatrix}$,

$$R_3 = \begin{pmatrix} -\cos\bar{\delta} & 0 & \sin\bar{\delta} \\ \sin\bar{\delta} & 0 & \cos\bar{\delta} \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad A = \alpha - \bar{\alpha}.$$

For the matrix of partial derivatives with respect to the E_k we now have, at once,

$$\begin{pmatrix} \frac{\partial\rho}{\partial E_1} & \cdots\cdots & \frac{\partial\rho}{\partial E_K} \\ \cos\delta\,\frac{\partial\alpha}{\partial E_1} & \cdots\cdots & \cos\delta\,\frac{\partial\alpha}{\partial E_K} \\ \frac{\partial\delta}{\partial E_1} & \cdots\cdots & \frac{\partial\delta}{\partial E_K} \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho^{-1} & 0 \\ 0 & 0 & \rho^{-1} \end{pmatrix} R_1 R_2 R_3 \begin{pmatrix} \frac{\partial r}{\partial E_1} & \cdots\cdots & \frac{\partial r}{\partial E_K} \\ r\cos\bar{\delta}\,\frac{\partial\bar{\alpha}}{\partial E_1} & \cdots & r\cos\bar{\delta}\,\frac{\partial\bar{\alpha}}{\partial E_K} \\ r\,\frac{\partial\bar{\delta}}{\partial E_1} & \cdots\cdots & r\,\frac{\partial\bar{\delta}}{\partial E_K} \end{pmatrix}. \quad (4)$$

It is convenient to leave the factor $\cos\delta$ with the α derivatives since, as already mentioned, the standard deviation of each measured α includes a $\sec\delta$ factor, so that elements $\sigma_\delta^{-1}\cos\delta\,\frac{\partial\alpha}{\partial E_k}$ are required in the matrix M of Section 2.2.

The partial derivatives of ℓ and m, observations in the third basic category, can be obtained from those of α and δ. Let β be the astronimical latitude of the observing station, λ_o its longitude (east) and S the sidereal time. Define λ by

$$\lambda = \lambda_o + S.$$

Then

$$\begin{pmatrix} \ell \\ m \\ n \end{pmatrix} = \begin{pmatrix} -\sin\beta\cos\lambda & -\sin\beta\sin\lambda & \cos\beta \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\beta\cos\lambda & \cos\beta\sin\lambda & \sin\beta \end{pmatrix} \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix},$$

where the third direction cosine, n, is introduced here only to give a complete expression. The 3×3 matrix corresponds to a change to left-handed axes but this is of no significance and in any case we are only concerned with the first two rows of the matrix.

On taking differentials,

$$\begin{pmatrix} \Delta\ell \\ \Delta m \\ \Delta n \end{pmatrix} =$$

$$\begin{pmatrix} -\sin\beta\cos\lambda & -\sin\beta\sin\lambda & \cos\beta \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\beta\cos\lambda & \cos\beta\sin\lambda & \sin\beta \end{pmatrix} \begin{pmatrix} -\sin\alpha & -\sin\delta\cos\alpha \\ \cos\alpha & -\sin\delta\sin\alpha \\ 0 & \cos\delta \end{pmatrix} \begin{pmatrix} \cos\delta\,\Delta\alpha \\ \Delta\delta \end{pmatrix}$$

$$+ \omega_E \begin{pmatrix} \sin\beta\sin\lambda & -\sin\beta\cos\lambda & 0 \\ -\cos\lambda & -\sin\lambda & 0 \\ -\cos\beta\sin\lambda & \cos\beta\cos\lambda & 0 \end{pmatrix} \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \Delta t.$$

Dropping the terms involving n or Δt and introducing the partial derivative notation, we have

$$\begin{pmatrix} \frac{\partial\ell}{\partial E_1} & \cdots & \frac{\partial\ell}{\partial E_K} \\ \frac{\partial m}{\partial E_1} & \cdots & \frac{\partial m}{\partial E_K} \end{pmatrix} =$$

$$\begin{pmatrix} -\sin\beta\cos\lambda & -\sin\beta\sin\lambda & \cos\beta \\ -\sin\lambda & \cos\lambda & 0 \end{pmatrix} \begin{pmatrix} -\sin\alpha & -\sin\delta\cos\alpha \\ \cos\alpha & -\sin\delta\sin\alpha \\ 0 & \cos\delta \end{pmatrix} \begin{pmatrix} \cos\delta\frac{\partial\alpha}{\partial E_1} & \cdots & \cos\delta\frac{\partial\alpha}{\partial E_K} \\ \frac{\partial\delta}{\partial E_1} & \cdots\cdots & \frac{\partial\delta}{\partial E_K} \end{pmatrix}. \quad \ldots (5)$$

Finally we consider derivatives of quantities from the last three categories, those which involve rates of change with time. It is clear that formulae for second derivatives are involved since, for example,

$$\frac{\partial\dot{\rho}}{\partial E_1} = \frac{\partial^2\rho}{\partial E_1\partial t} = \frac{\partial^2\rho}{\partial t\partial E_1} .$$

It is not difficult to extend the formulae of Merson[1] to cover the required second derivatives of geocentric co-ordinates. After doing so, however, it is still necessary to differentiate the right-hand sides of equations (4) and (5) to get the second derivatives of observed quantities. It was therefore decided that second derivatives should be computed numerically, using the available sub-routines for first derivatives.

Thus for an observation $\dot{\theta}$ we use the approximation

$$\frac{\partial\dot{\theta}}{\partial E_k} = \left(\frac{\partial\theta(E_k, t + \Delta t)}{\partial E_k} - \frac{\partial\theta(E_k, t)}{\partial E_k}\right)\Big/\Delta t , \qquad (6)$$

where Δt is some suitable fixed time increment. Unless told otherwise the programme takes $\Delta t = 1$ sec; for an orbit of long periodic time, a larger value of Δt would be more appropriate.

