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AN ELECTRIC ARC ACROSS AN AIR STREAM

by

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SUMMARY

The paper has three main aims: to describe the important features of the two-dimensional problem of an arc column across a low subsonic air stream; to present a simple model of the arc column which has proved qualitatively useful in the past; to demonstrate by analysing experimental results that the model is not quantitatively adequate.

A principal feature of the problem is that the cross-sectional area of the arc depends on the current passing through the arc, on the velocity of the undisturbed stream and on the ambient pressure and temperature. This feature is incorporated in the simple model, which employs an equivalent arc of circular cross-section carrying the same current and having no convection inside it. The experimental results, inferred from measurements on moving arcs at atmospheric pressure, are analysed in terms of Reynolds number, Nusselt number and drag coefficient. The results for the arc are found to differ greatly from results for solid circular cylinders, even when an allowance is made for loss of energy from the arc by radiation. It is therefore concluded that the equivalent arc, as here defined, is not a satisfactory model of an arc across an air stream, and some possible reasons for this, as well as possible lines of improvement, are given.

Departmental Reference: Aero 2714
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1 **INTRODUCTION**

When an electric arc is held, by means of a magnetic field, across an air stream, it not only creates a disturbance in the stream but it also reacts within itself. The size and shape of the arc, and the temperature inside it, depend on the velocity of the stream and on the ambient pressure and temperature as well as on the electric current passing through the arc. So, too, do the electric field required to maintain the current and the magnetic field required to prevent the arc from being blown along by the stream. In a theoretical treatment of arcs it is therefore necessary to take account of the situation inside the arc.

Of the many articles which provide the background to the theory of arc discharges, those by Sinkelburg and Maechler (1956), John and Bade (1961), Mills (1961), Cann (1961) and Ramona (1963) are particularly useful for their different contributions to the subject. The first four papers also contain extensive lists of references. The fifth paper is especially noteworthy since it regards arc theory from the points of view of heat transfer and fluid dynamics. From such papers as those cited may be extracted the following simplifying features which can be exploited in establishing a theory of arcs:

(i) that part of the arc, called the "column", which is not very close to the electrodes can be regarded as a plasma possessing charge neutrality;

(ii) if the pressure is sufficiently high and the electric field is not too high, the air inside the arc is in thermal equilibrium and can be treated as a continuum fluid with finite electric conductivity;

(iii) if the pressure is sufficiently high or the current is sufficiently high, the discharge is of a filamentary nature so that the arc column may be treated theoretically as having a definite boundary (which in this paper is called the "periphery") outside which the electric current is identically zero;

(iv) in many problems the arc column contains a portion (called the "uniform" column) which has no variation of properties along its length.

An example of a problem which possesses the above features is the one considered in this paper, namely the steady two-dimensional problem of the uniform column of an arc held at rest against an imposed subsonic flow by an applied magnetic field which is transverse to both the direction of the undisturbed flow and the direction of the current. This problem was one of several theoretical arc problems recently described by Lord (1963), and the present work is a summary and development of part of that paper. The general features of the problem are described in Section 2.
The approach to the solution of the problem is empirical, following a line originated by Suits and Foritsky (1959) and described more recently, for instance, by John and Bade (1961). The basic idea is to use the internal properties of a simplified equivalent arc in conjunction with experimental data. The properties of the equivalent arc used in this paper are described in Section 3.

Lord (1963) used empirical relations for the heat transfer and drag of solid cylinders to enable the theoretical treatment of the arc problem to be completed. The resulting trends appeared to be in rough qualitative agreement with observations, such as those made on moving arcs by Adams (1963) and by Spink and Guile (1964). However, it is possible to deduce experimentally the power put into unit length of an arc column and to estimate the column width visually, so that, neglecting radiation, the heat loss from the arc can be compared with that from the corresponding solid cylinder. A comparison made on the basis of Adams' work, for a particular pair of values of the flow velocity and the electric current at one atmosphere pressure, gave the astonishing result that the heat lost from the arc was about twenty-five times that for the cylinder. It is also possible to estimate the width of the arc from the internal properties of the equivalent arc, and the width obtained in this way was of the same order as that deduced visually and thus led to a confirmation that the heat lost from the arc was much greater than that from the solid cylinder. In order to investigate this discrepancy more closely, the properties of the equivalent arc have been used to calculate the heat transfer and the drag over the full range of Adams' results and to compare them with those for a solid body. The results of the calculations, which include an allowance for radiation, are described in Section 4.

2 GENERAL FEATURES OF THE PROBLEM

The main parameters of the problem are indicated in Fig. 1(a). The arc column is taken to have a finite cross-sectional area. The total current I through the arc is in the z-direction and is maintained by a uniform applied electric field $E_0$, which is also the total electric field everywhere since there is no induced electric field. The undisturbed air stream is in the x-direction and has a velocity $v_\infty$ which is small compared with the speed of sound appropriate to the ambient pressure $P_\infty$ and ambient temperature $T_\infty$. Although the effects of ambient pressure and temperature are important, we shall concentrate on the specific case in which the ambient pressure is atmospheric ($P_\infty = 10^5$ kilograms/metre second$^2$ in the m.k.s. system of units to be used throughout) and the ambient temperature is nominally 300$^\circ$K. The range of flow velocities will be between about 1 and 100 metres/second.
A uniform magnetic field $B_0$ is applied in the $y$-direction and this has the effect of holding the arc at rest. We assume that within the arc there is one fixed point which is the point of maximum temperature (and is called the "centre" of the arc although it may not be the geometrical centre). Thus the assumption is that for any values of the imposed velocity $v_\infty$ and the current $I$, the value of the applied magnetic field $B_0$ is such that the centre stays fixed. However, the position of the arc periphery depends on the values of $v_\infty$ and $I$ and also on the values of $R_0$ and $T_0$.

