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A SIMPLIFIED VERSION OF DANTZIG'S QUADRATIC PROGRAMMING ALGORITHM FOR SOLUTIONS USING A MEDIUM-SIZED DIGITAL COMPUTER

BY
ROY E. MURPHY

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INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
STANFORD UNIVERSITY
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A SIMPLIFIED VERSION OF DANTZIG'S QUADRATIC PROGRAMMING ALGORITHM FOR SOLUTIONS USING A MEDIUM-SIZED DIGITAL COMPUTER*

by

R. E. Murphy
Stanford University

1. Introduction

In a recent paper by G. Dantzig [1], an algorithm was introduced to obtain solutions for certain quadratic programming problems by a modified simplex program. This programming algorithm follows closely the methods employed by Wolfe and Markowitz [2]. In this introduction, we shall outline Dantzig's theorems for those unfamiliar with this method, and also develop the notation used in this paper.

1.1. The Criterion Function

We shall formulate the problem as the constrained maximization of a convex quadratic function in x. The criterion function (in vector/matrix notation) will be

$$\pi(x) = x^t b - \frac{1}{2} x^t A x, \tag{1.1}$$

where

$x \geq 0$,

$x^t A x \geq 0$ for all x, and A is a symmetric matrix.

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I wish to thank Mr. G. Kuykendall and Mrs. L. Owens of I.B.M., San Jose, California, for their help in introducing the author to 1401 Fortran and in testing this program.

Some of the columns of the A matrix may have zero elements. The activities associated with these zero rows and columns will be <u>linear</u> activities, not quadratic activities. Thus, by suitable construction of the A matrix, mixed quadratic/linear solutions may be obtained.

1.2. The Constraint Function

In addition to the positivity requirement for x, we require that the following equation (in vector/matrix notation) be satisfied:

$$Cx = d \; . \tag{1.2}$$

1.3. A Note on Slack Variables

The vector x is defined so that it contains the required slack components to make (1.2) hold as an equality.

1.4. The Kuhn-Tucker [3] Optimality Conditions

A solution to (1.1) and (1.2), say the vector x^*, can be segregated into two subsets. Let X^o be the subset of components of x which are zero, and X^+ the subset of components of x which are positive. The solution, x^*, is the optimal solution (leads to a maximum $\pi(x)$ under constraints, (1.2)) if

$$b - Ax^* - C^t\lambda^* = 0 \quad \text{for} \quad x^* \in X^+ \tag{1.3}$$

$$b - Ax^* - C^t\lambda^* < 0 \quad \text{for} \quad x^* \in X^o \; , \tag{1.4}$$

and

$$Cx^* = d \; . \tag{1.5}$$

The vector λ^* is defined as the Lagrangian multiplier vector.

1.5. The Complementary Vector

Suppose we define a new vector u^* such that

(1.6) $$u^{*t}x = 0 \ , \ u^* \geq 0 \ ,$$

and such that

(1.7) $$b + u^* - Ax^* - C^t\lambda^* = 0 \quad \text{for} \quad x^* \in X^o \ .$$

We can now write (1.3) and (1.4) as

(1.8) $$b + u^* - Ax^* - C^t\lambda^* = 0 \quad \text{for all} \quad x \geq 0 \ .$$

It can be seen then that the vector u is a special kind of slack variable which permits the Kuhn-Tucker inequality (1.4), together with (1.3), to be written as an equality (1.8).

1.6. Similarity Between These Equations and Linear Programming Equations

The similarity between the satisfaction of equations (1.5) and (1.8) and simplex method linear programming can be exploited if satisfaction of (1.6) can be ensured and some method of determination can be made of the efficient order in which to bring in new components of x (to take the place of the linear criterion function coefficients of linear programming). Dantzig has supplied such a selection criterion. A feasible, non-optimal solution, say x^o (which satisfies (1.5) but does not satisfy (1.3) or (1.4)), will be characterized by a u vector which contains negative components. The component of x^o corresponding to the most negative component of u^o is the activity to be introduced next into the basis (X^+). Dantzig has shown that such a selection rule will lead to a monotonically increasing $\pi(x)$.

2. Setting Up the Solution Matrix

2.1. The Complete Form of the Solution Matrix

Figure 2.1 shows the solution matrix for the quadratic criterion function (1.1), subject to the explicit constraints. Note the requirement

of equations (1.6), (1.2), and (1.8) is implicit in the computer instructions to follow.