No difficulty arises with weighting factors for observed quantities in the fourth and sixth categories. For the fifth category, however, there is some uncertainty as to the appropriate factor to associate with $\dot{\alpha}$. The natural factor is again sec δ on the basis that

$$\sigma_{\dot{\alpha}} = \sigma_{\dot{\delta}} \sec\delta$$

where $\sigma_{\dot{\delta}}$ is a given fixed standard deviation. But this relation would be derived from $\cos\delta\ \sigma(\dot{\alpha}) = \sigma(\dot{\delta})$ and it could very well be argued that one should preferably start from $\sigma(\cos\delta.\ \dot{\alpha}) = \sigma(\dot{\delta})$. The difficulty is associated with the fact that radars do not normally make measurements of angle rates independently of measurements of the angles themselves. (We contrast the situation with direction cosines when $\dot{\ell}$ and $\dot{m}$ may be regarded as essentially uncorrelated with ℓ and m.) Thus fifth category observations must be treated warily. For completeness, the programme permits their use and does so by computing, instead of (6),

$$\cos\delta \frac{\partial\dot{\alpha}}{\partial E_k} = \left(\cos\delta(t+\Delta t)\frac{\partial\alpha(E_k, t+\Delta t)}{\partial E_k} - \cos\delta(t)\frac{\partial\alpha(E_k, t)}{\partial E_k}\right)\Big/\Delta t . \qquad (7)$$

2.4 Operating instructions

In the rest of Section 2 it is assumed that the reader is familiar with the use of the Pegasus digital computer. We first give the simple operating instructions.

(i) Depress hand-switch 0 and the hoot key; depress switches 1, 2 and/or 3 as required (see below); clear the others.

Place the binary programme tape, 00.13.29, in the main tape reader.

START and RUN - the tape is read and a 77 (Z) stop reached.

(ii) Place the parameter tape (see Section 2.5) in the main reader.

RUN (or Start and Run) - the tape is read in, computing proceeds and, in due course, results are punched (see Section 2.6).

Computer action at the completion of a run (i.e. after punching results for a given parameter tape) depends on whether hand-switch 3 is set. If it is, further parameters are immediately read in without need for intervention; the machine can be left unattented while many sets of parameters, all on one tape, are dealt with. If switch 3 is clear the computer comes to a 77 stop in 0.4, permitting a change of parameter tape if further runs are to be made. The next parameters may be dealt with by repeating instructions (ii).

The interpretation of hand-switch 2 is explained in Section 2.5, and of hand-switch 1 in Section 2.6. It will be found that switch 2 is normally kept clear.

The computer time required to run the programme may be forecast roughly as follows; for each observation allow 20 seconds if its type does not include any rate measurement; if rate measurement is included allow 40 seconds.

Two amendments to the programme may be made, if desired, and fed into the computer after (i) above. They are referred to in Sections 2.6 and 3.6 respectively.

The value of Δt (see Section 2.3) may be changed from 1 second to any integral multiple, p, of 10 μsec. Two actions are necessary:- (a) the programme must be amended by feeding in the tape

T 266.2
+ p
Z

(b) the standard deviations of all rate measurements must be multiplied by p/100000 before they are punched on the parameter tape (see Section 2.5).

2.5 Parameter tape

The print out of an illustrative parameter tape is given below. The numbers on the right refer to blocks of punching, separated by black tape, and are introduced here for reference in the commentary which follows.

T233·7		(i)
+2		(no. of bursts)
J300·0		(ii)
16730		(semi-major axis)
0·0003		(eccentricity)
80		(inclination)
255		(RA of node)
0		(argument of perigee)
1964 Jan 1	0 0 0	(time at node)
0		
500		
*		
T227·0		(iii)
+15		(type)
-454159256		(X)
+427230370		(Y)
-133904980		(Z)
-0·211324797		(sin β)
+0·977415893		(cos β)
+0·685182992		(sin λ_o)
-0·728370968		(cos λ_o)
L		
1964 Jan 1	14 13 0	(iv)
11·5		
11		
+50		(v)
+0·000175		
+1		
+0·0000077		
*		
L		(vi)
1964 Jan 1	22 45 0	(vii)
11·5		
11		
+50		(viii)
+0·000175		
+1		
+0·0000077		
*		

(i) The number, b, of bursts of data must first be set in storage location 233.7; here b = 2.

(ii) The programme is entered at address 300.0 and a block of orbital parameters is at once read in. Quantities on consecutive lines are a_o, e_o, i_o, Ω_o, ω_o and t_o, the 0 and 500 being (arbitrary) dummy numbers. Polynomial coefficients may be incorporated in the parameter format, e.g. e_1 after e_o, n_1 and n_2 after the dummy 500. For complete details of this standard parameter format and punching rules, Section 2.4 of Ref.2 should be consulted.

Ref.2 makes a distinction between parameters which are "strict E type" and those which are not. This distinction is the same as the one made, in Section 2.1 of the present paper, between parameters which are 'unknown' and 'fixed' respectively. In the normal use of programme 00.13.29 there will be no fixed parameters. The programme can still be used when there are one or more, however, as follows:-

(a) the parameter type must contain an additional section; between Sections (ii) and (iii) (after the block of orbital parameters) must appear

+0.uvwxyz
L

(b) hand-switch 2 must be set (enabling the new section of tape to be read).

The interpretation of u,...,z is as in Ref.2, but it is noted that the additional punching occurs at different parts of the parameter tape in the two programmes.

(iii) Station data is always stored in block 227 of the computer. The first quantity is type number, 15 - designating ρ, ℓ, m, $\dot{\rho}$, $\dot{\ell}$ and $\dot{m}$ - in the case of the illustrative example. The next three quantities are station co-ordinates X, Y and Z in cm. The remaining four are sin β, cos β, sin λ_o and cos λ_o, where β and λ_o are latitude and longitude (east).

