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RESEARCH ON JET NOISE GENERATION
AND
SUPPRESSION

APRIL 1964

Prepared Under Department of the Navy,
Bureau of Naval Weapons

Contract N60-0887-d
Phase I

by

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GENERAL ELECTRIC COMPANY
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RESEARCH ON JET NOISE GENERATION AND SUPPRESSION

APRIL 1964

Prepared Under Department of the Navy,
Bureau of Naval Weapons

Contract NAV 62-0687-d
Phase I

by

K.R. Searson

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Flight Propulsion Division
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Cincinnati 15, Ohio
Experimental and analytical investigations on jet noise generation and suppression have resulted in a verification of the concept that jet noise is uniquely related to mean flow velocity distribution in the jet wake. Advances have been made in analytical techniques for calculating aerodynamic properties of free jet flow and the related acoustic properties of the flow and verification of the assumption that negligible attenuation of sound occurs during propagation through multi-element jet flows has been achieved.

Investigation of selective water injection as a jet noise suppression scheme has indicated little promise for in-flight noise reduction. Studies of noise generation by high temperature and high pressure ratio jets have extended the state-of-the-art in the area of high temperature flow noise suppression, and have resulted in a means for predicting total acoustic power beyond the limits of applicability of the Lighthill parameter.
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5.1 Introduction

5.2 Experimental Procedure

5.3 Discussion of Results

5.4 Conclusions
This is the final report covering the work performed under Phase I, "Jet Noise Suppression Research", Department of the Navy, Bureau of Naval Weapons, Contract No. NO0 ON 62-0687-a, in accordance with the Contractor's proposal entitled, "Jet Noise Suppression Research and System Studies", P61-86, dated March 1961.

The objectives of the Phase I program were: to extend the theory of jet noise suppression to high pressure ratio and high temperature flow by demonstrating the generality of the concept that jet noise is uniquely related to the flow velocity distribution in the wake; to further refine and develop analytical techniques applicable to computer solutions for jet flows and noise; to investigate significant variables influencing jet noise generation and suppressor performance; and to evaluate the feasibility of selective water injection as an in-flight suppression technique. The advanced knowledge achieved through the successful accomplishment of this research program, along with the findings of the Phase II program on operational noise problems and effects of suppression on mission performance, can be implemented toward the goal of developing a practical suppressor for military jet aircraft.

The problems dealt with in this research investigation have been divided into the five sections of this report, each of which is essentially self-contained for the convenience of the reader. However, the interrelation of the various aspects of the program is maintained by appropriate cross-references between the various sections, and by the condensation of the overall effort in the Summary.
Grateful acknowledgement is made to Commander A. Lee and Mr. R. Brown of the Bureau of Naval Weapons under whose guidance this program was carried out; to Dr. R. M. Kendall of Vidya, a Division of Itel Corporation, for his analytical contributions to the problem of jet mixing; to Mr. R. Lee, under whose technical direction this program was initiated at the Advanced Engine and Technology Department of the General Electric Company; and to Mr. D. L. Harshman, Manager of Installation Aerodynamics, Advanced Engine and Technology Department, whose guidance and encouragement contributed significantly to the accomplishment of the program objectives.
SUMMARY

Experimental and analytical investigations on jet noise generation and suppression have been carried out for the purpose of: 1) demonstrating and extending the generality of the concept that jet noise is uniquely related to jet wake mean flow velocity distributions; 2) further refining and developing analytical computer techniques for solution of jet flow and noise problems; 3) defining the effects of jet temperature and pressure ratio on noise characteristics for nozzles and nozzle suppressors; 4) determining the effect of the turbulent jet wake on the radiation of sound energy generated by multiple element suppressors; and 5) evaluating the feasibility of selective water injection as an in-flight suppression device.

The previously established technique for calculation of sound power spectra from aerodynamic properties of the jet wake is outlined in Section 1 of this report. It is demonstrated that the method is quite general, being applicable to nozzle-suppressors of various geometric designs. Furthermore, experimental evidence is presented which indicates that jet noise sound power spectra can be adequately predicted for the various nozzle configurations at flow pressure ratios up to 3.0, and flow temperatures up to 1800°F. Comparison between measured and predicted overall sound power shows that the prediction method used provides calculated levels within 4-2 db of acoustically measured levels for nearly all nozzle shapes and flow conditions tested.

In Section 2, the development of analytical techniques for jet flow field prediction is undertaken in order to provide a working tool for jet noise suppressor research. Starting from the approach suggested by the theory of Reichardt, computer programs are developed which represent an advancement of the state-of-the-art in the analysis of jet flow from complex nozzles. While it is shown that analytically derived mean flow velocity profiles for the jet mixing region in
general agrees satisfactorily with experiment, further work is required to bring computer techniques to their full state of usefulness.

In Section 3, the results of a study on jet noise generation and suppression at high flow temperatures, up to those representative of afterburning jet engine operation, are presented. Experimental data on noise generation by jets up to flow pressure ratio of 3.4 and temperature up to 3300°R has been obtained, indicating that for low pressure ratios and moderate temperatures little flow density effect exists, while over the full range of observation a temperature dependent flow density effect is found to occur. This effect is such that sound power varies with \( \left( \frac{e_j}{e_d} \right)^n \), where \( n \) is approximately a linear function of temperature. The results of scale model experiments have been used to establish an empirical relationship between nozzle energy flux and sound power generation, applicable to concentric flows as well as simple conical nozzles. Studies of suppressor nozzle acoustical performance at high flow temperature have been carried out experimentally, with results indicating that suppression is temperature as well as pressure ratio dependent. Jet noise suppression at high flow temperatures requires further study for the purpose of deriving generalized temperature scaling laws applicable to all nozzle geometries, since the experimental results presented indicate that temperature effect on suppression may be nozzle geometry dependent. The results of the high temperature jet noise investigation point out the need for flow temperature simulation in suppressor scale model development directed toward afterburning jet engines.

Section 4 presents the results of experimental investigations on dissipation of sound, generated by a jet, in the turbulent wake of the jet itself. No evidence of sound absorption has been found, and thus no modification of the correlation of aerodynamic and acoustic jet properties is required from this consideration. The effect previously referred to by other researchers as jet noise "shielding" is shown to be the result of directivity rather than sound power change.
Selective water injection as a jet noise suppression technique has been experimentally and analytically investigated, with results presented in Section 5. Studies indicate that the desired alteration of jet mixing cannot be achieved through limited water use, and that the suppression thus accomplished is less than that expected for uniform mixing of air and water. Adverse effects of high flow temperature and velocity on noise reduction capability of water injection have been experimentally demonstrated, and indicate that this suppression scheme is not practical for in-flight noise reduction.

The knowledge gained in the successful completion of this program on jet noise suppression research can be implemented, along with other existing technology, toward the development of a jet noise suppressor for application to military aircraft in flight. Thus, noise reduction for military jet aircraft might indeed be a practical reality.
1.0 Experimental Investigation on the Relationship Between Aerodynamic Properties of the Jet Flow Field and Resultant Noise Generation

1.1 Introduction

As a result of a previous Navy sponsored General Electric research program on jet noise generation and suppression (NOas 59-6160-c) an empirical formulation was developed for relating the aerodynamic properties of a free jet to its noise characteristics (Ref. 1). The hypothesis that the sound power generated at various axial positions in the jet mixing region depends only on the mean velocity distributions at these locations led to a general quantitative relationship for determining sound power spectra for simple conical and complex suppressor nozzles. It was demonstrated that, within certain limits of flow pressure and temperature, the manner in which the flow velocity distributions vary within the jet because of nozzle geometry variation does not influence the generality of the aero-acoustic correlation.

The usefulness of the previously established correlation between aerodynamic and acoustic jet properties depends to a great extent on its applicability to realistic flow temperatures and pressures, and to various nozzle configurations. While past verification of the theory was limited to cold flow, sub-critical pressure ratios and relatively simple nozzle designs consisting of a number of identical elements, the present study considered flow temperatures up to $1800^\circ R$, pressure ratios up to 3.0, and large variety of nozzle designs, in accordance with the program work statement in Reference 2. The generality of previously established empirical constants has been re-evaluated through correlation of aerodynamic and acoustic data, indicating that the velocity profile formulation can indeed be applied to heated, high pressure ratio jets, and essentially all nozzle geometries.
1.2 Prediction of Sound Power Spectra from Flow Profile Data

The previously established relationship between aerodynamic and acoustic properties of a free jet is restated from Reference 1:

\[ P(f) = K \rho_0 c_0^{-5} a_0 u_0^8 \left( \frac{1}{f} \right)^{1.2} \int_{a_x} \left( \frac{u}{u_0} \right)^8 d\left( \frac{a}{a_0} \right) \]

where:

- \( P(f) \) = Sound power per cycle at frequency \( f \)
- \( K \) = Empirical constant
- \( \rho_0 c_0^{-5} a_0 u_0^8 \) = Lighthill parameter
- \( f \) = Frequency
- \( u \) = Local mean flow velocity
- \( a \) = Flow cross-section area
- \( x \) = Axial location along the jet

The assumptions employed are: (1) No noise is generated within the flow potential core, and thus the integral must be evaluated over the region \( a(x) \) external to the core flow; (2) the axial distribution of scale of turbulence within the jet is linear, that is, \( L \sim x \); (3) the relationship between axial location and frequency of the sound source is given by:

\[ f \frac{d_e}{u_0} = \left( 1.25 \frac{x}{d_e} \right)^{-1.22} \]

where \( d_e \), nozzle equivalent diameter, is defined by:

\[ d_e = \sqrt{\frac{4}{\pi} \frac{a_0}{n}} \]
and \( n \) is the number of flow elements in the nozzle; \( (4) \) the total generated sound power is radiated.

Prediction of the sound power spectrum is accomplished by experimentally or analytically evaluating the integral in equation (1) at a sufficient number of \( x \)-locations along the jet so that the frequency range of interest is fully defined (Equation (2)). In this section of the report, the experimental evaluation of jet flow properties is discussed, while the analytical approach is developed in Section 2.0.

Under a previous Navy-sponsored research program (Reference 1) the validity of this method of sound power spectrum prediction was demonstrated for an eight-lobe suppressor nozzle, under cold flow and low pressure ratio conditions. For complex nozzle geometries employing elements of non-uniform size the nozzle equivalent diameter, \( d_e \), in the sound source location equation (Eq. (2)) must be suitably defined. The definition of nozzle equivalent diameter as previously stated in Eq. (3) is a logical choice in that it is an average elemental diameter, and the total noise is generated essentially in the region where the jet flow exhibits an individual element identity. Furthermore, the jet potential core length, which is related to the location of the region of maximum noise generation, correlates with the equivalent diameter as herein defined.

The assumption that the total sound power generated is radiated into the far field is considered in Section 4.0 of this report, and it is shown that no significant dissipation of acoustic energy in the audio frequency range occurs in the turbulent jet.
In order to determine the limits of applicability and to further develop the correlation described above, experimental investigation of flow velocity profiles was carried out for seven nozzle configurations. Flow temperatures up to 1800°F and nozzle pressure ratios up to 3.0 were tested, and aerodynamic data was correlated with measured sound power spectra, as described in the following section.

1.3 Experimental Determination of Flow Velocity Profiles and Correlation with Measured Sound Power Spectra

Aerodynamic measurements were made in the free jet wake for a number of nozzle geometries and flow conditions for the purpose of defining the mean flow properties of the jet. Flow velocity distribution was determined throughout the jet wake, and acoustic properties predicted from the flow measurements were compared with measured sound power spectra.

1.3.1 Aerodynamic Measurements

Detailed flow velocity data was obtained throughout the significant noise generating regions of the jet wake for the various nozzles tested using the research facility shown in Figure 1.1. The facility consists of a burner of the type commonly employed in aircraft gas turbines, followed by a flow mixer and pressure and temperature instrumentation for accurately determining inlet conditions to the nozzle under test. Air supply capacity for the facility is up to 9 lb/sec., and flow temperatures up to 1800°F are achieved by the burner. Nozzle pressure ratios up to 3.0 were possible for the size nozzles tested (up to 13 sq. in. exit area) with heated flow. For high pressure ratio cold flow testing, an indoor facility with higher airflow capability was employed, and test instrumentation used was similar to that for hot flow measurement. The
test nozzle was mounted on the facility, shown in Figures 1.1 and 1.2, and the wake was surveyed with a combination total pressure - total temperature probe, the position of which was remotely controlled by means of probe actuators mounted from a lathe bed. During the research program, an afterburner was added to the facility (Figure 1.2) increasing flow temperature capability to 3500°F, but instrumentation difficulty precluded velocity profile measurements above 1800°F, and the addition to the facility was used for studies of noise generation at high temperature as described in Section 3 of this report.

Temperature and pressure data was recorded automatically for probe traverses at each axial location in the jet wake, and point by point calculations of flow velocity were accomplished by computer. Since the velocity in the core region of the jet is not evaluated in the prediction of sound power spectra, the problem of measuring supersonic velocities and static pressure gradients in the core region of high pressure ratio jets was bypassed. For the pressure ratios under consideration, flow velocity in the noise producing mixing region is essentially always subsonic, while the expansion region with static pressure gradients and shock structure is assigned to the jet core in which no noise is generated. Thus, the computation of flow velocity was simplified in that the assumption of static pressure equal to ambient pressure could be made.

The prediction of sound power spectra from velocity data does not require knowledge of the velocity distributions at a given axial location in the jet; only the flow velocity-flow area relationship is required (Eq. 1). Therefore, the flow velocity data was reduced to the form of equivalent velocity profiles. That is, the flow was artificially reoriented to simulate flow from a conical nozzle for the purpose of simplifying the integration process and
providing a common means of presentation for all of the flow data for the various configurations. This equivalent data is presented in the next section, having been obtained from flow velocity maps at each location in the jet, by graphically accumulating flow area in the order of decreasing flow velocity to artificially create an equivalent conical nozzle profile for each nozzle geometry tested. From the equivalent profiles, the integration required for evaluation of Equation 1 was performed graphically.

1.3.2 Acoustical Measurements

Aerodynamically predicted sound power spectra were correlated with actual noise measurements obtained for each nozzle configuration. The outdoor test facility used for flow velocity measurements was also employed for sound power evaluation. Noise measurements were made under free-field conditions. A microphone attached to a remotely controlled boom (Figure 1.3) was used to survey the sound field at a radius of 15 feet from the nozzle. The microphone a Bruel & Kjaer Type 4133, has flat frequency response to 40,000 cps. A frequency analysis of the jet noise was obtained for a number of angular locations from the jet axis utilizing the instrumentation shown in Figure 1.4. From the directivity data thus obtained, sound power spectra were computed by the process of integration of sound pressures over the radiation area. The computation procedure assumes that the ground plane is semi-absorbing so that maximum error from this source is about 1 db, as the difference between assuming a totally absorbing and totally reflecting surface is 3 db. The location of the microphone at 15 feet from the jet permits measurements to a low frequency limit of 250 cps. The measurement radius was selected so as to provide far-field conditions for low frequency noise and yet not be so far from the source as to be influenced by reflections from nearby structures.
1.4 Discussion of Results

The aerodynamic and acoustical data obtained for the seven nozzle geometries tested are presented in this section, and correlation between predicted and measured sound power spectra is made.

1.4.1 Description of Nozzle Configurations

In order to determine the applicability of the aerodynamic-acoustic correlation to various nozzle geometries, a number of configurations were tested. A list of configurations and test conditions is presented in Table 1.1. The nozzles are all of approximately the same scale, with exit area ranging from 9.6 to 13 in². The geometric variations (as shown in Figures 1.5 thru 1.8) include simple circular exit, annular exit (plug nozzle), shrouded annular exit, multiple exit (segmented), and multiple exit with elements of unequal size. The variety of nozzle configurations provided a means of evaluating the applicability of the prediction technique to essentially all types of suppressors.

1.4.2 Correlation of Aerodynamic and Acoustic Data

From the measurements of total pressure and temperature in the jet wake for the various configurations and test points (as summarized in Table 1.1), mean flow velocities were calculated and flow velocity contours were plotted for each axial location within the jet wake. Velocity contours for the 19-tube and 18-segment nozzles are presented in Figures 1.9 and 1.10. The merging of the individual flows, which are quite distinct near the nozzle exit, can be readily observed as the flow progresses downstream. Equivalent flow profiles are derived from the velocity contours by accumulating the flow area graphically, in order of decreasing flow velocity, resulting in the profiles as shown in Figures 1.11 through 1.16. The equivalent flow profile curves enable the various
nozzles to be considered in terms of a fictitious simple circular exit geometry, but with flow profiles developing differently than for a circular exit nozzle. The normalized equivalent profiles and the curves showing the decay of maximum flow velocity as a function of axial location (Figure 1.17) provide the data required for aerodynamic prediction of sound power spectra by means of equations 1 and 2.

The spectra calculated from jet flow properties are compared with acoustically measured spectra in Figures 1.18 thru 1.23. Since the levels calculated are power spectrum levels (sound power per unit bandwidth), the values which were measured in percentage bandwidths have been converted to the same constant bandwidth reference for ease of comparison. Predicted and measured total acoustic power levels for all of the nozzle configurations and flow conditions for which power spectra have been determined are compared in Figure 1.24, where the differences between measured and predicted levels are plotted against flow exit velocity.

Several observations can be made concerning the results of this investigation. The shapes of the predicted sound power spectra agree very well with measured values, and the amplitudes agree, in general, within the limits of experimental accuracy. The spectrum shape results imply that the assumed distribution of sound sources within the jet wake is adequate, and that the evaluation of nozzle equivalent diameter by Equation 3 is valid. The degree of accuracy achieved in the prediction of absolute noise levels (for spectra and total acoustic power) verifies the applicability of the fully expanded flow velocity as the significant nozzle exit velocity, and furthermore indicates that the empirical constants employed in the correlation are applicable over
the range of flow conditions tested. Although somewhat greater amplitude deviation than can be attributed to experimental error is noted for the shrouded plug nozzle geometry, it is believed that the discrepancy is due to generation of noise (other than jet noise) by the shroud. The manner in which this noise is generated has not been investigated here, but the conclusion is in agreement with the majority of past experience which shows that addition of a shroud may, under some circumstances, result in production of a number of discrete frequency noise components. The possibility that noise generated in the inner regions of a complex jet is dissipated in the surrounding turbulent flow is more fully discussed in Section 4.0 of this report. However, the applicability of the aerodynamic-acoustic correlation to the variety of suppressor configurations evaluated confirms the result of that concurrent study that no significant absorption takes place.

1.5 Conclusions and Recommendations

The results of this investigation further confirm the hypothesis presented in Reference 1, which states that the noise generated by a jet is uniquely related to the mean-flow velocity distribution within the wake. The technique for calculating sound power spectra from aerodynamic properties of the jet wake is outlined, and the generality of the technique is demonstrated for: (1) nozzle geometry variations, including circular nozzles, plug nozzles, and suppressor designs whose individual elements are not all equal in exit dimensions; (2) flow pressure ratio up to 3.0; and (3) flow temperature up to 1800°K. Jet noise spectrum shapes and levels can be adequately predicted within the limits stated.
Extension of the applicability of the jet noise suppression theory to actual afterburner temperatures (beyond the scope of the present program) is discussed in Section 3.0 of this report, and it is probable that the aerodynamic prediction technique can be applied to this condition, though possibly with some modification. Toward that goal, the analytical techniques for flow profile prediction, presented in Section 2.0, could be employed within the limits of applicability to calculate flow data for correlation with noise data at afterburning temperatures.

Finally, the equivalent mean flow velocity profiles presented for the various noise suppressor nozzles may suggest other geometric designs, more applicable to specific problems, which would generate desired profiles, and thus, low noise levels.
REFERENCES:


SYMBOLS:

\[ a = \text{Flow Cross-section Area} \]

\[ a_0 = \text{Nozzle Exit Area} \]

\[ d_e = \text{Nozzle Equivalent Diameter} = \sqrt{\frac{4a_0}{\pi n}} \]

\[ f = \text{Frequency} \]

\[ n = \text{Number of Exit Elements in Suppressor Nozzle} \]

\[ P(f) = \text{Sound Power per Cycle} \]

\[ r_e = \text{Nozzle Equivalent Radius} = \frac{1}{2} d_e \]

\[ u = \text{Local Flow Velocity} \]

\[ u_o = \text{Effective (fully expanded) Flow Velocity at Nozzle Exit} \]

\[ x = \text{Axial Location, Measured from Nozzle Center Line} \]

\[ y = \text{Radial Location, Measured from Nozzle Center Line} \]
<table>
<thead>
<tr>
<th>Configuration</th>
<th>$d_o$</th>
<th>$F_o$</th>
<th>Flow Temperature</th>
<th>Pressure Ratio</th>
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<tbody>
<tr>
<td>Conical Nozzle</td>
<td>4.0&quot;</td>
<td>2.0&quot;</td>
<td>1660°R</td>
<td>1.2, 1.8</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>540°R</td>
<td>1.2, 1.8</td>
</tr>
<tr>
<td>Plug Nozzle</td>
<td>3.5&quot;</td>
<td>1.75&quot;</td>
<td>530°R</td>
<td>1.6, 2.0, 2.4</td>
</tr>
<tr>
<td>Plug Nozzle with Shroud</td>
<td>3.5&quot;</td>
<td>1.75&quot;</td>
<td>530°R</td>
<td>1.6, 2.0, 2.4</td>
</tr>
<tr>
<td>8 Lobe Nozzle</td>
<td>1.01&quot;</td>
<td>0.71&quot;</td>
<td>1800°R</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>530°R</td>
<td>1.8</td>
</tr>
<tr>
<td>19 Tube Nozzle</td>
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<td>0.45&quot;</td>
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<td></td>
<td></td>
<td></td>
<td>540°R</td>
<td>1.2, 1.8, 2.4</td>
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<tr>
<td>18 Segment Nozzle</td>
<td>0.825&quot;</td>
<td>0.413&quot;</td>
<td>530°R</td>
<td>2.4</td>
</tr>
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Figure 1.7a 19-Tube Nozzle
Figure 1.7b  End View of a 120° Sector of the 19-Tube Nozzle
Figure 1.6a 18-Segment Nozzle
Figure 1.8b 18 Segment Nozzle Exit Geometry

Scale: Full Size
Axial Location  \( X = 2 \) inches  

Flow Exit Velocity = 1000 ft/s

60° Symmetrical Section  

\( P_0/P = 1.8 \)  

\( T_0 = 540^\circ R \)

19 Tube Nozzles

Figure 1.9a - Flow Velocity Contours.
Axial Location $x = 4$ inches  
Flow Exit Velocity = 1000 ft/sec  

6*7 Symmetrical Section  
$P_o/P = 1.8$  
$T_o = 5100^oR$

19 Tube Nozzle

Figure 1.9b- Flow Velocity Contours.
Lines of constant velocity \( \text{ft/sec} \)

Axial Location \( X = 8 \) inches, Flow Exit Velocity = 1000 ft/sec

60° Symmetrical Section \( \frac{p}{\mu} = 1.5 \)

\( \frac{\n}{\mu} = 5\text{h} \)

29 Tube Nozzle

Figure 1.9c - Flow Velocity Contours.
Axial Location $X = 16$ inches  
Flow Exit Velocity $= 1000$ ft/sec  
60° Symmetrical Section  
$P_c/P = 1.8$  
$T_o = 540^\circ F$  

19 Tube Nozzle

Figure 1.9d - Flow Velocity Contours.
Figure 1.9e - Flow Velocity Contours.
Lines of Constant Velocity Ft./Sec.

200

600

900

1100

900

600

200

600

900

1120°

900

1100

1175

1160


Axial Location X = 2 inches  Flow Exit Velocity = 1190 ft/sec

60° Symmetrical Section

\[ \frac{P_0}{P} = 2.4 \]

\[ T_0 = 5100^\circ R \]

19 Tube Nozzle

Figure 1.9f - Flow Velocity Contours.
Lines of constant velocity Ft./sec.

Axial Location X = 0 inches  Flow Exit Velocity = 1190 ft/sec

P_0/P = 2.4  

60° Symmetrical Section  

19 Tube Nozzle

Figure 1.9g  Flow Velocity Contours.
Axial Location $x = 8$ inches  
Flow Exit Velocity = 1190 ft/sec

$60^\circ$ Symmetrical Section  
$p_0/p = 2.1$  
$p_0 = 31.0$ psia

19 Tube Nozzle

Figure 1.9h - Flow Velocity Contours.
Axial Location: $X = 16$ inches
Flow Exit Velocity: $1150$ ft/sec

60° Symmetrical Section

$P_{D}/P = 2.4$

$T_{O} = 510^\circ R$

19 Tube Nozzle

Figure 1.91 - Flow Velocity Contours.
Axial Location $X = 2$ inches

Flow Exit Velocity = 1200 ft/sec

$P_o/P = 2.4$  $T_o = 540^\circ R$

18 Segment Nozzle

Figure 1.10a- Flow Velocity Contours.
Lines of Constant Velocity ft/sec

Axial Location: \( X = 4 \) inches

Flow Exit Velocity = 1200 ft/sec

\( P_0/P = 2.4 \quad T_0 = 540^\circ R \)

18 Segment Nozzles

Figure 1.10b - Flow Velocity Contours.
Lines of Constant Velocity ft/sec

Flow Exit Velocity = 1200 ft/sec

Axial Location Z = 8 inches

P₀/P = 2.4  T₀ = 5400 R

12 Segment Nozzle

Figure 1.10c - Flow Velocity Contours.
Axial Location $X = 16$ inches

Flow Exit Velocity = 1200 ft/sec

$P_o/P = 2.4$  $T_o = 540^\circ R$

18 Segment Nozzle

Figure 1.10c– Flow Velocity Contours.