The most important features of the problem are the dissipation of electrical energy into heat within the arc and the consequent transfer of heat from the arc. There are three temperatures of particular importance: $T_\infty$, the ambient temperature of the imposed flow at infinity; $T_p$, the temperature of the arc periphery; $T_0$, the temperature at the centre of the arc. $T_\infty$ is a given parameter, whereas $T_0$ has to be found as part of the solution. The peripheral temperature $T_p$ is a theoretical parameter which serves to define the periphery of the arc and plays a decisive role in arc theory. We assume that $T_p$ has the same value at all points on the arc periphery. For a given gas at a given pressure $T_p$ is a constant which represents the highest temperature up to which the electric conductivity may be treated as negligibly small. In this paper, for an arc in air at atmospheric pressure, we take $T_p = 4000^\circ$K. Many workers on arcs, for example Goldenberg (1959) and Stine and Watson (1962), have used a value of the peripheral temperature equal to or near to the value chosen here. We assume that the arc current is in such a range that the central temperature $T_0$ lies between $4000^\circ$K and about $20000^\circ$K.

The temperatures and pressures involved in high pressure arc discharges are such that some effects of radiation must be taken into account. Here we include only the loss of energy from the arc column direct to infinity without any absorption by the external air stream. This heat-sink effect of radiation is represented by a net radiated power density per unit volume emitted from the arc.

The problem of the arc held in a stream is to find the electric field (or voltage gradient) $E_0$, the magnetic field $B_0$, and the central temperature $T_0$ in terms of $I$ for fixed values of $v_\infty$, $R_0$, $T_{\infty}$ and $T_p$. Of particular interest also are the arc cross-sectional area, the total amount of heat added to the stream and the drag on the periphery. The relations between the dependent and the independent parameters are known as the "arc characteristics". The first step in tackling this problem is to try and formulate a theoretical model. Such a model must incorporate simple yet realistic forms for the properties of air and for the boundary conditions at the arc periphery.
An accurate knowledge of the high-temperature properties of the gas inside the arc is very important; it is also very difficult to obtain. Detailed agreement between theoretical predictions and experimental measurements of the distributions of quantities within an arc column may only be expected when the most realistic data are used. (See, for instance, the theoretical calculations of King (1956) and the experimental results of Maeker (1962).) However, by using simple analytical formulae, it is possible to simplify the derivation of less-detailed features of the arc. We adopt this approach in this paper, and the formulae used are described in Appendix A.

The boundary conditions at the arc periphery are also very important. The introduction of the concept of the arc periphery means that a theoretical problem involving an arc column is resolved into two problems, one internal to the arc and the other external to it. These two problems must be solved separately and then their solutions matched at the periphery in order to give the solution to the whole problem. Only then can the arc characteristics be found. The size and shape of the arc periphery should be found as part of the solution, and in the determination of the arc characteristics, the properties of the periphery as a fluid boundary of very high temperature and of porous nature should be taken into account. The peripheral boundary conditions are greatly influenced by the assumptions made about the inside of the arc. For instance, in the arc model which is described in Section 3 it is assumed that the arc periphery is non-porous and moreover that there is no convection inside it, and this implies that the periphery of the equivalent arc is imbued with the (unlikely) property of being able to withstand tangential stresses. Also, if the shape of the periphery is not allowed to be wholly determined by the matching process it is not possible to match all the physical quantities at all points round the periphery. Again taking the model of Section 3 as an example, the assumption that the periphery is circular has the consequence that the internal properties of the equivalent arc and the external properties of the air stream may be related at only one or two points on the periphery.

3 THE MODEL OF THE EQUIVALENT CIRCULAR ARC WITHOUT INTERNAL CONVECTION

3.1 Properties of the equivalent arc

Corresponding to an arc column in an air stream we define an equivalent arc which is similar to that introduced by Lord (1963) and has the following properties:

(i) its centre coincides with the centre of the arc in the stream;
(ii) it carries the same current as the actual arc;
(iii) the electric conductivity $\sigma$ is constant inside it and equal to the value $\sigma_0$ appropriate to the temperature $T_0$ at its centre;

(iv) the radiated power density $\psi$ is constant inside it and equal to the value $\psi_0$ appropriate to the temperature $T_0$ at its centre;

(v) there is no convection inside it.

With these assumptions the variables of interest inside the arc, namely the temperature $T$, the pressure $p$, the current density $j_z$ and the magnetic field components $B_x$, $B_\theta$, must satisfy a set of equations comprising Ohm's law, the conservation of energy, the momentum or pinch equations which in the absence of convection relate the pressure gradient to the electromagnetic force, and Maxwell's equations. In the present context, however, the momentum and Maxwell's equations only serve to yield the overall result that

$$F = B_{\infty} I$$

where $F$ is the electromagnetic thrust acting on the inside of the arc, $B_{\infty}$ is the uniform applied magnetic field and $I$ the total current. This result, however, can be deduced from elementary principles whatever the distribution of current and induced magnetic field. Imagine the applied field $B_{\infty}$ to be removed while the current distribution is constrained not to change by the application of mutually-cancelling internal forces: then the arc can be regarded as being made up of filamentary currents all reacting with each other, but as all reactions are mutual $F = \sum F_i = 0$ where $F_i$ is the force on the $i$th filament. When $B_{\infty}$ is restored additional forces given by $F_i = j_i \times B_{\infty}$ are introduced where $j_i$ is the current in the $i$th filament. Since this equation is linear in $B$ and since $j_i$ is constant the two sets of forces can be superimposed so that after integrating over the cross-section equation (1) results.