(iv) The N_1 times of the observations in the first burst of data are specified by punching t_{11} (see Section 2.2), τ_1 (in minutes) and N_1 itself.

(v) Standard deviations for errors in the quantities observed are specified, viz. σ_ρ, σ_ℓ (= σ_m), $\sigma_{\dot{\rho}}$ and $\sigma_{\dot{\ell}}$ (= $\sigma_{\dot{m}}$) for the example given. Units for the six categories of observed quantities are: for ρ, metres; α and δ, seconds of arc; ℓ and m, -; $\dot{\rho}$, metres/sec; $\dot{\alpha}$ and $\dot{\delta}$, seconds of arc/sec; and

$\dot{\ell}$ and $\dot{m}$, -/sec; the standard deviations must be punched in the order of the categories which are included and they must be followed by an asterisk. (They are read in by the Pegasus Matrix Interpretive Scheme.)

(vi) The computer looks for station data for the second burst, similar to data under (iii). If there is no change, either in observation type or co-ordinates, it is only necessary to punch the L. This is a return link from initial orders, used as a sub-routine, to the programme.

(vii) Information for the second burst, - t_{21}, τ_2 and N_2 - is punched as under (iv).

(viii) The standard deviations for the second burst, even if identical with those for the first, must be punched in full.

NB Positive signs have been omitted in the punching of Sections (ii), (iv) and (vii), where they are optional. In Sections (i), (iii), (v) and (viii) they are compulsory.

2.6 Output

The regular output consists of a block of parameter standard deviations, each denoting the accuracy with which the corresponding orbital parameter could be computed from the given observational data. Thus for the illustrative parameters and data used in the previous section, the output was as follows:

+0·0478	(σ_a in km)
+0·000003	(σ_e)
+0·0008	(σ_i in degrees)
+0·0007	(σ_Ω in degrees)
+0·8461	(σ_ω in degrees)
0 00 00·334	(σ_{t_o} in h m s)
+0·0000	(associated with the
+0·00	dummy 0 and 500 of input).

If any parameter is of the fixed variety the appropriate standard deviation is printed as zero. If required, each quantity can be printed to three further decimal places by feeding in an amendment to the basic programme.

If the computer is operated with hand-switch 1 clear there is no other printing than that above. If this switch is set, however, the standard deviations are preceded by the punching of the complete covariance matrix, cov Z, from which they are derived. There is little point in giving the complete matrix for the example being considered, since the units of distance (a), angle and time are not the obvious ones. The first two columns are given for interest, however, in the standard floating point (argument exponent)

form. The element 1.073523×10^{-11} is, of course, the non-dimensional variance σ_e^2. The values of the covariance μ_{ae} from the first and second columns differ slightly due to rounding error. From σ_a^2, μ_{ae} and σ_e^2 may be obtained the correlations between errors in semi-major axis and eccentricity; its magnitude is +0.23.

1st col.		2nd col.	
	+3·030766 -16		+1·304585 -14
	+1·304584 -14		+1·073523 -11
	-5·627968 -14		+1·771246 -13
	+1·050549 -14		+1·227553 -11
	-9·670146 -12		-5·823236 -10
	-2·111723 -15		-9·643007 -14

The standard deviations are valuable in that they provide an immediate insight into accuracy attainable with a given set of observational data. The complete covariance matrix, particularly in its scaled floating point form, adds little to this quick picture. Thus even when the matrix is included in the computer output, it will not normally be sensible to print the tape. The necessity for the complete matrix arises if the accuracy (expressed as a standard deviation) with which some function of the computed orbital parameters is known is to be assessed. The obvious application is for estimating errors in an ephemeris based on the computed parameters; the computer programme already written for this purpose[2] requires the cov Z output tape as part of its input.

3 AN APPLICATION OF THE PROGRAMME

3.1 Background

The European Launcher Development Organisation (ELDO) have recently been interested in the accuracy with which parameters for certain orbits could be computed from tracking data generated by a single radar set. The orbits have been of the circular near-polar type, and such that the period, as seen from the earth, is a simple fraction of a day. Two particular cases are the '6 hour' and '8 hour' orbits, when the fraction is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. A 'day', here, must be taken to be that period, almost equal to a sidereal day but with allowance for satellite orbit precession, after which the track of the satellite on the earth repeats itself.

It was thought to be sensible that the first application of the new computer programme should be to one of these orbits of interest to ELDO. The 6 hour orbit was chosen. The radar was taken to be sited in the north-east of Australia.

For the particular orbit parameters considered (see Section 3.2) the satellite could be tracked twice a day, first when going south and second when going north an orbit and a half later. It was decided to look at the accuracy question for two cases; first, from the point of view of quick answers, how accurate an orbit could be obtained from data over the first (south-going) pass or, indeed, over less than this full pass; second, from the point of view of knowledge of the orbit after two or three days, what would be the accuracy when data from several passes were combined.

3.2 Orbital parameters, radar site and observation type

The orbital parameters are those listed in Section 2.5 for an illustrative parameter tape. For a 6 hour orbit at 80° inclination the effect of precession is to shorten the sidereal day by 0.24 min. The actual orbital period is thus $5^h 58^m 57^s$, to which corresponds the assumed semi-major axis of 16730 km. Though the orbit is nominally circular, eccentricity was taken to be 0.0003 to avoid any possible difficulty with e^{-1} factors. The argument of perigee is of course arbitrary and was taken zero; the standard deviation obtained for ω was inevitably going to be large, due to the small e, but this was of no importance. The parameter t_o, time at the node, was also arbitrary and taken at January 1.0 of 1964. The value of Ω was, of course, far from arbitrary; 255° was derived by making assumptions about the geography of launch and injection and by adding in the sidereal time at t_o.