-39-
Axial Location $X = 32$ inches

Flow Exit Velocity = 1200 ft/sec

$P_o/P = 2.4$  $T_o = 540^\circ F$

18 Segment Nozzle

Figure 1.10c- Flow Velocity Contours.
F_{o}/P = 1.2
To = 1660^\circ R

Conical Nozzle

Figure 1.11a - Normalised Equivalent Velocity Profiles
Plug Nozzle

Figure 1.12a - Normalised Equivalent Velocity Profiles
Normalized Equivalent Velocity Profiles

$P_o/P = 2.4$
$T_o = 530^\circ R$

Figure 1.12b: Normalized Equivalent Velocity Profiles
Figure 1.1.4 - Normalized Equivalent Velocity Profiles

Plug Nozzle with Shroud

\[ \frac{P_0}{P} = 1.65 \]
\[ \frac{T_0}{T} = 530 \text{ R} \]
Figure 1.3a Normalized Equivalent Velocity Profiles
8 Lobe Nozzle

Figure 1.14 - Normalized Equivalent Velocity Profiles
Figure 1.15a- Normalized Equivalent Velocity Profiles

19 Tube Nozzle

\[ \frac{P_0}{P} = 1.8 \]

\[ T_0 = 1670^\circ R \]

- \( \Delta - 17.8 \)
- \( \bigcirc - 71.0 \)
Figure 1.25c: Normalized Equivalent Velocity Profiles

29 Tube Nozzle
Figure 1.15a Normalized Equivalent Velocity Profiles

P₀/P = 2.4
T₀ = 560°R

19 Tube Nozzle
Figure 1.17c - Maximum Velocity Decay in Axial Direction.
Figure 1.7b - Maximum Velocity Decay in Axial Direction.
Figure 1.17c - Maximum Velocity Decay in Axial Direction.
Figure 1.17d - Maximum Velocity Decay in Axial Direction.
Figure 1.17c: Maximum Velocity Decay in Axial Direction.
Figure 1.17f - Maximum Velocity Decay in Axial Direction.
Figure 1.17g - Maximum Velocity Decay in Axial Direction.
Figure 1.17h - Maximum Velocity Decay in Axial Direction.
Figure 1.18a - Comparison of Predicted and Measured Sound Power Spectra.
Acoustical Data

\( P_o/P \)

Frequency - CR3

To = 530°R

Predicted Values

\( P_o/P \)

\( \circ \) - 1.2

\( \triangle \) - 1.4

\( \diamond \) - 1.8

* Predicted Values Calculated from Velocity Profiles from Reference 3.

Conical Nozzle

Figure 1.18b - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.19 - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.20 - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.21 - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.22a - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.22b - Comparison of Predicted and Measured Sound Power Spectra.
Figure 1.23 - Comparison of Predicted and Measured Sound Power Spectra.
2.0 ANALYTICAL STUDY ON THE PREDICTION OF AERODYNAMIC PROPERTIES OF FREE JET FLOWS

2.1 Introduction

In the preceding section of this report, as well as in Reference 1, the unique relationship between aerodynamic noise generation and jet flow field properties has been demonstrated. Thus, from knowledge of the flow field the jet noise sound power spectrum can be determined by analytical means. In order to relate jet noise to nozzle geometric parameters by the previously developed acoustical theory, a means is first required for prediction of the flow field.

The subject of free jet mixing and resultant noise generation has been the object of extensive experimental and analytical research for a number of years. In the field of free jets, the most complicated practical problem is found in the analysis of the free jet flow emanating from sound suppressor configurations. It has been found through research that good sound suppression performance requires a significant amount of geometric complexity. The analysis of the complicated jet flows produced by such configurations must depend heavily upon a very simple theory to be at all manageable. Such a theory is the approach suggested and used by Reichardt, and is the one the authors have demonstrated to be of practical usefulness to the analysis of the flow fields from jet suppressors.

In this study the development of analytical techniques for jet flow field prediction is undertaken in order to provide a working tool for suppressor research. The theory is applied to the problem of predicting aerodynamic properties of jet flows from suppressor nozzles through the use of a computer program. This section presents the analysis and computer programs that have
been developed around these theories toward the goal of analytically predicting the jet flow fields for complex nozzle geometries, from which the jet acoustical properties can be determined using existing techniques.

2.2 Theoretical Analysis and Computer Program Development

The analytical theory and method presented in this report are extensions of the theory and method originally developed in Reference 1. The original work produced satisfactory results in the prediction of the flow field and noise produced by elemental jet types (i.e., jets from circular and rectangular exits). That work was culminated by a computer program which provided the means for evaluating the theory for complicated jet flows. It is in the prediction of complicated jet flows (such as from the nozzle geometry shown in Figure 2.1) that the original work has been found to be marginally satisfactory. Predicted and experimentally determined profiles for the nozzle shown in Figure 2.1 are compared at a number of flow cross sections in Figure 2.2. It is the task of the study reported here to improve the original work in order that deficiencies can be removed.

2.2.1 Theoretical Analysis of the Jet Flow Field

This section of the report presents the analysis of an arbitrary jet flow field in which the solutions for velocity, shear stress, and other properties of the jet flow are considered for the most general case. The formal and detailed derivation of the equations, which supports the broad treatment given the analysis in this section, are presented in Section 2.6.

The principle problem area of this study has been the description of the manner in which jets that have different properties (i.e., velocity,
density, etc.) intermix. More specifically, the problem has been the proper analytical formulation for the rate of jet spreading (or mixing) under conditions of relative flow. Once the mixing process is accurately described, the determination of such properties as velocity, shear stress, and density is easily accomplished.

The original treatment of the jet flow problem (Ref. 1) used the Reichardt hypothesis which states that the spreading property of free jet-mixing is dependent only on the downstream distance (i.e., the x-coordinate). This hypothesis was used in the Reference 1 analysis even for complicated flows such as those emanating from sound suppressors. There is an extensive body of literature in existence, some of which is referenced in this report, which supports the theory that relative velocities determine the rate at which mixing regions spread. Physically, one may think of the mixing region as one composed of large eddies (of size comparable to the mixing region itself) which cause the transport of momentum, energy, and concentration. The size of these eddies is a function of the shear forces forming it and the time per unit distance available for their formation (convective effect). The first effect is often described by an expression involving the jet Mach number such as the one below:

\[ \frac{dx}{db_m} = C_{b_1} (1 + C_{b_2} M) \]  

This expression shows that, the higher the jet Mach number, the smaller the eddy size at a given station (Refs. 57 and 58).

Research at Stanford has shown that the velocity profiles for simple relative jet flows can be predicted by using a simple empirical
differential equation relating the growth rate to the relative velocity difference (Ref. 2). In this research activity, it has been found that the relative or convective velocity effect on mixing is well approximated by the equation given below:

\[
\frac{dx}{db} \sim \frac{U_1 + U_2}{U_1 - U_2}
\]  

(1b)

During the time when the work of this report was in progress, no information was available as to the manner in which these two effects were interrelated. As a consequence, for the purposes of this study, the two effects were geometrically superimposed to give the following equation:

\[
\frac{dx}{db} = C_{bl} (1 + C_{b2} H_c) \left( \frac{U_1 + U_2}{U_1 - U_2} \right)
\]  

(2)

Previous researchers have also found this relative velocity effect. In particular, Willis and Glassman (Ref. 3) found a strong relative velocity effect; however, they were not able to derive an empirical equation describing the effects they found. Similarly, Marshall and Bailey, and Weinstein (Refs. 4 and 5) report a relative velocity effect on the spread of the mixing region but again do not develop equations describing this effect. Kendall (Ref. 1) found that the centerline velocity distribution, using the Reichardt method, was well described using the rate of spread given by Equation (1) for the case of zero external flow velocity.

Section 2.2.3 gives the results of this analysis using the linearly superimposed solution (Eq. (2)) in a comparison of theory and data.
For simple jets, the relative flow velocity (or density) effect produces no difficulty so far as the theory of Reichardt is concerned. It is only when more than two simultaneous flows are concerned (such as might occur in a triple annulus of three different flows or in multi-element jet exit shapes) that difficulty with the Reichardt method is found. Before delving into a detailed discussion of this difficulty, it is perhaps well to give an example showing the practical significance of the problem. The most obvious example occurs in the superimposition technique for nozzle suppressors that is used in the Reichardt method itself. Consider the suppressor element shown in the sketch below surrounded in part by induced flow resulting from the mixing action of the jet:

![Sketch](image)

**Sketch (a). - Suppressor Flow Approximation.**

In the approximation technique used with the present theory (to be explained in subsequent sections), the induced flow is conveniently broken up into arbitrary fictitious jet flows, each having a unique velocity and other properties. The relative velocity results suggest that all three flows \( V_j, V_1, V_2 \) spread at different rates as they contact one another.
In the multitube type of suppressor, a similar situation arises as the mixing regions of each of the individual tubes interact. Since the interaction is not uniform, the convective velocity and hence the rate of mixing vary in all dimensions.

Another example is a multi-element suppressor having both primary jet flow and a sound suppressing low-energy secondary flow, all embedded in a finite external flow field. Such cases as these cause difficulty in the Reichardt method which in all other respects has proven to be extremely successful in dealing with the complicated flow fields found in suppressor nozzle configurations.

In his study of jet flows, Reichardt found that the simplified constant pressure axial momentum equation for turbulent flow

$$\frac{\partial \bar{P}u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{P} u v) = 0 \tag{3a}$$

could be satisfied by an elemental exponential equation (as ascertained from analyzing experimental data of conical free jet mixing).

$$\bar{P} u^2 \sim \frac{1}{b_m^2} e^{-r^2/b_m^2} \tag{3b}$$

if certain arbitrary conditions were satisfied, specifically,

$$\bar{P} u = \frac{b_m}{2} \frac{db_m}{dx} \frac{\partial \bar{P} u^2}{\partial r} \tag{3c}$$

In this elemental relation, there appears the variable $b_m$ upon which the rate of spread of the mixing region depends. It can be shown that the differential
equation for axial momentum (Eq. (3a)) is satisfied by Equation (3b) if, and only if, the parameter \( b \) is solely dependent upon the \( x \)-coordinate. Here the variable \( b \) is the same as that appearing in Equations (1a), (1b), and (2). That the variation of \( b \) in the other coordinate planes does lead to fallacious results can be readily shown as illustrated below:

Sketch (b).—Y dependent \( b \) conflict.

Consider the intermixing of two, two-dimensional jets, each spreading at a different rate (Sketch (b)). According to the superposition theory, which results from the linearization of the differential Equation (3a), the momentum in the mixing region, shown cross-hatched, is given by the sum of the momentum of the individual jets (which spread according to Equation (3b). In the potential core of the jet undergoing the least spread, there is found momentum flux from the other jet which is spreading faster. This region is shown by the double cross-hatching in Sketch (b). Thus it is found in this particular region that the momentum flux is the sum of the momentum flux of the potential core of
the first jet plus the momentum flux spread from the second jet - and this sum is greater than the momentum flux from the first jet alone. Such a result yields velocities in the potential core of the first jet (i.e., the jet that is spreading the least) which are greater than those found at the exit plane, a result contrary to physical reality. This discrepancy represents a limiting condition of prediction for jets having significant core interference.

The method used is an implicit solution for the $x$-coordinate using the spreading parameter $b_m$ as a dummy-dependent variable as suggested in the original work by Kendall in Reference 1. In this technique, the properties of the jet are determined for various $b_m$ planes, and the $x$-coordinate, which is now spatially dependent in the $b_m$ planes, is determined using Equation (2) above. In the $b_m$ planes, the momentum and energy equations are uniquely satisfied, and the contradiction demonstrated in Sketch (b) (which cannot occur in the $b_m$ planes) is necessarily avoided but the continuity of the $x$-plane is not guaranteed. Even if the $x$-plane is continuous, the satisfaction of momentum and energy conservation principles in a plane of constant $x$ is not necessarily guaranteed.

This particular problem area is a subtle one and perhaps not too important to the general class of free jet flows; and yet it is of utmost importance to the successful analysis of the acoustic suppression phenomena found in suppressor nozzles.

A closely related subject to the foregoing is the relative rate of spread of momentum and energy. Experiments of previous years led to the well-known result that energy spreads faster than momentum in free jet turbulent
mixing. The turbulent Prandtl number which relates the momentum transport to energy transport is commonly given the value of 0.7 to describe this phenomenon. The rate of spread of energy can be analytically specified in a manner quite analogous to the spread of momentum. Reference 15 shows that the Reichardt technique linearizes the energy and concentration equations in the same manner as the momentum equation. The analysis of concentration spreading, in this study is limited to concentration spreading at the same rate as energy (i.e., turbulent Lewis number of unity). There is a spreading parameter $b_h$ associated with the energy spreading as there is a $b_m$ in momentum spreading. In this study, the analysis was extended to relate $b_h$ to $b_m$ through Prandtl number. This was accomplished using the definition of Prandtl number and the equations for shear stress, etc., that are derived from the Reichardt method. Several approximations regarding shear stress derivatives must be made to achieve the stated result which has a simplicity in keeping with the state-of-the-art in spreading property knowledge. The approximate derivation is presented in Section 2.6.1 of this report, and the result is shown to be that the spreading parameters are related by the square root of Prandtl number:

$$\frac{b_m}{b_h} \propto \sqrt{Pr}$$  \hspace{1cm} (4)

This result has been programmed into the computer solutions contained in the report.

The solution for the properties of the jet flow downstream of the exit plane has been extended in generality from what was developed in Reference 1. The basic technique involved is the summation of momentum, energy,
and concentration flux using the same integral form as that developed in Reference 1. Only the general method of development will be presented here as the details of the derivation are presented in Section 2.6.2.

Using the superposition technique, the momentum, energy, and concentration flux at any point in space downstream of a jet exit contour(s) can be written as integrals having the form shown below in Equations (5), (6), and (7):

\[
M_x = \frac{1}{2\pi} \sum \left( \bar{f} u^2 \right) \int_{0}^{\theta} \left( 1 - e^{-r^2/b_m^2} \right) d\theta
\]

\[
H = \frac{1}{2\pi} \sum \left( \bar{f} u_x \right) \int_{0}^{\theta} \left( 1 - e^{-r^2/b_h^2} \right) d\theta
\]

\[
K_1 = \frac{1}{2\pi} \sum \left( \bar{f} u_x \right) \int_{0}^{\theta} \left( 1 - e^{-r^2/b_h^2} \right) d\theta
\]

By means of the energy equation, the velocity can be expressed in terms of these quantities as shown in Section 2.6.2. The resulting expression for velocity is a quadratic in which \( \approx \) the positive root has significance. The equation is given below.

\[
\frac{u}{c} = \frac{C_p}{2} \left( \frac{H^2}{4\xi^2} + \frac{M_x c_p T_0}{\xi} \right)^{1/2}
\]

where

\[
\xi = \frac{M_x}{2} + \frac{c_p T_0}{R}
\]
Once the velocity is known, the other properties of the jet such as temperature, density, enthalpy, etc., can be immediately obtained. The solution for the velocity and other jet properties has been generalized to the point of handling a number of different gases, each having its own molecular weight and specific heat relationship. This means that the solution is applicable to the study of hot turbojet exhausts in the analysis of sound suppressors. The only restriction that is made on the thermodynamics of the solution is that the jet flows at the exit be chemically frozen.

In Reference 1, it was found necessary to include two-dimensional effects in the equation for shear stress in order to obtain analytical results comparable with experimentation. At that time, the amended shear-stress equations were not included in the computer solution. This condition has been rectified and the equation for shear stress, in the course of this inclusion, has been refined. The exact derivation of the shear-stress equations stated below is given in Section 2.6.3. For the radial shear:

\[ \tau_r = \sum_{\text{contours}} \left( \frac{\rho u^2}{\tau} \right) \frac{db}{dx} \int \cos \theta \left[ \sqrt{\pi} \text{erf} \left( \frac{r}{b_m} \right) - \frac{1}{2} \text{erf} \left( \frac{r}{b_m \sqrt{2}} \right) - \frac{r}{b_m} \text{erf} \left( \frac{r}{b_m} \right) \right] d\theta \]  

(9)

and a similar relation for the circumferential shear stress with \( \sin \theta \) replacing \( \cos \theta \). The total shear at any flow-field point is given simply by the square root of the sum of the two component shears squared. A brief discussion of the results and comparison with the test data is given in Section 2.3. It is pertinent to note here that the computer programs can be used as research tools.
in this area as well as in the spreading coefficient problem previously mentioned. Here the effect of density on shear stress can be analyzed and the correctness of the present formulation for shear stress determined.

2.2.2 Analysis of Jet Flow Acoustical Properties

In this section, extensions to the acoustic theory presented in Reference 1 are considered. The refinement of the formulation for the generation of aerodynamic sound power proceeds from the point in Reference 1 where equation (A.36) is presented. The more involved derivations are presented in Section 2.6 of this report.

In the present work, more dependence is placed on the Reynolds shear stress parameter. This is a favorable development in that a suppressor is likely to have a bigger effect on the variation of shear stress than on any other aerodynamic parameter in the jet flow. The development is given in Section 2.6.4 where the result is shown to be

\[
\frac{d P_a}{d \ln f} = \frac{c_a U_c \alpha}{\left[ \frac{d (\ln U_c)}{d (\ln x)} - 1 \right] \rho_0 a_0^5} \int_A \left( \tau^7 \rho^{-3} \right)^{1/2} dA
\]

The analyses of this report show that the maximum power is generated at the terminus of the potential core in agreement with results of the majority of experimental researchers in this field.

The largest deficiency of analytical techniques for the prediction of jet acoustic power is the inability to accurately predict the reduction in acoustic power which is experimentally found with the multiple-exit or
contoured exit nozzles. This acoustic attenuation has been found to depend on both geometric and aerodynamic properties of the jet flow. Tests have shown that sound reduction can be produced even with simple jets by placing two simple exits, such as rectangles, in proximity to each other.

As a backup method, in case the refined aerodynamic techniques described above do not suffice to give the proper attenuation, an attenuation mechanism and an analytical method have been established in this study. The most obvious mechanism is the well-established attenuation of sound energy through viscous dissipation and is the one used here. In the case of a jet flow, the attenuation is presumed to occur in the mixing region where the apparent turbulent viscosity is the greatest. Such a mechanism has intuitively satisfying characteristics which explain several of the experimentally observed effects. These are:

(a) Sound suppressors which seem to work best are those having inner jet flows (e.g., the multi-tube suppressor). By the above notion, the sound energy produced by the inner jets is absorbed by the mixing regions surrounding these jets.

(b) As the jet velocity increases, more of the acoustic energy is passed through the mixing region by refraction. This plus the greater turbulent viscosity produces increased attenuation with increasing Mach number.

(c) Attenuation of sound energy produced by suppressors is known to be frequency dependent. The formulation that is
developed in this study is found to have frequency entering to the second power.

However, the theory of the viscous damping mechanism suffers in two areas. The first and most important is that the phenomenon is not in agreement with experimental evidence. Secondly, the mechanism does not lend itself to mathematical formulation since an exact analysis would detail the history of the wave front from each equivalent source in three-dimensional space. Because of the first deficiency, such an exact solution is not justified.

The development of the attenuation phenomenon presented herein is based on the Kirchhoff equation for plane waves (Ref. 6). In the Kirchhoff development used in this report, the viscous coefficient is replaced by the eddy viscosity, and the eddy Prandtl number is substituted into the equation. Further, a measure of the length over which the attenuation takes place is estimated using the mixing region width parameter, b_m. To account for the fact that more of the attenuation would seem to occur in the innermost portions of the jet (a logical consequence of the theory), a porosity factor is applied to the attenuation; this porosity factor relates the attenuation of each source in the acoustic integration to its location within the jet flow purely on a geometric basis. The developments of this theory and the equations below are given in Section 2.6.4. These equations are

(a) The attenuation formula is:

\[ \zeta = e^{-\alpha_x} \]

(11a)
where

\[ \frac{dx}{\gamma_2 x^2} = \frac{(b + \gamma - 1)}{3 \Pr} \]  \hspace{1cm} (11b)

(b) The mean length:

\[ X_e = C \sum_{k} E \frac{x}{\pi} \]  \hspace{1cm} (12)

(c) The porosity:

\[ \eta_f = \frac{R' A_n}{R_c A_c} \]  \hspace{1cm} (13)

The above set of Equations, (10) to (13), plus Equation (2), contain five arbitrary constants, the determination of which is discussed in Section 2.3.

Some preliminary results obtained using these formulas in the circular exit solution program are given in Section 2.3 of this report.

2.2.3 Computer Program

The equations of the foregoing section, along with many subordinate equations, have been written into two separate programs in the process of the development of the solution. The need for two programs has a historic background which can be briefly outlined at this point. The formulation of the equations was performed early in the study contract and it was at that time obvious that the determination of the five arbitrary coefficients involved in the solution could be obtained only through a trial-and-error procedure using a computer solution. Rather than modify the original program at this point, it was decided to create a small program which would fulfill the following three goals:
(a) Provide an economical means for obtaining the values for the five arbitrary coefficients.

(b) Provide means for ascertaining the correctness of the relative velocity effect on sound attenuation.

(c) Provide program logic for insertion into the main program during its revision.

It became obvious, however, as the programming of the "simple program" commenced that the problem had become complicated to the point where even a simple circular exit solution required sophisticated and extensive programming, especially if it was to be used in the revised main program. When it became obvious, moreover, that the simple solution was too large for the 1620 computer, several of the first goals were compromised and reliance, instead, was placed upon the use of the final major program. The circular exit program, then, as it has turned out, has had the function of performing (a) and part of (b) noted above. It has yielded preliminary values for the five arbitrary coefficients, and major portions of the program are being used in the main program itself.

The simple circular exit program cannot be used for multiple-exit configurations, that is, double annuli, but it can be used to evaluate the acoustic attenuation observed when a secondary flow of large expanse (compared to that of the jet itself) surrounds the circular jet. This encompasses by far the majority of the experimental data available on jet flows including those data available on mixing and the spread of mixing regions in jets surrounded by finite external flow fields.
In the following sections, the philosophy of the two programs, some of the programming methods used, and the broad physical features of the programs will be discussed. In Section 2.6.5, a more detailed description of the programs is given.

Preliminary Computer Program

The solution technique of the simple circular-exit program is, in general, parallel to that of the main program, and for this reason it will be discussed only briefly. In the simple circular-exit program, the inverse procedure is used wherein the solution is first performed for various $b_m$ planes and then $x$ is explicitly calculated using Equation (2). This technique offers the advantage that the flux integrations (which are $b_m$ dependent) need be performed but once for any exit configuration (in this case the exit radius); then the results of these integrations are used for all subsequent running of that exit configuration. Such a technique offers a factor of four saving in computer time compared to doing the integrations each time the program is run. In the main program a similar technique is used. The velocity is obtained through an iteration procedure as it is in the main program, and the sound power calculation is performed explicitly after the $x$'s (and therefore frequency) have been determined.

Main Computer Program

In this section is presented the philosophy underlying the computer programming of this project and an outline description of the program. This program combines the geometric generality of the previous computer program reported in Reference 1 with aerodynamic generality and sophistication established in this study. The aerodynamic generality stems from the fact that the
program can handle any number of gases having different compositions and heat capacities. At present, its only thermodynamic limitation is that the gas flows at the nozzle exit must be chemically frozen. Except for the "moderate compressibility limitation" on the development of the shear-stress equation, and the inherent thermodynamic limitations placed on the differential equation solution (i.e., the original simplified differential equation due to Reichardt), the program makes no restrictions as to the density, temperature, and the Mach number of the cases it can handle. Specifically, the program as it now stands should be capable of providing reasonably accurate solutions to actual turbojet suppressor problems. The only aerodynamic restriction that is placed on the solution is that the overpressure ratio of the jet at the exit plane be nearly unity. Actually, the program can be used in certain circumstances for the overpressure case and for providing approximate solutions to the ejector flow problem through clever input to the program. These techniques are discussed in Section 2.4.

The biggest stumbling block to the program development has been, as mentioned previously, the technique for the relative velocity effect. Another problem has been the development of a method for dividing the flow field so that the integrations in the sound-power calculation will have good accuracy. The program now integrates 200 points (approximately) in a typical quadrant of symmetry. For most suppressor configurations, this fineness is not sufficient to provide accurate sound-power integrations for frequencies higher than those found at the terminus of the elemental potential cores. For example, a typical multi-tube suppressor would require about an order-of-magnitude increase in the number of mesh points to obtain accurate acoustic integrations at the higher
frequencies. Such an increase is impossible to attain in the present program because of the memory storage limitations. It is thought, however, that the 200-point solution will provide sufficient accuracy to at least determine the maximum power output of these sound suppressors.