Ohm's law gives by (iii) above

$$j_z = \sigma_0 \sigma_0 = \text{constant} \quad (2)$$

The energy equation is

$$j_z E_z - \psi_0 = -\text{div} (\kappa VT) \quad (3)$$

where $\kappa$ is the thermal conductivity and depends only on $T$. Equation (3) is simplified by use of the heat flux potential $\phi$, defined by
\[ \phi = \int_0^T kdT \]  

so that \( \text{div}(k\nabla T) = \nabla^2 \phi \). It is convenient at this stage to introduce a further assumption,

(vi) that the periphery of the equivalent arc is a circle of radius \( \hat{r} \). Since it is a basic assumption that \( T \) is constant around the periphery this implies that a solution of (3) can be found for which \( T \), and hence \( \phi \), are functions of the radius \( r \) only. Equation (3) then becomes, by use of (4) and (2)

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\phi}{dr} \right) = -\left( \frac{R_o^2}{R_b} \sigma_o - \psi_o \right) . \]  

This equation is integrated to give

\[ \phi - \phi_o = -\frac{1}{2} r^2 \left( \frac{R_e^2}{R_b} \sigma_o - \psi_o \right) \]  

and in particular,

\[ \phi_o - \hat{\phi} = \frac{1}{4} \hat{r}^2 \left( \frac{R_e^2}{R_b} \sigma_o - \psi_o \right) \]  

where \( \hat{\phi} \) is the peripheral value of \( \phi \).

The total amount of heat \( \hat{Q} \) which is conducted to the arc periphery is given by

\[ \hat{Q} = 2\pi \hat{r} \left( -\frac{d\phi}{dr} \right)_{r=\hat{r}} \]  

\[ = \pi \hat{r}^2 \left( \frac{R_e^2}{R_b} \sigma_o - \psi_o \right) \]  

by (6), and since from (2) the total current \( I \) is given by

\[ I = \frac{R_e}{R_b} \sigma_o \pi \hat{r}^2 \]  

(9)
there follows
\[ \hat{Q} = E_\infty I - \psi_\circ \pi \hat{r}^2 \] (10)
or alternatively, from (7) and (8)
\[ \hat{Q} = 4\pi (\phi_\circ - \phi) . \] (11)

In order to complete the set of relations describing the overall internal properties of the equivalent arc, we need to augment equations (1), (9), (10) and (11) with the expressions connecting \( \sigma_\circ \) and \( \psi_\circ \) with \( \phi_\circ \). It follows from equations (A7) and (A9) of Appendix A that these are
\[ \sigma_\circ = \bar{\sigma} \left( \frac{\psi_\circ - \phi}{\phi} \right) , \] (12)
\[ \psi_\circ = \bar{\psi} \left( \frac{\psi_\circ - \phi}{\phi} \right)^2 . \] (13)

For the purpose of analysing experimental results we wish to express \( \hat{Q} \) and \( \hat{r} \) in terms of \( I \) and \( E_\infty \). (We shall not investigate the temperature \( T_\circ \) of the arc centre, which is represented above by \( \phi_\circ \), further in this paper.) It follows from equations (7) - (13) that
\[ \hat{Q} = \frac{E_\infty I \left[ 1 + \left( \frac{\psi}{\bar{\psi} \pi K_\circ} \right) \frac{I}{E_\infty} \right]^{-1}}{\pi^2 \left[ \frac{\bar{\sigma}}{\bar{K}_\circ} \right]^2} . \] (14)
\[ \hat{r} = \frac{1}{\pi^2 \left[ \frac{\bar{\sigma}}{\bar{K}_\circ} \right]^2} \frac{E_\infty}{\bar{\psi}} \left[ 1 + \left( \frac{\psi}{\bar{\psi} \pi K_\circ} \right) \frac{I}{E_\infty} \right]^{\frac{1}{2}} . \] (15)

We note that when radiation is neglected \( \hat{Q} = E_\infty I \), which is well known, and that \( \hat{r} \) is inversely proportional to \( E_\infty \); the result for \( \hat{r} \) depends, however, on the assumed variation of electric conductivity with temperature, whereas that for \( \hat{Q} \) does not. The effect of the particular form of radiation term used here is to introduce a factor which depends on \( I/E_\infty \) and which decreases the amount of heat conducted to the arc periphery and increases the radius of the arc.

3.2 "Matching" the equivalent arc and the air stream

The model of the equivalent arc in the air stream is illustrated in Fig. 1(b). Because the equivalent arc is over-specified by comparison with
the actual column of an arc in a stream, it is not possible to relate the internal properties of the equivalent arc and the external properties of the air stream at all points on the periphery. In fact, only the peripheral temperature $T$, which is a fundamental parameter of the theory, is continuous across the periphery at all points. The heat flux distributions on the inside and the outside of the periphery can satisfy only one common condition, and this is taken to be that the total amount of heat $\dot{Q}$ being conducted to the periphery from inside is equal to the amount of heat, $Q$ say, being convected away by the stream. The pressure distributions on the inside and the outside of the periphery can satisfy two common conditions, and these are taken to be that the pressure is continuous at the forward stagnation point and that the electromagnetic thrust force $F$ on the inside of the periphery is equal to the aerodynamic drag force $D$ on the outside. An indication of the extent to which the internal and external distributions of heat flux and pressure do not match at the periphery, for the simplified model, is given by Lord (1963) (see Fig. 5 of that paper).

