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FORM FUNCTIONS FOR DUAL COMPOSITION INCREMENTS OF PROPELLENT GRAINS

By

J. F. KOWALICK
And
M. S. SILVERSTEIN

PHILADELPHIA 37, PA.
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Prepared by

J. F. KOWALICK
Chemical Engineer
M. S. SILVERSTEIN
Chemist

Reviewed by

T. Q. CICCONI
Chief
Combustion & Detonation Section
W. J. KROGER
Director
Pitman-Dunn Institute for Research

Approved by

C. C. TAWCETT
Technical Director
Research and Development Group
R. E. BASSLER, Jr.
Lt. Colonel, Ordnance Corps
Chief
Research and Development Group

For

GEORGE H. PIERRE, Jr.
Colonel, Ordnance Corps
Commanding

RESEARCH AND DEVELOPMENT GROUP
FRANKFORD-ARSENAL
September, 1963
ABSTRACT

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It is shown that the pertinent parameters are the total burning distance of the propellant system, the burning distances of grains in each increment of propellant, the number of groups or increments, the mass of each group, and the number of groups which are burning simultaneously. The relationship of these parameters to the desired form function is expressed as a series of equations, each of which is an incremental form function. The sum of the series approaches, to any desired degree of accuracy, the specified form function.
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INTRODUCTION

Propellant devices generally produce a driving gas through chemical reaction. The number of moles of gas produced depends on propellant composition, while the mass consumption rate is, additionally, a function of propellant grain geometry and pressure. At constant pressure, constant linear burning rate makes the mass consumption rate a function of total burning-surface area. The relation between mass-fraction burned and distance-fraction burned is unique for every grain geometry. This relation is conventionally and appropriately called the "form" function. Thus, each grain configuration will exhibit a specific gas generation schedule.

It has been demonstrated* that most form functions can be satisfactorily represented by a quadratic equation of the form

\[ \frac{N}{C} = k_0 + k_1 Z + k_2 Z^2 \]  

(1)

where

- \( N \) = mass of propellant consumed
- \( C \) = total mass of propellant
- \( Z \) = distance fraction consumed
- \( k \) = constant.

As used here, the burning distance is defined as the shortest distance normal to the burning surface that a grain recedes until it either loses its structural integrity or is completely consumed - whichever comes first. The form function for a neutral geometry grain (such as a sheet propellant) is

\[ \frac{N}{C} = Z \]  

(2)

It will be demonstrated that any generalized form function, \( g(Z) \), based on a burning distance \( W \), can be approached by the incremental addition of several form functions of the form \( N/C = Z \), based on

groups of neutral geometry, sheet-type grains of burning distance \( w \), where \( w < W \). Neutral geometry grains are defined as having a constant burning surface during consumption.

**THE DISTRIBUTED PROPULSION SYSTEM**

The total propellant charge is composed of \( n \) groups (\( n \) is an integer) of neutral geometry grains. The mass per group is \( C_i \), where \( i \) runs from \( i = 1 \) to \( i = n \). All grains in any one group will commence burning simultaneously when the burning distance of the grains in the preceding group has receded to some fraction \( w/a \). The parameter, \( a \), is defined as the reciprocal of the distance fraction of the first group burned at the time group two begins to burn. Furthermore, the last \( (a) \) groups will be completely consumed simultaneously. Thus, the over-all distance burned, \( W \) (the total distance burned during consumption of the entire charge), is \( nw/a \).

Figure 1 is a graphical illustration of the mathematical model for the case of nine groups of grains (\( n = 9 \)) with each group commencing burning when \( 1/3 \) of the preceding group's burning distance has been consumed (\( a = 3 \)). Here, the abscissa represents the burning distance of grains in each group. When a group commences burning (for example, group 6), some groups may have been completely consumed (groups 1, 2, and 3), other groups partly consumed (group 4 is \( 2/3 \) consumed and group 5 is \( 1/3 \) consumed), and other groups will not have begun burning (groups 7, 8, and 9).