The program calculates flow properties and acoustic properties for a number of b planes, the number depending upon the length ratio and the acoustic bandwidth specified in the input. For example, if data are desired from the exit plane to a point downstream corresponding to \( x/R_{c,e} \) of about 30 and the acoustic spectrum is to be in one-third octave band widths, then about 19 b planes (exponentially spaced) will be used. This technique will assure a high probability of calculating an acoustic point for each acoustic band.

The program makes a "pass" through the 200 points of each b plane three times. On the first pass, it calculates the flow-field velocity and other properties such as density, temperature, etc. It places the velocity values into memory storage, and then puts it and other property values on the output tape. On the second pass (currently bypassed) the program interrogates the magnitude of the velocity of neighboring mesh points (surrounding the point in question) to determine whether or not to subdivide the existing mesh into much finer increments to provide a more accurate solution in a way which avoids the machine memory-capacity problem. The main mesh can be divided in one of three ways: a horizontal division by eight, a vertical division by eight, or a combined horizontal and vertical division by eight to provide 64 points within the existing mesh. The size of this inner mesh is increased as the program proceeds to calculate b planes further downstream. For example, at an \( x/R_{c,e} \) of approximately eight, the program divides the major meshes into 16 points.
instead of 64, the general idea being, of course, that the mixing regions at this point are so large that 16 points provide as much accuracy as 64 points provide near the exit plane. There is also a practical reason for this decrease in mesh size. As the program proceeds to $b_m$ planes located further downstream, the number of meshes increases and would soon exceed the capacity of the 7094 memory. By increasing the mesh size, the mesh solutions can actually be nested in the machine logic. Figure 2.3 shows a $b_m$ plane and some typical major mesh points having inner mesh divisions at several physical locations in the $b_m$ plane, and shows how these inner meshes are nested to conserve memory storage.

On the third pass, the program interrogates the velocity points, which are stored in memory, within "the cone of influence" from the point in question. This cone of influence has a physical size approximately equal to twice the size of the typical mixing region at that point in space. Figure 2.4 shows the cone of influence interrogation on a figurative basis. The program determines the relative maximum and minimum of the velocity points in the $b_m$ plane within the cone base circle and uses these relative maximum and minimum in a calculation of spreading properties of the jet flow and the acoustic properties. From these properties, a unique value of $x$ and frequency is obtained for each of the major mesh points.

At each point, the sound power which is calculated is assigned a frequency index which is related to the frequency calculated at that point. This sound-power calculation (which at this time is in terms of energy) is added to all other energy calculations in the $b_m$ plane in question having like frequency indices. These frequency indices have a bandwidth tolerance of one-tenth octave. At the termination of a $b_m$ plane, the sound-power calculations
are ordered in decreasing levels of sound power and those sound-power sums having frequency indices within one-third of an octave of each other are added and the frequency of this sum is normalized. Figure 2.5 presents a basic flow chart for the program, outlining the discussion just presented. It in no way represents the entire logic of the program which is far more extensive. This flow chart is keyed to the program listing in the more complete discussion of the program in Section 2.6.5.

2.3 Discussion of Results

This section will discuss the results that have been achieved in this study program. Included in this discussion are the evaluation of the coefficients, comparison of theoretical velocity profiles with the data, comparison of theoretical shear stresses with data, and comparison of the theoretical sound-power production with data.

Evaluation of Constants

The analytical theory developed in this study involves five arbitrary constants which must be determined from the data existing in the literature. Two of these coefficients are in the equation which relates the rate of spread of the mixing region to the axial coordinate and the convective Mach number (Eq. (3)) and one of each are in the equations for sound-power generation, attenuation, and frequency. In the evaluation that has been conducted thus far, the constants have been found to be coupled so they cannot be determined independently, thus increasing the difficulty of obtaining values for the constants.
The constants for the rate of spread equation have been determined largely from the work of Lawrence (Ref. 7) following the work originally done in this area (Ref. 1). Kendall based the value of the coefficients in Equation (1) upon the spreading of the flux on the jet centerline where the cylindrical solution is completely analytic. In the present study, the velocity profiles for all space have been compared with the data of Lawrence and the best fit of the velocity profiles has yielded the coefficient value. It has been found that the coefficients are only slightly different from the values presented in Ref. (1). These values are:

\[ C_{b1} = 13.5 \]  \hspace{1cm} (14a)

\[ C_{b2} = 1.5 \]  \hspace{1cm} (14b)

The above constant values produced reasonably good predictions of the velocity profile data at all points in the flow field for the two cases that were run (refer to Section 2.3). The acoustic prediction at Mach 0.7 is extremely good concurrently. However, the acoustic prediction at Mach 1.05 was found to be 2db lower than the data (Fig. 2.6). This result is directly attributable to the reduction in mixing predicted by Equation (3) for the higher jet Mach number. With \( C_{b2} \) set equal to zero, the prediction of jet noise is seen to be much better.

The work done by researchers in the field of acoustics has generally supported the contention that the frequency of the sound produced at a given \( x \) location is proportional to the eddy velocity. On this basis, the value of the coefficient would be expected to have a value of 2.0. Using the
sound-power profiles such as Figure 4-11 in Reference 1 and the results of the circular solution, the value of the coefficient has been found to be

\[ C_\tau \approx 4.5 \]  

(15)

These same data have yielded values for the sound-power coefficient and the attenuation coefficient of

\[ C_a = 1410 \]  

(16)

\[ C_x = 250 \]  

(17)

If the analysis were precisely correct, these values for the coefficients would apply for all manner of configurations and conditions. However, since the analysis is at best approximate, it may be found in the course of running other configurations that minor changes in the value of these coefficients will yield better overall results.

Velocity Profiles

Figures 2.7(a) through 2.7(e) present the results of the theoretically predicted velocity profiles determined with the theory presented herein as compared with the data of Reference 7. It is seen that the theory indicates less spread of momentum flux at both high and low values of the axial distance. Since the theory conserves momentum, the shape of the velocity profile suggests that the data are in error since the data definitely do not satisfy the conservation of momentum condition at axial distances near the exit.
Shear Stress

The comparison of theoretical and experimental determination of the shear stress found in simple flows is made in Figure 2.8. It is seen that the theoretical prediction is in reasonable agreement with the data (in keeping with the prediction of Ref. 1).

Sound-Power Prediction

Figure 2.6 presents the acoustic predictions obtained with the theory and the values of the constants given previously. At Mach 0.7, the prediction is by all standards excellent. The premature dropoff in sound power at the low frequencies found previously has been eliminated. At Mach 1.0, the results have not been quite as satisfactory for reasons suggested before. Note that the difference between theory and data is quite uniform and that here again the theory seems to be predicting the correct profile and frequency shift.

2.4 Additional Approximations for Ejector, Overpressure, and Induced Flow

Ejector Approximation

Because of the problem encountered in the mixing rate difficulty discussed in Section 2.1, the extension of the analysis to encompass ejector (i.e., shrouded nozzle configurations) was not performed in this study. However, sufficient work was done to establish an approximate procedure that can be used with the existing program to determine the flow-handling characteristics of ejector-type sound suppressors. The method is approximate because the theory behind it assumes constant pressure mixing and because it uses a fictitious geometry surrounding the primary flow. This pseudo-geometry is shown in Figure 2.9, wherein a hexagonal outer nozzle surrounds the primary nozzle, the
hexagonal configuration replacing the actual circular ejector, (such a configuration is of little use in those sound suppressors utilizing an ejector surrounding a complicated exit configuration such as the petal suppressor shown in Sketch (a). The hexagonal configuration is necessary to deceive the program into generating reflected velocity profiles at the hexagonal boundaries which then approximate a shearless wall velocity profile. Such a solution for simple ejector geometries should produce qualitative flow-rate ratios for those ejector configurations in which the secondary shroud length is quite short.

**Overpressure Approximation**

The overpressure approximation that can be used is again restricted to simple configurations such as a multitube suppressor and/or to low overpressure ratios. The criterion is the degree of flow interaction and the attendant shock structure strength in the downstream flow field. Such shock structures obviously cannot be handled by the existing program. The approximation that can be made for these simplified cases is to increase the exit area to one corresponding to the exit Mach number based upon one-dimensional expansion of the exhaust gases to the ambient static pressure. The modified area coordinates are then substituted for the physical exit coordinates in the input to the program.

**Induced-Flow Approximation**

A possible cause of the suppression of jet noise found with complicated jet exit configurations is the inducement of a flow in the mixing regions. Such flow inducement theoretically reduces the shear stress and thereby the noise level. To duplicate this effect analytically, it is necessary to input to the computer program a fictitious exit shape, modifying the actual
suppressor exit shape. In the modified shape, estimated velocity values are used to produce the induced flow. Referring to Sketch (a), which is a partial view of a petal-type suppressor exit, the fictitious exit shape that is added is shown by the dashed lines and the approximate flow in this portion is shown by the cross hatching. Such a technique allows the computer to consider the added jet flow as the entrained flow. This technique remains to be verified using the new computer solution of this study.

2.5 Detailed Analyses and Computer Programs

2.5.1 Derivation of Prandtl Relation

The Prandtl number relates the transport of energy to the transport of momentum. By definition, the turbulent Prandtl number is given by the following equation:

$$\text{Pr} = \frac{\tau}{\frac{\partial u}{\partial r}}$$

(15)

The definition of the eddy viscosity and heat-conduction coefficients in this equation are as follows:

$$\eta_e = -\frac{\tau}{\frac{\partial u}{\partial r}}$$

(19)

$$k_e = -\frac{\rho c_p \nu^2}{\frac{\partial u}{\partial r}}$$

(20)

Now as shown in Section 2.5.3, the shear stress is given by

$$\tau = \left(\frac{b_m}{2}\right) \left(\frac{d b_m}{dx}\right) \left(\frac{\partial (\rho u^2)}{\partial r}\right)$$

(21)

Substitution into Equation (19) yields

$$\eta_e = \frac{b_m}{2} \left(\frac{d b_m}{dx}\right) \left(\frac{\partial (\rho u^2)}{\partial r}\right) \frac{\partial u}{\partial r}$$

(22)
To put the expression for $k_e$ in a similar form, it is necessary to make the following approximation:

$$\overline{\rho v_i h_i^0} = \overline{\rho v_i h_i^0}$$

which will hold reasonably well for moderately incompressible mixing. By analogy to the linearization of momentum flux (which leads to (21), one can show that

$$\overline{\rho v_i h_i^0} = -\frac{b_h}{2} \left(\frac{d b_h}{dx}\right) \left[\frac{\partial (\tilde{\rho}/h_i^0)}{\partial r}\right]$$

where the use of the enthalpy spreading parameter $b_h$ can be noted. Substitution into (20) produces the following equation:

$$k_e = c_p \frac{b_h}{2} \left(\frac{d b_h}{dx}\right) \left[\frac{\partial (\tilde{\rho}/h_i^0)}{\partial r}\right]$$

Substituting the relations for the transport of momentum and energy (22) and (25) into (16) yields the following equation:

$$Pr = \frac{b_m \left(\frac{d b_m}{dx}\right) \left[\frac{\partial (\tilde{\rho}/h_i^0)}{\partial r}\right]}{b_h \left(\frac{d b_h}{dx}\right) \left[\frac{\partial (\tilde{\rho}/h_i^0)}{\partial r}\right]}$$

Equation (2) shows that $db_m/dx$ is approximately constant, at least up to the end of the potential core. The presumption made here is that the $b_h$ derivative will behave similarly. If such is the case, the derivative ratio is proportional to the ratio of the spreading parameters. With this approximation and further expanding the derivatives in (26) (neglecting radial gradients of density), the following result obtains:
This expression relates the square of the spreading parameter ratio to the Prandtl number and another term containing derivatives of velocity and enthalpy. This latter term can be written in terms of logarithmic derivatives.

\[ \left( \frac{b_n}{b_m} \right)^2 = \frac{1}{Pr} \left[ \frac{2}{1 + \frac{\partial (\ln \bar{u})}{\partial (\ln \bar{h})}} \right] \]  

The approximation which is made at this juncture is that the logarithmic derivative ratio is approximately unity. This approximation leads to the final result that the spreading coefficients are related to the square root of the Prandtl number.

\[ \frac{b_m}{b_n} \approx \sqrt{Pr} \]  

The validity of the above approximation would seem to be most open to question under isoenergetic mixing conditions where theoretically there is no gradient in stagnation enthalpy.

2.5.2 Derivation of Velocity and Related Expressions

At any point in the jet flow field the flux of momentum, energy, and gas concentration are known as a consequence of the theory used in this study (see Eqs. (5), (6), and (7)). These three flux quantities will be termed \( M_x \), \( H \), and \( K \) in the subsequent development. It can be shown that the mean of the product of the unsteady quantities in these flux functions, namely,
can be well approximated by the product of the means. The error is generally less than 10 percent. Reference 15 presents an exact treatment of this problem and it will not be considered further here. Also, the concentration expression commonly used for the mixing of two dissimilar gases is extended in this analysis to an indefinite number of gases without the benefit of a formal proof. Thus

\[ k_o = 1 - \sum k_i \]

where \( k_o \) is the total concentration of ambient gases at a point and the sum represents the calculated total concentration of jet gases.

From the energy equation, using the Crocco relative enthalpy for boundary condition satisfaction, one obtains

\[ h^0_r = h - h^0_o + \frac{u^2}{2} \]

Now using the concentration expression given above, the local specific heat for an arbitrary gas mixture is given by

\[ c_p = c_p_i \cdot k_i + \left( 1 - \sum k_i \right) \cdot c_p_o \]
As will be seen, it is convenient to define a mean specific heat in the derivation of the velocity expression such as

$$\overline{C_p} = \int_{T_1}^{T_2} C_p \, dt / (T_2 - T_1) \tag{36}$$

If the specific heat is related to temperature through a series expansion of the type

$$C_p = \lambda_1 + \lambda_2 T_2 \tag{37}$$

then by (35) there results

$$C_p = k_1 (\lambda_{i,1} + \lambda_{i,2} T) + (1 - \sum k_i) C_{p0} \tag{38}$$

and substitution into Equation (36) yields the following equation for the mean specific heat:

$$\overline{C_p} = \frac{k_1 \lambda_{i,1} (T_2 - T_1) + k_1 \lambda_{i,2} (T_2^2 - T_1^2) / 2}{T_2 - T_1}$$

$$+ \left[ 1 - \sum k_i \frac{\lambda_{0,1} (T_2 - T_1) + \lambda_{0,2} (T_2^2 - T_1^2) / 2}{T_2 - T_1} \right] \tag{39}$$

Taking the base enthalpy point at absolute zero reduces the above equation to the one used in this study

$$\overline{C_p} = \frac{k_1 \lambda_{i,1} + k_1 \lambda_{i,2} T}{2} + (1 - \sum k_i) \left( \frac{\lambda_{0,1} + \lambda_{0,2} T}{2} \right) \tag{40}$$
Substitution of this mean specific heat into the energy equation produces

\[
\rho \bar{u} \frac{\vec{v}}{x} = \rho \bar{v} \left( \frac{c_p m T - c_p o T^0 + \bar{u}^2}{2} \right) = H
\]

(41)

The following algebraic manipulations and substitutions can be performed to produce a quadratic expression in the desired velocity variable:

\[
\rho = \frac{H - \bar{u} c_p m P/\text{Rg}(k)}{u^{3/2} - \bar{u} c_p o T^0}
\]

(42)

\[
\rho \bar{u}^2 = u^2 \left( \frac{H - \bar{u} c_p m P/\text{Rg}}{u^{3/2} - \bar{u} c_p o T^0} \right) = M_T
\]

(43)

\[
\bar{u}^2 \left( H - \frac{u c_p m P}{\text{Rg}} \right) = M_T \left( \frac{u^2}{2} - \bar{u} c_p o T^0 \right)
\]

(44)

\[
\left( \frac{M_T + \frac{c_f m}{\text{Rg}}} \right) \frac{\bar{u}^2}{2} - \bar{u} M_T c_p o T^0 = 0
\]

(45)

In this quadratic, only positive roots have physical significance and the final equation for the velocity is

\[
\bar{u} = \frac{H}{\bar{u}} + \left( \frac{\bar{u}^2 + M_T c_p o T^0}{\bar{u} \xi^2} \right)^{1/2}
\]

(46)

where:

\[
\xi = \frac{M_T + \frac{c_f m}{\text{Rg}}} \]

(47)

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For the general case involving gas mixtures of different molecular weights and specific heat coefficients, the solution is iterative around the variables, \( c_p \) and \( R_g \). This is done through the solution for concentration

\[
k_1 = \frac{k_1}{P_u}
\]

(48)

the gas constant

\[
R_g = R_g^0 k_1 + (1 - \sum k_1) R_g^0
\]

(49)

and Equation (40) for the mean specific heat, \( c_p \). When the molecular weight and specific heat are constant and equal for all the gases entering into the problem, then the solution does not involve iteration. In running the programs, the convergence of the velocity solution has been rapid.

2.5.3 Shear-Stress Derivation

The derivation has as its basis, the representation of shear stress by the well-known equation for Reynolds stress

\[
\tau_{ij} = P \overline{u_i'v_j'}
\]

(50)

Now it can readily be shown that the mean of the fluctuating quantities in (1) can be replaced with differences of mean quantities. Thus,

\[
\overline{u_i'v_j'} = \overline{uv} - \overline{u} \overline{v}
\]

(51)

If the presumption of moderate incompressibility made in Reference 1 is made again here, then the results of (51) substituted into (50) yields the fundamental...
relation used for the shear stress in the derivation

\[ \tau_{ij} = \tau_{ij} = F_{uv} - \overline{P} \overline{u} \overline{v} \quad (52) \]

Reichardt found a relationship between the first term of (52) and the radial gradient of momentum flux in his mixing solution. For an elemental jet, this relation is

\[ \overline{F}_{uv} = - \frac{b_m}{2} \left( \frac{db_m}{dx} \right) \frac{\partial (\overline{P} u^2)}{\partial r} \quad (53) \]

The momentum flux at any point in space for an elemental jet from the Reichardt solution is given by

\[ \overline{P} u^2 = \left( \overline{P} u^2 \right)_e A_e e^{-x^2/b_m^2} \overline{b_m}^2 \quad (54) \]

and the derivative of (54) with respect to \( r \) yields the following relation:

\[ \frac{\partial (\overline{P} u^2)}{\partial r} = -2 \left( \overline{P} u^2 \right)_e A_e r e^{-x^2/b_m^2} \overline{b_m}^4 \quad (55) \]

Substitution of this result into (3) gives a relation for the first term in quantities which can be evaluated through a computer integration.

The second term is reduced by applying the continuity equation to the flow field of the elemental jet. In polar coordinates, this expression is

\[ \frac{P}{r} \frac{\partial (\overline{P} \overline{v})}{\partial r} + \frac{\partial P \overline{u}}{\partial x} = 0 \quad (56) \]
At a given x-plane, the gradient of mass flux with respect to x can be considered to be a function of r only. Thus, the above differential equation becomes a simple first-order differential equation in one unknown, having as its solution:

$$\rho \bar{v} = \frac{1}{r} \int_{0}^{r} \frac{\partial (\rho \bar{u})}{\partial x} r' dr'$$  \hspace{1cm} (57)

where the constant of integration is zero. Multiplication through by $\bar{u}$ leads to the result:

$$\rho \bar{v} \bar{u} = \frac{\bar{u}}{r} \int_{0}^{r} \left. \frac{\partial (\rho \bar{u})}{\partial x} \right|_{r'} r' dr'$$  \hspace{1cm} (58)

Using again the elemental jet momentum flux relation (54) and taking the differentiation outside of the integration, the following sequence of equations can be written:

$$\rho \bar{u} \bar{v} = \frac{\bar{u}}{r} \left. \frac{\partial \rho \bar{u}^2}{\partial x} \right|_{r'} r' dr' \left( \int_{0}^{r} \rho \bar{u}^2 \frac{\partial \bar{u}}{\bar{u}} r' dr' \right)$$  \hspace{1cm} (59)

$$\rho \bar{u} \bar{v} = \frac{\bar{u}}{r} \left. \frac{(P \bar{u}^2)}{\bar{u}} \right|_{r'} \frac{\partial}{\partial x} \left( \int_{0}^{r} e^{-r'^2/b_m^2} \frac{1}{\bar{u}} r' dr' \right)$$  \hspace{1cm} (60)

Introducing (54) (with the mean of the product assumed equal to the product of the means) $\bar{u}$ can be eliminated, namely,
Presuming that the x-directed gradient of density is nearly zero justifies the insertion of the density square root into the derivative. The density expression may be removed altogether if it is presumed that the local density is approximately equal to the mean density over the integration. Such simplification reduces Equation (61) to the following (after integration):

\[

\rho \overline{u'v'} = \frac{e^{-r^2/2b^2} (\rho u^2)}{\sqrt{\rho}} \frac{A_e}{b_m r} \left( \frac{1}{b_m} \int_0^r \frac{e^{-r'^2/2b^2_m}}{\sqrt{\rho}} \frac{d r'}{r'} \right) \left( \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} \right) \left[ \frac{b_m}{r} \left( 1 - e^{-r^2/2b^2_m} \right) \right] \right)

(62)

Performing the indicated partial differentiation and combining terms leads to the following final expression for the second term of the shear-stress equation for an elemental jet in terms of quantities which can be evaluated:

\[

\overline{u'v'} = \frac{e^{-r^2/2b^2} (\rho u^2)}{\sqrt{\rho}} \frac{A_e}{b_m r} \left[ e^{-r^2/2b^2} - \left( \frac{r^2}{b^2_m} + 1 \right) e^{-r^2/b^2_m} \right]

(63)

Addition of the two terms produces this result for the elemental jet shear stress:

\[

\tau = \frac{e^{-r^2/b^2_m} (\rho u^2) A_e}{b_m \frac{d}{dx}} \left( 1 + \frac{r^2}{b^2_m} - e^{-r^2/2b^2_m} \right)

(64)

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The shear stress at a point in the flow field of a finite jet is obtained by the linear superposition of the solutions obtained for the elemental jet. (The exit is decomposed into a matrix of elemental jets.) In the computer program, the superposition is replaced by an integration. The radial and tangential components of the shear stress are obtained by multiplying the integral relation by the cosine or sine of the angle between lines connecting the point in question to the origin and to the elemental jets being integrated. (See Fig. 2.10).

Again utilizing the superimposable nature of the equations, integration over the exit plane produces the total shear in the two coordinate directions (s and $\phi$):

$$
\tau_s = \int_A \frac{(\rho u^2) e}{\eta b} \frac{d b}{dx} e^{-r^2/b_m^2} \cos \theta \left[ \frac{r^2}{b_m^2} - e^{r^2/2b_m^2} + \left( \frac{r^2}{b_m^2} + 1 \right) \right] dA_e \tag{65}
$$

Integrating in polar coordinates ($dA = r' dr' d\theta$) with respect to $r'$ yields

$$
\tau_s = \sum_{\text{contours}} \frac{(\rho u^2) e}{\eta r} \frac{d b}{dx} \int \cos \theta \left[ \sqrt{\pi} \text{erf} \left( \frac{r}{b_m} \right) \right.
- \left. \sqrt{\frac{\pi}{2}} \text{erf} \left( \frac{r}{b_m \sqrt{2}} \right) - \frac{r}{b_m} e^{-(r/b_m)^2} \right] d\theta \tag{66}
$$

and a similar expression for $\phi$ with $\sin \theta$ replacing $\cos \theta$. 

