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GYROSCOPIC EFFECT IN BALL BEARINGS OF FUZE ARMING VANE

23 JULY 1963

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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ABSTRACT: The rotating section of a fuze armed by a vane spinning in a windstream due to free fall was variously mounted with steel ball bearings, nylon ball bearings, and teflon O-rings. Wind tunnel tests were run and theoretical analyses made to determine the rise time and the steady state speed of the vane as a function of the air speed. An expression for the steady state velocity of the vane when mounted with ball bearings was developed which showed that gyroscopic action of the balls determined the maximum angular velocity of the vane; the theoretical results are in good agreement with experimental data.
This report covers a study of the motion of a fuze arming vane mounted with ball bearings carrying a thrust load when the vane is exposed to a high velocity airstream. Its purpose is to disseminate the information obtained for the edification of engineers concerned with similar situations. This work was performed under Task No. RM37 73001/212 1/W114 00 003.

Appreciation is expressed to Mr. Arnold S. Munach who was responsible for the wind tunnel tests and who supplied the data for comparison with the theoretical results.

R. E. ODENING
Captain, USN
Commander

J. H. ARMSTRONG
By direction
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References

(a) Palmgren, A., Ball and Roller Bearing Engineering
INTRODUCTION

1. When the ROCKEYE I fuze (Mk 258 Mod 0) is exposed to the airstream, its arming vane rotates and engages a clutch after reaching some predetermined velocity. The rotational energy of the vane is then used to raise a pin which unlocks the rotor. For the sake of safety in the case of an accidental drop while landing, it is desirable that the speed for clutch engagement be greater than the maximum velocity of the vane when exposed to an airstream which has a velocity comparable to the landing speed of jet aircraft. In order to determine the minimum angular velocity at which the clutch should be engaged, wind tunnel tests were run which allowed the vane to spin freely to its maximum velocity. High speed movie photography was used to continuously record the vane motion.

2. Initially the rotating assembly in the fuze was mounted by means of a thrust ball bearing with steel balls. During development of the fuze, it was decided that steel ball bearings could not be used due to Brinelling of the races in vibration tests. Consequently, it was decided to try teflon O-rings for both thrust and radial bearings. The O-ring type bearing introduces sliding friction instead of rolling friction; thus, the maximum angular velocity is significantly less when O-ring bearings are used instead of ball bearings. This decrease in the range of the angular velocity may in turn complicate the air speed discrimination. In an effort to eliminate the Brinelling and simultaneously to attain a high steady state angular velocity, ball bearings with nylon balls were investigated. Three units were assembled with nylon ball bearings (see Figure 1) and wind tunnel tests were run. Data from these tests as well as from similar tests using teflon O-rings and from earlier tests with the steel balls are included herein. In none of these tests were the bearings lubricated. The purpose of this report is to present a theoretical analysis of the motion of the vane and to compare its predictions with the test results.
3. In developing an equation of motion for the vane it was assumed that the friction torque has two components, one proportional to load (Coulomb friction) and one proportional to velocity (viscous friction). This assumption gives good agreement for the range where the velocity is increasing, but the calculated maximum velocity is much higher than that which was observed in the tests involving ball bearings. Experimentally it was found that steady state angular velocity is reached rather abruptly, indicating that a new phenomenon becomes dominant at high speed (see Figure 2). This corresponds to the velocity where slipping of the balls in the bearing occurs due to a gyratory effect. Therefore the equation of motion is valid only to the point where slipping begins; here a cut-off velocity is reached.