Note that the over-all burning distance, \( W \), is equal to \( n/a \) (or three) times the burning distance, \( w \), of the first group. Note also in Figure 1 that, for the last \( (a) \) groups to be consumed simultaneously, the burning distances of the latter \( (a - 1) \) groups (groups 7 and 9) must be less than the burning distance, \( w \), of the first group. The reason for using neutral geometry grains is that their form function is linear and, thus, when individual groups of grains are burned as described above, the over-all form function can be expressed as a series of linear segments.
Figure 1. Graphical Illustration of the Programmed Burning Model
Mathematical Description of the Model

Consider a propellant system with \( n \) groups of scheduled grains, such that the \( i \)th group is commencing burning. If \( i > a \), the first \((i - a)\) groups will have been completely consumed. Any other groups preceding group \( i \) will have been partly consumed. All other groups, including group \( i \), will not have commenced burning. Figure 1 illustrates the case where \( a = 3 \), and \( i \) is some intermediate group (e.g., \( i = 6 \)).

The total mass fraction of core propellant consumed as group \( i \) commences burning can be expressed in summation form as

\[
N = \frac{\Phi}{C} = \frac{1}{C} \left\{ \sum_{j=1}^{n+1-i} C_j + \sum_{k=1}^{(a-1)/a} C_{i-a+k} \right. \\
+ \left. \sum_{q=1}^{i+a-(n+2)}/(a-q) C_{n+q-a+1} \right\}
\]

where \( \Phi = \) the form function for the distributed propulsion system.

The subscripts on the \( C \)'s refer to the integral number of the group. The first summation term represents the first \((i - a)\) groups that are entirely consumed. Restrictions on \( i \) and \( j \) are that \( i \) must be greater than \( a \), and \( j \) greater than one. The second summation term represents those \((n+1-i)\) groups preceding group \( i \) that are partially consumed and whose burning distance fractions consumed can be expressed in integral multiples of \( 1/a \). The restriction on this second term is that \( k \) must be greater than \((a - i)\). The last summation term represents those groups preceding group \( i \) that are partially consumed and whose burning-distance fractions consumed cannot be generally expressed in terms of integral multiples of \( 1/a \). Three restrictions on this term are that \( a \) be greater than one, and that \( q \) be greater than \((a - n - 1)\), and less than \( a \).

Statement of the Problem

A solid propellant device will have associated with it a specified propellant form function, \( g(Z) \), and a burning distance \( W \). The problem, in terms of a distributed propulsion system, can be stated as
follows: based on a burning distance of \( W \), using \( n \) groups of neutral geometry, sheet type grains such that a maximum of \( (a) \) groups burn at a time, generate form function, \( g(Z) \); then, determine the mass and burning distance for each group based on a core propellant composition which is identical with that required in the propellant system.

Solution of the Problem

Since both \( n \) and \( (a) \) have been specified, the burning distance \( w \), for grains in all groups except the latter \( (a - 1) \) groups can be determined from the relation

\[
w = \frac{(a/n)W}{(4)}
\]

This relation becomes more evident if, in Figure 1, a comparison is made between the over-all burning distance \( W \) and the individual burning distance \( w \). Also, from Figure 1, the burning distance for grains in the latter \( (a - 1) \) groups are, successively,

\[
\frac{(a - 1)W}{n}, \frac{(a - 2)W}{n}, \ldots, \frac{[a - (a - 1)]W}{n}
\]

The total mass fraction burnt can be calculated from Equation 3 at \( n \) values of \( i \), namely 1, 2, 3, \ldots, i, \ldots, n. These values correspond to \( n \) definite values of \( Z \), by the equation \( Z = (i - 1)/n \), namely

\[
0/n, 1/n, 2/n, \ldots, (i - 1)/n, \ldots, (n - 1)/n
\]

From Equation 3, the \( n \) values of mass fraction consumed are

\[
\Phi_1, \Phi_2, \Phi_3, \ldots, \Phi_i, \ldots, \Phi_n.
\]

For each value of \( Z \), a corresponding value of \( g(Z) \), the specified mass fraction consumed, can be generated:

\[
g(0/n), g(1/n), g(2/n), \ldots, g[(i - 1)/n], \ldots, g[(n - 1)/n]
\]
Upon setting the specified and the scheduled mass fractions equal to each other for correspondence values of \( Z \) and \( i \), a set of \( n \) independent equations with \( n \) unknowns is generated.

\[
\begin{align*}
(n-1)/n \\
\begin{array}{c} 
Z=0 \\
\end{array} \\
\begin{array}{c} 
g(Z) = \\
\end{array} \\
\begin{array}{c} 
g(i/n) = \\
\end{array} \\
\begin{array}{c} 
\phi_{i+1} \\
\end{array} \\
\end{align*}
\] (9)

which can also be expressed as

\[
\begin{align*}
n-1 \\
i=0 \\
g(i/n) = \alpha_{i+1,1} C_1 + \alpha_{i+1,2} C_2 + \cdots + \alpha_{i+1,i+1} C_{i+1} \\
+ \cdots + \alpha_{i+1,n} C_n
\end{align*}
\] (10)

where the \( \alpha \) terms are the coefficients of the \( C \)'s in Equation 3.