$$
\begin{array}{|cccc|cccc|c|}
\hline
b_1 & b_2 & \cdots & b_m & d_1 & d_2 & \cdots & d_n & \\
\hline
a_{11} & a_{12} & \cdots & a_{1m} & c_{11} & c_{21} & \cdots & c_{n1} & x_1 \\
a_{21} & a_{22} & \cdots & a_{2m} & c_{12} & c_{22} & \cdots & c_{n2} & x_2 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mm} & c_{1m} & c_{2m} & \cdots & c_{nm} & x_m \\
\hline
-1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & u_1 \\
0 & -1 & \cdots & 0 & 0 & 0 & \cdots & 0 & u_2 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 & u_m \\
\hline
c_{11} & c_{12} & \cdots & c_{1m} & 0 & 0 & \cdots & 0 & \lambda_1 \\
c_{21} & c_{22} & \cdots & c_{2m} & 0 & 0 & \cdots & 0 & \lambda_2 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
c_{n1} & c_{n2} & \cdots & c_{nm} & 0 & 0 & \cdots & 0 & \lambda_n \\
\hline
\end{array}
$$

Fig. 2.1.1

2.2. The Reduced Form of the Solution Matrix

We shall employ an abbreviated notation for the solution matrix, so that Fig. 2.1.1 may be represented as

(2.1)

$$
\begin{array}{|c|c|c|}
\hline
b & d & \\
\hline
A & C & x \\
\hline
-I & \emptyset & u \\
\hline
C^t & \emptyset & \lambda \\
\hline
\end{array}
$$

One practical disadvantage of the simplex quadratic programming algorithm is the very large size of the solution matrix. With m activities and n constraints, the solution matrix is a $(m+n) \times (2m+n)$ matrix, where $m > n$. This large matrix limits the application of the method to either the solution of small problems or the use of large computers.

We note that n of the m activities are <u>slack</u> activities, the rest being <u>productive</u> activities. We may partition the x vector as follows:

$$x = \langle x_p ; x_s \rangle \tag{2.2}$$

where x_s contains the components of x which represent slack activities,

x_p contains the components ($m-n$ in number) of x which represent productive activities.

In the same way, we write

$$u = \langle u_p ; u_s \rangle \ . \tag{2.3}$$

Furthermore, we will assume that the criterion function is restricted only to the productive activities; that is, the slack activities are not costly or productive. Thus, we have

$$b = \langle b_p ; \emptyset \rangle \ , \tag{2.4}$$

where b_p contains only the coefficients corresponding to the productive activities.

Also, we have

$$A = \left[\begin{array}{c|c} A_p & \emptyset \\ \hline \emptyset & \emptyset \end{array} \right] \tag{2.5}$$

where A_p is the matrix whose elements are associated only with the productive activities, and

$$C = \left[\begin{array}{c} C_p \\ \hline I \end{array} \right] \tag{2.6}$$

where C_p is the matrix associated with the productive activities, and

I is the unit matrix associated with the slack activities.

The whole solution matrix of (2.1) may be rewritten as

(2.7)
$$\begin{array}{c|c|c|c}
b_p & 0 & d & \\
\hline
A_p & \emptyset & c_p & x_p \\
\hline
\emptyset & \emptyset & I & x_s \\
\hline
-I & \emptyset & \emptyset & u_p \\
\hline
\emptyset & -I & \emptyset & u_s \\
\hline
c_p^t & I & \emptyset & \lambda
\end{array} .$$

We note from the last two rows of (2.7) that u_s and λ are related in a very useful way. An increase in a component of u_s implies an equal increase in a corresponding component of λ, and vice versa. Thus, for any component in u_s and λ, we may write

(2.8)
$$u_j + \delta_j = \lambda_j + \delta_j, \quad j = 1,2,\ldots,n ,$$

where δ_j is some change to u_j or λ_j. For all practical purposes, any selection criterion based (in part) on u_s could be based also on λ. The solution matrix (2.7) may then be rewritten in a reduced form. We have

(2.9)
$$\begin{array}{c|c|c}
b_p & d & \\
\hline
A_p & C_p & x_p \\
\hline
\emptyset & I & x_s \\
\hline
-I & \emptyset & u_p \\
\hline
C_p^t & \emptyset & \lambda
\end{array}$$

Where we have m activities and n constraints, the general solution matrix of dimension $(m+n) \times (2m+n)$ may be reduced to a $m \times 2m$ matrix. Such a

reduction leads to a marked saving in computer memory elements.