For a high orbit there is no need to admit an unknown parameter t_o represent the effect of air drag; so no parameter, other than the basic six, was introduced. For a similar exercise with a low orbit it would be necessary to introduce a parameter n_1, possibly also n_2; the value of n_1 would not have to be known and could in fact be set to zero, the important thing being the presence of the additional parameter in the model.

The position of the radar, in Australia, was taken to be at about latitude 12° S and longitude 137°E. The station parameters used for the programme are those given by the illustrative parameter tape of Section 2.5.

The radar was assumed to make observations of type 5, i.e. to measure the following six quantities; slant range, ρ; direction cosines relative to ground plane axes, ℓ and m; $\dot{\rho}$; $\dot{\ell}$ and $\dot{m}$. Accuracies were taken to be given by the following expressions for standard deviations; $\sigma_\rho = 50$m, $\sigma_\ell = \sigma_m = 1.75 \times 10^{-4}$; $\sigma_{\dot{\rho}} = 1$ m/sec, $\sigma_{\dot{\ell}} = \sigma_{\dot{m}} = 7.7 \times 10^{-6}$/ sec.

3.3 Observation bursts

For the given orbit, as looked at from the given radar site, the satellite would appear above the horizon four times a day, twice going south and twice going north. One of the north passes and one of the south may be ruled out at once, however, on the ground that the satellite would never climb to an elevation greater than 10°. Of the other two passes the satellite would reach about 50° on the south-going and about 60° on the north-going. Taking 10° as (an arbitrary) radar horizon, the approximate times of visibility are as follows:

south going	14^h13^m	to	16^h8^m,
north going	22^h45^m	to	24^h40^m,

an interval of 115 minutes in each case. These are the times for the first day (1964 January 1); they become 4^m12^s earlier, each successive day.

It was decided, as stated in Section 3.1, to carry out the computer runs under two heads. The first covered the use of data from just the first possible pass, starting with an observation at 14^h13^m; successive observations were at intervals of 5 minutes (chosen arbitrarily), totals up to a maximum possible of 24 observations being considered. The second head covered runs using more than one burst of data; each burst corresponded to one pass and was divided into 10 intervals of 11.5 minutes, giving 11 observations over the burst. Under the second head runs were carried out for one burst only, two, three, four and six; the final case corresponded to use of data from all passes over a three day period.

3.4 Results for data using first pass only

Let N be the number of observations used, a suffix being redundant in the case of a single pass. The observations are assumed to be made at 5 minute intervals so that the last corresponds to a time 5(N-1) minutes from the (effective) beginning of the pass. The maximum value of N is 24. For values of N less than the maximum the programme shows how the accuracy of orbit determination, using data up to a given point reached in the pass, varies as that point advances in time.

Results were obtained for a number of values of N from 5 to 24. The corresponding standard deviation estimates σ_a, σ_e, σ_i, σ_Ω, $e\sigma_\omega$ and σt_o are plotted against N in Figs.1-6. The reason for plotting $e\,\sigma_\omega$, with $e = 0.0003$,

has been implied in Section 3.2. It can be seen clearly if it is observed that standard deviations in the non-singular orbital elements $c(=e\cos\omega)$ and $s(=e\sin\omega)$ are given, when $\omega = 0$, by

$$\sigma_c = \sigma_e \quad \text{and} \quad \sigma_s = e\,\sigma_\omega .$$

It is of interest to know how far the decrease in each σ for increasing N, indicated in Figs.1-6, is caused by the fact that an increasingly long arc of the orbit becomes available and how far by the mere fact that there are more observations. Since the standard deviation of any measured quantity is in general inversely proportional to the square root of the number of observations made, we can compute an 'accuracy per unit observation' by multiplying each σ by the appropriate $\sqrt{N}$. For semi-major axis, Fig.7 has been obtained in this way from Fig.1. It confirms a result that might be expected intuitively, namely, that increasing coverage is important until about half the pass has been seen, while data from the second half gives small return.

Results from just one pass - less than two hours worth of data - are thus surprisingly good. From merely 24 observations semi major axis is known to about 250 metres. Hence orbital period is known to about 0.5 sec. This in turn means that the suspected drift of the satellite after a full (modified sidereal) day is known to about 2' arc or, after a year, to about 12°. It is interesting to know to which of the four sorts of quantities being measured - ρ, ℓ and m, $\dot{\rho}$, $\dot{\ell}$ and $\dot{m}$ - the good accuracy is due. To throw light on this point the following repeats of the 24 point run were carried out, with different assumptions as to the observation type:- (a) type 1 (ρ only), (b) type 3 (ℓ and m), (c) type 4 ($\dot{\rho}$ only), (d) type 6 ($\dot{\ell}$ and $\dot{m}$), (e) type 11 (ρ and $\dot{\rho}$), (f) type 13 (ℓ, m, $\dot{\ell}$ and $\dot{m}$). The results are listed in Table 1. An immediate conclusion is that measurement of $\dot{\ell}$ and $\dot{m}$ is of little value and adds nothing to measurement of ℓ and m. In fact to "pull their weight" the accuracy of angular rate observations would have to be improved by a factor of about 20. Further features of Table 1 are discussed in Section 3.6.

3.5 Results for data using several passes

Results were obtained in a similar way to that of the previous section. Complete data over each of several passes being assumed, the number of passes used was increased from one to six. To restrict the use of computer time the number of observations per burst was cut to 11 leaving, even then, 66 observations to be dealt with in the case of six passes analysed. For the analysis of the first pass only, of course, nothing was added to previous knowledge.