An additional equation applies after complete consumption of all groups of grains:

\[
g(n/n = 1) = \frac{1}{C} \left( \sum_{p=1}^{n} C_p \right)
\] (11)

where \( p \) denotes the group number. This additional equation is necessary for solution of the set, since the solution to the first equation in the set, \( g(0/n) = \phi_1 \), is trivial.

The general solution of the set is

\[
\begin{bmatrix}
\alpha_{21} & \alpha_{22} & \cdots & g(1/n) & \cdots & \alpha_{2,n} \\
\alpha_{31} & \alpha_{32} & \cdots & g(2/n) & \cdots & \alpha_{3,n} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{i+1,1} & \alpha_{i+1,2} & \cdots & g(i/n) & \cdots & \alpha_{i+1,n} \\
\vdots & \vdots & \ddots & \vdots \\
C_i & = & 1 & 1 & \cdots & 1 \\
\end{bmatrix}
\] (12)
where $D$ is the determinant of the coefficients.

Graphical Description of the Model

Figure 2 illustrates how the incremental addition of the form functions of five groups of grains results in some selected form function. This figure represents a comparison of the over-all form function, where $a = 2$, with individual form functions for each group. The mass fraction consumed at any point on the over-all form function equals the summation of mass fractions of all the groups either totally or partially consumed at that point. Note that the total mass fraction consumed at any jointure point (points marked X) depends on the mass fraction consumed at the preceding jointure point, plus the contributions of the (a) groups (here, $a = 2$) burning simultaneously in the preceding interval of distance fraction consumed. This interval is $w/a$ (or, in Figure 2, $w/2$) in length.

Excluding the latter $(a-1)$ groups, it is generally true that the slope of the form function for each individual group is proportional to the mass of that group. Thus, groups three and four, which have identical slopes in Figure 2, can be expected to have the largest masses of the first four groups.

Illustration of the Use of the Distributed Propulsion System

Assume that it is desired to duplicate the form function, $g(Z) = Z^2$, for a large gun system consisting of 4.0 pounds of propellant divided into 1334 grains of 0.2-inch burning distance, with a distributed propulsion system consisting of 10 groups of sheet type grains of identical composition. Let the maximum number of groups burning at one time vary from one to five. The problem is to calculate the mass and burning distance of each group for each value of $(a)$ ($a$ varies from one to five, and $n$ equals 10). Solution: for $n = 10$, $a = 1$, Equation 3 becomes

$$C \Phi = \sum_{j=1}^{i-1} C_j + \sum_{k=1}^{11-i} (1-k) C_{i-1+k} + \sum_{q=1}^{i-11} \left( \frac{i-10-q}{1-q} \right) C_{10+q}$$
Figure 2. Incremental Addition of Form Functions

Mass Fraction Consumed

- Ordinate Fraction Consumed

- Ordinate Fraction Produced

- Ordinate Fraction Consumed

- Ordinate Fraction Produced
Corresponding values for Z and $g(Z)$ when group $i$ commences burning are presented in Table I. Note that $Z$ is determined from the relation, $Z = (i-1)n$, and is independent of the value of $(a)$.

**TABLE I. Values of the Form Function, $g(Z) = Z^2$**

<table>
<thead>
<tr>
<th>Group $i$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>0</td>
<td>.1</td>
<td>.2</td>
<td>.3</td>
<td>.4</td>
<td>.5</td>
<td>.6</td>
<td>.7</td>
<td>.8</td>
<td>.9</td>
</tr>
<tr>
<td>$g(Z) = Z^2$</td>
<td>0</td>
<td>.01</td>
<td>.04</td>
<td>.09</td>
<td>.16</td>
<td>.25</td>
<td>.36</td>
<td>.49</td>
<td>.64</td>
<td>.81</td>
</tr>
</tbody>
</table>

An additional equation is applicable after complete consumption:

$$\frac{N}{C} = 1.0 = C_1 + C_2 + \ldots + C_{10}.$$  

Upon equating the $g(Z)$'s to the $\phi$'s for corresponding values of $Z$ and $i$, and solving the set of simultaneous equations so formed, the following group masses (Table II) are obtained.