Notation

4. For convenience in reference all symbols used in subsequent paragraphs are defined here.

\[ \begin{align*}
A &= \text{frontal area of vane, ft}^2 \\
\text{d} &= \text{diameter of ball, in.} \\
D &= \text{pitch diameter of bearing, in.} \\
F &= \text{axial force on vane due to airstream, lb} \\
g &= \text{acceleration of gravity, } 32.6 \text{ in./sec}^2 \\
I &= \text{weight moment of inertia of rotating assembly, lb-in.}^2 \\
J &= \text{mass moment of inertia of ball, slug-in.}^2 \\
m &= \text{mass of ball, slugs} \\
M &= \text{gyratory reaction moment, in.-lb} \\
M_f &= \text{friction torque per ball, in.-lb} \\
N &= \text{rotational speed of moving race, rpm} \\
N_b &= \text{rotational speed of ball center about axis of bearing, rpm} \\
r &= \text{moment arm from axis of rotation of assembly to bearing surface, in.} \\
t &= \text{time, sec} \\
T &= \text{torque exerted on vane by airstream, in.-lb} \\
T_f &= \text{total friction torque, in.-lb} \\
T_g &= \text{velocity dependent component of friction torque, in.-lb} \\
T \mu &= \text{load dependent component of friction torque, in.-lb} \\
\end{align*} \]
\[ V \text{ = velocity of windstream, ft/sec} \]
\[ V_e \text{ = velocity of point on ball in contact with stationary race, in./sec} \]
\[ V_i \text{ = velocity of point on ball in contact with moving race, in./sec} \]
\[ V_m \text{ = velocity of center of ball, in./sec} \]
\[ V_{m}^{x} \text{ = velocity of windstream in test run, ft/sec} \]
\[ V_{x} \text{ = velocity of windstream for calculated conditions, ft/sec} \]
\[ Z \text{ = number of balls in the bearing} \]
\[ \alpha \text{ = angle between radial plane of bearing and direction of load through the balls (0° for pure radial bearings; 90° for pure thrust bearing)} \]
\[ \beta \text{ = constant of proportionality, ft-lb/sec} \]
\[ \beta_o \text{ = constant of proportionality, lb-sec} \]
\[ \gamma \text{ = density of ball, slug/in.}^3 \]
\[ \delta \text{ = angular displacement of vane, rad.} \]
\[ \dot{\delta} \text{ = angular velocity of vane, rad/sec} \]
\[ \ddot{\delta} \text{ = angular acceleration of vane, rad/sec}^2 \]
\[ \mu \text{ = Coulomb friction factor of bearing} \]
\[ \mu_t \text{ = friction factor resisting sliding of balls} \]
\[ \rho \text{ = density of air, slugs/ft}^3 \]
\[ \omega \text{ = angular velocity of ball around its own axis, rad/sec} \]
\[ \Omega \text{ = angular velocity of ball center around the bearing axis, rad/sec} \]

Equation of Motion

5. The net torque on the vane is

\[ T = T_A - T_F = \frac{I}{j} \ddot{\theta} \]  \hspace{1cm} (1)

Assume the friction torque is the sum of two components

\[ T_\mu = \mu \ F \ r \]  \hspace{1cm} (2)

and

\[ T_\beta = \beta_o \ \dot{\theta} \ r \]  \hspace{1cm} (3)

3
For a given moment arm and loading, $T$ is constant and

$$I/q \dot{\theta} = \tau - \beta_0 \dot{\theta} r$$

$$\dot{\theta} + \frac{\beta_0 r}{I} \dot{\theta} = q \left( \frac{\tau - \tau}{I} \right) \quad (4)$$

$$\theta = A + B e^{-\frac{\beta_0 r}{I} t} + \left( \frac{\tau - \tau}{\beta_0 r} \right) t$$

After evaluating the constants $A$ and $B$ for the initial conditions ($t = 0$, $\theta = 0$, $\dot{\theta} = 0$), one finds

$$\theta = \left( \frac{\tau - \tau}{\beta_0 r} \right) \left[ \frac{I}{\beta_0 r} \left( e^{-\frac{\beta_0 r}{I} t} - 1 \right) + t \right] \quad (5)$$

$$\dot{\theta} = \left( \frac{\tau - \tau}{\beta_0 r} \right) \left( 1 - e^{-\frac{\beta_0 r}{I} t} \right) \quad (6)$$

Before equation (6) can be used to calculate the angular velocity of the vane, the unknown parameters $(\tau - \tau)$ and $\beta_0$ must be determined or eliminated.