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2.5.4

**Sound-Power Derivation**

In Reference 1, the following equation was derived from the theory of Lighthill:

\[ P_a(x) \sim \left( \frac{\Delta V}{\kappa} \right)^4 f_e^4 \left( \int \frac{\tau^2}{\kappa} \, dV \right) \frac{1}{\rho_0 a_0^5} \]  \hspace{1cm} (67)

This equation relates the sound power produced within a volume, \( \Delta V \), about a point in the flow field to the eddy oscillation frequency and the shear stress generating the eddy. The volume at the point is, as in Reference 1, assumed to have as its characteristic dimension, the mean size of the eddy, \( \sigma \). Utilizing the relation below (again from Ref. 1) yields the following equation for the volume in question. The frequency equation derived in Reference 1 is also used here.

\[ \overline{\Delta V} \sim \sigma^3 \]  \hspace{1cm} (68)

\[ f_e \sim \frac{1}{\sigma} \left( \frac{x}{\rho} \right)^{1/2} \]  \hspace{1cm} (69)

The product of these two quantities is then:

\[ \overline{\Delta V} f_e^4 \sim \frac{U_c}{x} \left( \frac{x}{\rho} \right)^{3/2} \]  \hspace{1cm} (70)

By placing the shear stress and density variables into the integrals of (67) and simplifying, there results

\[ P_a(x) \sim \frac{U_c}{\sigma^5} \left( \frac{\tau^7 x^{-3}}{\kappa} \right)^{1/2} \int_V \, dV \]  \hspace{1cm} (71)
This volume integral can be replaced with an area integral by considering the sound power to be produced in unit length:

\[
\frac{dP}{d\ln x} \sim \frac{U_c}{\rho_0 a_0^5} \int_{A} \left(\tau^7 e^{-3}\right)^{1/2} dA
\] (72)

From the frequency relation

\[
f \sim \frac{U_c}{x}
\] (73)

the following useful equation can be derived:

\[
\frac{d\ln f}{d\ln x} = \frac{d\ln U_c}{d\ln x} - 1
\] (74)

Substitution of the last relation into Equation (72) yields the sound power per cycle of frequency (or bandwidth). Introducing the attenuation coefficient and the arbitrary constant into the equation produces the equation for sound-power production used in this study

\[
\frac{dP}{d\ln f} = \frac{C_a U_c}{(d\ln U_c - 1)} \frac{\alpha}{\rho_0 a_0^5} \int_{A} \left(\tau^7 \rho^{-3}\right)^{1/2} dA
\] (75)

The acoustic power developed by free jets (sound suppressors in particular) has two characteristics: (1) a Mach number effect and (2) a shielding effect wherein the sound produced by eddies internal to the overall jet structure are apparently masked, shielded, or attenuated in some manner by the surrounding flow field. An exact solution would describe the attenuation, refraction, and convection effects of the sound produced at each point in the
flow field. To avoid the difficulties of performing such a solution, two parameters will be artificially introduced into the standard viscous dissipation formula, which (it is hoped) will provide the desired effects. These are: (1) the jet porosity which simply relates the point to the jet flow field in a geometric fashion and (2) an extent effect which should provide a measure of the time the acoustic energy is under the influence of the viscous dissipation.

Forming an attenuation coefficient

\[ \alpha = e^{-\frac{\alpha}{k}x} \]  

(76)

where \( x_e \) is given by

\[ x_e \sim b_m E_r M_e \]  

(77)

and where \( \alpha_k \) is obtained from the Kirchhoff viscous dissipation equation

\[ \alpha_k = \frac{\gamma_c r^2}{\rho_0 \delta^3} \left( \frac{4}{3} + \frac{\gamma - 1}{Fr} \right) \]  

(78)

This equation is slightly different from the form usually found. Here the eddy viscosity replaces the kinematic viscosity and has been factored out of the expression through the use of the Prandtl relation. The eddy viscosity in Equation (78) is taken to be some mean viscosity over the mixing region. To obtain this effect, the following arbitrary equation has been formed.

\[ \frac{\overline{\gamma_e}}{x_e} \sim \frac{2}{\delta x} \left( \gamma_e \right) \frac{x_e}{2} + \gamma_e \]  

(78)

The local eddy viscosity is given by the standard definition
\[ \eta_e = \tau / (2 \overline{u} \partial x) \]  

(80)

and using the approximate relation for the shear stress neglecting density effects produces

\[ \eta_e = \frac{\rho U_c b_m}{2} \frac{d b_m}{d x} \]  

(81)

It should be noted that the above relation for the eddy viscosity is an approximation for the elemental jet and not for the general flow field which would involve integration of such quantities (as in Section 2.6.2).

In light of the preceding approximations in the attenuation development, such an integration is not warranted.

Finally, the arbitrary equation for the porosity effect that has been tentatively used in this study is given below:

\[ E_f \sim 1 - \frac{R' A_n}{R_c A_e} \]  

(82)

In effect, this equation arbitrarily de-emphasizes the sound energy produced by mixing regions close to the jet axis.

Clearly, the use of such arbitrary expressions reflects the complete lack of experimental knowledge in this area which could direct such analytical efforts and, moreover, the complexity of the situation, which discourages a more detailed and fundamental treatment.
2.5.5 Computer Program Description

This section of the report presents the description of the two programs that were developed in the course of the study contract. Of the two programs, the second or main program will receive the most attention since it is the one having the most significance to the analysis of suppressors and, indeed, is the desired product of the study. These programs are presented in this section and the reader is asked to refer to these sections during the descriptions which follow. As part of the program description typical input and output for the two programs and the manner in which the programs are used will be discussed.

Single Circular Exit Program

The single circular exit program and its two companion subroutines are presented in this section. The first of these subroutines (Subroutine Flux) is used once for a given exit radius and the second subroutine is used thereafter. The exception to this procedure is if different bandwidths in the acoustic spectrum are desired or if a different starting point in the \( b_m \) dummy coordinate is desired. These cases require that the longer, first-mentioned subroutine be used (again for the first run only). The running time for the program using the second subroutine is a factor of four less, so that there is a considerable advantage to using it. A typical running time for the program using the second subroutine is about 1 hour on the 1620 computer (evaluating 19 \( b_m \) planes). The second subroutine (Subroutine Short Run) is used in conjunction with the output from the first subroutine (the output used is a part of the input).
The following brief paragraphs give a functional resume of the major portions of the program.

Preceding the exit-conditions calculation, input, and normal program preliminaries, is the sound-power function which is the main equation for the noise prediction. It is basically the same as Equation (10). In the exit-conditions calculation, the properties at the exit plane such as velocity, temperature, and the fluxes of mass momentum and energy are determined. The flux values are necessary, of course, in determining properties in the jet flow.

The determination of initial conditions is primarily for the various integrations which take place later in the program. Of particular note is the frequency initialization procedure. Here the frequency bands are specified and the sound power in them cleared to zero.

Next follows the determination of the number of radial planes which will be calculated and the specification of limits for the various loops in program logic.

Entering into the radius loop (wherein the points along a radial ray are calculated) the flux calculation loop is first found and it is at this point that either of the two subroutines to the program are called. From the flux calculation, the velocity is determined.

The calculation of the local aerothermal acoustic properties is followed sequentially by the calculation of sound power. The sound-power calculation procedure is more complex than need be for this program since, in the case of a simple circular exit, there is only one mixing region and hence
only one characteristic frequency in each \( b_m \) plane. The procedure shown is used in the main program when more than one characteristic frequency is found.

The radius loop is concluded with the integration of the momentum energy and mass flux in the \( b_m \) plane. These integrations are used primarily to check the program's accuracy since energy and momentum are theoretically conserved. The radius loop is terminated when the velocity difference (jet minus free stream) approaches zero and the program is completed with the calculation of sound power in decibels and the final output.

Because the program is quite simple, the only procedure that might require a little explanation is the manner in which the two subroutines are used. The program also has an idiosyncrasy in the subroutine transfer which also bears some explanation.

Presume that the program has been run once with the first subroutine and it is now desired to run it with the short flux subroutine. It is necessary, then, to take the output cards from the first running and remove all acoustic output data. This is easily accomplished since these data are readily distinguished blocks of output. The modified output should then consist of a sequence of two output data cards followed by a blank card etc., from beginning to end. In this form, it can be considered input for the short-flux subroutine running of the program. Figure 2.11 shows the respective positions of the input as it would be loaded in the course of running the program. Notice that the modified output-input follows the regular input to the program.

A peculiarity may be found in running the short flux program. Since the output does not have the significant figure accuracy carried in the
machine, the termination of the radius loop previously mentioned does not occur for precisely the same radius with the second subroutine because of a minor difference in velocity values. This naturally destroys the correct sequence of operation if the machine overruns on a $b_m$ plane calculation. The machine continually checks the sequence so that such an error automatically halts the machine. To correct this situation (which is found to occur only for the last few $b_m$ plane calculations), it is only necessary to set (artificially) the flux value in the output to zero on the last card of the particular troublesome $b_m$ plane output. (In Figure 2.13 of the typical output, these quantities are circled).

Figure 2.12 shows some typical input to the program, which, for the most part, is largely self-explanatory. The units are in ft-lbs-seconds for all quantities including those for specific heat and the gas constant. As the program now stands, the proper value for the characteristic radius is the same as the radius itself. The output is presented in Table I which again is largely self-explanatory. The program has been modified so that the decibel calculation is performed for as many bandwidths as in the output to the $b_m$ planes (19 in the case shown in Figure 2.13).
Main Program

The Compressible Flow Mixing Profile and Acoustics Program and its subroutines (presented in this Section) is a greatly expanded version of the original computer program developed in the study reported in Reference 1. The major contributor to this expansion has been the logic required by the routine for the determination of the spreading characteristics of the jet flow under relative conditions, and the routines required to avoid exceeding the memory-storage capacity of the machine. Much of the flux integration logic for complicated exit shapes that was developed in the original program has been retained and can be recognized in the major subroutine to the program - Subroutine FLUX. It was originally thought that computational time could be significantly reduced by integrating the flux from simple exits with simplified methods such as used in the Single Circular Exit program. In particular, the intent was to calculate circular and rectangular exit shapes in this fashion. It has turned out though that only the circular exit can be so treated, due to the complexity of integrating the shear equations for the rectangular exit in closed form. Subroutine CFLUX is this routine. It is somewhat different than the subroutine used for the Single Circular Exit program, in that the former recognizes the circular exit as a point source at sufficient removed distances from the exit (which again increases the speed of the program). For this reason, subroutine CFLUX will provide greatly reduced computational times for suppressor exit shapes such as the multiple tube nozzle.

The following discussion will consider first the main program and its self-contained subroutine and then the principle subroutines to the program itself. As in the preceding program discussion, the emphasis will be
placed on the functional aspects of each of the major portions of the program and little or no mention will be made of the programming logic involved.

Skipping past the two inner subroutines for the moment, the first section of technical importance are the exit area and exit conditions calculation procedures. In the first, the total exit area of the nozzle configuration under consideration is calculated. In addition, the area of each of the elements making up the nozzle (as in a multiple tube-type suppressor nozzle) is stored for later use in the flux integral checks. In the conditions calculation procedure, there are two techniques and one iteration involved. The iteration is necessary because in this program only the exit stagnation pressure and temperature are specified, and all properties are obtained from these two properties. In the case where variable gamma is being considered, an iteration on the energy equation is required to determine the exit static conditions. In the first method (constant and identical gamma and molecular weight for all gases of the system), the iteration procedure is not required since the exit static properties can be obtained from the standard one-dimensional Mach number relations.

As in the previous program, certain initial calculations and zero settings must be performed and these are found next in this program as well. The frequency initialization is much the same as before. The velocity is set equal to the free-stream value to assure correct interrogation results.

The determination of the indices on the major loops in the program logic follow the initial condition determinations, and this in turn is followed by the major loop of the program which determines the set of $b_m$ planes
in which the values for the acoustic and aerodynamic properties of the jet flow are calculated.

It is at this point, that the control for the $b_m$ plane pass is found. The program makes three passes in each $b_m$ plane as discussed in Section 2.2.3. The pass is controlled by the value of a special program signal term - in this case the coded word is IBPAS. In every pass, the program logic proceeds to the flow-field matrix which determines the two-dimensional point matrix in the flow-field $b_m$ planes. These points are determined by means of radius and angle loops which specify points approximately at the corners of rectangles (refer to the flow chart, Fig. 2.5).

The Section entitled "No Mesh Set on NMDOM" is artificially included in the program on a temporary basis to insure that the second pass of the program is bypassed. As noted previously, this section must be bypassed because the companion subroutine to the matrix subdivision has not yet been completed.

The flux loop procedure follows next, and it is at this point that the subroutines are called into the main program. The various fluxes are summed over the number of individual elements that exist in the suppressor configuration. Several bypasses are included in the flux calculation to reduce the program running time. The first of these, entitled "X zero shear and iteration bypass", is used at the first $b_m$ plane calculation which is coplanar with the exit of the suppressor nozzle and which is used to associate the exit velocity with corresponding mesh points in the first $b_m$ plane. The second bypass is on the shear and velocity iteration calculations which are not needed for those matrix points lying within the potential core of the jet flux.
The velocity iteration which follows is the same as that used in the preceding program with the addition of a CP and PGC (perfect gas constant) loop which provides a solution for gases of different molecular weight and variable specific heat. Two coefficients are used to describe the functional relationship between specific heat and temperature.

The next major section of importance is the interrogation loop wherein those points lying within a circle whose radii are twice the width of the mixing region are compared to determine the relative maximum and minimum of this set of points. Since the major mesh points are not subdivided in the present program the inner mesh points cannot be interrogated, and for this reason this portion of the program is now bypassed as indicated in the listing. Once the relative maximum and minimum velocities in a given mixing region are known, then properties such as the convective velocity, the local value of the x-coordinate, the eddy viscosity, and the acoustic properties can be determined. The attenuation calculation procedure is identical to that used in the preceding program, and only slight modifications have been made to the x determination and the eddy viscosity calculation procedures.

The matrix subdivision section of the program, which is bypassed for reasons previously discussed, contains three separate routines depending upon the manner in which the major mesh points are subdivided.

The first two sections beginning with statements numbered 167 and 168, respectively, divide the major matrix either circumferentially or radially, and the section beginning with statement 169 performs the division both ways. In all three cases the actual subdivision is carried out by one of
the self-contained subroutines (subroutine mods). This subroutine tells the main program what the mesh size is in the subdivision - for \( b_m \) planes near the exit, this division is 8 by 8; for \( b_m \) planes near the end of the potential core, the division is by 4; and downstream of this point, the division is by 2. For \( b_m \) planes three diameters downstream of the nozzle exit plane, there are no inner mesh sub-divisions.

Following this calculation procedure, are the ratio calculations for the output, sound-power calculations (which are similar to those in the preceding program), the mass momentum and energy integrations, and the termination of the mesh point angle and radius loops. Next is found the acoustic search and addition routine which is another major portion of the program logic and the one which uses the other internal subroutine for the program - subroutine orders. It is here that the program searches through the frequency indices that have been determined at a given \( b_m \) plane (one frequency index being associated with each mixing region having different relative maximum and minimum velocities). Those sound-power values having frequency indices less than one-third octave apart are added together to form the acoustic output of the program. The band assignment and shift routine on the following page is currently not used in this program due to memory limitations. In it, the machine compensates for the fact that the sound power is being calculated at discrete points and not integrated throughout the whole flow field. In the band assignment and shift routine, the program searches through the bands that have been specified in the output after the final \( b_m \) plane has been calculated to make sure that each major frequency (i.e., the major mixing regions) contribute to each bandwidth once and only once after they appear and before they
disappear. Finally, all the sound power from the various mixing regions are
added at each bandwidth to give the total sound power generated. At present,
since the band assignment and shift is not operational, this function will have
to be performed by hand, using the output from the acoustic bandsearch and
addition routine.

Following the main program listing, is the listing for sub-
routine FIAA and its subordinate subroutines. This subroutine is a complicated
bit of programming logic which is necessary to avoid exceeding 7090 machine
memory storage. It also serves to keep the cost of running the program down to
a reasonable level. In this subroutine, the geometric integrations that are
performed for each matrix point are stored along an axial ray so that for each
position in space the geometric integrations need to be performed but once.
Since there are approximately 19 b plane calculations, this technique alone
presents a 19 to 1 saving in computational time. This subroutine also stores
the value of the flux integrations themselves so that subsequent runs with this
particular geometry can be made without re-doing the time-consuming integrations
which (as in the preceding program) will result in a considerable time reduction.

Despite its complexity, the main program is easier to use than
the first. Any number of cases can be considered at the same time, since the
program uses the correct subroutines automatically. Thus, any sequence of
nozzle configurations and aerodynamic cases for each configuration can be
handled in any sequence desired. It is important that a correct case number
for each configuration be included since it is the case number which tells the
machine which subroutine it is to use. For instance, case No. 1 is reserved
for the first-run integrations which are stored on a master tape. This master
tape must be used for all subsequent running of this configuration.
2.6 Conclusions

In the course of conducting this study, several significant areas worthy of further analytical and experimental research were uncovered. These areas, of course, are refinements to the existing knowledge and, as a consequence, will be less rewarding for the same amount of work than has been experienced in the past. This is particularly true in the one area that has caused the most concern in the present study - the relative velocity effect on the rate of spread of jet mixing.

The conclusions of this study are as follows:

(1) Reichardt's mixing theory is not adequate in fully describing the relative rate of mixing in complicated flow situations.

(2) The jet Mach number effect observed primarily in the data of Reference 10, are in contradiction to the observed acoustic results from Reference 1. (Presuming the present acoustic theory is essentially the correct one). As a consequence, the value for the second rate of spread coefficient (constant $C_{b2}$) is found to be zero at transonic jet Mach numbers.

(3) Experimental evidence indicates that the rate of spread in the mixing region is to some degree $x$ dependent. This effect is not included in the present analysis, and the acoustic predictions for circular exits indicate that such a dependence is desirable for a more accurate prediction.
(4) The modified shear-stress equations successfully predict experimentally measured shear stresses.

(5) The analytical velocity profiles, in general, satisfactorily agree with experiment. In this regard, it is noted that the experimental data of Reference 7 seem to have a deficiency in that the data are at odds with the conservation of momentum principle.

(6) In general, the computer programs presented herein are an advancement in the state-of-the-art in the prediction of fluid dynamic and acoustic properties of interacting free jets. Further work is required, however, to bring these programs to their full state of usefulness.
SINGLE CIRCULAR EXIT AERO-AcouSTIC SOLUTION  RDG6-8-3  8380.01

FORMAT AND DIMENSION STATEMENTS

DIMENSION RAD(100)*
1  EVIS(100)  DSOPL(25)*FREQ(25)* DB(25)* REXMAX(2)* REMIN(2)* GAN
2AE(2)*RGAE(2)* CPE(2)* TPE(2)* TTE(2)* AVELE(2)* UE(2)* RHOE(2)*
3NLPE(2)* EXOM(2)* EMAS(2)*
4 70: FORMAT (6F16.5)
5 703 FORMAT (110.4F10.5)
6 70S FORMAT (4F10.4, E10.3,3110)
7 71C FORMAT (12F10.4, 2110, 2E10.3 /I)
8 711 FORMAT (6F10.4, 5F10.4, 2110/1H )
9 712 FORMAT ( 4F10.4, 110, 2E10.3/ ( <0X+110, 2E10.3))
10 713 FORMAT (110.4E10.4)
11 COMMON NK, NS, PI, CL, SKTIL

FUNCTION

DSPF (A+B+C) = A*CA=UCCONN* ATTN=SORTF ((SHEAR**7)/(RHO*3)) *RADJ
1 *DERAD/(6*(C**5)*1.0-DLNUC)

INPUT

571 READ 703, NNMNZ, CDIM, RADC, REXMAX, REXMAX(1), REXMAX(2), REMIN(1), REMIN(2)
572 READ 701, PRADL, CX, CA, CFREQ, GB1, CB2, P, BMGR, BMIH, (GAMAEL, RGEL, L, CPE(L), TPE(L), TTE(L), L, L = 1, 2)
1 FREQ = 0.
2 PI = 3.14159
3 TPI = 6.28318
4 AREAC = PI*RADC*RADC
5 RDL = RADC
6 SEMAS = 0.
7 SEEOM = 0.
8 SENLP = 0.
9 EANZQ = 0.

EXIT CONDITIONS CALCULATION

NK = NNMNZ + 1
DO 601 K = 1, NK
1 GAMAI = (GAMAEL(K) = 1.)/(GAMAEL(K)
2 EMACH = 2**(1(DPE(K)/P)**GAMAI) -1.)/(GAMAEL(K)=1.0) 
3 TTE = TTE(K) / (1.0 + (GAMAEL(K) = 1.0) * EMACH/ 2.0) 
4 AVELE(K) = SORTF ( GAMAEL(K) * RGEL(K) * TE) 
5 EMACH = SORTF ( EMACH) 
6 UE(K) = EMACH * AVELE(K)
7 RHOE(K) = P / ( RGEL(K) * TE ) 
8 TNLPE(K) = CPE(K) * TTE(K) - CPE(NK) * TTE(NK) 
9 EMAS(K) = RHOE(K) * UE(K) 
10 EOKP(K) = EMAS(K) * UE(K)
11 EMOK(K) = EMAS(K) * EKTA * SEMAS 
12 EMOK(K) = EMAS(K) * EXTA - SEMAS 
13 SENLP = EXTA + TNLPE(K) * TNLPE(K) = EXTA + SENLP + .00001 
14 UDIV = UE(1)
15 TDIV = TTE(1)
RHODV = RHOE1
SHDIV = RHOE1 * UE1 * UE1
601 EAN0Z = EAN0Z + exta

END EXIT CALCULATION

UMAX = UE1
UMIN = UE(NKP)
AVELC = AVELC1
URAT = (UMAX + UMIN) / 2.0
UCONV = URAT
BM = BMINT
BMOLD = 0.0
DRADI = RADC / 20.

INITIAL CONDITIONS LOOP

RAD(1) = 0.
DO 610 KJP = 1, NKP
KJ = KJP + 1
RAD(KJP) = RAD(KJP) + DRADI
610 EVIS(KJP) = 0.
EVIS(1) = 0.
XOLD = 0.
DUCNV = 0.0
NSC = 0

END INITIAL CONDITIONS LOOP

FREQUENCY NEST INITIALIZATION LOOP

DSPL(1) = 0.
DB(1) = 0.
FREQ1(1) = 125.
NOCT = 24
FREQJ = 1.259921
DO 560 JFREQ = 2 * NOCT
FREQ1(JFREQ) = FREQ1(JFREQ - 1) * FREQJ
DB(JFREQ) = 0.
560 DSPL(JFREQ) = 0.

END FREQUENCY INITIALIZATION

DSL = 0.
BMRT = BMOR + RADC / BMINT
NMBM = 3 * LOGF(BMRT) / 0.69315

BM LOOP

DO 540 I = 1, NMBM

CONVECTIVE VELOCITY CALCULATION

UCONN = (UMAX + UMIN) / 2.0 + DUCNV * (BM - BMOLD)
CMACH = UCONN / AVELC

-125-
SPREADING COEFFICIENT CALCULATION

\[
DXBM = \left( CB1 * (1 + CB2 * (1 + EXPF (-BM/(0.2*RADC)))) \right) * \\
\text{ICMACH} \times \text{UMAX} / \text{UMIN} / \text{UMAX} - \text{UMIN} \\
\text{IF} (1 - 1) \ 100 + 591.592
\]

591 \ DXBMD = DXBM

592 \ X = XOLD + (DXSM + DXBMD) * (BM - BMOLD) / 2.

\[
BM = BM / \text{SORTF(PRADI)} \\
XRA = X / (XOLD + 0.0001) \\
\text{UCONR} = \text{UCONN} / \text{UCONV} \\
\text{DLNUG} = \text{LOGF} (\text{UCONR}) / \text{LOGF}(XRA) \\
\text{DUCNV} = (\text{UCONN} - \text{UCONV}) / (X - XOLD) \\
\text{UCONV} = \text{UCONN} \\
\text{ISET1} = BM / \text{RADC} * 0.15 \\
\text{NMDOJ} = 2 \ ** \ ISET1 \\
\text{IF} (\text{NMDOJ} - 8) \ 593.594.594
\]

594 \ NMDOJ = 4

593 \ DOJ = NMDOJ

\[
\text{DERAD} = \text{RADI} \ * \ DOJ \\
\text{NMJMX} = (\text{RADC} + 2.5*BM) / \text{RADI} + 3. \\
\text{TMT} = 0. \\
\text{TMST} = 0. \\
\text{TNTPT} = 0. \\
\text{FNSS} = 1. \\
\text{TYPE} 705 * X * \text{AVELE}(1) / UE(1) * BM, TNLPE(1) \ , NMBM, NMJMX, NMDOJ
\]

RADIUS LOOP

DO 510 \ J = 1, \ NMJMX + NMDOJ

DLI = CDIM

RADJ = RADIJ

END COEFFICIENT CALCULATION

FLUX CALCULATION LOOP

\[
CL = RADJ / \text{REMAX}(1) \\
\text{BMTIL} = BM / \text{REMAX}(1) \\
SJ = -1. \\
\text{CALL FLUX} (\text{SHK}, SJ) \\
\text{BMTIL} = BM / \text{REMAX}(1) \\
SJ = 0. \\
\text{CALL FLUX} (\text{SF}, SJ) \\
\text{IF} (\text{SF} = 0.001) \ 522, 522, 523
\]

523 \ TM = EMOM(1) * \text{SF} \\
H = EMAS(1) * TNLPE(1) * SHK \\
\text{SHEAR} = EMOM(1) * SJ / DXBM

VELOCITY ITERATION

\[
\text{RG} = 1700. \\
U3 = 500. \\
U2 = 0. \\
U1 = 100000. \\
\text{GO TO} 97
\]

91 \ AP = TM / Z.0 + CP * P / KG \\
\text{UN} = H / (2 * 0 + AP) + \text{SORTF} (H + H / (4 * 0 + AP * AP) + (TM * CPE(N}
IXP)*TTE(NKP)/AP )
UD = UN = U3
IF ( ABSF (UD) = 1 ) 96: 96: 92
92 U2 = (U2+U3 -U1+UN )/ (U2 +U3 -U1-UN )
95 U1 = U3
U3 = U2
U2 = UN
GOTO 98
97 RHO = TM / (U3*U3)
GO TO 99
98 RHO = TM / (UN*UN)
99 T= P/RG * RHO
82 RG = RGE(1)
CP = CPE(1)
GO TO 91