The continuity of the pressure at the stagnation point has already been implied in this paper by restricting the ambient pressure to be atmospheric and taking properties of the air inside the arc at one atmosphere. The remaining conditions $I = \dot{Q}$ and $D = F$ lead, by using equations (14) and (1) respectively, to the results

$$\dot{Q} = B_{bo} I \left( 1 + \left[ \frac{\omega}{\Delta \rho c_{p}} \frac{I}{B_{bo}} \right] \right)^{-1},$$

$$D = B_{bo} I.$$  \hspace{1cm} (16) \hspace{1cm} (17)

It is not possible to proceed further with the theoretical treatment of this problem without introducing relations connecting $\dot{Q}$ and $D$ with the stream velocity $v_{o}$ and the arc radius $R$. Instead of assuming such relations on the basis of information for solid cylinders (as done previously by Lord (1963)), we now use equations (15), (16) and (17) to analyse some experimental results for arcs.

4. APPLICATION OF THE MODEL TO THE ANALYSIS OF EXPERIMENTAL RESULTS

4.1 Description of the results

It is extremely difficult to perform the experiment of holding a uniform arc column at rest against an air stream and there do not seem to be any
published experimental results for a direct current arc in a subsonic stream. Therefore, in seeking some evidence on the heat transfer and drag of an arc it is necessary to use results obtained for a moving arc driven by a magnetic field, and then to translate the results to an arc at rest. The results used here are those given by Adams (1963) for an arc rotating between annular electrodes.

These tests were made in air at room pressure and temperature and the electrode material was carbon. Adams performed two series of experiments with the inner electrode, the cathode, having a fixed radius. In the first series of tests (Part I) the gap between the inner and outer electrodes was fixed and the arc was examined through ranges of current and applied magnetic field. In the second series of tests (Part II) the current and the magnetic field were fixed and the electrode gap was varied. The variation of the electrode gap in the second series of tests enabled some definite information on the shape of the arc centreline to be obtained. As a consequence, it is possible to describe some of the properties of a rotating arc in the following highly-simplified way:

(i) the arc as a whole appears to rotate with uniform angular velocity;
(ii) each element of the arc moves perpendicular to itself with a constant velocity which is the same for all elements;
(iii) the shape of the centreline of the arc is an involute with the inner electrode as the base circle (see Adams (1963) and also Jedlicka (1964));
(iv) as the electrode gap is increased the velocity of an arc element decreases until at a certain critical gap the velocity takes a value which it then retains for all larger gaps;
(v) for all gaps equal to or greater than the critical value the gradient of the arc voltage with respect to the length of the arc, calculated on the basis that the arc centreline is an involute, is constant, so that a uniform arc column appears to be established.

It is deduced from these results that provided the electrode gap is sufficiently large a rotating arc may be regarded as a straight arc moving perpendicular to its centreline and having an effective length of that of the appropriate involute. The results for velocity and voltage gradient may therefore be taken to apply to an arc driven in a straight line by a magnetic field. The results for the moving arc can then be transformed to give results for an arc held at rest, and provided the applied magnetic field and the
current are not too large the differences between the results in the two cases are negligible.

In this way it is possible to get the following spot point which connects corresponding values of current I, magnetic field \(B_\infty\), velocity \(v_\infty\), and electric field (voltage gradient) \(E_\infty\) from Figs. 5 and 3 of Part II of Adams' paper:

\[
\begin{align*}
I &= 360 \text{ amps}; \\
B_\infty &= 0.0470 \text{ webers/metre}^2; \\
v_\infty &= 47.0 \text{ metres/second}; \\
E_\infty &= 1200 \text{ volts/metre}.
\end{align*}
\]

However, we require variations of \(B_\infty\) and \(B_\infty\) with \(v_\infty\) and I, and Adams' paper does not include tests with variable I and \(B_\infty\) for the larger electrode gaps. We therefore assume that the variations for the small electrode gaps, where a uniform column was not established, can be taken to apply for the larger gaps. (Recent unpublished results for larger gaps indicate that this procedure is justified.) In equations (3) and (4) of Part I Adams quotes relations of the form \(v_\infty \propto B_\infty^{0.50}I^{-0.33}\) and \(v_\infty \propto B_\infty^{-0.27}\), where it is assumed that \(B_\infty\) is proportional to the voltage gradient with respect to the electrode gap. Adams does not quote an index for the variation of \(v_\infty\) with I at constant \(B_\infty\), but the evidence of his graphs is that this variation is small and we neglect it within the range of Adams tests, namely \(120 \text{ amps} < I < 720 \text{ amps}, 0.0059\) webers/metre\(^2\) < \(B_\infty\) < \(0.0940\) webers/metre\(^2\). It then follows that the variations of \(B_\infty\) and \(B_\infty\) in terms of \(v_\infty\) and I are \(v_\infty \propto v_\infty^{0.45}I^{-0.15}\) and \(B_\infty \propto B_\infty^{1.67}I^{-0.55}\), where the relevant range of \(v_\infty\), calculated from the ranges of \(B_\infty\) and I, is \(9.42 \text{ metres/second} < v_\infty < 59.6 \text{ metres/second}\). By combining these variations with the spot point values we finally arrive at the following relations:

\[
\begin{align*}
\frac{B_\infty}{1200 \text{ volts/metre}} &= \left(\frac{v_\infty}{47.0 \text{ metres/second}}\right)^{0.45} \left(\frac{I}{360 \text{ amps}}\right)^{-0.15}, \\
\frac{B_\infty}{0.0470 \text{ webers/metre}^2} &= \left(\frac{v_\infty}{47.0 \text{ metres/second}}\right)^{1.67} \left(\frac{I}{360 \text{ amps}}\right)^{-0.55}.
\end{align*}
\]