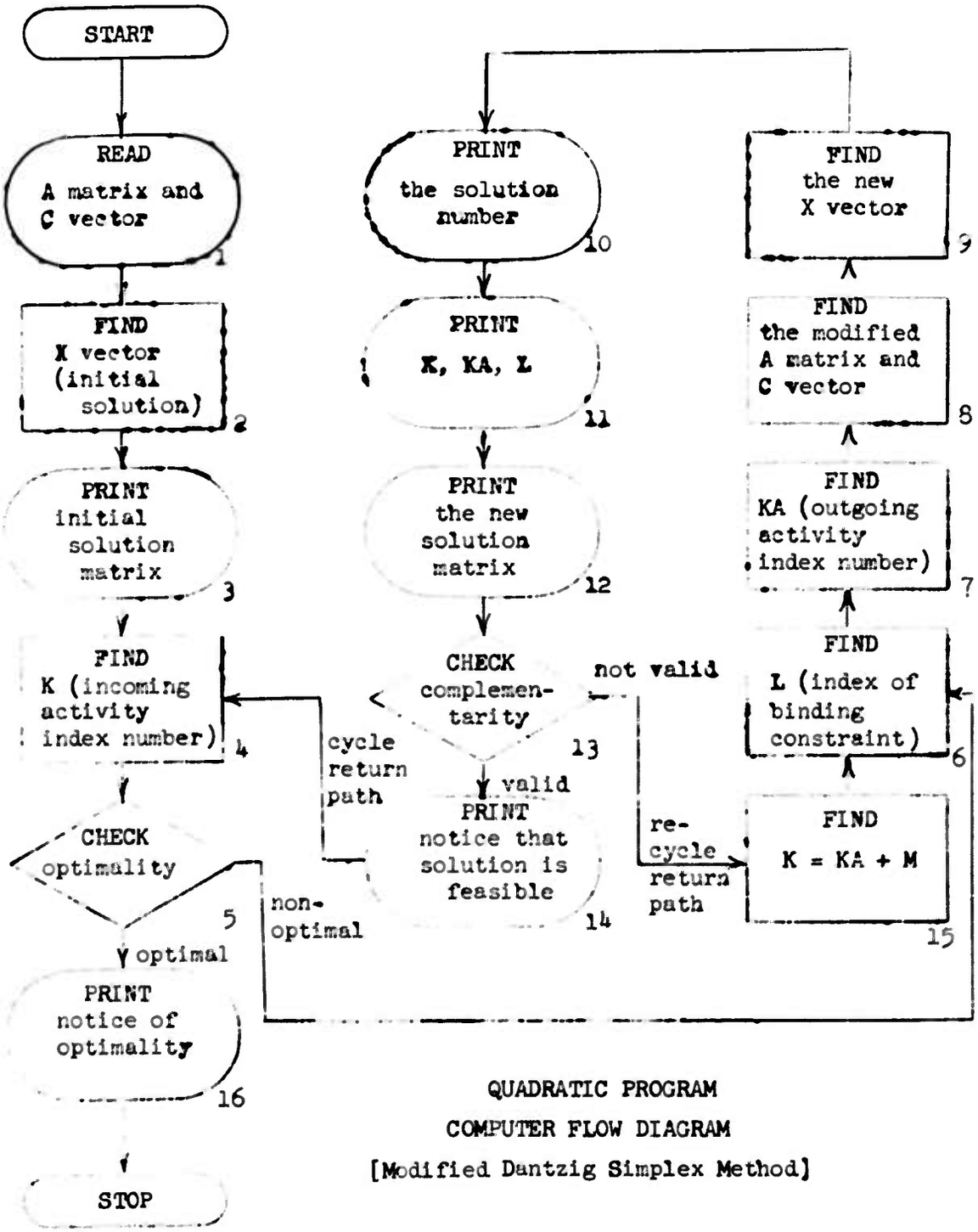
3. The Computer Program

3.1 General Information

The computer program is written in Fortran language for the I.B.M. 1401 computer [4]. The 1401 computer is a medium-sized machine which is quite commonly found in use as an accounting machine in many thousands of business firms in the United States and abroad. Since quadratic programming is a valuable aid to profit analysis in business, we feel that this program will find many uses in these firms. Approximately 8,000 memory positions are required to store the program and the necessary fixed routine instructions for the computer. Thus, if 12,000 memory positions are available, sufficient memory positions remain to store a 15 x 30 solution matrix with an accuracy of six significant figures (this includes space for the vectors x_p, x_s, u_p, λ, b_p, and d). This solution capacity should be adequate for most business problems. One solution cycle requires approximately 30 seconds of machine time and, on the average, about 10 cycles are required to find an optimal solution. Thus, a solution may be obtained, on the average, in 5 minutes of computer time. No magnetic tape units are required, the problem being solved completely in the magnetic core storage areas.

3.2 Description of the Block Diagram

It is necessary to make the computer Fortran program notation different from the descriptive notation used in the previous sections. The x vector, u vector, and λ vector are known to the computer as a single X vector. The assigned index numbers are the only clues to identification of the x, u, and λ vectors within the X vector. Also, the



QUADRATIC PROGRAM

COMPUTER FLOW DIAGRAM

[Modified Dantzig Simplex Method]

Fig. 3.2.1

c and d vectors are known to the computer as a single C vector, from which the c and d vectors may be identified only by the assigned index numbers. The solution matrix is known to the computer as the A matrix. The submatrices described in previous sections are only identified by the assigned index numbers.

With these definitions in mind, Fig. 3.2.1 may be examined. Figure 3.2.1 is a simplified block diagram of the modified Dantzig simplex method. Each block will be described in detail below.