LOCAL PROPERTIES
522 TMA1 = 0.
TMI = 0.
TNTP1 = 0.
RADJ = RADJ/RADC
SHERR = 0.
URAT = 0.
VMACH = 0.
GO TO 505
96 GAMMA* CP / (CP - RG)
TNTLP = H /(RHO * UN)
TT = (TNTLP + CPE(NKP) * TTE(NKP))/CP
AVEL = SQRTF ( GAMMA * RG * T)
VMACH= UN /AVEL

EDDY VISCOSITY AND DERIVATIVE CALCULATION
EVISN = RHO * UCONN * BM /(2 *O *DXBM )
DEVIS = (EVISN - EVIS(J))/ (X-XOLD)
EVIS(J) = EVISN

RATIO CALCULATIONS
RADJR=RADJ/RADC
XR=X/RADC
SHERR = SHEAR/SHDIV
RHROR = RHO/ RHOVDV
TTR = TT/TDIV
URAT = UN * /UDIV
BMR=BM /X
UCONR = UCONN/UDIV

ATTENUATION CALCULATIONS
IF (RADL - RADJ) 1: 1: 3
1 RADP = RAOL
GO TO L
3 RADP = RADJ
2 EFUNC = 1:O = RADP * EAN0Z / (RADL * AREAC)
XEFEC = CX * BM * EFUNC * CMACH
EVISM = EVIS(J)+DEVIS*XEFEC/2.0
FREQ = FREQ
FREQ = CFREQ * UCONN / X
C
FREQUENCY NEST INDEX
IFREQ = 3.0 * LOGF (FREQ / 125. ) / 69315 + 1.
C
SOUND POWER CALCULATION
ATKRK = FREQ * FREG * EVISM / {4.0 + (GAMMA-1.0) / PRADL} / (RHOE(NKP) * AVELE(N
1KP) ** 3.)
ATTN = EXPF (-ATKRK * XEFEC)
IF(1-1) 100, 487, 494
487 IF (J-1) 100, 493, 494
493 IFREQ = IFREQ
GO TO 5
494 FREQ = IFREQ - IFREQ
IF (ABS(FRED) - 0.05) 6, 6, 5
6 GO TO (17.17.17.8)*NSC
7 SC = 1.
NSC = NSC + 1
DDSPL = DSPF(SC, RHOE(NKP) * AVELE(NKP))
DSL = DSL + DDSPL
GO TO 521
8 SC = 0.5
DDSPL = DSPF(SC, RHOE(NKP) * AVELE(NKP))
DSL = DSL - 0.5 * DDSPL
DSPL0 = - 0.5 * DDSPL
DSL = DSL + DDSPL
GO TO 521
5 DSPL (IFRED) = DSPL (IFRED) + DSL
PUNCH = 710, XR, RADJR, IFRED, NSC, DSPL (IFRED), FREQ
SC = 1.
DDSPL = DSPF(SC, RHOE(NKP) * AVELE(NKP))
DSPL = DDSPL
DSL = DDSPL
NSC = 2
IFRED = IFRED
C
MASS MOMENTUM AND ENERGY INTEGRATIONS
C
521 TMAST = TP1 * RADJ*FNSS * RHO * UN
TM1 = TP1 * RADJ*FNSS*TM
TNTP1 = TP1 * RADJ*FNSS * RHO * UN * TNTLP
TMAST = TMAST + TMAS1
TMIT = TMIT + TM1
TNTP = TNTP + TNTP1
IF (J-11) 100, 483, 484
483 UMAX = UN
AVELC = AVEL
FNSS = 2.
484 FNSS = 6.0 - FNSS
GO TO 510
C
TERMINATION OF RADIUS LOOP
C
505 RADL = RADJ
XOLD = X

-128-
TMIT = (TMIT - TMI/2) * DERAD / 3
TMAST = (TMAST - TMAST/2) * DERAD / 3
TNTPT = (TNTPT - TNTPT/2) * DERAD / 3

INTEGRAL CHECKS

FEMOM = (SEMO M - TMIT) / SEMOM
FEMAS = (SEMAS - TMAST) / SEMAS
ENRG = (SENLP - TNTPT) / SENLP
PUNCH 711: XR, RADJR, URAT, SHERR, SNK, VMACH, SF, SJ, DL1, UCNR, BMR, DXBM, T
TRAINS NK
PUNCH 712: XR, FEMOM, ENRG, SJ, (KJP, FREQI(KJP), DSP(KJP), KJP = 1, NO
ICT)
GO TO 520

510 PUNCH 711: XR, RADJR, URAT, SHERR, SNK, VMACH, SF, SJ, DL1, UCNR, BMR, DXBM, T
TRAINS NK

END RADIUS LOOP

520 BMOLD = BM
DXBMO = DXBM
540 BM = 1.259921*BM
DSPL(IFREQ) = DSPL(IFREQ) + DSL
DO 530 KJ = 1, NQCT
DRAT = DSPL(KJ) / 0.000000000001 + 1.
530 DB(KJ) = 10. * LOGF(DRAT) * 0.434
PUNCH 713: K, FREQI(K), DSP(K), DB(K), K = 1, NQCT

END LOOP

803 IF (SENSE SWITCH 2) 801, 802
801 GO TO 571
802 GO TO 572

100 PAUSE
GO TO 803
END
SUBROUTINE FLUX CALCULATION PROCEDURE

SUBROUTINE FLUX (PSI, TAU)

COMMON NK,NS,PI,DL,CL,BMTI1,BMTIL*,PNS,XSO,EXPOC,TPSI

IF (TAU) 205, 201, 201

205 NSGO = 2
GO TO 202

201 NSGO = 1

TAU = 0.

202 XSO = 1.0/(BMTIL*BMTIL)
EXPOC = (1.-CL)*(1.-CL)*XSO
PNS = 4.
PSIP = 0.
PSI = 0.
TPSI = 0.
TTAU = 0.
DL = 0.

IF (CL-1.) 203, 204, 204

204 VL=CL-1.
NS = 1.0/DL + 0.000001
VN=NS
DL = 1.0/DL
GO TO 206

203 IF (EXPOC - 20.0)* 222, 221, 221

221 PSIP = 1.
GO TO 208

222 PSIP = 1. - EXPF(-EXPOC)

208 NS=CL/DL+/000001
IF (NS) 231, 231, 224

224 VN=NS

DL = CL/( VN)
VL = 1.0/CL
PSIP = PI*VL*(1.-PSIP)*0.000001*DL + PSIP

206 NS=2*NS
DO 212 IS = 1, NS
NK = IS

233 VL = VL + DL
TARG = (CL/VL + VL/CL - 1.0/(CL*CL)) / 2.
PHI = ATANF(SQRT((1.0 - TARG*TARG)/TARG)
IF (PHI) 214, 215, 215

214 PHI = PI + PHI
215 EXPAR = VL*VL*XSO
IF (25.-EXPAR) 231, 231, 216
216 PSI = PNS*EXPF(-EXPAR)*PHI*VL
GO TO (217, 218) + NSGO

217 TAU = PNS*EXPF(-EXPAR)*(1.+2.*EXPAR - EXPF(EXPAR/2.))*SINFCPH1)
218 TPSI = TPSI + PSI
TTAU = TTAU + TAU
PNS = 6.0 - PNS

212 CONTINUE

231 PSI = PSIP + (TPSI*2.-PSI)*XSO*DL/(3.*PI)
GO TO (219, 220) + NSGO

219 TAU = ABSF((TTAU- TAU/2.)*DL/(3.*PI*BMTIL))

220 RETURN

END
C  SHORT RUN SUBROUTINE
SUBROUTINE FLUX ( PSI, TAU)
COMMON NK, NS, PSI, CL, BMTIL
220 FORMAT (5F10.6)
   NS = 0
   NK = 0
   RADI = 0.
   BMD = 0.
   IF (TAU) 213 206 206
213 READ 220, XR, RADJR, SF, SJ, SHK
   RAD1 = RADJR - CL
   IF (ABS(RAD1) - 0.01) 205 205 202
206 BMD = XR - BMTIL
   IF (ABS(BMD) - 0.005) 201 201 202
202 PAUSE
   TYPE 220, RADI, BMD, XR, RADJR, CL
   PSI = 1.0
   TAU = 0.0
   GO TO 210
201 PSI = SF
   TAU = SJ
   GO TO 210
205 PSI = SHK
   TAU = -1
210 RETURN
END
COMPRESSIBLE FLOW MIXING PROFILE AND ACOUSTICS PROGRAM - VIDYA-63-4

SUBROUTINE ORDERS(N*V*IPH)
DIMENSION V(100)*IPH(100)
LN = N
DO 1 I=1+LN
1 IPHI(I) = I
K = 0
DO 4 J=2+LN
  IF (V(J-1)*V(J)) 3,4,4
3 VJ = V(J-1)
  V(J) = VT
  IT = IPHI(J-1)
  IPHI(J-1) = IPHI(J)
  IPHI(J) = IT
4 CONTINUE
  IF (K) 6,6,2
6 RETURN
END

MATRICES INDEX AND MODULAR OVERRIDE SUBROUTINE

SUBROUTINE MODS(MFRST)
COMMON PI,P1,P2,PRAD,PRAS,FREQ,IMH,BM,XE,NEST,NUM,LEAV,PHI,ALPO,R,SA,SIG,CDM,ISAP,BINT,RADEC,5F,SH,SR,SJ,MODG,NUMDOM,MODP
MFRST = 1
IF (MODP-(NUMDOM-1)1218,1218,217
217 NUMDOM*NUMDOM+2
218 MFRST=MFRST+MODP
MODX=MODS=25*MODP
RETURN
END

LABEL
LIST
XEQ
CARDS COLUMN
LIST
DIMENSION XE(50),ALPO(50),LEAV(50),NUM(50),KN
1(50),ISAP(50),XITA(50),GMW(50),PTE(50),TTE(50),ENTLP(50),PGCE(50),
2CE(50),GAMK(50),EMOE(50),EE(50),ACK(50),EMAS(50),EH(50),SF
3(50),SH(50),SJR(50),SJC(50),SK(50),GKHON(50),SPL(25),DB(25),FREQ(2
45),ISSH(20),NREJ( ),KAPF(25),ISPL(85),O5PS(85),ETA(2,50),UN(20,20
5),IFEX(20,20),UZ(20,20),XBMQ(20,20),UOMN(20,20),EVIS(20,20),DSFL
6(20,20),XOLD(20,20),UN( ),UZ( ),UN( ),UZ( ),UN( ),UZ( ),UN( )
7(1),O5PS(25,25,25).
1SA,SIG,CDM,ISAP,BINT,RADEC,5F,SH,SR,SJ,MODG,NUMDOM,MODP
COMMON PI,P1,P2,PRAD,PRAS,FREQ,IMH,BM,XE,NEST,NUM,LEAV,PHI,ALPO,R

-12-
1 FORMAT (6F10.4)
2 FORMAT (2F10.4)
3 FORMAT (2F10.4, 3I10)
4 FORMAT (80H)

18 FLOW MIXING PROFILE PROGRAM

2 INPUT DATA

1 COMPRESSIBLE FLOW MIXING PROFILE PROGRAM

3 INPUT DATA**#**

4 FORMAT (ISH OPFB*5,1714

5 IPEII*5*ZAW

6 I

7 P111.5.1!H

8 IPEI

9 P~or

10 FORMAT I1gHO

11 NCIIaI3*134

12 NCFsIa,124

13 LPIIm

14 ZSYMsIZ1*14H

15 I

16 NESTaI3913H

17 ZN0XuZ3
eoi9

18 I

19 RZADCmF6s394

20 COIM*F6o3984

21 FORMAT 13140 XE(I24214)aPS.2,

22 7H4

23 ALPO

24 I(12v214)wF7s4s

25 714 LEAV112t214)mt3o 714

26 NMJM(12s214)ft3*74 i4(44I

27 9

28 OMA(3H ALPtl2,I14,lZ2.21)mp7*4%7 A(1241

29 II2,2HumF7s4*74 DALP(12vl12ZN1)nFl*,*?H RA(12,atl.12*zH)*F?*.s

30 FORMAT (81141 EXIT

31 CONDITIONS/P/,e

32 9514

33 CONTOUR STATIC VELO

34 MACH MOMENTUM ENTHALPY/ 93H

35 TEMPERATURE NUMBER FLUX

36 FLUX// 95H4

37 DEG RANKINE FT

38 5/SEC

39 LB/SQ FT

40 LB/SQ FT//119H

41 12 H 7H

42 LEAV(12:2H)=13; 7H NUM(12:2H)=13.7H KNI1

43 22;12H=11:7H 15x9(12.2H)=13)

44 FORMAT(123H)

45 DALP(12:1H)=F7:4,7H RA(12:1H)=F7:41

46 RA(12:1H)=F7:4,7H DALP(12:1H)=F7:4,7H RA(12:1H)=F7:4)

47 10 FORMAT (76H)

48 OPERATING CONDITIONS FOR

49 1 CASE NUMBER 14'

50 11 FORMAT (61H)

51 EXIT CO

52 NITIONS:// 95H

53 CONTOUR STATIC VELO

54 CITY MACH

55 MOMENTUM ENTHALPY/ 93H

56 TEMPERATURE NUMBER FLUX

57 FLUX// 95H4

58 DEG RANKINE FT

59 5/SEC

60 LB/SQ FT

61 LB/SQ FT//119H

62 12 FORMAT(110H)

63 RADIUS ANGLE VELOCITY TEMPERATURE M

64 MENTUM ENTHALPY RADIAL TANGENTIAL TOTAL /110H -

65 FLUX FLUX

66 3 SHEAR SHEAR SHEAR 111H

67 4 RADIAN S FT/SEC DEG R

68 LB/SQ FT

69 LB/FT SEC

70 LB/SQ FT//119H

71 5 LB/SQ FT

72 LB/SQ FT//119H

73 13 FORMAT (9H)

74 IPEI0+3.5H

75 OPF3.5H

76 F6.1+4H

77 F6.1+IP

78 15E12.3/1

79 16 FORMAT (57H)

80 1=1PE11+.4//

81 15 FORMAT (612 )

82 16 FORMAT (5F10.4)

83 17 FORMAT (69H)

84 EXIT AREA OF CONTINUE NUMBER 12:2H = F10.4 )

85 18 FORMAT (94H)

86 TEMPERATURE TOTAL PRESSURE/ 91H

87 2 DEGREES RANKINE

88 3/5 FT//119H

89 12; 30M

90 4 F8.2+ 24H

91 89141

92 CONTINUE CONTOUR MOLECULAR WEIGHT

93 1 LAMBDA ONE LAMBDA TWO /89H

94 2 FT/SZ-DEGR (FT/S-DEGR)ISO//20X13-16

95 3XF6.3,12X1PE10.3,10XE10.3 )

96 20 FORMAT (9XF10.3,5XF5.3,5XF6.4,4XF6.4,1P5E12.3 )

97 23 FORMAT (40H)

98 SOUND POWER SPECTRUM ///29X H F

99 1 FREQUENCY FREQUENCY SOUND POWER

100 2 H INDEX REF 10-12W

101 3 13 IPE10.3

102 300 FORMAT (40H)

103 PSEUDEO AXIAL LOCATION B SUB M=F9.5+25M FT

104 1A5E NUMBER- 13 //1

105 302 FORMAT (11H)
303 FORMAT(176H) S mies POWER
1 INDEX AND MASS FLUX// 115H I SUEO AXIAL LOCATION S
2 SOUND POWER LEVEL INDEX MASS FLUX VELOCITY TO THE 8TH
2 SOUND POWER LEVEL INDEX FREQUENCY INDEX VELOCITY TO THE 8TH
3- INTEGRAL/106H FEET (FT LB/SQ )
4 SEC50 . FT=10/SEC#8///
304 FORMAT (76H) COMPRESSIBLE FLO
1W MIXING PROFILE PROGRAM///
234H CASE NUMBER=12/ 40H
3 41DN = F9.5+3H FT/// 79H
5 ANGLE TOTAL PRESSURE/ 80H RADIUS
6 FEET RADlANS INCHES MG GAUGE
305 FORMAT(23H) F9.3 13H F6.3 19H
1
307 FORMAT(213)3F10+5*4F10+2 1
309 FORMAT(6H X(12,2H)=F8+5+9H SPL1(12,2H)=1PE11+4+8H FM(12
1+2H)=1PE11+4 8H U61(12,2H)=1PE11+4///
329 FORMAT (10X+F10+2*F10+2)
330 FORMAT (110) DSPF(A+B = A + B *SORF(TU2RO*TU2RO*TU2RO*TAU)
PI=3.1415927
PI2=6.2831853
NT2 = 2
C
INPUT DATA READIN AND PRINTOUT
C
READ INPUT TAPE NT2 .15+NT3+NT4+NT5+NT6+NT7+NT8
READ INPUT TAPE NT2*1+NCI+NCF+INDX+NEST*LPFI+ISYM*RADC+CDIM
READ INPUT TAPE NT2*3+IXE(K)+ALPO(K)+ISAP(K)+LEAV(K)+NUM(K)+KN(K)
1+K=1+NEST)
FLPHI=LPHI
WRITE OUTPUT TAPE NT3*4 -
WRIT.E OUTPUT TAPE NT2*5+NCI+NCF+LPFI+ISYM+NEST*INDX+RADC+CDIM
SEMAS=0
SEKOM=0
SEMLO=0+00001
AREAC=P1*RADC+RADC
EAEN=0
DO 319 K=2+NEST
NUMK= NUM(K)
READ INPUT TAPE NT2*2+(DALP(N+K)+RA(N+K)+N=1+NUMK)
ISAPE=ISAP(K)
FLEV=LEAV(K)
WRITE OUTPUT TAPE NT3+8*K*KE(K)+K+ALPO(K)+K+LEAV(K)+K+NUM(K)
1+K+KN(K)+K+ISAP(K)
NUMK=NUM(K)
WRITE OUTPUT TAPE NT3*9+(N+K+DALP(N+K)+N+K+RA(N+K)+N=1+NUMK)
C
EXIT AREA CALCULATION
C
GO TO (315,316,317)*1SAP
315 XITA(K) = .25*(RA(1+K) + RA(NUM(K)+K)*DALP(1+K)
DO 313 N = 2+ NUMK
313 XITA(K) = XITA(K)+.25*(RA(N+K) + RA(N-1+K))*DALP(N+K)
XITA(K) = XITA(K)*FLEV
GO TO 319

-131-
316 \text{XITA}(K) = \text{PI} \times \text{RA}(1, K) \times \text{RA}(1, K)
\text{GO TO 319}
317 \text{S1} = \text{RA}(3, K) \times \text{RA}(3, K) + \text{RA}(1, K) \times \text{RA}(1, K) - 2 \times \text{RA}(1, K) \times \text{RA}(3, K) \times \cos(\text{DA} \text{LP}(1, K))
\text{S2} = \text{RA}(1, K) \times \text{RA}(1, K) + \text{RA}(2, K) \times \text{RA}(2, K) - 2 \times \text{RA}(1, K) \times \text{RA}(2, K) \times \cos(\text{DA} \text{LP}(2, K))
\text{XITA}(K) = \text{S1} \times \text{S2}
\text{WRITE OUTPUT TAPE NT3+17, K, XITA(K)}
319 \text{EANOZ} = \text{EANOZ} + \text{XITA(K)}

\text{C}
\text{EXIT CONDITIONS}
\text{DO} 999 \text{ NC} = \text{NC}1 + \text{NCF}
\text{READ INPUT TAPE NT2=307, OCTD=NOCT, XINT=XOR, PRADL=FREDG, CMW=PS, CP}
\text{READ INPUT TAPE NT2=16+PTE(K), GMW(K), ETA(K), ETA(K+2),=1+NEST}
\text{WRITE OUTPUT TAPE NT3=10, NC, OCTD, NOCT, XOR, PRADL=FREDG, CMW=ICP}
\text{DO 119 K=1, K+NEST}
\text{WRITE OUTPUT TAPE NT3=18, K, PTE(K), TTE(K)}
\text{IF ETA(K+1)=120, K+121}
121 \text{DO 120 K=1, K+NEST}
\text{WRITE OUTPUT TAPE NT3=19, K, GMW(K), ETA(K), ETA(K+2)}
\text{CONTINUE}
\text{ENTLP} = (\text{ETA}(1, 1) + \text{ETA}(1, 2) + \text{TTE}(11) + \text{TTE}(1))
\text{LBPA} = \text{S1}
\text{DO 113 K=1, K+NEST}
\text{IF ETA(K+1)=111, K+112}
112 \text{ENTLP} = (\text{ETA}(1, 1) + \text{ETA}(1, 2) + \text{TTE}(1, 1)) + \text{TTE}(K)
\text{PGCE(K) = 49690/CMW(K)}
\text{PGC = PGCE(K)}
\text{IF PTE(K)=PS} 321, 321, 322
\text{TEK=286} \times \text{TTE(K)} \times \text{PS/PTE(K)}
115 \text{CPE(K)=ETA(K+1)+ETA(K+2)+TEK}
\text{CP=CPE(K)}
\text{STEN1}=\text{ETA}(1, 1)+\text{ETA}(1, 2)+\text{TTE}(1) \times \text{TEK}
\text{LBPA} = \text{S2}
\text{GO TO 37B}
111 \text{PGCE(K)=49690/CMW}
\text{PGCE(K)=PGC}
\text{ENTLP(K)=TTE(K) \times CP}
\text{CPM=CP}
\text{ENTLP} = \text{CP} \times \text{TTE}(1)
378 \text{GAM} = 1/(1-\text{PGC}/\text{CP})
\text{GAM(K)=GAM}
\text{ACH2} = 2 \times ((\text{PTE}(1) / \text{PS}) / \text{GAM}) - 1 / (1 - \text{ACH2}(1) - 1)
\text{GO TO (407, 408)} \times \text{LBPA}
407 \text{TEK} = \text{TTE}(1) / (1 + (\text{GAM} - 1) \times \text{ACH2}(1))
408 \text{PHOE(K)}=\text{PS} / (\text{PGC} \times \text{TEK})
\text{SV2} = \text{GAM} \times \text{PGC} \times \text{TEK}
\text{GO TO (116, 381)} \times \text{LBPA}
381 \text{STEN2} = \text{ENTLP(K)} / (1 + \text{ACH2}(1) \times \text{GAM} \times \text{PHOE(K)}) \times \text{PS}
\text{DSTEN} = \text{STEN2} - \text{STEN1}
\text{TEK} = \text{DSTEN} / \text{CPE(K)} \times \text{TEK}
\text{IF (ABSF(DSTEN-10)=116.116.115}
321 \text{CPE(K)=ETA(K+1)+ETA(K+2)+TTE(K)}
\text{CP=CPE(K)}
\text{LBPA=1}
GO TO 378
116 EM(K) = ACH2 * GAM * PS
    UE(K) = SQRTF(ACH2 * SVZI)
    SVE(K) = SQRTF(SVZI)
    ACH(K) = UE(K)/SVE(K)
    EMAS(K) = RHOE(K)*UE(K)
    ENTLP(K) = ENTLP(K) - ENTL0
113 EH(K) = EMAS(K) * ENTLP(K)
    GAM = GAMK(1)
    PGC=PGCE(1)
    ASR= PS * GAM * PGC * SQRTF(GAM * PGC * TE(1) ) *TE(1)
    UDIV = UE{INDX}
    TDIV = TTE{INDX}
    RHODV=RHOE{INDX}
    SMIV = RHOE{INDX} * UE{INDX} + UE{INDX}
WRITE OUTPUT TAPE NT3:11, (K:TEK *)UE(K) + ACH(K) * EM(K) * EH(K)
1) *K=1, NEST)
WRITE OUTPUT NT3 *14 * ASR
DO 122 K=2, NEST
REMAS=RHOE(K) *(UE(K)-UE(1))
SEMAS=SEMAS + REMAS*XITA(K)
SEMM=SEMW+REMAS*XITA(K)*(UE(K)-UE(1))
SENT=SENLP+REMAS*ENTLP(K)*XITA(K)

C END EXIT CALCULATION
C FREQUENCY NEST INITIALIZATION LOOP

C IF ( NOCT ) 513:513:514
513 NOCT = 10
514 IF( OCTD ) 515:515:516
515 OCT = 1
516 SPL(1)+0
      DB(1) = 0
      FREQ(1)= FREQB
      NOCTD = 10 * NOCT
      OCT = NOCT
      NOCT = OCT*OCTD
      FREQ = EXPF(0.69315/OCT)
      DO 561 JFREQ=2*NOCO
          FREQ(1)=FREQ(1)+FREQ(JFREQ-1)*FREQ
          DB(1)=FREQ(1)=0.
      561 SPL(*FREQ) = 0.0
C Not Operational
C SOUND POWER INITIALIZATION
C
C DO 151 N=1,NOCT
C DO 151 K=2,NMBM
C DO 151 M=1,MIXT
151 DSMPL(N+K+M) = -1*
C VELOCITY INITIALIZATION LOOP
C
C DO 511 M=1,20
C DO 511 I=1,40
C 511 UN(1+M) = 0.
INITIAL CONDITIONS

IF (XINT) 94,94,95
94 XINT = CDIM / 4.0 + 0.01
95 IF(XOR) 517,517,518
517 XOR = 10.
518 BMAT = XOR * RADC / XINT
NMBM = OCTD * LOGF (8MRAT) / 0.69315 + 2.0
BMINT = 0.075 * XINT
BM = 0.0
DSIGZ=RADC / 10.