It is meaningless to give the results in graphical form. Instead, they are set out in Table 2; the results for the case of two passes have been given also as the illustrative output in Section 2.6. The table agrees well with intuitive notions. Thus as soon as data from a second pass are included, this pass being some eight hours later than the first and covering a very different part of the orbit, the accuracy of all parameters is improved, a factor of about 5 or 6 being involved in each case. Addition of a third pass, however, affects the accuracies of the elements in quite different ways. Eccentricity and the angular elements - i, Ω and ω - are scarcely improved since the third pass merely repeats the first. But semi-major axis is improved by a factor of nearly 20, because for the first time a really accurate measure of orbital period becomes available. The effect on the sixth element, t_o, is nearly as good. The improvement in semi-major axis as up to six passes are included fits in with the view that the standard deviation of the error with which orbital period may be measured is inversely proportional to the total time over which observations are used. A standard deviation for semi-major axis of less than 1 metre (after 3 days) is, of course, remarkably good. It corresponds to a drift in satellite position of no more than $0^\circ.05$ at the end of a year.

3.6 Further discussion

It is wished to draw attention to three points which arose in connection with the present application of the computer programme but which have general significance.

The first point concerns the importance of the non-diagonal terms of the covariance matrix, cov Z. It is well brought out by reference to Table 1, introduced in Section 3.4. It may be wondered why it is that use of observations from all of the four relevant categories (ρ, ℓ and m, $\dot{\rho}$, $\dot{\ell}$ and $\dot{m}$) yield so much better accuracy ($\frac{1}{4}$ km) for semi-major axis, in particular, than observations from any single category (15 km at best).

If C_1, C_2, C_3, C_4 denote the covariance matrix for observations restricted to each of the four relevant categories in turn, then the overall covariance matrix is given by

$$(\text{cov } Z)^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + C_4^{-1}.$$

Hence if non diagonal terms of the matrices were neglected we should expect, from Table 1,

$$\sigma_a^{-2} = 38^{-2} + 15^{-2} + 32^{-2} + 496^{-2} = (13)^{-2},$$

so that the actual σ_a is about 50 times better than expectation.

The explanation lies in the relevant magnitude of the non-diagonal terms, i.e. in the correlations between semi-major axis and the other five elements. These correlations are large for all the presently considered matrices C_1, C_2, C_3, C_4 and cov Z. For cov Z (for which $\sigma_a = \frac{1}{4}$ km) the correlation between a and e is -0.75 and between a and t_o -0.964.

The second point also relates to the presence of large correlation elements in the covariance matrices. As seen from equation (2), the final operation in the determination of cov Z is the inversion of the matrix $(M^T M)$. If this matrix is ill-conditioned there may be considerable loss of accuracy in the performance of the operation.

The matrices leading to the results listed in Table 1 were all rather badly conditioned. The outstanding case, surprisingly, was for the first line of the table, corresponding to orbital determination from range data only. The standard deviations given for this case had to be found by inverting $(M^T M)$ with the aid of the Double Length Matrix Interpretive Scheme for Pegasus. Use of the normal scheme (incorporated in the programme) had previously led to results which underestimated all the standard deviations by the same factor, 0.42; σ_a, for example, had been estimated at 16 km.

One reason for ill-conditioning, in the case under consideration, may be seen at once. The sixth element in the orbital model, t_o, is a time which is 14 hours before the beginning of the 2 hour burst of data used to determine the elements. The situation is analogous to the difficulty encountered if one tries to fit a straight line $y = mx + c$ to a set of points (x,y) which are all bunched well to one side of the y-axis.

If ill-conditioning is suspected when the programme is to be used, the remedy is to feed in an amendment to the programme such that the matrix $(M^T M)$ is output instead of (or in addition to) standard deviations and cov Z. This matrix may then be inverted by means of a short, specially written programme which uses double-length arithmetic with a working length of 22 significant figures instead of only 9.

The final point concerns the choice of dynamic model. Let us suppose that, for a particular orbit, no parameter other than semi-major axis, say, is of interest. It is tempting then to argue: "since we have nominal (e.g. launch) values of e, i, Ω, ω and t_o, and since we are not interested in improving these, let us keep them fixed and see how accurately we can determine the one parameter we want to know".

In the case of the example being considered this would involve use of a parameter tape containing +0.100000 (see Section 2.5); values for σ_a far better than those quoted in this paper would be obtained.

The snag, of course, is that if e etc. are held fixed they are assumed known exactly, whereas they actually will contain errors which react on a. This reaction will be important unless e etc. are known to greater accuracy than that to which they could be computed if treated as unknowns. Such knowledge will not normally exist, though it has been assumed as a justification for omitting the parameter n_1 (see Section 3.1) in the present exercise. Summarising for the general case, of which the present example is typical: an assessment study must assume the necessity to estimate all parameters, even though the answers for only one or two may be actually looked at.

4 CONCLUSIONS

A programme has been written which fills an important gap in the Pegasus computer library of satellite orbit programmes. It should be useful in assessment work on the accuracy with which hypothetical orbits can be determined from hypothetical observations.

A first application of the programme has demonstrated that accurate determination of circular near-polar six-hour orbits can be made using radar data from a single station in Australia. This example has, at the same time, focussed attention on the danger which is inherent in the operation of inverting ill-conditioned matrices.

Table 1

ACCURACY OF ORBIT DETERMINATION FROM SINGLE-PASS DATA FROM OBSERVATIONS OF VARIOUS TYPES

Observation type	σ_a (km)	σ_e	σ_i (deg)	σ_Ω (deg)	$e\sigma_\omega$ (deg)	σ_{t_o} (sec)
1 (ρ only)	38	0.0034	0.798	0.656	0.121	249
3 (ℓ and m)	15	0.0007	0.005	0.004	0.014	73
4 ($\dot{\rho}$ only)	32	0.0022	1.398	0.391	0.075	162
6 ($\dot{\ell}$ and $\dot{m}$)	496	0.0234	0.442	0.616	0.586	2502
11 (ρ and $\dot{\rho}$)	21	0.0018	0.439	0.352	0.064	137
13 (ℓ,m,$\dot{\ell}$,$\dot{m}$)	15	0.0007	0.005	0.004	0.014	73
15 (all)	$\frac{1}{4}$	10^{-5}	0.004	0.002	0.001	1