**TABLE II. Group Masses**

<table>
<thead>
<tr>
<th>Group</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (lb)</td>
<td>.04</td>
<td>.12</td>
<td>.20</td>
<td>.28</td>
<td>.36</td>
<td>.44</td>
<td>.52</td>
<td>.60</td>
<td>.68</td>
<td>.76</td>
</tr>
</tbody>
</table>

Burning distances can be calculated from Equations 4 and 5. Table III offers a comparison of the masses and burning distances of each group for various values of $(a)$. Note that the latter $(a - 1)$ groups have smaller burning distances than the other groups.
### TABLE III. Mass (lb) and Burning Distance (in.) of Each Group for a = 1 to a = 5

<table>
<thead>
<tr>
<th>Group</th>
<th>Value</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(C_4)</th>
<th>(C_5)</th>
<th>(C_6)</th>
<th>(C_7)</th>
<th>(C_8)</th>
<th>(C_9)</th>
<th>(C_{10})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>W</td>
<td>.04</td>
<td>.12</td>
<td>.20</td>
<td>.28</td>
<td>.36</td>
<td>.44</td>
<td>.52</td>
<td>.60</td>
<td>.68</td>
<td>.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
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<td>.02</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>.08</td>
<td>.16</td>
<td>.24</td>
<td>.32</td>
<td>.40</td>
<td>.48</td>
<td>.56</td>
<td>.64</td>
<td>.72</td>
<td>.40</td>
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<tr>
<td></td>
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<tr>
<td>3</td>
<td></td>
<td>.12</td>
<td>.24</td>
<td>.24</td>
<td>.36</td>
<td>.48</td>
<td>.48</td>
<td>.60</td>
<td>.72</td>
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<td>.06</td>
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<td>.06</td>
<td>.04</td>
<td>.04</td>
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<tr>
<td>4</td>
<td></td>
<td>.16</td>
<td>.32</td>
<td>.32</td>
<td>.32</td>
<td>.48</td>
<td>.64</td>
<td>.64</td>
<td>.48</td>
<td>.40</td>
<td>.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.08</td>
<td>.08</td>
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<td>.08</td>
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<td>.08</td>
<td>.06</td>
<td>.06</td>
<td>.04</td>
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</tr>
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<td>5</td>
<td></td>
<td>.20</td>
<td>.40</td>
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<td>.40</td>
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<td>.60</td>
<td>.64</td>
<td>.48</td>
<td>.32</td>
<td>.16</td>
</tr>
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<td>.10</td>
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<td>.10</td>
<td>.08</td>
<td>.06</td>
<td>.04</td>
<td>.04</td>
<td></td>
</tr>
</tbody>
</table>

**M** - Mass (lb)

**W** - Web (in)

**Limitations on a and n**

Certain limitations exist on the values of \(n\) and \(a\). From Equation 4, it is evident that \(w\) decreases as \(n\) increases. Thus, the limiting (smallest practical) value of \(w\) determines the largest value of \(n\). The smallest value of \(n\) is determined from the desired degree of approach of the scheduled form function to the specified form function. As \(n\) increases, the distributed form function approaches the specified form function more closely.

Other factors that may impose limitations on \(n\) and \(a\) are grain formulating and forming variables encountered in manufacturing and certain aspects of the propellant system, such as chamber geometry, ignition, and loading density. These factors will vary with the propellant used and the individual test setup.
Grain Preparation Techniques

It is envisioned that an outer layer of propellant with a much slower burning rate than that of the core propellant, completely covers all available grain surface area. The thickness of the outer propellant for a specific grain group will depend on the scheduling time to the commencement of burning for that group. Hereafter, this outer propellant layer will be referred to as the scheduling propellant.

Two grain-forming techniques are being employed in current experimental work. The first technique utilizes preformed sheets of two double base propellants having widely different burning rates. A "sandwich" grain is formed, with the faster burning sheet lying between two sheets of the slower burning scheduling propellant. Uncured core propellant is employed as the adhesive. Thus, successive groups of grains will be characterized by increasing thicknesses of the scheduling propellant. As the thickness of the outer layer increases, the burning time to reach the core propellant increases at constant pressure.