INDEX REGISTER DETERMINATION

DO 185 KA=1,NMBM
185 HIPAS = 1
IF (KA=1) 99,99,100
100 HIPAS = 1
GO TO 118
99 HIPAS=2
118 RACO = 2.0*BM
DSIG=DSIGZ*SET2
ID 727 11=1,NOCFD
727 150.111=0.
DSIG = DSIG/8.0
IMAT = 0.
IMIT = 0.
IF NDL=1
NU1=0.
N1=0.
WHILE OUTPUT TAPE NT3=300*BM+NC
WHILE OUTPUT TAPE NT6=300*BM+NC

-137-
WRITE OUTPUT TAPE NT7*300*BM,NC
LINE3=6
LINE6=8
LINE7=8
WRITE OUTPUT TAPE NT3
WRITE OUTPUT TAPE NT6+12
WRITE OUTPUT TAPE NT7*

FLOW FIELD MATRIX

998 SIG=0.001*DSIG
1BPAS = 1BPAS+1
BUC = SIG

RADIUS LOOP

DO 126 M=1,NMJMX
DPHI = PI2/FLOATF (LPHI+IS)
DPHI = DPHI*SET2
ISSY = IPIL*(M-1)+1-1SYM
ISSM(M) = ISSY
DPHIP=DPHI/8.0
PHI=0.

ANGLE LOOP

DO 152 I=1,ISSY
IM=0
MM=0
MAT = I
IFEX = 0
NVPAS = 1
DERAD = DSIG
DEPHI = DPHI

NO MESH SET ON NMDON
NMDON = 8

BYPASS REGISTER -FLUX SUMMATION

GO TO (153,205,505)+1BPAS
505 GO TO (204,288)+MBPAS

FLUX LOOP

153 SFT = 0.
SHEAR = 0.
SHEAC = 0.
SHEAT = 0.
TM = EM(1)
H = EH(1)
CALL FLMA(NC,1+M+KA)
DO 971K=2+NEST
K=2+NEST-1K
IF (SF(K) = .999) 411,411+412
412 KNK = 1
GO TO 413

411 KNK = KN(K)

413 TM = TM + (EM(K) - EM(KNI)) * SF(K)

H = H + (EM(K) - EM(KNI)) * SH(K)

GK(K) = EMAS(K) + SH(K)

X ZERO SHEAR AND ITERATION BYPASS

GO TO (328,349), M3PAS

328 SHEAR = SHEAR + (EM(K) - EM(KNI)) * SJR(K)

SHEAC = SHEAC + (EM(K) - EM(KNI)) * SJR(K)

SHERT = SORTF(SHEAR + SHEAR + SHEAC + SHEAC)

349 IF (SFT(K) = 0.999) THEN 97, 324

97 SFT = SFT + SF(K);

K = 1

324 IF (SFT = 0.999) THEN 332, 523, 523

POTENTIAL CORE BYPASS

GO TO (328,349), M3PAS

272 UN(1, M) = UE(K)

TNTLP = TNLTP(K)

RHO = ROEP(K)

GAM = GAM(K)

SHEAR = 0

SHEAC = 0

SHEAT = 0

TS = TTE(K)/(1 + (GAM-1)/2 * ACHM * ACHM)

ACHM = ACH(K)

GO TO 382

523 GO TO (102, 334), M3PAS

END BYPASS

VELOCITY ITERATION

334 U3 = 500

IFEX(I, M) = 1

GO TO 98

102 U3 = UN(I, M)

98 U2 = 0

U1 = 100000

GO TO 103

104 AP = TM/(U2 + CPH*PS/Pgc)

UN(I, M) = H/(2 + A*AP) + SORTF(M*I/(4 + O*AP*AP) + (TM*ENTLO + O))/4

UO = UN(I, M) - U3

ABS(TD-1) = 105, 105, 107

107 U2 = (U2 + U3 - UN(I, M)) / (U2 + U3 - U1 - UN(I, M))

U1 = U3

U3 = U2

U2 = UN(I, M)

GO TO 108

103 RHO = TM/(U3 + U3)

TS = PS/(1700 * RHO)

GO TO 109

108 RHO = TM / (UN(I, M) + UN(I, M))

TS = PS/(Pgc + RHO)
109 IF(ETA(1,1)) 104, 104, 114
C
C CP AND PGC LOOP
C
114 TOTLK = 0.
CPM=0.
CP=0.
PGC = 0.
DO 521 NK = 2, NEST
GKON (NK) = GK(NK)/(RH0*UN(I+M))
PGC = PGC1(NK)*GKON(NK)+PGC
CP = GKON (NK) * (ETA(NK+1) + ETA(NK+2)*TS + ETA (NK+3)/TS)
ITSL1 + CP
CPM= GCON(NK) * ETA(NK+1) + ETA(NK+2)*TS + CPM + PGC
521 TOTLK = TOTLK + GCON (NK)
PGC = PGC + (1. - TOTLK) * PGC (1)
CP = CP + (1. - TOTLK) * ETA(NK+2) * ETA (1+2) * TS
CPM = CPM + (1. - TOTLK) * ETA(NK+1) + ETA(NK+2)*TS
GO TO 104
C
C LOCAL PROPERTIES
C
105 GAM = CP / (CP-PGC)
TN=TN/L/(RH0*UN(I+M))
SV = SORTF (GAM * PGC * TS)
TT=TN/L/CP
ACHM = UN(I+M) / SV
302 TPG = PS * ((GAM-1)/2 * ACHM + ACHM) * (GAM/(GAM-1)) - TS
TP = TPG + PS
HGIN = TPG/70.727
WRITE OUTPUT TAPE NT6, 13, SIG, PHI + UN(I+M), TS, TM, H, SHEAR, SHEAR, SHED
IF (SENSE SWITCH 5) 213, 214
213 WRITE OUTPUT TAPE NT6, 305, SIG, PHI, HGIN
214 GO TO (414, 415), M, M
415 UN(I+M) = UN(I+M)
X BYPASS AND MESH BYPASS

GO TO (562,172,205,207):MAT

INTERROGATION LOOP

204 MXPLN = 1
UMAX = 0.
UMIN = UN (IP+1)
MDM = RACO/DSIG + 1
IDI = RACO / (SIG+DPHI) + 1
MIN = MAXOF (M-MDM+1)
MAX = MINOF (MDM-MRAM+1)

255 DO 181 MP = MIN+, MAX
11 = ((MIN -1)*(MP-1))/(M-1) + 1
12 = ((MAX -1)*(MP-1))/(M-1) + 1

DO 181 IP = 11,12
GO TO (883,882):MXPLN

882 UMAX = MAX1F(UZ(IP,MP),UMAX)
UMIN = MIN1F(UZ(IP,MP),UMIN)
GO TO 881

883 UMAX = MAX1F(VZ(IP,MP),UMAX)
UMIN = MIN1F(VZ(IP,MP),UMIN)
GO TO 881

881 IF (NNDOM-B) 333,181,181

This section now bypassed

INNER MESH INTERROGATION

333 IFX = (IFEXO(IP,MP)*I)/100 + 1
INDI = IFEXO(IP,MP)
GO TO (192,193,194,194)*IFX

192 MODSG = INDI
CALL MOD(MFRST)
DO 195 MPP=MFRST+B,NNDOM
GO TO (195,195,195,195):MXPLN

885 UMAX = MAX1F(VZ1(MODX,IP,MP),UMAX)
UMIN = MIN1F(VZ1(MODX,IP,MP),UMIN)
GO TO 195

884 UMAX = MAX1F(VZ1(MODX,IP,MP),UMAX)
UMIN = MIN1F(VZ1(MODX,IP,MP),UMIN)
195 CONTINUE
GO TO 181

193 MODSG = INDI
CALL MOD(MFRST)
DO 196 IPP=MFRST+B,NNDOM
GO TO (196,196,196,196):MXPLN

887 UMAX = MAX1F(UZ1(MODX,IPP,MP),UMAX)
UMIN = MIN1F(UZ1(MODX,IPP,MP),UMIN)
GO TO 196

886 UMAX = MAX1F(UZ1(MODX,IPP,MP),UMAX)
UMIN = MIN1F(UZ1(MODX,IPP,MP),UMIN)
196 CONTINUE
GO TO 181

194 MODSG = INDI
CALL MOD(MFRST)
CONVECTIVE VELOCITY CALCULATION

DO 198 MPP=MFRST+8*NMODX
DO 198 IPP=MFRST+8*NMODX
GO TO (608,889,1MXPLN
889 UMAX=MAX1F(UZ2(MODX,IPP,MPP),UMAX)
UMIN=MIN1F(UZ2(MODX,IPP,MPP),UMIN)
GOTO 198
888 UMAX=MAX1F(UN2(MODX,IPP,MPP),UMAX)
UMIN=MIN1F(UN2(MODX,MODX,IPP,MPP),UMIN)
198 CONTINUE
181 CONTINUE

CONVECTIVE VELOCITY CALCULATION

187 GO TO (281,282,1MXPLN
281 UCONN=(UMAX+UMIN)/2*0
CMACH=UCONN/SVE(1)
DXBM=(CB1*(1+CB3)*(1-EXP(-XOLD/(2*RADC)))*CMACH)*(UMAX+UMIN)/
1*(UMAX-UMIN)
IF (KA-3) 675.873.873
673 IF (XOLD) 674.874.875
874 MXPLN=2
GO TO 255
282 UCONP=(UMAX+UMIN)/2*
CMAP=UCONP/SVE(1)
DXBMD(I+M)=(CB1*(UMAX+UMIN))/(UMAX-UMIN)
X DETERMINATION
X=(DXBM+DXBMD(I+M))/2*0*BM
GO TO 876
876 X=XOLD(I+M)+(DXBMD(I+M)+DXBM)/2*0*(BM-BMOLD)
876 TAUX=SQR(T(I+M)/DXBM
DXBMD(I+M)=DXBM

EDDY VISCOSITY AND CONVECTIVE CALCULATIONS

XRA=X/(XOLD+0.00001)
IF (KA-2) 345,345,346
345 UCONV(I+M)=UCONN
346 UCOVR=UCONN/UCONV(I+M)
DLNOC=LOGF(UCOVR)/LOGF(XRA)
UCONV(I+M)=UCONN
EVISN=RH0*UCONN*BM/(2.0*DXBM)
DEVIS=(EVISN-EVIS(I+M))/(X-XOLD)
EVIS(I+M)=EVISN
ATTENUATION CALCULATION

IF (PADL-SIG) 199,199,201
199 RADP=RADL
GO TO 202
201 RADP=SIG
202 EFUNC=1.0-RADP*EAND2/(RADL*AREAC)
XEFC=CMACH*BM*EFUNC+CMACH
EVISN=EVISN+DEVL*XEFC*2.0
FREQ=FREQ
FREQ=C/UCONN/X
ATRKR = FREQ * FREQ * EVISH * (4/3 + (GAM - 1.0)) / PRADL / (RHOE(1)*SVE)

ATTN = EXPF(-ATRKR*XEFEC)

FREQUENCY NEST INDECES

IFREQ = OCTO * LOGF (FREQ / FREOB) / 0.69315 + 1.0
NFREQ = 10 * LOGF (FREQ/FREOB)/0.69315+1.0
NFREQ = 10 * LOGF (FREQ/FREOB ) / 0.69315 + 1.0
GO TO 562
GO TO 215

This section now bypassed

MATRX SUBDIVISION

205 IF (NDOM - 8) 355, 355, 125
356 IF(1FEX(I+M) -200) 352, 392, 125
352 IF (IFEX(I+M)) 354, 354, 353
353 NREJ = INREJ + 1
    NREJ(INREJ) = MODF(IFEX(I+M)-1,100) + 1
354 IF (I-1) 125, 125, 129
129 IF (I-1) 125, 125, 133
133:NDI = 1
    I= ((I-1)+(M-2))/(M-1)+1
    SFSG = SFT(I+M)-SFT(I+M-1)
    IF(ABS(SFSG)-0.1) 152, 152, 132
135 INDI = INDI + 1
152 SFSGP = SFT(I+M)-SFT(I-1+M)
    IF(ABS(SFSGP)-0.1)125, 125, 158
156 INDI = INDI + 2
    NVPS = 2
GO TO 125
337 GO TO (125, 167, 168, 169)*NDI
167 IFIN = IFIN+1
    IFEX(I+M) = IFIN
    MODG = IFIN
    SIG = SIG+SETI-7.5)*DSIGP
    MAT = 2
    DERAD = DSIGP * DEL
    CALL MODS(MFRST)
    IF (NDOM-8) 336, 562, 562
336 DO 203 WH=MFRST, 8+NDOM
GO TO 153
172 UNI(MODX1+MM)=UNI(I+M)
GO TO (416, 417)*MBPAS
417 UZI(MODX1+MM) = UNI(I+M)
416 SIG = SIG + DERAD
GO TO 125
203 CONTINUE
GO TO 562
168 IFIN = IFIN + 1
    IFEX(I+M)=IFIN+100
    MODG=IFIN
    PHI=PHI +(SETI-7.5)*DPHIP
    DEPHI = DPHIP * DEL
    MAT = 3
    CALL MODS(MFRST)
    IF (NDOM-8) 229, 562, 562
DO 209 IM=MFAST +8*NMDOM
   GO TO 153
206 UNI(MOEX:IM,M)=UNI(M,M)
   GO TO (418,419),MBPAS
419 UZI(MOEX:IM,M)=UNI(M,M)
418 PHI = PHI + DEPHI
   GO TO 125
209 CONTINUE
   GO TO 562
169 IFIN2=IFIN2+1
   IFEX(I+M)=IFIN2+200
      MODSIG=IFIN2
      SIG = SIG + (SET1-7*5)*DSIGP
      PHI = PHI + (SET1-7*5)*DPHIP
      DERAD = DSIGP *DEL
      DEPHI = DPHIP *DEL
      MAT=4
   CALL MODS(MFAST)
   IF(NMDOM-8) 223,562,562
223 DO 211 MM=MFAST+B*NMDOM
   DO 212 IM=MFAST+B*NMDOM
   GO TO 153
207 UN2(MOEX(MOEX :IM,M)=UNI(M,M)
   GO TO (501,502),MBPAS
502 UZ2(MOEX, IM,M)=UNI(M,M)
501 PHI = PHI + DEPHI
   GO TO 125
212 CONTINUE
   SIG = SIG + DERAD
211 CONTINUE
   MAT = 1
   GO TO 562

C       SOUND POWER FACTOR
C
125 TU2RO=TAU/RHO*TAU
   DSFL(I+M)=DSFL(I+M) + DSFP(DERAD,DEPHI)
   GO TO (562,344),NVPAS.
C
C       RATIO CALCULATIONS
C
562 RR = SIG/RADC
   SR1=SHEAR/SHDIV
   SR2 = SHEAC/SHDIV
   SR3 = SHEAT/SHDIV
   RHOR = RHO/RHODIV
   URAT = UNI(M,M)/UDIV
   TTR=TT/TDIV
   TR=T5/TDIV
   BHR = BM/X
   WRITE OUTPUT TAPE NT3:20*RR*PHI*RHOR*URAT*TR*TTR*SR1*SR2*SR3
C
C       INTERMEDIATE BYPASS REGISTER
C
   GO TO (152,541,215),IBPAS
541 GO TO (344,152),NVPAS
C
SOUND POWER CALCULATION

215 FRED = NFREQ - NFREQ
IF(ABS(FRED) > EPS, 226 + 226.357
357 MIX=0
FR= FREQ*(KA*MIX)*UCONNX/XOCDL*(1+M)
IFR= 10^LOGF(FREQ/FREQ(1/FREQ))/*69315 +1
IDIF = IFR-NFREQ
IF(IDIF) 358, 362, 363
358 IF(DPSL(NFREQ+KA+MIX+1)) 362, 362.363
IF(MIX-20) 363, 364, 365
363 MIX = MIX +1
GO TO 357
362 DPSL(NFREQ+KA+MIX)*DPSL(NFREQ+KA+MIX)*DPSL*CA/P2*FLPH/A5R
364 DPSL(NFREQ)*DPSL(NFREQ)+DPSL*CA/P2*FLPH/A5R
DPSL=0
GO TO (356+506)*LFINL
356 DDFSPL = DDFSPL(1+M)*DXBM=7/(1-DSLNUC)*UCONNX*ATTN
NFREQ = NFREQ
226 DDFSPL = DDFSPL(1+M)*DXBM=7/(1-DSLNUC)*UCONNX*ATTN
IF(I=11) 273, 273, 276
275 IF(I=15SY) 276, 273, 273
273 DDFSPL=DDDFSPL/2
276 DSL = DSL+DDDFSPL
U1=UN1+M**8
U8=U81+U8*DEPHI*DESIG
WRITE OUTPUT TAPE NTG+10+R+PHI+TAU+DXBM*ATTN+DLPNC+DDDFSPL+FREQ*X
GO TO 166

MASS MOMENTUM AND ENERGY INTEGRATIONS

344 TAS1=SIG*RHO*UN1(1+M)*DESIG*DEPHI
TM1=TMAS1+UN(1+M)
TNTPI=TNTLP+TMAS1
GO TO (284+278)*NVPAS
284 IF(I=11) 277, 277, 283
273 IF(I=15SY) 276, 277, 277
277 TMAS1=TMAS1/2
TM1=TM1/2
TNTPI=TNTLP/2
278 TMAST = TMAST + TMAS1
TMIT = TMIT + TM1
TNTPT = TNTPT + TNTLP
IX=X/CDIM**2
233 GO TO (166+203+209+212)*MAT

TERMINATION OF ANGLE LOOP

166 XOCDL(1+M) = X
LINE3=LINE3+1
LINE6=LINE6+1
LINE7=LINE7+1
IF(LINE3=40) 241, 241, 244
241 WRITE OUTPUT TAPE NT3+300+BM+NC
244 IF(LINE6=40) 242, 242, 245
242 WRITE OUTPUT TAPE NT6+300+BM+NC
245 IF (LINE7=40) 243, 243, 152
243 WRITE OUTPUT TAPE NT7=300,BM,NC
152 PHI=PHI+DPHI

C  TERMINATION OF RADIUS LOOP
SIGNAL+DSIGNAL-BUG
286 BUG=0
   GO TO (998,998,288)+BPAS
288 RADL = SIG
       MRADL = M
   BMADL = BM
   GO TO (326,325)+BPAS
325 BM = BMINT
   GO TO 327
326 BM= +FRADL + BM
   LFIM=2
   GO TO 357

C  ACOUSTIC BAND SEARCH AND ADDITION

C  CALL ORDERS (NOCFD+DSPS+ISPL)
  JK=1
  JMJ=1
  JEND=0
  JDSEQ=10/OCTD
  NOJ=NOCFD-1
  NOJP=NOCFD
459 DO 453 J MJ=JMJ+NOJ
   IF (DSPS(JM)+453+458
458 JJP=JMJ+1
   DO 454 JJP=NOJP
      GO TO (457+454)+JK
457 IF (DSPS(J)+454+455
454 NSIGNAL=NOJP
      GO TO 456
455 ISPCLUSION(JM)=ISPCLUSION(JM)
   IF (DSPS(JM)+452+451
452 DSPS(JM)*DSPS(JM)+DSPS(J)
   ISPCLUSION(JM)=DSPS(JM)*FLOATF(ISPL(J))+DSPS(JM)*FLOATF(ISPL(JMM))+DSPS
1(JM)+DSPS(JM)
   ISPCLUSION(JM)=99
   DSPS(JM)=1
   JEND=1
451 CONTINUE
   JSIGNAL=JMJ
   IF (JEND) 374:374,453
453 CONTINUE
   GO TO 374
456 NOJ=NSIGNAL-1
   NOJP=NSIGNAL
   JMJ=2
   JK=2
   GO TO 459

374  INNER INDEX TERMINATION

C  327 NMJ=M+(20.*BM/RADC+10.1)+SET2+1
     IF (NMJ=X-20) 600,600,601
601 SET2 = SET2*2
ISET = ISET + 1
MRADL = (MRADL+1)/2
NMDOJ = SET2
GO TO 327
NMDOJ = 2 ** ISET
600 ISETI = LOGF (BMINCDIN * 0.151)
SET2=2**ISETI
IF (NMDOJ=8) 531.532.532
531 NMDOJ= SET2
532 DEL = NMDOJ
SET2=ISETI
TERMINATION OF BM LOOP
C
WRITE OUTPUT TAPE NT3.303
* DO 141 IFREQ=1.NOCT
WRITE OUTPUT TAPE NT3.5BM.5PS(IFREQ).IFREQ=U81
141 CONTINUE
185 CONTINUE
This section not operational
BAND ASSIGNMENT AND SHIFT
C
383 DO 386 IF=1.NOCT
KSIG = 0
MIXT=20
DO 386 M=1.MIXT
CALL ORIGERS (NMCM+DSPL*KAPP)
DO 386 KA=2.NMCM
MIXT=MIXT(M1)
NDRA=1
IF (DSPL(IF+KAM) 391.395.395
391 IF(IF=1) 395.395.665
665 IF(DSPL(IF+1+KA-M) 393.394.394
393 IF(IF=2) 395.395.666
666 IF(DSPL(IF+2+KA-M) 395.396.396
396 NDRA=2
394 IF (IF=NOCT-1) 667.395.395
667 IF(DSPL(IF+1+KA-M) 397.398.398
397 IF(IF=NOCT-2) 668.395.395
668 IF(DSPL(IF+2+KA-M) 395.399.399
395 GO TO 392
398 GO TO (661.666.1) NDRA
661 DSPL(IF+KA-M) = (DSPL(IF+1+KA-M) + DSPL(IF+1+KA-M))/2
662 DSPL(IF+KA-M) = (DSPL(IF-2+KA-M) + DSPL(IF-1+KA-M))/2
399 GO TO (663.664.1) NDRA
663 DSPL(IF+KA-M) = (DSPL(IF+1+KA-M) + DSPL(IF+2+KA-M))/2
664 DSPL(IF+KA-M) = (DSPL(IF-2+KA-M) + DSPL(IF+2+KA-M))/2
KAD = KAPP(KA+1)=KAPP(KA)
IF (ABSF(KAD)=1) 385.385.386
385 KAPP = KAPP(KA+1)
KSIG = 1
IF (DSPL(IF+1+KAPP+M) 387.387.388
387 DSPL(IF+1+KAPP+M) = DSPL(IF+KAPP+M)
388 IF (DSPL(IF-1+KAPP+M) 389.389.391
389 DSPL(IF+1+KAPP+M) = DSPL(IF+KAPP-1+M)
392 DSPL(IF+KAPP-1+M) = (DSPL(IF+KAPP-1+M) + DSPL(IF+KAPP+1))/2
386 CONTINUE
IF(KSIG-1) 390.383.383

