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Analytical Approximations
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By Cecil Hastings, Jr. and J. P. Wong, Jr.
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Analytical Approximation

Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than 0.0017 over \((0, 2)\),

\[ q(2, x) \approx 0.135 + 0.566 \left( \frac{x}{2} \right)^2 - 0.096 \left( \frac{x}{2} \right)^4. \]

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{x}{2} + \frac{R^2 + x^2}{2}} I_0\left(\frac{R}{x}\right) dR$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0008 over $(0,3)$,

$$q(3,x) = .011 + .231 \left(\frac{x}{3}\right)^2 + .654 \left(\frac{x}{3}\right)^4 - .329 \left(\frac{x}{3}\right)^6.$$
Analytical Approximation

Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0011 over \((1, \infty)\),

\[ q(R, R) \approx \frac{R + 0.183}{2R - 0.388} \]

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0004 over \((0,1)\),

\[ q(R, R) = \frac{1}{2} \left[ 1 + e^{-R^2} I_0(R^2) \right] \]

\[ = 1 - .4921 R^2 + .3212 R^4 - .0966 R^6 \]

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Analytical Approximation

Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-f/2} (f^2 + x^2) I_0(f x) \, df \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0013 over (0,4),

\[ q(4,4-y) \approx \frac{.551}{\left[ 1 + .187 y + .055 y^2 + .051 y^3 \right]^4} \]

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