We treat these relations as being representative of results for an arc held at rest. The particular forms of the relationships are not regarded as having any theoretical significance whatever. The relations merely enable some idea to be obtained of the sizes of \(B_\infty\) and \(B_\infty\) and of their variations with \(v_\infty\) and I, within the given ranges. The relations are displayed graphically for three particular values of \(v_\infty\) in Figs. 2(a) and 2(b). It is seen that the electric field \(E_\infty\) increases markedly with increase of velocity and decreases slightly with increase of current, while the magnetic field \(B_\infty\) varies in the same manner but to a more pronounced degree.
4.2 Analysis of the results

By using the expressions for \( f \), \( Q \) and \( D \) given in equations (15), (16) and (17) respectively, in combination with the experimental results expressed by equations (18) and (19), it is possible to calculate \( f \), \( Q \) and \( D \) in terms of \( v_\infty \) and \( I \). The results of the calculations, for the particular values of \( v_\infty = 9.42 \), 47.0 and 89.6 metres/second and the range 120 amps < \( I < 720 \) amps are displayed in Fig. 3. The results for \( f \) and \( Q \) depend upon the assumed radiation law whereas those for \( D \) do not. Although the effects of radiation are important, they do not alter the general trends of the results within the quoted ranges. These trends are: the size of the arc (Fig. 3(a)) increases with increase of current and decreases with increase of velocity (it is of interest that the calculated value of \( f \) at the spot point is about twice as big as the visual estimate made from Fig. 12 of Part II of Adams (1963)); the total heat from the arc to the stream (Fig. 3(b)) increases with both increase of current and increase of velocity, although there appears to be a tendency for the rate of increase with current to decrease rapidly; the drag of the arc (Fig. 3(c)) increases with both increase of current and increase of velocity, so we note that the reduction in the size of the arc with increase of velocity is by no means great enough to reduce the drag.

From the heat transfer and fluid dynamic points of view we wish to know how \( D \) and \( D \) vary with \( v_\infty \) and \( I \). We express the results in terms of the following non-dimensional forms of \( v_\infty \), \( f \), \( Q \) and \( D \): the Mach number \( \mathcal{M} = v_\infty/a_\infty \), where \( a_\infty \) is the speed of sound in the undisturbed stream (the Mach number is used here only because a more significant non-dimensional form of \( v_\infty \) is not apparent and its use is not meant to imply that any effects of an approach to the sound speed are considered), the Reynolds number \( Re = \rho I (2f)/(\frac{D}{2}) \), where \( \rho \) and \( \eta \) are the density and viscosity based on the free stream pressure and on the mean temperature \( T_\infty \) between the arc periphery and stream (\( T_\infty = \frac{1}{2}(T_{\text{arc}} + T) \)); the Nusselt number \( Nu = \left(\frac{\gamma}{2\pi}\right)(2f)/(\hat{\phi} - \phi_\infty) \), where the more accurate \( (\hat{\phi} - \phi_\infty) \) replaces the term \( x_\infty (\hat{\phi} - \phi_\infty) \) generally used; the drag coefficient \( C_D = \frac{D}{2} \rho_\infty v_\infty^2(2f) \), where \( \rho_\infty \) is the density of the undisturbed stream. In order to evaluate these non-dimensional parameters it is necessary to make assumptions about the properties of air at temperatures below 4000°K additional to those given in Appendix A. We assume that: the gas is perfect so that \( \rho = p/RT \), where \( R \) is the gas constant for unit mass; the specific heat at constant pressure \( c_p \) is constant and equal to \( (7/2)R \); the specific heat ratio \( \gamma \) is constant; the coefficient of viscosity \( \eta \) is such that the Prandtl number \( Pr = \eta c_p/k \) is constant. These assumptions are only approximately true for air since they neglect the effect of vibrational relaxation and oxygen.
dissociation. After some manipulation, and using the values $R_e = 10^5$ kilograms/metre second$^2$, $T_e = 300^\circ$K, $R = 280$ metres$^2$/second$^2$K, $\gamma = 1.4$ and $Pr = 0.730$, the non-dimensional parameters in a form suitable for analysing the experimental data become:

\[
M = \frac{v_{\infty}}{3.46 \times 10^2 \text{metres/second}}
\]

\[
Re_{\infty} = \frac{v_{\infty} \rho}{2.50 \times 10^{-4} \text{metres}^2/\text{second}}
\]

\[
Nu = \frac{\rho}{1.25 \times 10^3 \text{watts/metre}}
\]

\[
C_D = \frac{D/v_{\infty}^2 \rho}{1.16 \text{kilograms/metre}^3}
\]

Then $Nu$ and $C_D$ are easily calculated in terms of $M$ and $Re_{\infty}$ by using the data given in Fig.3. The results for $Nu$ are shown in Fig.4(a); also plotted in this figure is the empirical relation appropriate for hot solid cylinders as given by Knudsen and Katz (1953). The results for $C_D$ are shown in Fig.4(b) which also contains the empirical results for (cold) solid cylinders as given by Knudsen and Katz (1958). Scales of $Re_{\infty}$, the Reynolds number based on the ambient temperature $T_e$, are also shown in Figs.4(a) and 4(b).

Inspection of Fig.4 leads to the following observations.