Table 2

ACCURACY OF ORBIT DETERMINATION FROM MULTI-PASS DATA

No. of passes used	σ_a (metres)	$10^7\sigma_e$	$10^5\sigma_i$ (deg)	$10^5\ \sigma_\Omega$ (deg)	$10^5\ e\sigma_\omega$ (deg)	σ_{t_o} (sec)
1	376	164	514	334	162	1.7
2	48	33	79	72	25	0.3
3	2.5	30	57	70	9	0.024
4	1.8	25	43	58	7	0.021
6	0.9	20	35	46	6	0.016

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No.	Author	Title, etc.
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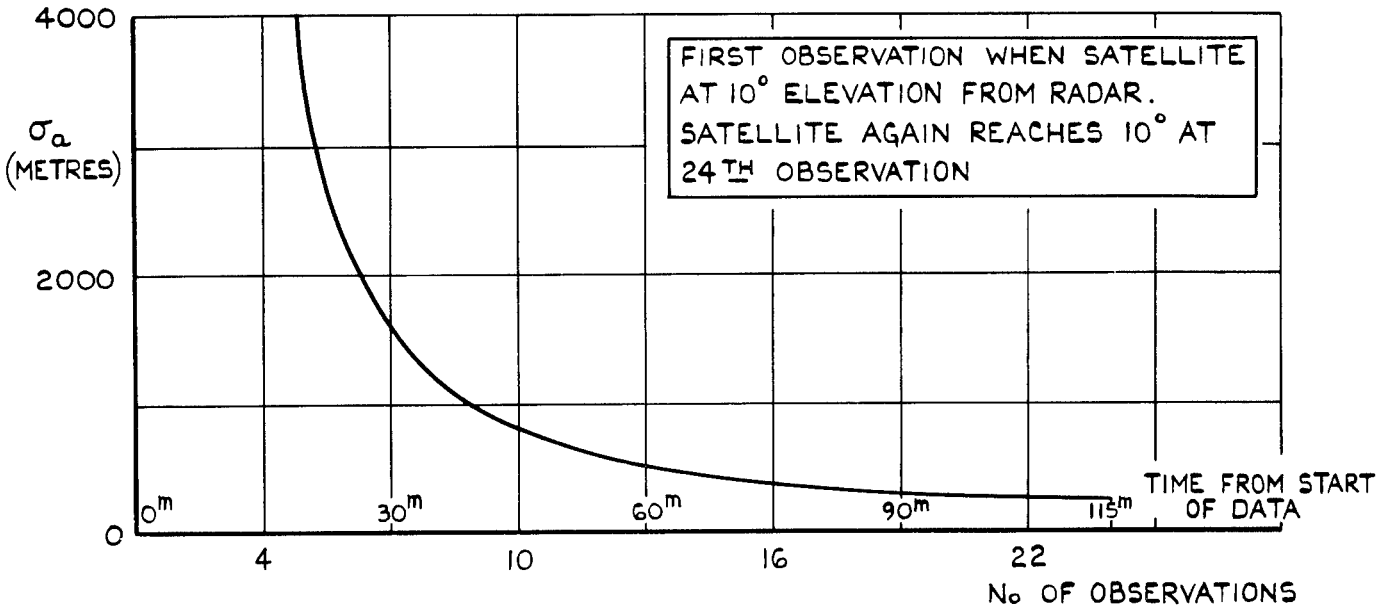

FIG. 1 ACCURACY OF SEMI-MAJOR AXIS DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

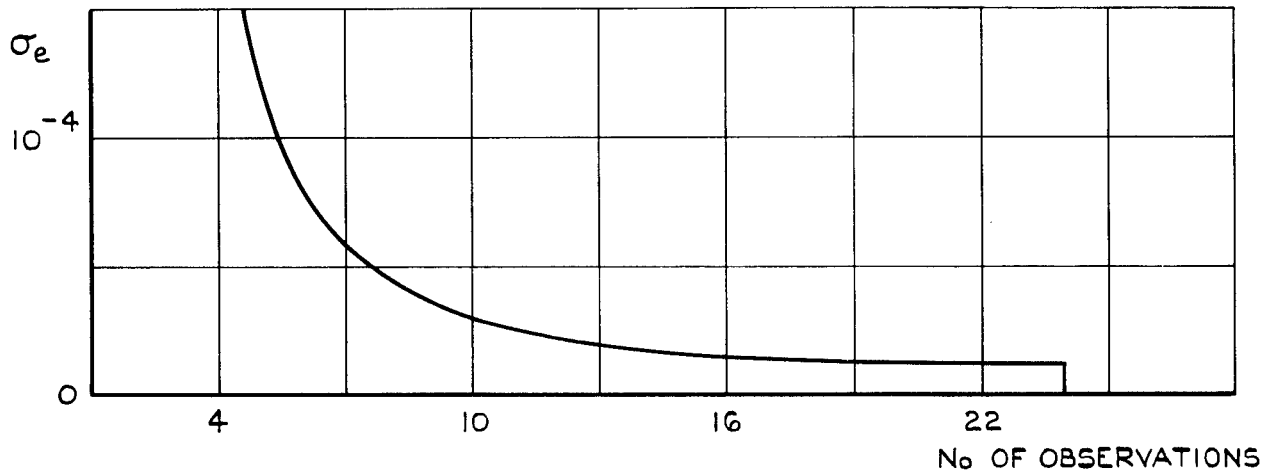

FIG. 2 ACCURACY OF ECCENTRICITY DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