**Evaluation of $\beta$**

6. Defining a new constant $\beta = \frac{\beta_0}{\tau}$ and using wind tunnel data for the vane angular velocity at two times and for a given air speed, one can eliminate $(\tau - \tau)$ as follows:

$$\frac{\dot{\theta}_1}{\dot{\theta}_2} = \frac{1 - e^{-\frac{\beta_0 r}{I} t_1}}{1 - e^{-\frac{\beta_0 r}{I} t_2}} \quad (7)$$
Now only $\beta$ must be evaluated. By using the Taylor series expansions for the exponentials and collecting like powers of $\beta$, one can form a power series in terms of $\beta$.

$$
\sum_{m \geq 1} (-1)^{m+1} \left[ \frac{\dot{\theta}_2 t_1^m - \dot{\theta}_1 t_2^m}{m! (I/r)^m} \right] \beta^m = 0
$$

The first two terms give a first approximation to the value of $\beta$ as

$$
\beta_1 = 2 \frac{I}{r} \left( \frac{\dot{\theta}_2 t_1 - \dot{\theta}_1 t_2}{\dot{\theta}_2 t_1^2 - \dot{\theta}_1 t_2^2} \right) \tag{8}
$$

If one puts equation (7) in the following form

$$
\frac{1 - e^{-\frac{\beta r t_1}{I}}}{1 - e^{-\frac{\beta r t_2}{I}}} - \frac{\dot{\theta}_1}{\dot{\theta}_2} = 0
$$

or

$$
1 - e^{-\frac{\beta r t_1}{I}} - \frac{\dot{\theta}_1}{\dot{\theta}_2} \left( 1 - e^{-\frac{\beta r t_2}{I}} \right) = 0,
$$

then one may define $f(\beta)$ and $f'(\beta)$ as

$$
f(\beta) = 1 - \frac{\dot{\theta}_1}{\dot{\theta}_2} - e^{-\frac{\beta r t_1}{I}} + \frac{\dot{\theta}_1}{\dot{\theta}_2} e^{-\frac{\beta r t_2}{I}} = 0 \tag{9}
$$

$$
f'(\beta) = \frac{\dot{\theta}_1}{I} e^{-\frac{\beta r t_1}{I}} - \frac{\dot{\theta}_1}{\dot{\theta}_2} \frac{r t_2}{I} e^{-\frac{\beta r t_2}{I}} = 0. \tag{10}
$$
Now a better value of $\beta$ can be obtained numerically by using Newton's method where

$$\beta_m = \beta_{m-1} - \frac{f(\beta_{m-1})}{f'(\beta_{m-1})}. \quad (11)$$

7. When the value of $\beta$ has been determined, one can use it in equation (7) with a known value of $\dot{\phi}$ and $t$ to find $\dot{\phi}$ corresponding to any other $t$. To obtain a relation between $\dot{\phi}$ and $t$ for an air velocity other than the one that was used in the wind tunnel, solve for $(T_A - T\mu)$ in equation (6) using a known $t$ and $\dot{\phi}$ from the test. Since the momentum of the flowing air and therefore the torque on the vane is proportional to the velocity, one may obtain the torque term for any other air velocity as

$$(T_A - T\mu) \Big|_{v = v_x} = \left(\frac{v_x}{v_e}\right) (T_A - T\mu) \Big|_{v = v_e}.$$ 

Since the other parameters in equation (6) are independent of the air flow, one can use this equation with the torque expression calculated above to obtain $\dot{\phi}$ as a function of $t$, i.e.,

$$\dot{\phi} = \left(\frac{v_x}{v_e}\right) \left(\frac{T_A - T\mu}{\beta \cdot r}\right) \bigg|_{v = v_e} \left(1 - \frac{\sqrt{\beta_e r}}{r} t\right).$$