Block 1

During this operation the computer accepts input information from cards and/or tape. This information is placed in the reserved memory positions and the assembly of the A matrix is completed. In addition, the C vector is also assembled in the appropriate memory locations.

Block 2

In Block 2, the A matrix and the C vector are used to determine the initial value of the X vector. This initial solution is formed by running the slack activities at a level which consumes all the available resources. Furthermore, the complementary productive activities are also introduced (since the productive activities are all at zero) so that the C vector is completely satisfied. This initial solution, although a feasible solution, is far from optimal since all the resources are "thrown away" by the slack activities.

Block 3

The printer now prints out the initial solution. First, the C vector is printed out. Then the A matrix and X vector (in the far right column) are printed out row after row, until the entire matrix and vector are

complete. This matrix may then be examined to assure that the data fed into the computer has been stored in the proper positions.

Block 4

If the value of this least-valued complementary activity is positive or zero, the optimality theorems indicate that the present solution is the optimal solution and the computer passes on to Block 17. However, this is rarely the case, especially on the first solution. Ordinarily, the least-valued complementary activity is negative. If it is negative, the computer passes on to Block 6.

Block 6

In Block 6, the Kth (most profitable) activity is increased from zero until the first of the C vector constraints coefficient becomes binding. Which constraint is the first to bind, is determined by dividing the K row of A matrix elements, one at a time, into the appropriate C vector coefficients. Note that only certain C vector coefficients (columns) are used during this process. These certain coefficients are in the columns associated with the activities which are restricted to zero or positive values only; that is, only the productive activities and the slack activities. Thus, only the C vector coefficient associated with a sign-restricted activity can possibly bind the incoming activity. The complementary activities and the Lagrangian multipliers are not sign-restricted for feasible solutions. One exception exists: The C vector coefficient associated with the complementary activity for the Kth activity is sign-restricted during this solution cycle. It may not be permitted to go negative at this time. This is because it is very desirable for the incoming activity, X(K), to be bound by the C vector coefficient which

formerly bound its own complementary activity. The reasons behind this statement will become more apparent later in Block 13.

Block 7

Since the binding C vector constraint, L, formerly was the bound for some other activity that was formerly in the basis, this other activity must be identified and made to drop out of the new basis (go to zero) following the introduction of the Kth (most profitable) activity. This activity is found by finding the index number of the row with a 1. in column L, which also has a positive, non-zero X vector component. This index number is recorded as KA.

Block 8

Now that K, L, and KA are known, the A matrix and the C vector are modified according to the simplex rules. Following this operation, the Kth row will have 0. everywhere except in the Lth column where the number 1. will appear. The KA^{th} row, which formerly had 0.'s everywhere but the Lth column (where it had a 1.), will now generally contain coefficients of various values. Everywhere else the A matrix coefficients will be modified by the simplex process to recognize the existence of a new basis (containing the Kth, instead of the KA^{th}, activity).

Block 9

At this stage, the new X vector is assembled. The incoming activity, X(K), will become positive and equal to the new C(K), while the outgoing activity, X(KA), will become 0. . Since the C vector has been modified, the other activity levels will change somewhat.

Block 10

The printer will print the sequence number of the new solution.

Block 11

The printer will print the index numbers of the incoming and outgoing activities and the binding constraint.

Block 12

The printer will now print the new C vector. Following this, the A matrix and X vector components will be printed out, row by row, until the entire solution has been printed.

Block 13

The complementarity check is now made. The complementary activity of the incoming activity is checked to see if it is 0. . If it is, this can happen in the following way: Suppose that the binding C vector coefficient was the former binding coefficient for the complementary variable of the incoming activity. In this case, as the incoming activity comes in, its own complementary activity goes to 0. . Most of the time the complementary activity falls but does not go to zero (this is because some activity other than the complementary one is dropped). If the complementary activity associated with the incoming activity becomes zero, the complementarity test passes. If not, the test fails. The optimality theorems indicate that if the test passes, the solution is feasible. If the test fails, the solution is non-feasible.

If the complementarity test passes, the computer moves on to Block 14. If the test fails, the computer moves on to Block 15.

Block 14

The computer enters this block only when the complementarity test passes. The printer will print out a notice that the solution (printed formerly in Block 12) is a feasible solution.

The computer now returns to Block 4 and determines the conditions for another solution.

Block 15

The computer enters this operation only when the complementarity test fails and the solution is non-feasible. According to the optimality theorems, the complementary activity associated with the binding C vector constraint coefficient must be brought into the basis. Therefore, K is set equal to the index of this activity and the computer attempts to pass the complementarity test again by going back to Block 6, with this new value of K.

Block 16

This block is entered only if the optimality test of Block 5 is passed. In this block the printer prints out a message indicating that the solution (printed last) is the optimum solution. Following this message the computer stops and the problem is solved.