390 DO 384 IF=1,NDCT
   DO 384 M=1,20
      DO 384 KA=2,NMBM
      SPL(IF) = DSPL(IF,KA,M) + SPL(IF)
      SAFD = SPL(IF)/0.0000000000000001 +10
      DB(IF) = 10*LOGF(SAFD)/0.268
   384 WRITE OUTPUT TAPE NT3,23,1F,FRE1(IF),DB(IF)
   CALL COPYNT6+1
   CALL COPYNT7+1
   CALL COPYNT8+1
      END FILE NT4
      999 CONTINUE
   REWIND NT4
   CALL EXIT
      END
SUBROUTINE FLMA(NC,IMS,KAI)
COMMON PI*PI2,PADL,PRDQ,FREQ1,IMH,BM,KE,NEST,NUM,LEAV,PHI,ALPD,
IRASI,CDM1,ISAP,BMINT,RADC,SP,JR,SJC
DIMENSION MML(8),KAL(8),SF(30),SH(30),SRJ(30),SJC(30),SFR( ",8)
IMI(8),SJCmI(8),SRP(8)
IF(NC-1)99920,50
20 IF(I-I)99+21,25
21 IF(M-I)99+22,28
22 IAP=1
IF(KA-I)99+23,27
23 FL=LOGF(FREQ)
RAB=RADC/BMINT
SET2=1
MML(1)=1
IFG=1
MAXM=50
MIT=1
24 KAL(MIT)=LOGF(RAB)*SET2+.5)/FL+O+9999
IF(KAL(MIT)-NMBH)25,40,26
25 MIT=MIT+1
SET2*SET2+SET2
GO TO 24
26 KAL(MIT)=NMBH
GO TO 40
27 REWIND NT5
IJU=NMD0J
MO=1
GO TO 34
28 MIT=(M-1)*NMD0J
IF(MIT-MAXM)30,30,29
29 MAXM+MIT
MML(ISET)+M
IFC=1
GO TO 40
30 MLO=MLO+MJU
IF(MLO<MML(IAP))32,32,31
31 IJU=MJU/2
MLO=ILO-IJU+(M-1)
MJU=IJU*(3-M)+(MML(IAP)-1)/2
IAL+IAL+1
32 MIT=MJU+1
CALL FILE(NT5,MIT)
34 MJU=IJU
IFC=0
GO TO 38
35 IF(IFIC)99+36
36 DO 37 MAT=2,1IJU
37 READ TAPE NT5, MIT
38 READ TAPE NT5,MIT,MAT,(SF(KAP,K),SHH(KAP,K),SJC(KAP,K),SJRP(KAP,
K),KAP=MIT,MAT,K=2,NEST)
33 DO 39 K=2,NEST
SF(K)=SF(KAP,K)
SJR(K)=SJRP(KAP,K)
SJC(K)=SJC(KAP,K)
39 SH(K)=SHH(KAP,K)
WRITE TAPE NT4, (SF(K),SH(K),SJR(K),SJC(K),K=2,NEST)
RETURN
SUBROUTINE FLUX(MIT,SFP,SHP,SJCP,SJRP)
COMMON P1,P2,PRAD,PRASQ,FREQ,IMH,BM,XE,NEST,NUM,LEAV,PHI,ALPO,
1RA,SIG,CDIM,ISAP
DIMENSION SIC(*),SIR(*),VI(*),TI(*),CMX(*),VAO(*),TAO(*),SA
1DO(*),SACO(*),XE(*),NUM(*),LEAV(*),ISAP(*),ALPO(*),DALP(*),SFP(*),
230,RA(*),SHP(*),SJCP(*),SJRP(*),REF(*),KAP(*),KAPC(*),KAPC(*),KAPC(*),
302,NEST
DO 193 K=2,NEST
NODE=2
BO=BM
DO 201 IB=1,MIT
SIC(IB)=0.0
SIR(IB)=0.0
VI(IB)=0.0
TI(IB)=0.0
CMX(IB)=BO+XE(K)
201 BO=BO+FREQ
NUMK=NUM(K)
LEAF=LEAV(K)
ISAP=ISAP(K)
30 PIAL=PHI-ALPO(K)
31 ABPA=ABS(PHAL)
1F(ABPA-P1)34.3432
32 PHAL=PHAL-SIGNF(P12PHAL)
33 GO TO 31
34 COSPA=COSF(ABPA)
RADO=SQRTF((RA(NUMK+K)-SIG)*RA(NUMK+K)-SIG)*2.*RA(NUMK+K)*SIG*
1(COSPA))
1F(RADO-0.0005*CDIM)35.3536
RADO=0.0
35 NODE=1
GO TO (44.36.44)*ISAFE
36 COSTO=(SIG-RA(NUMK+K)*COSPA)/RADO
1F(ABSF(COSTO)-1.0138.37.37
37 THD=.(PI-SIGNF(P1+COSTO))/2.
SINTO= 0.0
GO TO 39
38 SINTO = SIGNF(SQRTF(1.0 -COSTO*COSTO)*PHAL)
40 THO=ATANF(SINTO/COSTO)
   IF(COSTO) 151,152,152
151 THO=THO+PI
GO TO 154
152 IF(THO) 153,154,154
153 THO=THO+PI2
154 GO TO (39,143,39,1)SAPE
   CL=RADO/RA(1+1)
   CRA=RA(1+1)
   CALL CFLUX(MIT,VI,TL,SIC,CL,CRA)
   DO 210=1+1,MIT
      S1C(IB)=SIC(IB)*COSTO
      S1C(IB)=SIC(IB)*SINTO
      S1R(IB)=S1R(IB)*SINTO
      S1R(IB)=S1R(IB)*COSTO
   GO TO 143
39 DO 203 IB=1,MIT
   RADX = RADO/CMX(IB)
   VAO(IB)=1.0-EXPF((-RADX*RADX)
   GO TO (41,42,1,1)
41 TAO(IB)=1.0-EXPF(-(RADX*PRADL)*(RADX*PRADL))
42 SA =1.0-GRAY2*ERRORF(RADX) + RADX * (VAO(IB)=1.0)*CM
   SARO(IB)=SA *COSTO
   SACO(IB)=SA *SINTO
203 CONTINUE

C LEAF INTEGRATION

44 DO 134 J=1,LEAF
DO 134 N=1,NMK
46 PHAL = PHAL - DALP(N+K)
47 ABPA = A65F(PHAL)
   IF (ABPA - P1) 50,50+AB
48 PHAL = PHAL - SIGNF(P12+PHAL)
49 GO TO 47
50 COSPA = COSF(ABPA)
   RAD = SQRTF((RA(N,K)-SIG)*(RA(N,K)-SIG)+2.0*RA(N,K)*SIG*(1.0-
   1/COSPA))
   IF (RAD < 0.0005 * SIG) 51, 51, 52
51 NODE = 1
GO TO 132
52 COST = (SIG-RA(N,K)*COSPA)/RAD
   IF(ABS(COST)-1.0)53,53,53
53 TH = (P1-SIGNF(P1*COST ))/2*
   SINT = 0.0
GO TO 62
54 SINT = SIGNF(SQRTF(1.0 -COST * COST)*PHAL)
56 TH=ATANF(SINT/COST)
   IF(COST) 161,162,162
161 TH=TH+PI
GO TO 59
162 IF(TH) 163,59,59
163 TH=TH+PI2
59 GO TO (60,62,1)NODE
60 NODE = 2
61 DTH = 0.0
62 GO TO 121
63 ABOTH = ABS(DTH)
64 IF(ABOTH = DTH)121,121,65
65 IF(ABOTH = PI*166+66*101
66 0 - 360 CORRECTION
67 THO = THO + SIGNF(P12* DTH
68 GO TO 62
69 INITIATION OF AUXILIARY INTEGRATION
70 60 = ABOTH/DTHM+1
71 Q = LO
72 DTH=DTH/Q
73 RCRC = RADO*COSTO - RAD*COST
74 RCRC = RCRC + SIGNF(0.00000001, RCRC)
75 70 ABLE=(RADO*SINTO-RAD*SINT)/RCRC
76 71 RAR=RADO*SINT=ABLE*RADO*COSTO
77 AUXILIARY INTEGRATION
78 DO 92 L=1:LO
79 TH=THO + DTH
80 COST = COSF(TH)
81 SINT=SINF(TH)
82 RADO=KRO/(SINT-ABLE*COST)
83 DO 204 IB=1:MIT
84 RADO=RADO/CMX(1B)
85 VA = 1.0 - EXPF (-RADX * RADX)
86 SA = (0.88623 * ERRORF(RADX) + RADX * (VA-1*0)) *CM
87 SAR = SA * COST
88 SAC = SA * SINT
89 VI(1B)=VI(1B)+(VA+VA0(1B))*DTH
90 SIC(1B)=SIC(1B)+(SAC+SACD(1B))*DTH
91 TIR(1B)=TIR(1B)+(SAR+SARD(1B))*DTH
92 GO TO (90+91)*IMM
93 TAO(1B)=TA
94 VA=1B*VA
95 SACD(1B)=SAC
96 SARD(1B)=SAR
97 CONTINUE
98 THO = TH
99 GO TO 132
100 MAIN LINE INTEGRATION
101 DO 205 IB=1:MIT
102 RADO=RADO/CMX(1B)
103 VA = EXPF (-RADX * RADX) + 1
104 SA = (0.88623 * ERRORF(RADX) + RADX * (VA-1*0)) *CM
105 SAR = SA * COST
106 CONTINUE
107 -52-
124 SAC = SINT * SINT
   VI(IB) = VI(IB) + (VAO(IB)) * DTM
126 SIR(IB) = SIR(IB) + (SAR + SARO(IB)) * DTM
   SIC(IB) = SIC(IB) + (SAC + SACO(IB)) * DTM
   GO TO (127, 128) TH
127 TA = EXPF1 - (RAX*PRADL) * (RAX*PRADL) + 1.0
   TI(IB) = TI(IB) + (TA + TAO(IB)) * DTM
   TAO(IB) = TA
128 VAO(IB) = VA
   SACO(IB) = SAC
   SARO(IB) = SAR
205 CONTINUE
   THO = TH
132 RADO = RAD
   COSTO = COST
134 SINT0 = SINT
143 SPF(IB,K) = VI(IB)
   GO TO (191, 192) IMH
192 TI(IB) = VI(IB)
191 SMF(IB,K) = TI(IB)
   SJP(IB,K) = SIC(IB)
193 SJRP(IB,K) = SIR(IB)
RETURN
END

SUBROUTINE CFLUX (MIT, VS, TS, SSC, CL)
COMMON P1, P12, PRADL, PRADQ, FREQ, IMH, IM, XE
DIMENSION ARR(40), VS(30), TS(30), SSC(3), XE(20), SFL(40)
   ISJ = 1
   DL1 = 20.
   DL2 = 4.
   EXPD = (1. - CL) * (1. - CL)
   CLJ = 1. / CL
   CL2 = CL + CL1
   CL1 = CL - CL1
   VLA = ABSF(CL - 1.0)
   DL = MAX(0.1, CL, CL1, (CL - 1.0) / DL2)
   VLD = VLA
   B = 0.16666667
   A = 0.33333333
   SFO = 0.
   SFL(1) = 0.
   INCREMENTS AND AREAS
   IF(CL - 1.0) 203, 205, 204
204 NS = 1. / DL + 9999
   VN = NS
   DL = 1. / VN
   ARR(1) = 0.
   GO TO 206
203 NS = CL / DL + 9999
   IF(NS) 231, 231, 224
204 VN = NS
   DL = CL / VN
   ARR(1) = 1. * EXPOC
206 NS = NS + NS + 1
   DC 215 15 = 2 + NS
VL=VL+DL
VLS=VLS+VL
CPH12=CL1/VL*VL/CL
SPH12=SQR(4-CPH12*CPH12)
PHI=ATAN(SPH12/CPH12)
IF(PHI) 214+215+215
PHI=PHI+PI
CTHE2=CL2-VLS/CL
THE=ATAN(SQR(4-CTHE2*CTHE2)/CTHE2)
IF(THE) 217+275+275
THE=THE+PI
275 SFL(IS)=(B+SOF+A*SPH12)*DL +SFL(IS-1)
SFO=SPH12
C=A
A=B
C=C
215 ARR(IS)=VLS*PHI+THE-VL/CL3*SPH12
C FLUX INTEGRATIONS AT SEQUENTIAL B PLANES
BO=BM
DO 233 IB=1,MB
VI=0*
TI=0*
SI=0*
BO=BO+XE(I)
IF(BO) 280,280,281
280 XSO=1000000.
GO TO 296
281 XSO=CA*BV /BD
XSO=XSO*XSQ
280 BO=BO+XREGJ
IS=15+1
ISO=1
DLJ=DL+FLOBT(1JS)
VL=ABS(CL-1
VL=VL+DLJ/2
272 VLS=VLS+VL
EXPAR=VLS*XSO
EXPV2=EXP(-EXPAR/2*)
EXPV=EXPV2*EXPV2
ARD=ARR(IS)-ARR(IS0)
VI=VI+ARD+EXPV
GO TO (270+271+1)MH
270 TI=TI+ARD+EXPV(-EXPAR+PRASO)
271 SIC =SIC +EXPV*(1+EXPAR+EXPV-EXPV2..*(SFL(IS)-SFL(IS0))
ISO=IS
VL=VL+DLJ
IS=XMINOF(IS+ISJ+NS)
IF(IS-NS) 272+296+296
296 VS(IB)=VI/P1*XSO
TS(IB)=TI/P1*XSO
SSC(IB)=SIC/P1*XSO
IF(ARR(I)) 233+233+298
298 VS(IB)=1-EXP(-EXPOC*XSO ) +VS(IB)
GO TO (299+295)+1MH
295 TS(IB)=VS(IB)
GO TO 233
299 TS(IB)=1-EXP(-EXPOC*XSO ) +PRASO+TS(IB)
233 CONTINUE
RETURN
END
LIST OF SYMBOLS

A
area

A_c
area of circle having a radius equal to \( R_{c,e} \)

A_n
nozzle exit area

C_a
acoustic constant

C_{b1}
constants

C_{b2}

C_f
frequency constant

C_x
length constant

E_f
porosity parameter for attenuation

H
enthalpy flux function (Eq. (7))

K
concentration flux function (Eq. (7))

M
Mach number

M_C
convective Mach number

M_f
momentum flux function (Eq. (5))

P
pressure (static)

P_a
acoustic power

Pr
Prandtl number

R
geometric radius (circular nozzle exit radius)

R'
flow-field coordinate

R_c
largest radius of the mixed flow field

R_g
gas constant

T
temperature
- bounding velocity
- convective velocity
- volume
- attenuation length measure
- acoustic velocity
- momentum spreading parameter
- enthalpy spreading parameter
- coefficient
- specific heat
- mean specific heat
- frequency
- frequency at point of maximum noise production
- frequency determined by Strouhal relation
- enthalpy
- relative enthalpy (Eq. (A.17))
- local concentration
- eddy heat-conduction coefficient (Eq. (A.3))
- radial coordinate (polar) (for elemental jet flows) (Fig. 10)
- variable of integration (Fig. 10)
- polar coordinate (radial) (Fig. 10)
- temperature (variable of integration)
- local mean velocity, (x-directed)
- fluctuating component of the x-directed velocity
- local mean velocity (r-directed)
\( v' \) fluctuating component of the \( r \)-directed velocity

\( w \) watts

\( x \) axial coordinate

\( \overline{\Delta V} \) space correlated mean eddy volume

\( \alpha \) attenuation energy ratio, coordinate angle (Fig. 10)

\( \alpha_k \) Kirchhoff attenuation parameter

\( \gamma \) ratio of specific heats

\( \lambda \) specific heat coefficients

\( \eta_e \) eddy viscosity coefficients

\( \rho \) density

\( \tau \) shear stress

\( \theta \) angle coordinate (Fig. 10)

\( \xi \) velocity parameter (Eq. (8b))

\( \sigma \) eddy characteristic dimension

Subscripts

1 adjacent gas flows

2 exit
e exit
i gas (concentration) index

\( i,j \) tensor components

o ambient

m maximum

n exit element index

r relative (enthalpy) (Eq. (A.17))

s vector directions

\( \phi \) vector directions

Superscripts

\( o \) stagnation

\( - \) mean
REFERENCES


22. Mixing of Compressible Fluid Streams, United Aircraft Corporation, Research Department, East Hartford, Conn., Report R-12006-02, Reported by: K. K. Klingensmith, Approved by: Reeves Morrison, February 17, 1948, Copy No. 34.


24. Rosensweig, Konrad E.: Measurement and Characterization of Turbulent Mixing, Fuels Research Laboratory, Department of Chemical Engineering, Massachusetts Institute of Technology, May 18, 1959.


Figure 2.1 - End View of a 180° Sector of the 12 Tube Nozzle
Figure 2.2(a) - Mean Velocity Profiles for the 19 Tube "Ozzle"
Figure 2.2(b) - Mean Velocity Profiles for the 23-inch Nozzle

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Figure 2.2(c) - Mean Velocity Profiles for the 1/2 Tube Nozzle
Figure 2.2 (d) - Mean Velocity Profiles for the 19 Tube Nozzle
1. Vertical division by four.
2. 4 by 4 division
3. Nesting of two 4 by 4 division in memory storage.
4. Horizontal division by four.
5. No division

Points interrogated for the 4 by 4 inner mesh sub-division.

Note: Cross hatching indicates area of integration.

Figure 2.3 - Matrix division, integration, and superposition for b_2 plane.
Figure 2.7 - Velocity profile comparison.

(a) $x/R = 1.14$

$C_{b1} = 12.5$

$C_{b2} = 1.5$

Data Reference 7
Figure 2.7 - Continued.

(b) \( x/R = 4.58 \).
Distance from centerline, $r/R$ vs. Velocity ratio, $\bar{u}/\bar{u}_e$.

(c) $x/R = 8.0$.

Figure 2.7 - Continued.
-176-
(d) $x/R = 16.0$.

Figure 2.7 - Continued.
(e) \( \frac{x}{R} = 32.0 \).

Figure 2.7 - Concluded.
Data reference 26

Figure 2.8 - Shear stress prediction.
Figure 2.9 - Hexagonal ejector approximation.
Figure 2.10 - Flux integration coordinate nomenclature.
I. Case data

- Modified input deck
  - Short flux subroutine deck
    - Machine subroutine deck
      - Main program deck

Figure 2.11 - Loading sequence for circular exit program.
### Table

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<th>Nozzle Minimum Radius</th>
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### Figure 2.12a
- Typical input.
Figure 2.12b - Typical input. (Concluded)
Figure 2.13 - Typical Output

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<tr>
<th>Distance Ratio</th>
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<th>Enthalpy Flux</th>
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3.0 Investigation of the Effects of Flow Temperature and Pressure on Jet Noise Generation and Suppression

3.1 Introduction

In order to apply jet noise suppression concepts to afterburning engines, an understanding of jet noise generation and suppression at high flow temperatures must be developed beyond the present state-of-the-art. The validity of the correlation between free jet mean-flow velocity profiles and noise generation and suppression by heated jets has been demonstrated in Section 1 of this report, but the flow temperature range investigated does not extend to levels achieved by afterburners. This section describes the results of acoustical investigations, carried out in addition to the original scope of the research program, for the purpose of providing information on noise generation by high temperature jets, for both simple circular nozzles and suppressor configurations. The results of this study provide basic information on noise generation by high temperature jets, and give further insight to the problem of evaluating noise suppressor performance by means of cold flow scale model tests.

Experimental investigations have been carried out to evaluate the jet noise generated by a conical nozzle over a range of flow temperatures and pressure ratios, extending up to a temperature of 3300°F and pressure ratio 3.4. The jet noise spectra and overall sound power levels are presented, and the effect of jet density on noise generation is described. The results of the high temperature scale model conical nozzle tests, as well as concentric jet and full scale engine data have been used to establish an empirical relationship between nozzle energy flux and total sound power generated, applicable to concentric jet flows of unequal velocities and temperatures as well as to
conical nozzles at all pressure ratios and temperatures.

Scale model acoustical evaluations of suppressor-nozzle noise reduction performance at high temperatures have been accomplished, with results indicating that jet noise suppressor action is temperature as well as pressure ratio dependent. Data obtained indicates that noise reduction may be more difficult to achieve at high flow temperature, and that suppressor scale model evaluations must be carried out with heated flow or suitable substitute gas. Further experimental activity in the area of jet noise generation and suppression at flow temperatures in the afterburner range is recommended for the purpose of defining generalized temperature scaling laws applicable to all nozzle configurations.

3.2 Noise Generation by High Temperature Jets

Experimental investigation on noise generation by jets operating at temperatures up to 3300°R have been carried out utilizing a free-field acoustic test facility (Figure 3.1), described in Section 1.3.1 of this report. Tests were conducted with a conical nozzle 3.85 inches in diameter at the exit, operating at pressure ratios of 1.7, 2.0, 2.5, 3.0, and 3.4, varying the flow total temperature from 560°R to 3300°R for each pressure ratio condition. Sound pressure spectra were measured along a circular arc at 15 feet from the nozzle exit, and at various angles from the discharge direction. Sound power spectra were computed by integration of the sound pressures over the hemispherical radiation surface.

3.2.1 Instrumentation

The flow conditions which were controlled and monitored during the experimental investigation were total pressure and total temperature at the inlet to the nozzle. An 8 inch diameter flow section preceding the nozzle
inlet contained a static pressure tap and three IR vs IR-Rh thermocouples which were continuously monitored during the tests. The area ratio of approximately 4:1 from the measuring section to the nozzle throat provided an effective plenum upstream of the nozzle inlet.

Acoustic measurements were made using a Brüel and Kjaer type 4133 condenser microphone system which was calibrated by means of a B & K pistonphone. The microphone system frequency response was flat to 40 kcps, covering the entire frequency range of measurements without need of response correction. Microphone output was filtered through a B & K 1/3 octave spectrometer, over a range of band center frequencies from 200 to 32,000 cps. Graphic representation of the sound pressure spectra was obtained by recording spectrometer output with a B & K 2-level recorder. Sound power spectra and overall PWL were computed from the pressure level spectra obtained at a distance of 15 feet from the nozzle and at angles of 20, 30, 40, 50, 60, 75, 90, 110, 130, and 150° from the flow direction.

3.2.2 Test Results

The measured total acoustic power levels and sound power spectra are presented in Figures 3.2 and 3.3a through 3.3d, for the range of nozzle flow conditions investigated. In Figure 3.2 the overall sound power levels are plotted against flow velocity for the purpose of comparison with the Lighthill eighth-power parameter: $k_p A U^{0.8} c_o^{-5}$. The flow velocity used is a calculated value, assuming that the flow fully expands isentropically from the measured total pressure to the ambient static pressure of 30 in. Hg. As would be expected, excellent agreement with the Lighthill equation is obtained at moderate pressure ratios (up to 2.0) and relatively low temperatures (up to
approximately $1700^\text{R}$. The limits of applicability of the Lighthill parameter are clearly visible in the figure, and a flow temperature influence, apart from velocity, is apparent. Thus it is seen that for estimates of jet noise total power at high flow pressure ratio and/or temperature the Lighthill relationship is inadequate. Figure 3.2 shows that at low flow temperature ($560^\text{R}$) sound power generation increases as $u^{20}$ over the range of pressure ratios from 2.0 to 3.1, while at a flow temperature of $3300^\text{R}$ the overall sound power is proportional to approximately $u^6$. Examination of the increase in sound power resulting from velocity increase due to flow temperature rise at constant pressure ratio shows that at a pressure ratio of 3.0 sound power increase is proportional to $u^2$ from $1100$ to $3300^\text{R}$.

This variation of the velocity exponent from 2 to 20 suggests that the Lighthill parameter might take the form: $k\left(\frac{\rho_j}{\rho_a}\right)^n u^5 c_o^{-5}$. In Figure 3.4 the effect of jet density on sound power generation is shown. Over the entire range of flow conditions investigated the sound power is observed to increase in proportion to jet density squared, considering the data in a rather gross manner. A detailed observation, however, indicates that for a given flow temperature the sound power varies with $(\rho_j/\rho_a)^n$, where $n$ is temperature dependent. The magnitude of this temperature dependence is demonstrated by the substantial change in slope of the sound power vs jet density curve, progressing from temperature of 560 to $3300^\text{R}$. The jet density ratio exponent $n$ is plotted as a function of flow total temperature in Figure 3.5. Values of $n$ vary from +12 to -6, and it is observed that the relationship between $n$ and flow temperature is approximately: $n \propto T_o^{-3}$. For low pressure ratio jet flow at moderate temperatures the flow density effect on jet noise generation is negligible. However, the results of the investigations described herein
extend knowledge of the behavior of jet noise generation to a broader range of flow conditions, including high temperature and high pressure ratio flow which is likely to be experienced in advanced propulsion systems.

3.3 Correlation of Jet Noise with Nozzle Energy Flux

Results of the scale model investigations on noise generation by simple circular jets operating at temperatures up to 3300°R have been used to establish a general empirical relationship between total acoustic power and jet stream power.

Figure 3.6 shows normalized power level, \( (PWL - 10 \log_{10} A) \), plotted as a function of nozzle total energy flux. The scale model conical nozzle data presented is that described in Section 3.2, and turbojet engine noise data is included to demonstrate the generality of the relationship. For simple conical nozzles the energy flux, \( \varepsilon \), is given by:

\[
\varepsilon = \frac{h_t}{A} (W/A)
\]

where \( h_t \) is the flow total enthalpy per pound of fluid.

A tentative method for including noise generation by concentric flows in this correlation has also been developed, and three concentric flow examples are included in Figure 3.6. A table of nozzle flow conditions is also given in the figure. The dual stream concentric flow nozzle utilized in obtaining the noise data is shown in Figure 3.7. Flow parameter variations tested included a heated high velocity outer flow with cold inner flow, and a case with nearly equal flow velocities but with substantial temperature difference between the two streams. The confirmation of the validity of the correlation for concentric flow requires additional experimental investigation, but results obtained thus far are extremely encouraging. The correlation developed is as follows:
Let:
- $A$ = flow area, ft$^2$
- $h_t$ = total enthalpy, Btu/lb
- $h$ = static enthalpy, Btu/lb
- $W$ = weight flow, lb/sec
- $N$ = stream number
- $K$ = relative velocity correction factor
- $U$ = flow velocity, ft/sec
- $U^2/2gJ$ = flow kinetic energy, Btu/lb
- $E_t$ = stream total power, ft-lb/sec
- $E$ = energy flux, Btu/ft$^2$/sec

The stream number, $N$, is defined by the following sketch of typical concentric nozzles:

For each stream ($N = 1, 2, \ldots$)

$$h_{\Delta N} = h_N + (U_N^2/2gJ)K$$  \hspace{1cm} (2)

where

$$K = (U_N - U_{N+1})^2/U_N^2$$  \hspace{1cm} (3)

Now, the total stream power is

$$E_t = \frac{1}{N} \sum h_N W_N$$  \hspace{1cm} (4)

and

$$E = E_t/\sum A_N$$  \hspace{1cm} (5)

By proper substitution:

$$E = \frac{\sum [h_N - (1 - K)U_N^2/2gJ] W_N}{\sum A_N}$$  \hspace{1cm} (6)
For a single stream, Eq. (6) reduces to the simple form of Eq. (1). The significance of Eq. (6) is that for consideration of concentric flows it is necessary to sum the energy flux for each of the jets. The inner jet flux is defined herein as the relative flux with respect to the surrounding jet; thus the inner flux is based upon the static enthalpy plus the relative velocity head.