(i) The inclusion of the radiation factor has an important effect. It decreases both $Nu$ and $C_D$ and increases $Re_{\infty}$ and hence significantly alters the shapes of the $Nu$-$Re_{\infty}$ and $C_D$-$Re_{\infty}$ curves. It appears that an allowance for radiation is an essential preliminary to a detailed examination of arc results. We shall concentrate on the "radiation-included" results henceforth in this paper. A detailed investigation of the effects of radiation on arc voltage-current characteristics in particular is given by Lord (1964).

(ii) The overall ranges of $Re_{\infty}$ involved are quite narrow. This is because the radius of the equivalent arc decreases with increase of the velocity of the stream.

(iii) The relation between the Nusselt number and the Reynolds number for the equivalent arc is quite different from that for the corresponding solid cylinder. In the first place, the magnitude of $Nu$ is about ten times as big. Secondly, $Nu$ varies much more rapidly with $Re_{\infty}$ at constant $M$ for the
arc than it does for the cylinder. Thirdly, Nu for the arc depends on M as well as on Re, whereas in this range of Re, it is independent of M for the cylinder.

(iv) The relation between the drag coefficient and the Reynolds number for the arc is also much different from that to be expected for the solid cylinder. The drag coefficient for the arc varies with both Re and M, unlike that for a solid cylinder; the actual magnitude of CD is close to the value which might be expected for a cold solid cylinder at the same Reynolds number, but this cannot be regarded as of particular significance. (Note that the value of CD for the spot point using the "visual" arc radius is not close to that for a solid cylinder.)

There is one way of plotting the present data which gives a surprisingly good collapse and seems to hold promise for future investigations. To avoid interrupting the comparison between the equivalent arc and the solid cylinder, a discussion of this aspect of the analysis of the experimental results is given separately in Appendix B.

5 CONCLUSION

The model of an arc in an air stream as a circular arc without internal convection does not lead to a satisfactory quantitative analysis of the experimental results. The following factors may be contributing to this situation.

(i) The quoted results for the heat transfer and the drag of a solid cylinder may be inappropriate. One reason why some modification to the empirical relation for heat transfer from solid cylinders is desirable is to be found in the different nature of the boundary layer for the arc and the cylinder. A simple laminar boundary layer analysis gives a relation that Nu is proportional to Re, which would be represented by a straight line on Fig.4(a), but in fact the slope of the empirical curve increases with Reynolds number owing to the spread of turbulence through the boundary layer. Any boundary layer surrounding the arc, however, is likely to be fully turbulent because of unsteadiness in the arc boundary, and this implies that a better empirical curve would be one based on turbulent boundary layers for all Reynolds numbers. A rough allowance can be made for this effect and leads to an increase in Nu for the empirical curve by a factor that is about 2 for the Reynolds numbers appropriate to the arc results. A further reason for uncertainty in the empirical curve lies in the fact that the arc temperatures are well above the range covered by the tests on hot cylinders.
(ii) The interpretation of the results for a rotating arc to an arc held at rest could be erroneous. Possible sources of error could be: the use of data for small electrode gaps; the neglect (in transforming from the rotating arc to the arc at rest) of the induced swirl of the air in the annular gap, the possibility that the arc is rotating in its own wake, and the departures of the shape of the arc centreline from an involute; the actual unsteadiness of the arc (which is inevitable in all experiments of arcs in air streams because it is not possible to eliminate entirely fluctuations in the stream velocity and ripples in the direct current supplied to the arc); the contamination of the air inside the arc by carbon vapour ejected from the electrodes. Of these possibilities the last two appear to be the most likely to reduce the discrepancy between the results for the arc and the cylinder.

(iii) The limitations of the theoretical model are too severe. Contributing factors here could be: the over-simplified treatment of radiation; the failure to take into account the variation of electric conductivity within the arc; the pre-selection of the arc periphery as a circle; the neglect of motion within the arc and the attributing to the arc periphery of the improbable property of withstanding viscous stresses; the assumption that the periphery is non-porous. The last three possibilities are likely to be the most powerful.

(iv) The concept of the arc periphery as an isothermal boundary of fixed temperature independent of the stream parameters and the current may be too restrictive. An alternative method of choosing the periheral temperature could be to define it so that the minimum field principle is obeyed, as discussed by Peters (1962). A change of viewpoint of this nature could produce very large reductions in the non-dimensional form of the heat transfer from the equivalent circular arc. It would, however, alter the theoretical concept of the periphery more drastically than might be desirable.

Although the idea of treating an arc in a stream as an impervious circular conductor does lead to a useful insight into the qualitative overall behaviour of the arc, and also enables all the stages of the calculation of arc characteristics to be carried through, it appears from the evidence of this paper that it is not adequate for quantitative calculations nor for providing an understanding of the important processes involved in the heat transfer. The equivalent circular arc without internal convection is itself capable of refinement, but it seems more likely that in a better model it may be necessary to take account of the departure of the shape of the arc from a circle, the probable porous nature of the arc periphery (as, for instance, in the treatment
of flow through an arc by Broadbent (1965), and the existence of fluctuations in the shape of the arc cross-section. It will require a careful blend of experiment and theory to achieve a proper understanding of this problem.
Appendix A

PROPERTIES OF AIR AT ATMOSPHERIC PRESSURE

The high temperature properties of air and nitrogen are sufficiently similar for information on nitrogen to be used in the calculation of the overall characteristics of arcs in air (see Shepard, Watson and Stine (1964)). Accordingly, we mix data for air and data for nitrogen in deriving approximation formulae for the properties of air. While the formulae given display the appropriate trends there is, however, much room for improved approximations.