3.3. The Fortran Statement of the Solution Algorithm

Appendix I is a reproduction of the Fortran statement list for the solution algorithm. The essential heart of the algorithm starts at statement sequence number 30. The operations preceding this point are the instructions required to read in the initial data and form the initial solution matrix. These instructions can sometimes be modified to suit the particular application of the algorithm as a component of a larger program. In this case, these statements have been made general enough to permit the program to be used for most applications.

3.4. The Layout of Input Data Cards.

In the assignment of coefficients to be punched in the input data cards, some arbitrary, but prescribed, index order of the constraints and activities is assumed. This index order, once set, is adhered to throughout the assignment.

3.4.1. The Index Card

The index information is placed on one card with the following format:

Card No.	Column No.'s	Coefficient
1	1-2	NT (the number of productive activities)
1	3-4	MT (the number of constraints)

The following equations must be satisfied when selecting NT and MT:

$$1 \leq NT \leq 10$$

$$1 \leq MT \leq 10$$

$$2 \leq NT + MT \leq 15$$

3.4.2. The C Vector Cards

The C vector is placed on two cards with the following format:

Card No.	Column No.'s	Coefficient
2	1-8	C(1)
2	9-16	C(2)
2	17-24	C(3)
2	25-32	C(4)
2	33-40	C(5)
2	41-48	C(6)
2	49-56	C(7)
2	57-64	C(8)

Card No.	Column No.'s	Coefficient
2	65-72	C(9)
2	73-80	C(10)
3	1-8	C(11)
3	9-16	C(12)
3	17-24	C(13)
3	25-32	C(14)
3	33-40	C(15)

All unused components of C are left blank.

3.4.3. The Quadratic Coefficient Cards

We have NT cards of the following format:

Card No.	Column No.'s	Coefficient
4	1-8	A(1,1)
4	9-16	A(1,2)
4	17-24	A(1,3)
4	25-32	A(1,4)
4	33-40	A(1,5)
4	41-48	A(1,6)
4	49-56	A(1,7)
4	57-64	A(1,8)
4	65-72	A(1,9)
4	73-80	A(1,10)

..... The above sequence is repeated for NT cards, the last of which is .....

3 + NT	73-80	A(NT,10)

All the unused A elements are left blank.

3.4.4. The Constraint Coefficient Cards

We have MT cards of the following format:

Card No.	Column No.'s	Coefficient
3 + NT + 1	1-8	C(1,1)
3 + NT + 1	9-16	C(2,1)
3 + NT + 1	17-24	C(3,1)

Card No.	Column No.'s	Coefficient
3 + NT + 1	25-32	C(4,1)
3 + NT + 1	33-40	C(5,1)
3 + NT + 1	41-48	C(6,1)
3 + NT + 1	49-56	C(7,1)
3 + NT + 1	57-64	C(8,1)
3 + NT + 1	65-72	C(9,1)
3 + NT + 1	73-80	C(10,1)

.....The above sequence is repeated for MT cards, the last of which is.....

3 + NT + MT	1-8	C(1,MT)
3 + NT + MT	9-16	C(2,MT)

..... and so on until .....

3 + NT + MT	73-80	C(10,MT)

All the unused elements of C are left blank.

3.4.5. Special Instructions on Punching These Cards

The decimal point must always fall four digits from the end of the coefficient's field.

The decimal point is not punched in the card; the computer is programmed to provide it internally.

Minus signs can be punched before the first significant digit but under no circumstances can minus signs be placed outside the field occupied by the coefficient. Thus, negative coefficients can have only three significant figures before the decimal point.

In cases where the significant digits stop before the end of the field, zeros must be filled in until the remainder of the field is full of digits. No zeros are required before the first significant digit in order to fill the field.

Examples:

a) For the coefficient -436.23, punch

|-|4|3|6|2|3|0|0|

b) For the coefficient 0.0025, punch

|_|_|_|_|_|_|2|5|

c) For the coefficient 3261., punch

| 3 | 2 | 6 | 1 | | | | |

d) For the coefficient 0., punch

| | | | | | | | |

3.4.6. Order of the Cards

The data cards are placed, in the order described above, behind the Fortran program deck and processed as a standard Fortran compilation/computation sequence with either the 1401 Fortran compilation deck or 1401 Fortran tape.

3.5. The Output Format

Appendix II shows a sequence of solution sheets to a typical example of a quadratic program used to maximize profits in a firm. The first sheet shows the initial data in the solution matrix and the initial X vector. The last sheet shows the feasible optimal solution and the final solution matrix.

By suitable modifications to the Fortran program, the feasible optimal solution may be placed on tape, to be used in further processing by another program (such as a bookkeeping routine). Thus, the quadratic program may be used as a "subroutine" for a more involved problem.