The efficiency of conversion of jet stream power to acoustical power is seen in Figure 3.6 to be uniquely related to nozzle energy flux. Good correlation between model and full scale engine data has been demonstrated, thus indicating that the nozzle energy flux parameter is quite general and valid in application to the problem of jet noise estimation.

3.4 Effects of Flow Temperature on Jet Noise Suppressor Action

The value of cold flow scale model investigations in the development of jet noise suppressors is well recognized, both from the standpoint of time requirements and hardware expense and complexity. Up to the present time, jet noise suppressor investigations have been carried out under the assumption that noise reduction achieved in cold flow tests can be directly related to expected full scale suppressor noise reduction by Strouhal number scaling. Furthermore, experience has shown that jet noise suppressors in general perform poorly at low pressure ratios, with noise reduction performance (compared to an equal area conical nozzle) improving as pressure ratio increases. In order to apply scale model techniques to the development of a suppressor for afterburning turbojet engines, it was deemed necessary to investigate the effects of high temperature flow on suppressor performance in order to determine the validity of scaling laws and the Mach number effect on suppression.
Scale model suppressor noise investigations were carried out utilizing the nozzles shown in Figures 3.8, 3.9, and 3.10. These suppressor nozzles are of approximately the same flow area as the conical nozzle described previously in this section, and are described in detail in section 1 of this report. Measurements of total acoustic power were made in the same manner as for the conical nozzle, though over a somewhat more limited range of flow pressure ratio and temperature.

Comparisons of total acoustic power generation have been made for the suppressors and conical nozzle. For convenience, the conical nozzle data is replotted in terms of $PWL$ vs pressure ratio, for various temperatures, in Figure 3.11. Figure 3.12 presents corresponding data for the three suppressor models: (1) 8-lobe, (2) 19-tube, and (3) 18-segment with plug. In order to determine the effect of flow temperature on suppression of jet noise, the data of Figures 3.11 and 3.12 has been cross plotted in Figure 3.13, which shows overall $PWL$ as a function of flow temperature for the four nozzles operating at three pressure ratios: 1.8, 2.4, and 3.0. It can be readily observed that for each of the suppressor nozzles the noise reduction (the difference between the conical nozzle and suppressor nozzle curves) at constant pressure ratio is temperature dependent. Furthermore, this temperature dependence appears to be different for each suppressor. Note that at pressure ratio 1.8 little noise reduction is shown for cold flow, and that as temperature is increased the suppression capability increases and then finally declines. At flow temperatures near 1000°F, only slight Mach number (pressure ratio) effect is observed, and for pressure ratio 2.4 little temperature effect is evidenced. It is fortuitous that turbojet engine suppressor development up to the present time has been for engines operating at pressure ratios near 2.4 and temperatures near 1000°F.
since noise reduction at pressure ratio 2.4 shows little temperature dependence. However, at low pressure ratio (1.8) cold flow test results would not provide a satisfactory estimate of sound power reduction experienced at increased temperatures.

While no general means of correlating suppressor performance for all types of suppressors, for various temperatures and pressures, has been developed, the results of the investigation have several important implications. First, evidence has been presented to show that jet noise reduction is not simply pressure ratio dependent for a fixed configuration. Furthermore, the variation in noise reduction performance as a function of flow temperature for different suppressor configurations indicates that model suppressor evaluation must be ultimately performed experimentally at flow temperatures and pressures representing full scale conditions. Finally, the need for further research into normalization of suppressor performance for temperature and pressure effects is emphasized.

3.5 Conclusions and Recommendations

Results of scale model experimental investigations on noise generation by jets from conical and suppressor nozzles have been presented for flow temperatures ranging up to 3300°F. The following important conclusions are summarized:

1) Noise generation by conical jets is not adequately described by the Lighthill equation except for low pressure ratio and moderate temperature flow.

2) A temperature dependent flow density effect on noise generation by conical jets has been found to occur. Total sound power varies with \((p_j/p_a)^n\), where \(n\) is temperature dependent.

3) An empirical relationship between sound power and nozzle energy flux has been developed for conical nozzles, including concentric flows, for the
range of pressure ratios from 1.4 to 3.4, and temperatures up to 3300°R.

Jet noise suppressor acoustical performance is temperature as well as pressure ratio dependent, so that scale model suppressor development must include final experimental evaluation under flow conditions representing full scale engine temperature as well as pressure ratio.

It is recommended that further research effort be directed toward extending the generality of the energy flux concept to complex nozzle designs, including further verification of applicability to concentric flows. Jet noise suppression research should be continued for the purpose of developing general laws for scaling temperature and pressure ratio effects on noise reduction performance.
SYMBOLS:

A - Nozzle throat area, ft²

c_o - Ambient sonic velocity

E_t - Jet stream total power, Btu/sec.

h - Static enthalpy, Btu/lb.

h_t - Total enthalpy, Btu/lb.

k - Constant in Lighthill parameter; relative velocity correction factor in energy flux correlation.

n - Density ratio exponent

N - Stream number for concentric jet flow

P - Static or ambient pressure

P_o - Total pressure

PWL - Sound power level, db re 10⁻¹³ Watts

T_0 - Total temperature

U - Flow velocity, or fully expanded flow velocity for supercritical pressure ratio

W - Weight flow, lb/sec.

ε - Energy flux, Btu/ft² sec.

ρ_a - Ambient air density

ρ_j - Jet static density
Figure 3.2 - Total Acoustic Power Generated by Scale Model Conical Nozzle (Dia. = 3.85 in.) as Compared to Levels Predicted by Lighthill Parameter.
Figure 3.3a - Conical Nozzle Sound Power Spectra
Nozzle Diameter 3.85"
Figure 3.3b - Conical Nozzle Sound Power Spectra
Nozzle Diameter = 3.85°
Figure 3.3c - Conical Nozzle Sound Power Spectra
Nozzle Diameter = 3.85"
Figure 3.3d  - Conical Nozzle Sound Power Spectra

Nozzle Diameter = 3.85°.
Figure 3.4 - Effect of Jet Density on Sound Power Generation.
Figure 3.5 - Effect of Jet Total Temperature on Density Exponent $n$. 

$$PWL = 10 \log_{10} A + 10 \log_{10} \left(\frac{u}{c_o}\right)^6 + 10 \log \left(\frac{c_j}{c_a}\right)^n + K(T)$$
Figure 3.6 - Correlation of Jet Noise Total Acoustic Power with Nozzle Energy Flux.
Figure 3.7  Concentric Flow Nozzle
Figure 3.8  Eight Lobe Nozzle (296)
Figure 3.10  19 Tube Nozzle
Figure 3.11- Conical Nozzle Jet Noise Generation.
Figure 3.12 - Supersonic Nozzle Jet Noise Generation.
Figure 3.13 - Effect of Flow Temperature and Pressure Ratio on Suppressor Acoustic Performance.
4.0 EXPERIMENTAL STUDY ON TRANSMISSION AND DISSIPATION OF SOUND THROUGH A TURBULENT JET WAKE

4.1 Introduction

A major assumption of the jet noise suppression theory relating the aerodynamic and acoustic properties of jet flow is that all of the stream-generated noise radiates, without significant loss, into the far field. For complex suppressor-nozzle designs, the validity of this assumption can be questioned, since sound sources associated with the inner flow elements are surrounded by high velocity, turbulent flow. There exists some experimental evidence that acoustic energy is dissipated in the highly turbulent jet mixing region. In Reference 1, jet noise data is presented showing that sound levels measured in the far field of a rectangular array of nozzles is higher when the measuring location is oriented along the long side of the multiple tube configuration as compared to the short end of the array (Figure 4.1). This effect, referred to as shielding, indicates that perhaps the sound generated by innermost elements of a suppressor is unable to propagate through the turbulent regions created by surrounding suppressor elements. Although theoretical justification for significant absorption of audio frequency sound in a jet wake is lacking, it is important that the possibility be investigated since positive evidence of sound dissipation would require modification of the correlation between acoustic and aerodynamic properties of the jet wake. Furthermore, absorption of acoustic energy in the jet wake could explain in part the increase in noise reduction by a suppressor as pressure ratio is increased.

Experiments have been conducted in an effort to determine whether or not dissipation of sound does occur, or whether the noise reduction effect
referred to as shielding is a directivity rather than power level change effect.

4.2 Experimental Procedure

The determination of the magnitude of sound energy absorption in turbulent flow, as related to jet noise suppression, requires that the power level of a source be determined after propagation through a turbulent jet wake compared to the source power level in the absence of the turbulent jet. Ideally, a discrete frequency sound source, which could easily be distinguished from broad-band jet noise, could be located within a jet wake, and sound power level change could be determined in a reverberant room for various jet velocities. However, the problem of locating such a sound source within a jet wake without altering its acoustic output is quite difficult.

An alternative method for evaluating sound absorption by the jet wake involves placement of the sound source outside the jet boundary while positioning the acoustic pickup within the wake. While this scheme eliminates the source variation problem mentioned above, it presents the difficult problem of extracting the pure tone source signal from the jet noise and jet pressure fluctuations especially for high Mach flow. Even more difficult would be the evaluation of convective effects on the sound propagating through the jet flow. Since the sound path through the wake would depend on flow velocity, comparison of levels measured with and without flow would be exceedingly difficult.

In a third possible approach to the experimental investigation, the high intensity pure tone sound source can be located outside the jet wake, and a survey of sound pressure level is made by traversing a microphone along the jet boundary, but not in the flow, on the opposite side of the wake.
field acoustic environment is required, and effects of changing sound path length
due to convection of the signal in the jet flow can be minimized by locating the sound source sufficiently far from the jet wake. Because of the decreasing flow velocity in the jet in the downstream axial direction, a focusing of the sound propagating through the flow would occur as described schematically in Figure 4.2. The sound waves entering the wake in the high flow velocity region near the nozzle exit are convected further downstream than waves entering in the lower velocity regions of the jet, thus forming the shadow zone near the nozzle exit and somewhat focusing the rays at a downstream location. In order to evaluate the possibility of sound dissipation, the intensity of the transmitted sound must be measured over a suitably long region of the jet so that a valid average sound pressure can be determined, considering both the shadow zone and region of apparent amplification.

Experimental investigations were conducted following the three previously described procedures, but concentrating on the first and third methods. The second scheme, with the sound pickup within the jet wake, presented considerable difficulty in extracting the pure tone sound from the jet pressure fluctuation and thus it was abandoned.

The acoustic signal for which attenuation effects were to be evaluated was produced by a Hartman generator (Figure 4.3). The device consists of a 3/8 inch diameter convergent nozzle which discharges at pressure ratio of 4.5 into a 1/2 inch diameter cavity. The generator produces a high intensity discrete tone. The frequency of the whistle is a function of the cavity depth which can be varied by a piston in the cavity. The acoustic power level produced is on the order of 150 dB (re 10^{-13} watts) over a frequency range of 1200 to 9000 cps.
Experiments on the dissipation of sound with the source within the jet were carried out using the apparatus shown in Figure 4.4. The Hartman generator was located in a cylinder within an annular jet so that the jet flow would not affect its acoustic output. Measurements of reverberation sound pressure level were made in a reverberant room, for various source frequencies and nozzle pressure ratios. Average pure tone sound levels were determined by slowly moving the microphone within the room and reading its output, separated from jet noise by means of a narrow band filter, with an acoustic integrator (Ref. 2). Static pressure was measured within the Hartman generator enclosure, and the maximum variation from ambient was 2.5" H₂O at nozzle pressure ratio P/P₀ = 2.2. This small change in pressure could not affect the whistle performance. Results of the investigation are shown in Figure 4.5, where reverberation sound pressure level for the whistle tone is plotted against nozzle pressure ratio for various pure tone frequencies. A small decrease in sound level is observed as the jet flow begins, but this reduction does not appear to increase as flow pressure ratio is raised. If the sound energy is dissipated in the turbulent flow, then the more turbulent higher pressure ratio flow should provide increased noise reduction. Since this is not the case, a plausible explanation for the slight decrease in sound pressure level is that the supply pressure to the Hartman generator decreases slightly as nozzle airflow is initiated. Since rather small supply pressure variation can affect the whistle output, this is believed to be the source of the noise reduction. In the succeeding experiment, supply pressure was monitored much more closely so as to preclude such an occurrence.
Further experiments were conducted to determine if sound in the audible frequency range is attenuated during propagation through the turbulent flow exhausting from a nozzle. As in the previously described work, a high intensity, pure tone sound source (Hartman Generator) provided the acoustic signal, and a 3-1/2 inch diameter conical nozzle was utilized to create the turbulent wake. The experimental setup was as shown in Figure 4.6. The sound source was located 4-1/2 nozzle diameters downstream of the nozzle exit, and several diameters to the side of the stream. On the opposite side of the jet wake a microphone was traversed over a distance of approximately 4 nozzle diameters. Thus, by positioning the turbulent flow between sound source and acoustic pickup its effect on the acoustic signal could be determined. In order to measure only the direct sound and eliminate reverberation, the Acoustics Laboratory anechoic room was utilized. The room is capable of handling the airflow from the 3-1/2 inch diameter nozzle operating at pressure ratio up to 2.4. To test whether attenuation through the stream could be measured in this facility, a mechanical sound barrier 6" in diameter was positioned between sound source and microphone in the approximate location of the jet wake. The attenuation measured using the mechanical barrier in place of the jet wake was 13 db; sufficient to indicate that reflected and refracted sound is at a level low enough so as not to obscure attenuation through the stream, which might be of lesser magnitude.

Source frequencies of 2.5, 5.0, and 9.0 kcps were used, and nozzle pressure ratios up to 2.4 were tested. So as to obtain a usable signal to noise ratio, a Panoramic Sonic Analyzer with 25 cps filter band width was used as a signal filter for the microphone output.
A plot of sound pressure level along the microphone traverse path, for three nozzle pressure ratios, is shown in Figure 4.7. Note that "shielding" occurs in the region near the nozzle exit, and amplification is experienced further downstream due to convection of the signal in the jet wake. Results indicate that the peak amplitude measured is independent of flow velocity, but that the flow has a pronounced influence on directivity. Thus, if dissipation of sound does occur, it is insignificant in magnitude in the audible frequency range. The results indicate that the "shielding" of noise from one jet by the flow from another as reported by other researches is a directivity effect, and that the decrease in noise experienced at one position is accompanied by an increase where the sound is convected downstream.

4.3 Conclusions

The results of the experiments described indicate that little, if any, sound is absorbed by the turbulent jet wake, in agreement with the work in Ref. 3. Thus, no modification of the correlation of aerodynamic and acoustic jet properties is required from this consideration.

The shielding effect of a jet wake has been shown to be a result of directivity rather than sound power change. The problem of measuring the sound power of a jet in the presence of a shielding wake, requiring exceedingly precise acoustical measurements, has been bypassed by substitution of a pure tone sound source for the jet noise source.
REFERENCES


Figure 4.1 - Shielding of Jet Noise by Multi-Tube Nozzle Arrangement.

From Ref. 1
Figure 4.2 - Schematic Description of Focusing of Sound by a Jet Wake.
Figure 4.4 - Experimental Arrangement for Test of Dissipation of Sound through Turbulent Flow; Sound Source Located within Jet Wake.
Figure 4.5 -- Sound Dissipation Test Results for Noise Source within Jet Stream
Figure 4.6 - Experimental Arrangement for Test of Dissipation of Sound through Turbulent Flow.
Figure 4.7 - Effect of Flow Velocity on Amplitude and Directionality of Sound Propagating Through Jet Wake.
5.0 AN EVALUATION OF SELECTIVE WATER INJECTION AS A JET NOISE SUPPRESSION TECHNIQUE.

5.1 Introduction

Conventional techniques for jet noise suppression basically rely on subdivision of the main nozzle flow into a number of separate streams in such a manner as to produce favorable jet mixing through interaction of the elemental jets. The velocity profile theory indicates that the sound power generated by a free jet depends only on the mean-flow velocity distribution in the mixing region of the wake. If indeed the noise generated at a given axial location along the jet is primarily dependent on the velocity profile at that position and sufficiently independent of past history of the flow and the means employed to achieve the velocity distribution, then a mixer-type suppressor could effectively be replaced by another scheme which would alter the mean-flow velocity profiles without first subdividing the main jet. By injecting water sprays into a heated jet, the temperature and velocity of the flow can be lowered in selected regions of the wake, altering the velocity profiles. While it is known that spraying large quantities of water into the jet wake will reduce the sound power, generated (Ref. 1) the quantity of water required is such that this means of suppression cannot be considered as practical for in-flight use. However, by proper utilization of limited water flow, it is possible to alter the mean-flow velocity profiles rather than accomplish a gross velocity reduction for the entire jet. It is the suppression effect of this means of water injection which has been investigated and is reported here in fulfillment of the proposal work statement (Ref. 2).
The objective of the investigation was to determine feasibility of the selective water injection concept for jet noise suppression. As previously stated, the noise reduction achieved by non-uniform introduction of water sprays should be greater than reduction by thorough mixing and evaporation of the water in the jet wake, if the scheme is to be successful. Thus, as a standard for comparison, a theoretical calculation of overall power level reduction was made for a jet with completely mixed and evaporated water. The results are shown in Figure 5.1, and are based on the assumption that acoustic power depends on the eighth power of the flow velocity. The change in flow velocity was determined from heat balance calculation for the air and water mixture.

5.2 Experimental Procedure

The experimental approach adopted was to measure noise reduction for a heated jet with water injection as compared to an unsuppressed jet. Rather than make a complete survey of the sound field for sound power determination, it was decided to measure the more significant acoustic parameter of sound pressure level reduction at the angle of maximum jet noise radiation (40° from the jet axis). On the basis of noise directivity studies reported in References 3 and 4, it is seen that the one point sound pressure spectrum measurement at the angle of maximum noise can be used to estimate the sound power level spectrum.

The noise measurement system consisted of a Brüel and Kjær type 4133 microphone (with flat frequency response to 40 kcps) located 15 feet from the jet nozzle at an angle of 40° from the jet axis. Microphone output was frequency
analyzed by means of a Bruel and Kjaer spectrometer, and recorded on a B & K level recorder.

The test facility used was an outdoor free field test stand, with no significant reflecting surfaces except the ground, and capable of continuous heated flow. The test nozzle employed was a 4 inch diameter convergent design, and is shown with water spray tube configurations in Figure 5.2. Flow pressure and temperature were measured, as well as water and air mass flow. The airflow measurements were used to monitor the effective nozzle area, thus providing a rough indication of water evaporation upstream of the nozzle exit. A number of spray tube configurations were employed, with water inlet locations at two diameters upstream of the nozzle exit for initial tests and twelve diameters upstream for subsequent runs. The tubes were immersed radially into the airstream, and were equally spaced around the circumference of the air duct.

Eight tubes, with an orifice 0.050 inches dia. in the end of each, were used in the configuration with tube location 2 diameters upstream. Immersion depths of 0 (tube ends flush with nozzle wall), 2 inches, and alternate tubes at 0 and 1 inch were tested. Water flow was varied from 0 to 3.5 gallons/minute for airflow conditions of: (a) nozzle pressure ratio = 1.8, total temperature = 1400°F, and (b) pressure ratio = 2.7, total temperature = 1240°F. Sound pressure level reduction at the position 15 feet from the nozzle and at an angle of 40° from the jet axis was determined by comparing runs with and without water flow for a fixed airflow condition and spray tube geometry.
For the tests with water injection at 12 nozzle diameters upstream of the exit, 6 tubes were used. Water was introduced through a single 1/8" diameter hole in the end of each tube, and in an alternate scheme water was sprayed through 8 holes 0.045" dia. located along the length of each tube. Water flow rates up to 27% of the airflow (by weight) were achieved for flow temperatures up to 1340°F and over a range of pressure ratios from 1.35 to 2.15.

5.3 Discussion of Results

Initial investigations of noise reduction by means of selective water injection, utilizing the spray tubes located 2 diameters upstream of the nozzle exit, indicated that little evaporation of water occurred in the flow region near the nozzle exit. Even at the maximum water flow rate (13% of the airflow) only 4 db sound pressure level reduction was achieved, as shown in Figure 5.3.

Based on the theoretical curves in Figure 5.1, for complete mixing of the water with the heated airstream, a 6 db sound power level reduction would be expected. While it appears that this amount of suppression at the source might be obtainable in practice by complete mixing of water and air, the configuration tested did not achieve noise reduction as intended by alteration of flow velocity profiles. No change in airflow occurred as water flow was increased, indicating that only a limited amount of evaporation could have taken place in the nozzle, since formation of steam would effectively decrease the flow area available for the heated air.

Examination of the sound pressure reduction spectra (Figure 5.3) reveals that at the highest pressure ratio the noise reduction was shifted to a lower
frequency. This is explained by consideration of the increased jet velocity, since the water droplets were transported further downstream (toward the location of lower frequency sound source) before evaporation and resultant cooling and deceleration of the flow.

In order to achieve more complete vaporization of the water injected into the flow, further tests were conducted using the spray tubes located 12 nozzle diameters upstream of the exit. Comparison of suppression for the two spray tube designs (one hole in the end of each tube compared to 8 smaller holes spaced along the tube) showed no significant difference between the coarse and fine sprays. Rather, noise reduction was found to depend essentially on water/air ratio for a given airstream pressure and temperature. Overall sound pressure level reduction as a function of water/air ratio is shown in Figure 5.4, and it is apparent that suppression is a linear function of water/air ratio over the range tested. Comparing the curves for equal pressure ratios but varying temperatures it is evident that noise reduction increases as the flow temperature rises, and a maximum reduction is achieved, after which the additional temperature rise adversely affects suppression. Interpolated data from Figure 5.4 for water/air ratio of 15% is replotted in Figure 5.5 to indicate the effects of water residence time in the airstream (flow velocity effect).

It is the combined effects of increase in flow temperature causing: 1) more rapid vaporization of water, 2) decreasing droplet residence time, and 3) providing less noise reduction capability (as shown in Figure 5.1) which cause the maximization of suppression noted in Figure 5.4.
The calculation adverse effect of temperature results from the fact that for a given water air ratio the flow temperature change accompanying complete mixing and evaporation is essentially independent of initial temperature and thus, percentage-wise, the cooling effect decreases with increase in initial temperature.

The effects of water injection on the jet noise sound pressure level spectrum are shown in Figure 5.6. Reduction is achieved over a wide range of frequencies, and it is significant that even at low pressure ratio suppression is accomplished in the high frequency region (for the test results shown, the sound pressure level spectrum peak occurs at 1250 cps).

5.4 Conclusions

Results of the selective water injection investigation indicate that the desired alteration of jet mixing cannot be achieved by this means in order to affect noise reduction. Although the mean-flow velocity profiles can be altered through use of selective sprays, the jet mixing is not greatly influenced and the suppression accomplished is less than would be possible through uniform mixing of water and air.

The adverse effect of high temperature and flow velocity on noise reduction capability of water injection and the large water/air ratios required indicate that this scheme might not be practical for application to existing aircraft and engines.

Effects on nozzle aerodynamic performance have not been measured, but losses would be small if water evaporation and resultant cooling of the flow
occurs primarily external to the nozzle, as is indicated by test results.

Jet noise suppression by means of water injection is unusual in that high frequency noise reduction can be achieved, and furthermore, suppression can be accomplished for low pressure ratio flow.
REFERENCES


Figure 5.1 - Calculated Reduction of Sound Power Level for Complete Evaporation and Mixing of Water with Jet Exhaust.
\[ \Delta \text{db} = \text{SPL with H}_2 \text{O mists} - \text{SPL w/o H}_2 \text{O} \]

<table>
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<th>Pressure Ratio</th>
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<th>Tube Immersion</th>
<th>Water/Air Ratio</th>
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<td>10.3%</td>
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<tr>
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<td>1860°R</td>
<td>0&quot;, 1&quot;</td>
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<td></td>
<td></td>
<td>(alternately 1/4 each)</td>
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<tr>
<td>2.7</td>
<td>1700°R</td>
<td>2&quot;</td>
<td>6.6%</td>
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</tbody>
</table>

Note: Spray tubes located 2 dia. upstream of nozzle exit.

Figure 5.3 - Sound Pressure Level Reduction of Water Injection Nozzle Measured at 45° from Jet Axis.
Figure 5.4 - Water Injection Nozzle Overall Sound Pressure Level Reduction at Angle of Maximum Noise ($^{\circ}$ from Jet Axis)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$V$ (ft/sec)</th>
<th>$P_e/P_{in}$</th>
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</table>
Figure 5.5 - Water Injection Nozzle Overall SPL Reduction at Angle of Maximum Noise
Figure 5.6 - Water Injection Nozzle Noise Reduction at Angle of Maximum Noise