We require expressions for the thermal conductivity $\kappa$, the electric conductivity $\sigma$ and the radiated power density $\psi$. In general these depend on the temperature $T$ and on the pressure, but with the pressure fixed at one atmosphere they are all expressible as functions of $T$ only. Now a useful simplification is achieved in arc theory if the temperature and the thermal conductivity are combined into the heat-flux potential $\varphi$, which when arranged to be zero at zero temperature is defined by

$$\varphi = \int_0^T \kappa dT \quad \text{(A1)}$$

We take advantage of this fact by ultimately expressing the temperature dependence of $\sigma$ and $\psi$ in terms of $\varphi$.

Temperature range $0 < T < \hat{T} = 4000^\circ K$

For the thermal conductivity $\kappa$ we assume that

$$\frac{\kappa}{\kappa_0} = \left(\frac{T}{\hat{T}}\right)^{3/4} \quad \text{(A2)}$$

where $\kappa_0 = 0.175$ watts/metre $^0K$; a comparison between this approximation and the values of $\kappa$ given by Kaecker (1959) for nitrogen is shown in Fig.A1. It follows that the corresponding approximation for $\varphi$ is

$$\frac{\varphi}{\varphi_0} = \left(\frac{T}{\hat{T}}\right)^{7/4} \quad \text{(A3)}$$

where $\varphi_0 = (4/7)\kappa_0 \hat{T} = 400$ watts/metre. For the electric conductivity $\sigma$ and the radiated power density $\psi$ we take...
Temperature range \( 4000^\circ K \leq T \leq 20000^\circ K \)

For the heat-flux potential \( \psi \) we assume that

\[
\frac{\psi - \bar{\psi}}{\bar{\psi}} = \left( \frac{T - \bar{T}}{\bar{T}} \right)^{7/2}
\]

where \( \bar{T} = 12800^\circ K \) and \( \bar{\psi} = 10^5 \) watts/metre; a comparison between this approximation and the values of \( \psi \) given by Goldenberg (1959) for nitrogen is shown in Fig.A2 (it is noticeable that the values of \( \psi \) in this temperature range are very much larger than \( \bar{\psi} \)). For the electric conductivity we assume that \( \sigma \) is given directly in terms of \( \psi \) by

\[
\frac{\sigma}{\bar{\sigma}} = \frac{\psi - \bar{\psi}}{\bar{\psi}}
\]

where \( \bar{\sigma} = 10^4 \) mhos/metre; a comparison between this approximation and the data given by Goldenberg (1959) for nitrogen is shown in Fig.A3. For the radiated power density \( \dot{\psi} \) we take

\[
\dot{\psi} = \left( \frac{T - \bar{T}}{\bar{T}} \right)^{7}
\]

where \( \bar{\psi} = 10^{10} \) watts/metre\(^3\); a comparison between this approximation and the data given by Kivel and Bailey (1957), used in conjunction with the data of Hilsenrath and Beckett (1956) and Hilsenrath, Green and Beckett (1957), is shown in Fig.A4. It follows that \( \dot{\psi} \) is given in terms of \( \psi \) by

\[
\frac{\dot{\psi}}{\bar{\psi}} = \left( \frac{\psi - \bar{\psi}}{\bar{\psi}} \right)^2
\]
Appendix B

A PROMISING ASPECT OF THE ANALYSIS OF EXPERIMENTAL RESULTS

It follows from equation (17) and the definitions of $C_D$ and $Re_1$ in Section 4.2 that, for fixed ambient and peripheral temperatures, $B_{\infty}/v_{\infty}$ is proportional to $C_D Re_1$. Also, it follows from equations (15) and (16) and the definition of $Nu$ in Section 4.2 that, for fixed ambient and peripheral temperatures and fixed ambient pressure, $v_{\infty}^2 I/\beta_{\infty}$ is proportional to $Nu Re_1^2$. If both $C_D$ and $Nu$ depended only on $Re_1$ (as they would for a solid cylinder at low Mach number) then it would follow that $B_{\infty}/v_{\infty}$ would depend on $v_{\infty}^2 I/\beta_{\infty}$ only. Both $B_{\infty}/v_{\infty}$ and $v_{\infty}^2 I/\beta_{\infty}$ can be determined from the experimental results, without involving any assumptions about the radiation loss, and to plot one against the other does not imply any assumptions whatever about the structure of the arc or the properties of the air inside it. The graph of $B_{\infty}/v_{\infty}$ against $v_{\infty}^2 I/\beta_{\infty}$ is given in Fig.B1. The collapse of the data is noticeably good. However, the full significance of this plot is not yet evident. The term $v_{\infty}^2 I/\beta_{\infty}$ is proportional to a non-dimensional parameter introduced by Lord (1964) (and denoted by $K$) which appears to play an important role in arc theory in general. When variation of the ambient pressure $p_{\infty}$ is taken into account, the parameter $K$ is proportional to $p_{\infty}^2 v_{\infty}^2 I/\beta_{\infty}$. It is therefore suggested here that, in future analyses of experimental results for arcs under the influence of imposed flows and applied magnetic fields, it may prove illuminating to plot $B_{\infty}/v_{\infty}$ against $p_{\infty}^2 v_{\infty}^2 I/\beta_{\infty}$. 