Gyroscopic Cut-Off Velocity

8. The equation of motion which has been set up enables one to calculate $\dot{\phi}$ as a function of $t$ as long as the force system assumed for the derivation is effective. However, at high speeds of rotation the balls in a thrust ball bearing experience an appreciable gyroscopic force due to the changing direction of their spin axes. This force will cause slipping of the balls when it exceeds the frictional resistance. Therefore, to determine the velocity at which slipping will occur, the gyroscopic effect due to the change of direction of the axis of rotation of a ball in the bearing is considered. The vector equation for a gyroscopic moment is
The magnitude of the moment is
\[ M = J \omega \Omega \sin \alpha, \quad (13) \]
and the mass moment of inertia of the ball is
\[ J = \frac{md^2}{12} = \frac{\pi \delta d^5}{60}. \quad (14) \]

To determine the value of \( \Omega \) use a diagram of instantaneous velocities of points on the ball.

\[ \text{Then} \]
\[ v_i = \frac{\pi DN}{60} \]
\[ v_e = 0 \]
\[ \vec{v}_m = \frac{v_i + v_e}{2} \]
Hence,
\[ \vec{v}_m = \frac{\pi DN}{60} = \frac{v_i}{2} \]
and
\[ \frac{\pi DN}{60} = \frac{\pi Dn}{120}. \]
Thus, \[ N = \frac{n}{2} \]
Remembering that linear velocity is the product of the radius and the angular velocity, one can also express the velocity of the ball centers as
The angular velocity of the ball center about the bearing axis can be found in terms of $\omega$ by dividing the velocity of the rotating ball by the distance it travels in one circuit of the bearing.

$$\Omega = \frac{\pi D}{30} = \frac{\pi n}{60} \quad (15)$$

Thus

$$\omega = \frac{D - \Omega}{d} = \frac{\pi D n}{60d} \quad (16)$$

Now substituting (14), (15) and (16) into (13) yields

$$M = \left( \frac{\pi x d^5}{60} \right) \left( \frac{\pi D n}{60d} \right) \left( \frac{\pi m}{60} \right) \sin \alpha$$

or

$$M = \frac{\pi^3 x D d^4 n^2}{216, 000} \sin \alpha \quad (17)$$

The friction torque per ball which resists slippage is

$$M_F = \frac{\mu_s F d}{Z} \quad (18)$$

Therefore sliding will occur when $M \geq M_F$

$$\text{or} \quad \frac{\pi^3 x D d^4 n^2}{216, 000} \sin \alpha \geq \frac{\mu_s F d}{Z}$$

and the maximum angular velocity can be found from

$$n^2 = \frac{695 c \mu_s F}{Z \pi D d^3 \sin \alpha} \quad (19)$$

9. In reference (a) Palmgren states that for high speeds the maximum friction torque per ball which can be expected to resist slippage is

$$M_F = 0.03 F d / Z$$
Therefore for steel ball bearings the maximum velocity is found from

$$m^2 = \frac{1.908 \times 10^5 F}{2 \pi d^3 \sin \alpha} \quad (20)$$

In the case of nylon balls the cut-off angular speed is determined from

$$m^2 = \frac{1.304 \times 10^6 F}{2 \pi d^3 \sin \alpha} \quad (21)$$

where the friction factor is again assumed to be 0.02. In the test with the steel ball bearings the pressure angle $\alpha$ was 90°. The arrangement of the nylon bearings is shown in Figure 1; the airstream loaded only the forward bearing. It is apparent that the load line through the balls is nominally at 45° with respect to the plane of the race. With these facts and equations (20) and (21), it is now possible to calculate the theoretical cut-off angular speed at a given air speed for the ball bearings. A numerical example is given in the appendix.

10. The value 0.02 for $\mu_s$ is satisfactory for the bearing materials used here, but for special bearings it may be necessary to determine the friction factor from the experimental data. For example, one could put the observed value of the steady state velocity into equation (17) and solve for $M$. Then set $M = M_F = \mu_s F d / \pi$. Now one may solve for $\mu_s$ and then use this value to obtain a new expression for $n$ as a function of $F$.