4. Summary

a) The optimal use of limited resources, where constant returns to scale cannot be assumed, is a major concern in the theory of the firm.

b) Quadratic programming is a practical answer to this problem.

c) Medium-sized digital computers are available to a great many firms. By a modification of the Dantzig quadratic algorithm, these machines may be used to solve a great many of the optimization problems in business firms.

d) This paper describes the modified Dantzig quadratic program.

e) A Fortran digital computer program is included, with instructions for its use, and a practical example.

REFERENCES

[1] Dantzig, G. B., "Quadratic Programming: A Variant of the Wolfe-Markowitz Algorithms," Research Report No. 2, Operations Research Center, University of California, Berkeley, April 14, 1961.

[2] Wolfe, P., "The Simplex Method for Quadratic Programming," Econometrica, Vol. 27, No. 3 (1959), 382-398.

[3] Kuhn, H. W., and A. W. Tucker, "Nonlinear Programming," Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. Berkeley: University of California Press, 1951: 481-492.

[4] "Fortran Specification for the IBM 1401," 1401 Data Processing System Bulletin J24-1455-1, International Business Machines Corporation, White Plains, New York, 1961.

Appendix I

```
PARAMI9R0206PS
      READ 5, NT, MT
      JB=NT+1
      M=NT+MT
      N=M+M
      DIMENSION A(30,15),C(15),X(30)
      PRINT 8
      DO 3 I=1,30
      X(I)=0.
      DO 2 J=1,15
    2 A(I,J)=0.
    3 CONTINUE
      READ 6,(C(J),J=1,10)
      READ 7,(C(J),J=11,15)
      DO 5 I=1,NT
    5 READ 6,(A(I,J),J=1,10)
      DO 60 J=JB,M
      READ 6,(A(I,J),I=1,10)
      X(J)=C(J)
      MR=J+M
      DO 50 I=1,NT
   50 A(MR,I)=A(I,J)
   60 CONTINUE
      DO 70 J=1,NT
      I=M+J
      X(I)=-C(J)
   70 A(I,J)=-1.
      PRINT 600,(C(I),I=1,15)
      DO 200 I=1,N
  200 PRINT 615,(A(I,J),J=1,15),X(I)
C     FIND INCOMING ACTIVITY, K
      JB=0
```

```
      MR=M+1
  105 E=0.
      K=0
      DO 120 I=MR,N
      IF(E-X(I)) 120,120,110
  110 E=X(I)
      K=I-M
  120 CONTINUE
      JA=M+K
C     OPTIMALITY CHECK
      IF(X(JA)) 205,700,700
C     FIND BINDING CONSTRAINT, L
  205 L=0
      E=999999.
      DO 230 J=1,M
      IF(A(JA,J)*C(J)) 219,204,219
  204 IF(X(J)) 219,230,219
  219 IF(A(K,J)) 212,230,209
  212 IF(C(J)) 210,230,230
  209 IF(C(J)) 230,230,210
  210 D=C(J)/A(K,J)
      IF(E-D) 230,230,220
  220 E=D
      L=J
  230 CONTINUE
      IF(L) 235,235,240
C     FIND DROPPING ACTIVITY
  240 DO 243 I=1,N
      IF(X(I)) 241,243,241
  241 IF(A(I,L)) 242,243,242
  242 KA=I
  243 CONTINUE
```

```
C     MODIFY A MATRIX AND SOLVE FOR NEW CONSTRAINTS, C(J)
      E=A(K,L)
      DO 300 I=1,N
  300 A(I,L)=A(I,L)/E
      C(L)=C(L)/E
      DO 320 J=1,M
      IF(J-L) 310,320,310
  310 D=A(K,J)
      C(J)=C(J)-D*C(L)
      DO 315 I=1,N
  315 A(I,J)=A(I,J)-D*A(I,L)
  320 CONTINUE
      JB=JB+1
      PRINT 630,JB,K,KA,L
      PRINT 600,(C(I),I=1,15)
C     SOLVE FOR NEW ACTIVITY LEVELS
      X(K)=C(L)
      X(KA)=0.
      DO 400 I=1,N
      IF(X(I)) 330,400,330
  330 E=0.
      DO 340 J=1,M
  340 E=E+C(J)*A(I,J)
      X(I)=E
  400 PRINT 615,(A(I,J),J=1,15),X(I)
C     COMPLEMENTARITY CHECK
      IF(X(JA)) 510,595,595
  510 K=KA+M
      GO TO 205
  595 PRINT 640
      GO TO 105
  700 PRINT 710
```

```
      STOP 222
  235 STOP 888
    5 FORMAT (2I2)
    6 FORMAT (10F8.4)
    7 FORMAT (5F8.4)
    8 FORMAT (32H1WOLFE SIMPLEX QUADRATIC PROGRAM,//)
  600 FORMAT (1X,15F8.4,//)
  615 FORMAT (1X,16F8.4)
  630 FORMAT (12H1IN SOLUTION,I4, 9H,ACTIVITY,I4,6HIS IN,,I4,21HDROPS, AN
     1D CONSTRAINT,I4,6HBINDS.)
  640 FORMAT (28H0THIS IS A FEASIBLE SOLUTION)
  710 FORMAT (21H AND OPTIMAL SOLUTION)
      END
```