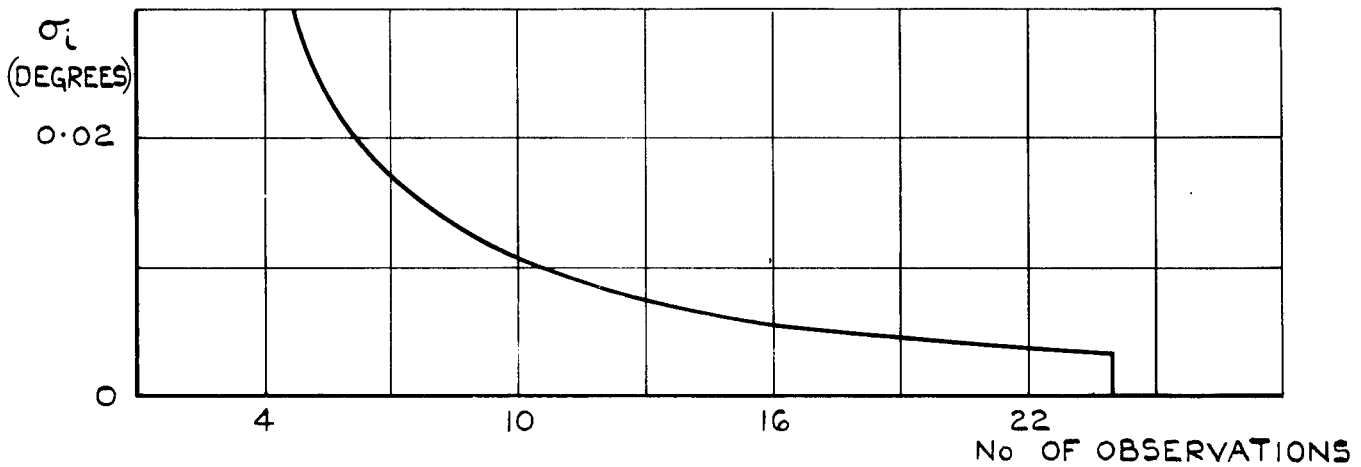

FIG. 3 ACCURACY OF INCLINATION DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

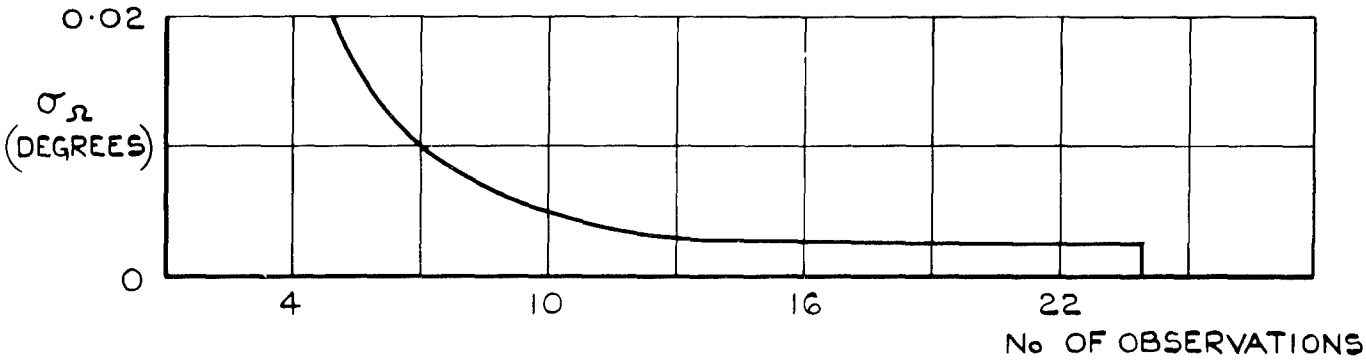

FIG.4 ACCURACY OF NODAL RIGHT ASCENSION DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

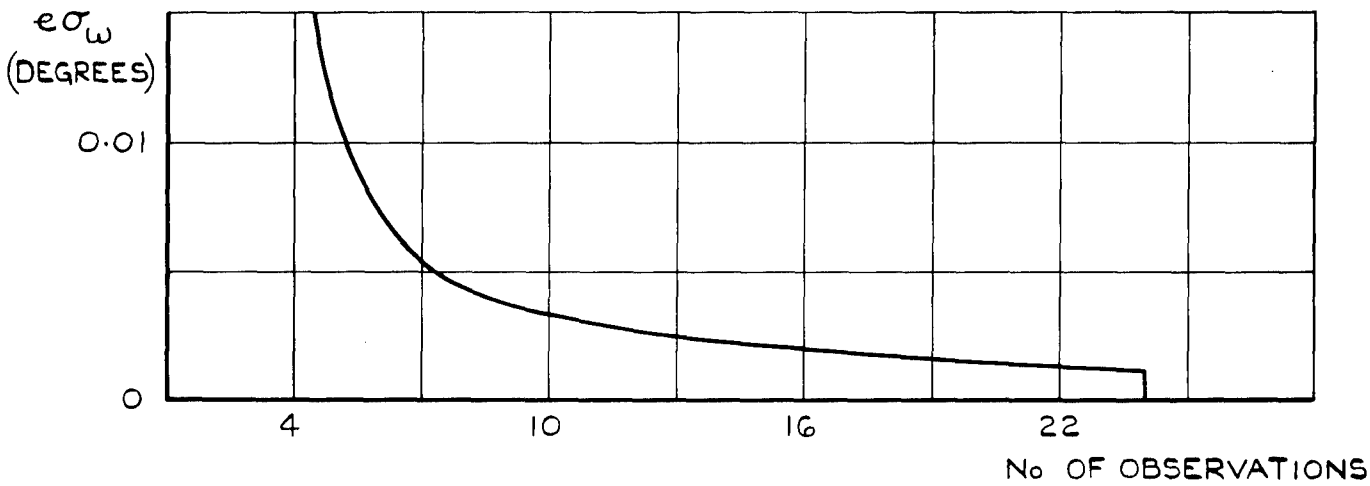

FIG.5 ACCURACY OF ARGUMENT OF PERIGEE DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