The second technique involves the application of a nitrocellulose lacquer to a sheet of core propellant. The basic variables to be considered here are the relative percentages of added resins, plasticizers, active and/or inactive solvents, and diluents in the lacquer, application techniques, and nitocellulose layer thickness.

It has been estimated that when the burning rates at constant pressure of the two propellants differ by an order of 10 : 1 (with the outer layer obviously having the slower burning rate), the outer, or scheduled, propellant composition is only seven percent by weight of the propellant system. Thus, properties of the gas formed are largely determined from the properties of the core propellant alone.

One criterion for the dual compositions in any system operating over a range of pressures is that the ratio of their burning rates be constant and independent of pressure. Interpreted mathematically, this criterion states that the pressure exponents in the burning rate expression for each of the two propellant compositions should be equal.

The masses \( C_1, C_2, \) etc., represent masses of the core propellant only. The method of calculation of the mass of scheduling
propellant in each group, for a specified form function, will depend on the properties of the scheduling propellant - specifically, on the burning rate-pressure relationship and the density of the propellant.

Results on the burning behavior of the dual composition grains in a closed bomb and a vented vessel apparatus will indicate the effectiveness of these techniques in providing scheduled burning in accordance with the model. These results will basically consist of pressure-time data and dP/dt vs time data.

CONCLUSIONS

1. Any desired form function can be approached by the scheduled burning of a number, \( n \), of increments of dual composition, neutral geometry, solid propellant grains. The form function so achieved is dependent on the mass of the inner (core) propellant for each increment, the thickness for each increment of the outer (scheduling) propellant covering grains of uniform size in that increment, and the ratio of burning rates of the two propellants.

2. The degree of approach of the scheduled form function to the desired form function increases as \( n \), the number of increments (groups) is increased. As the number of increments burning simultaneously \( (a) \) is increased, the burning distances of the core propellant becomes more nonuniform from increment to increment. In general, the latter \( (a - 1) \) increments have smaller burning distances than the other increments.

RECOMMENDATIONS

It is recommended that tests be carried out to test the basic model. It is expected that the results of such tests will reveal which of the two-grain preparation techniques - laminating propellant layers or applying nitrocellulose lacquer films - is better from a practical standpoint.
1 - Hq., U.S. Army Materiel Command
   Attn: AMCRD-DE-W
   Mr. C. L. Thulin
   Washington 25, D.C.

1 - Attn: AMCRD-RS-CM
   Mr. M. Miller

5 - Attn: AMCSA-SC

1 - Attn: AMCRD-RS-CM
   Mr. J. Crellin

1 - Attn: AMCRD-RS-CM
   Mr. J. Chalmers

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   Mr. J. E. Rainer
   Picatinny Arsenal
   Dover, N.J.

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   Mr. R. W. Reed

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   Attn: Technical Library
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   Applied Physics Laboratory
   The Johns Hopkins University
   8621 Georgia Ave.
   Silver Spring, Md.

2 - Bureau of Weapons
   Department of the Navy
   Attn: EMM03
       Mr. C. P. Nicholson
       Washington 25, D. C.

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   Attn: Lt. Commander Arsenault
   U. S. Naval Propellant Plant
   Indian Head, Md.

10 - Defense Documentation Center
     Cameron Station
     Alexandria, Va.

4 - British Defense Staff
    Attn: Scientific Information Officer
    British Embassy
    3100 Massachusetts Ave.,
    Washington, D. C.
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**FORM FUNCTIONS FOR DUAL COMPOSITION INCREDENTS OF PROPEL-**
**LANT GRAINS** by J. F. Kowalick and M. S. Silverstein

It is demonstrated mathematically that any desired form function can be approached by the burning of increments of groups of dual composition propellant grains of neutral geometry (constant surface during burning). This schedule burning is achieved by varying the thickness of each of the outer layers of propellant, with the outer layers having the lower burning rate.

It is shown that the pertinent parameters are the total burning distance of the propellant system, the burning distance of grains in each increment of propellant, the number of groups or increments, the mass of each group, and the number of groups which are burning simultaneously. The relationship of these parameters to the desired form function is expressed as a series of equations, each of which is an incremental form function. The sum of the series approaches, to any desired degree of accuracy, the specified form function.

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FORM FUNCTIONS FOR DUAL COMPOSITION INGREDIENTS OF PROPELLANT GRANS
by J. F. Kowalick and M. S. Silverstein

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