WOLFE SIMPLEX QUADRATIC PROGRAM

0.0500	0.0380	0.0100	0.0400	-0.3000	-0.2000	-0.0450	2.3990	4.4640	52.7370	0.	0.	0.	0.	0.	
0.0011	0.	0.	0.	0.	0.	0.	0.0490	0.0810	1.0320	0.	0.	0.	0.	0.	0.
0.	0.0015	0.	0.	0.	0.	0.	0.0520	0.1340	0.3920	0.	0.	0.	0.	0.	0.
0.	0.	0.0010	0.	0.	0.	0.	0.0740	0.1100	0.1990	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.0050	0.	0.	0.	0.0480	0.1240	0.2980	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	2.3990
0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	4.4640
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	52.7370
-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0500
0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0380
0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0100
0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0400
0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.3000
0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2000
0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.0450
0.0490	0.0520	0.0740	0.0480	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0810	0.1340	0.1100	0.1240	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1.0320	0.3920	0.1990	0.2980	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.

IN SOLUTION 1.ACTIVITY 115 IN. BOUNDS. AND CONSTRAINT BOUNDS.

0.0051	0.0380	0.0100	0.0400	-0.3000	-0.2000	-0.0450	48.9592	0.4933	2.2111	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	48.9592
-0.0012	0.0015	0.	0.	0.	0.	0.	1.0612	0.0480	-0.7032	0.	0.	0.	0.	0.	0.
-0.0017	0.	0.0010	0.	0.	0.	0.	1.5102	-0.0123	-1.3595	0.	0.	0.	0.	0.	0.
-0.0011	0.	0.	0.0050	0.	0.	0.	0.9746	0.0447	-0.7129	0.	0.	0.	0.	0.	0.
0.0224	0.	0.	0.	0.	0.	0.	-20.4082	1.6531	21.0613	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.
-0.0224	0.	0.	0.	0.	0.	0.	20.4082	-1.6531	-21.0613	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.4933
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	2.2111
-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0051
0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0380
0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0100
0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0400
0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.3000
0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2000
0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.0450
0.0490	0.0520	0.0740	0.0480	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0810	0.1340	0.1100	0.1240	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1.0320	0.3920	0.1990	0.2980	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.

```
IN SOLUTION   2,ACTIVITY  181S IN,  11DROPS, AND CONSTRAINT   131NDS.
 0.1254  0.0315  0.0007  0.0340 -0.1746 -0.2000 -0.0450 48.9592  0.4933  2.2111  0.      0.      0.      0.      0.

 0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      0.      0.      48.9592
-0.0238  0.0027  0.0018  0.0011 -0.0238  0.      0.      1.0612  0.0480 -0.7032  0.      0.      0.      0.      0.      0.
-0.0339  0.0018  0.0035  0.0016 -0.0339  0.      0.      1.5102 -0.0123 -1.3595  0.      0.      0.      0.      0.      0.
-0.0220  0.0011  0.0016  0.0061 -0.0220  0.      0.      0.9796  0.0447 -0.7129  0.      0.      0.      0.      0.      0.
 0.4581 -0.0238 -0.0339 -0.0220  0.4581  0.      0.    -20.4082  1.6531 21.0613  0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.
-0.4581  0.0238  0.0339  0.0220 -0.4581  0.      0.     20.4082 -1.6531-21.0613  0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      0.      0.4933
 0.      0.      0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      2.2111
-20.4082  1.0612  1.5102  0.9796-20.4082  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
 0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      -0.0315
 0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      -0.0007
 0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      -0.0340
 0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.1746
 0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.2000
 0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.0450
 1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.1254
 1.6531  0.0480 -0.0123  0.0447  1.6531 -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
21.0612 -0.7032 -1.3595 -0.7129 21.0612  0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.

THIS IS A FEASIBLE SOLUTION
```

```
IN SOLUTION   3,ACTIVITY   41S IN,  14DROPS, AND CONSTRAINT   43INDS.
 0.2488  0.0251 -0.0084  5.6115 -0.0512 -0.2000 -0.0450 43.4623  0.2477  5.2117  0.      0.      0.      0.      0.

 0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      0.      0.      43.4623
-0.0197  0.0025  0.0015  0.1888 -0.0197  0.      0.      0.8752  0.0396 -0.5685  0.      0.      0.      0.      0.      0.
-0.0280  0.0015  0.0031  0.2687 -0.0280  0.      0.      1.2470 -0.0243 -1.1679  0.      0.      0.      0.      0.      0.
 0.      0.      0.      1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      5.6115
 0.3783 -0.0197 -0.0280 -3.6315  0.3783  0.      0.    -16.8508  1.8152 18.4722  0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.
-0.3783  0.0197  0.0280  3.6315 -0.3783  0.      0.     16.8508 -1.8152-18.4722  0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      0.      0.2477
 0.      0.      0.      0.      0.      0.      0.      0.      0.      1.0000  0.      0.      0.      0.      0.      5.2117
-16.8508  0.8752  1.2470151.7680-16.8508  0.      0.    158.4670 -7.2234115.3310  0.      0.      0.      0.      0.      0.
 0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      -0.0251
 0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.0084
-3.6315  0.1888  0.2687165.1370 -3.6315  0.      0.    161.7670  7.3739117.7330  0.      0.      0.      0.      0.      0.
 0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.0512
 0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.2000
 0.      0.      0.      0.      0.      0.     -1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.0450
 1.0000  0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.2488
 1.8152  0.0396 -0.0243  7.3739  1.8152 -1.0000  0.     -7.2234 -0.3293  5.2571  0.      0.      0.      0.      0.      0.
18.4722 -0.5686 -1.1679117.7330 18.4722  0.     -1.0000115.3300  5.2571-83.9364  0.      0.      0.      0.      0.      0.

THIS IS A FEASIBLE SOLUTION
```