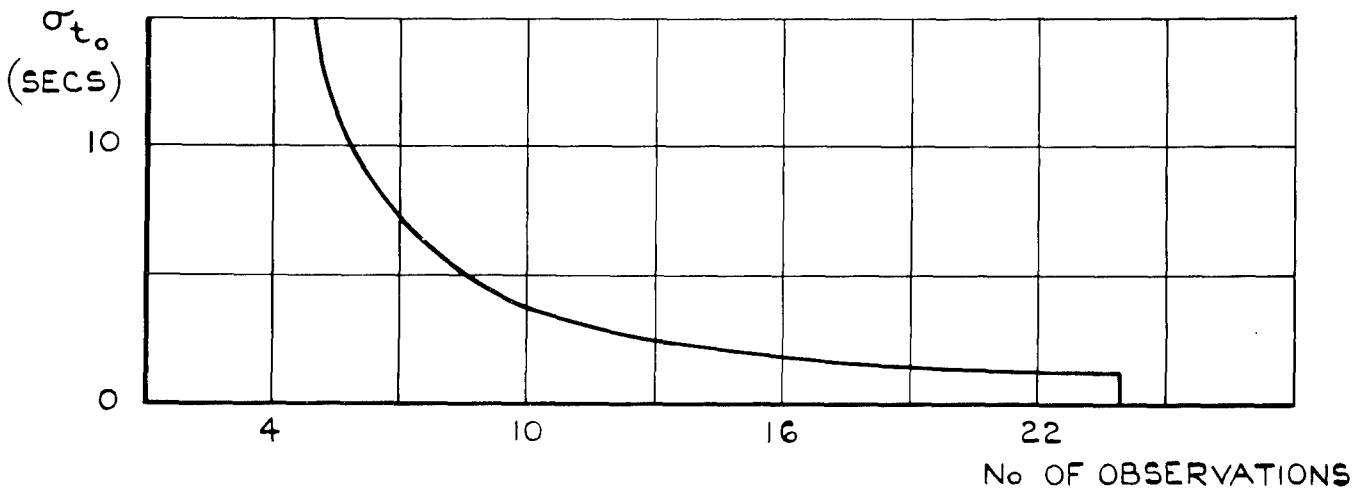

FIG.6 ACCURACY OF NODAL EPOCH DETERMINATION V. NUMBER OF OBSERVATIONS 5^m APART

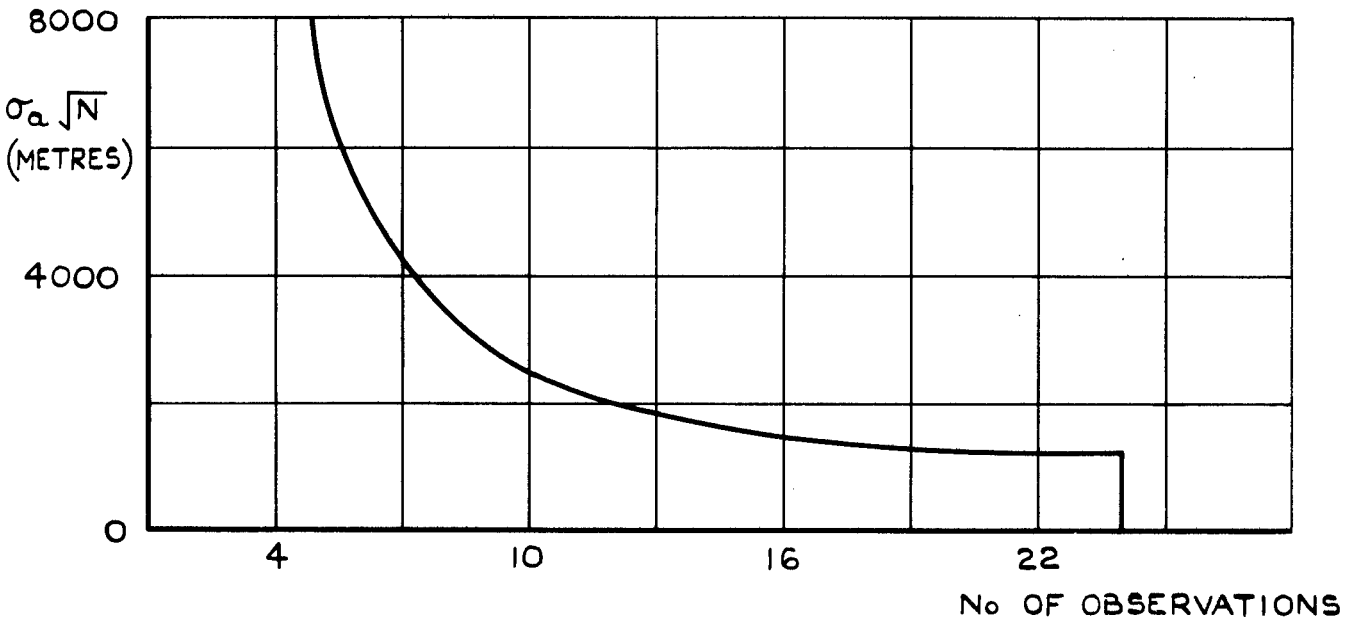

FIG. 7 RESULTS OF FIG. 1 EXPRESSED WITH $\sqrt{N}$ FACTOR INCLUDED

Gooding, R. H. 521.3:
519.283

A PEGASUS COMPUTER PROGRAMME FOR THE ASSESSMENT OF THE ACCURACY OF SATELLITE ORBIT DETERMINATION

Royal Aircraft Establishment Technical Report 65092 April 1965

A computer programme is described by means of which one may estimate the potential accuracy of the elements of a given satellite orbit, if determined from observational data of specified type and assumed accuracy. An application of the programme is made to an orbit of six hours period determined from radar observations at a single station.

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DETACHABLE ABSTRACT CARDS