---


SYMBOLS

\( a_{\infty} \)  speed of sound in undisturbed stream

\( c_p \)  specific heat at constant pressure

\( j_z \)  current density

\( p \)  pressure

\( r \)  radial distance measured from centre arc

\( r \)  radius of equivalent arc

\( v_{\infty} \)  velocity of undisturbed stream

\( B_r, B_\theta \)  components of total magnetic field

\( B_{\infty} \)  applied magnetic field

\( C_D \)  drag coefficient of arc

\( D \)  drag per unit length on outside of arc column

\( E_{\infty} \)  applied electric field (voltage gradient)

\( F \)  thrust per unit length on inside of arc column

\( I \)  current

\( M \)  Mach number

\( N_u \)  Nusselt number

\( P_r \)  Prandtl number

\( Q \)  heat convected from arc by air stream

\( Q_i \)  heat conducted internally to arc periphery

\( R \)  gas constant for unit mass

\( Re \)  Reynolds number

\( T \)  temperature

\( \gamma \)  ratio of specific heats

\( \eta \)  coefficient of viscosity

\( \theta \)  circumferential angle measured from rear of arc

\( \kappa \)  thermal conductivity

\( \rho \)  density

\( \sigma \)  electric conductivity

\( \varphi \)  heat-flux potential

\( \psi \)  radiated power density

Superscripts:

*  values at arc periphery

-  constant reference values used in describing gas properties
Subscripts:

\[ \infty \] values in undisturbed stream
\[ 0 \] values at centre of arc
\[ \frac{1}{2} \] values calculated at mean temperature between undisturbed stream and arc periphery
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FIG.1(a) FEATURES OF PROBLEM: MAIN PARAMETERS

GIVEN: $P_\infty, T_\infty, V_\infty, I, \hat{T}$

UNKNOWN: $E_\infty, B_\infty, T_0$
Fig. 1 (b) Features of Problem: Model for Empirical Treatment

Given: $P_\infty, T_\infty, \nu_\infty, i, \hat{r}$

Unknown: $E_\infty, B_\infty, \hat{r}, Q, D$

THERMAL BOUNDARY LAYER

PERIPHERY

CENTRE

WAKE
FIG. 2(a) EXPERIMENTAL RESULTS: ELECTRIC FIELD $E_\infty$
FIG. 2(b) EXPERIMENTAL RESULTS: MAGNETIC FIELD $B_\infty$
Fig. 3(a) Derived results for equivalent arc: radius $\hat{r}$.
Fig. 3

Derived Results for Equivalent Arc: Total Heat Flux Q

- Radiation Neglected
- Radiation Included
Fig. 3

INDEPENDENT OF RADIATION

$X$ SPOT POINT

$D_{kg/sec^2}$

$V_{∞} = 89.6$ METRES/SEC

$V_{∞} = 47.0$ METRES/SEC

$V_{∞} = 9.4$ METRES/SEC

FIG. 3 (c) DERIVED RESULTS FOR EQUIVALENT ARC: DRAG D
FIG. 4 (a) COMPARISON BETWEEN CIRCULAR STATIC ARC AND CIRCULAR SOLID CYLINDER: NUSSELT NUMBER Nu AGAINST REYNOLDS NUMBER Re^{1/2}

(FROM KNUDSEN AND KATZ)
P.505, FIG.17-16

x SPOT POINT (USING CALCULATED RADIUS)
+ SPOT POINT (USING VISUAL RADIUS)

CIRCULAR SOLID CYLINDER

EQUIVALENT ARC:
--- RADIATION INCLUDED
- - - RADIATION NEGLECTED
FIG. A1 ASSUMED PROPERTIES OF AIR AT ONE ATMOSPHERE PRESSURE: THERMAL CONDUCTIVITY $k$ FOR $0 \leq T \leq \hat{T}$
**Fig. A.2**

**Assumed Properties of Air at One Atmosphere Pressure:** Heat Flux Potential $\Phi$ for $T \geq \hat{T}$

\[
\begin{align*}
\Phi &= 400 \text{ Watts/Metre} \\
\tilde{\Phi} &= 10^6 \text{ Watts/Metre} \\
\hat{T} &= 12800^\circ K \\
\hat{\tilde{T}} &= 4000^\circ K
\end{align*}
\]
Fig. A.3 assumed properties of air at one atmosphere pressure: electric conductivity $\sigma$ for $T \geq 1$.
Fig. A.4 $\psi$ WATTS/METRE$^3$

$\psi = 10^{10}$ WATTS/METRE$^3$

$\bar{\psi} = 12800^\circ K$

$\hat{\psi} = 4000^\circ K$

Figure A.4 Assumed properties of air at one atmosphere pressure: radiated power density $\psi$ for $T > \hat{T}$
FIG.B1 ANALYSIS OF EXPERIMENTAL RESULTS: $B_{\infty}I / v_{\infty}$ AGAINST $v_{\infty}^2 I / E_{\infty}$
The paper has three main aims: to describe the important features of the two-dimensional problem of an arc column across a low subsonic air stream; to present a simple model of the arc column which has proved qualitatively useful in the past; to demonstrate by analysing experimental results that the model is not quantitatively adequate.

A principal feature of the problem is that the cross-sectional area of the arc depends on the current passing through the arc, on the velocity of the undisturbed stream and on the ambient pressure and temperature. This feature is incorporated in the simple model, which employs an equivalent arc of circular cross-section carrying the same current and having no convection.

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inside it. The experimental results, inferred from measurements on moving arcs at atmospheric pressure, are analysed in terms of Reynolds number, Nusselt number and drag coefficient. The results for the arc are found to differ greatly from results for solid circular cylinders, even when an allowance is made for loss of energy from the arc by radiation. It is therefore concluded that the equivalent arc, as here defined, is not a satisfactory model of an arc across an air stream, and some possible reasons for this, as well as possible lines of improvement, are given.

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