IN SOLUTION 4,ACTIVITY 215 IN, 90DROPS, AND CONSTRAINT 9BINDS.

0.3718	0.0093	-0.0175	4.4304	0.0718	-0.2000	-0.0450	37.9818	6.2545	9.7677	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	37.9818
0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	6.2545
-0.0401	0.0030	0.0040	0.3847	-0.0401	0.	0.	1.7851	-0.6142	-1.5171	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	4.4304
1.2798	-0.1353	-0.0947	-12.2857	1.2798	0.	0.	-57.0074	45.8285	44.5279	0.	0.	0.	0.	0.	0.
-0.4966	0.0637	0.0367	4.7676	-0.4966	0.	0.	22.1222	-25.2458	-14.3540	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.
-1.2798	0.1353	0.0947	12.2857	-1.2798	0.	0.	57.0074	-45.8285	-44.5279	0.	0.	0.	0.	0.	0.
0.4966	-0.0637	-0.0367	-4.7676	0.4966	0.	0.	-22.1222	25.2458	14.3540	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	9.7677
-0.4381	1.3353	1.5124	196.2060	-20.4381	0.	0.	1.3313	182.3690	11.6453	0.	0.	0.	0.	0.	0.
0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0093
0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.0175
0.0305	-0.2808	-0.0023	200.2920	0.0305	0.	0.	-1.3592	186.1670	-11.8879	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.0718
0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2000
0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.0450
1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.3718
1.6517	0.0606	-0.0122	8.9437	1.6517	-1.0000	0.	0.0607	-8.3129	0.5308	0.	0.	0.	0.	0.	0.
21.0830	-0.9634	-1.3611	142.7970	21.0830	0.	-1.0000	-0.9693	132.7200	-9.4752	0.	0.	0.	0.	0.	0.

IN SOLUTION 5,ACTIVITY 1915 IN, 120DROPS, AND CONSTRAINT 28BINDS.

0.1188	0.1532	-0.0155	3.0599	-0.1813	-0.0458	-0.0450	37.9775	7.5233	9.5864	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	37.9775
0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	7.5233
-0.1220	0.0495	0.0046	-0.0584	-0.1220	0.0495	0.	1.7821	-0.2013	-1.5434	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	3.0599
4.9683	-2.2332	-0.1220	7.6874	4.9683	-2.2332	0.	-55.8719	27.2641	45.7134	0.	0.	0.	0.	0.	0.
-2.2332	1.0514	0.0495	-4.6358	-2.2332	1.0514	0.	22.0584	-16.5057	-14.9121	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.
-4.9683	2.2332	0.1220	-7.6874	-4.9683	2.2332	0.	55.8719	-27.2641	-45.7134	0.	0.	0.	0.	0.	0.
2.2332	-1.0514	-0.0495	4.6358	2.2332	-1.0514	0.	-22.0584	16.5057	14.9121	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.0000	0.	0.	0.	0.	0.	9.5864
-55.8725	22.0587	1.7821	-1.0804	-55.8725	22.0587	0.	-0.0072	1.0034	-0.0642	0.	0.	0.	0.	0.	0.
27.2643	-16.5058	-0.2018	147.5320	27.2643	-16.5058	0.	1.0016	137.2200	8.7623	0.	0.	0.	0.	0.	0.
0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.0156
7.6874	-4.6357	-0.0589	158.3310	7.6874	-4.6357	0.	-1.0779	147.5300	-9.4271	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1813
0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.0458
0.	0.	0.	0.	0.	0.	-1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.0450
1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1188
0.	1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1532
45.7135	-14.9123	-1.5435	-9.4259	45.7136	-14.9123	-1.0000	-0.0644	8.7611	-0.5592	0.	0.	0.	0.	0.	0.

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