RESULTS

11. For the case of steel balls and a 200-knot airstream the calculated maximum angular velocity is only 150 rpm greater than that determined in the wind tunnel tests (22,650 rpm). This indicates that the friction factor which produces the moment to resist sliding is very close to the assumed value of 0.02. Figure 2 is a plot of angular velocity versus
time for both wind tunnel data and equation (7) for a vane in a 200-knot windstream. Similar theoretical curves for velocities of 150, 250 and 525 knots are given in Figure 3. The theoretical curve is in close agreement with the wind tunnel data for the 200-knot case. It is hypothesized that the 150 and 250-knot curves are also valid; however, at 525 knots there may be damage to the bearing before the vane can spin up to the velocity calculated above.

12. Of the three fuzes with nylon ball bearings, one failed during the test; hence no data from this fuze are included. Figure 4 shows the data from the other two fuzes and also presents the calculated steady state vane angular speed as a function of air speed for 0°, 45° and 90° pressure angles in nylon ball bearings. The data cross from the calculated 45° pressure angle curve to the calculated 90° pressure angle curve between 225 knots and 325 knots. This may appear at first to be unreasonable. However, if one looks again at the fuze assembly in Figure 1 it is apparent that the pressure angle need not be 45° at all times and for each assembly. For example, suppose that the dimensions are such as to give a pressure angle of exactly 45°. Then the axial load is low, then the cut-off speed should follow the theoretical curve of Figure 4 for $\alpha = 45^\circ$. However, when the air flow loads the bearings a significant amount there is a tendency for the nylon balls to deform. This would cause a change in the pressure angle and consequently a change in the gyroscopic force. This would explain the drop of the data from the curve for $\alpha = 45^\circ$ to the curve for $\alpha = 90^\circ$. It should be possible to calculate the change in pressure angle from a knowledge of the deformation characteristics of nylon and the contour of the bearing surfaces, but this would be difficult in practice.

13. There is another condition which could cause the pressure angle $\alpha$ to be even more indeterminate and could in fact cause a greater dispersion of data than that of the two assemblies which were run. This would be the condition caused by a tolerance build-up. For example, suppose that the tolerances for the inside race, which is on the nut, add up such that the inside race is as large as possible; and conversely that the tolerances for the outside race, which is inside the housing, add up so that this race is as small as possible; then the radial space for the balls would be
minimized. As a consequence of this condition the pressure angle could be approximately 0° and there would be no gyroscopic forces. With suitable contours of the bearing surfaces this could be counteracted by a deformation of the balls when the force of the airstream becomes large enough. The pressure angle could change from 0° to 45° and perhaps even to 90° if the axial force is sufficiently large. Consequently, the gyroscopic torque would go from zero to a maximum.

14. The curves for $\alpha = 0^\circ$ and $\alpha = 90^\circ$ on Figure 4 show the respective limits of possible cut-off speeds (i.e., steady state angular velocity) for the nylon ball bearings used in an assembly such as is shown in Figure 1. This leaves the possibility of a very wide dispersion of data. For example, from Figure 4 it would theoretically be possible to get about 26,000 rpm from one assembly and only 9,000 rpm from another assembly while both are experiencing a 100-knot air speed. (In practice the upper limit of 26,000 rpm would probably never be reached due to the deformation of the nylon balls.) Also, the assembly which spins at only 9,000 rpm in a 100-knot airstream would require a 300-knot airstream to reach 26,000 rpm. This sort of behavior is very undesirable for designing a fuze with air speed discrimination.

15. Figure 5 presents the angular speed as a function of time for an assembly with nylon ball bearings and for one with steel ball bearings. The air speed for both runs was 250 knots. It should be noted that the pressure angle $\alpha$ is different for the two assemblies. Consequently the curves are presented only for qualitative study. As indicated from Figure 4 the cut-off speed would be 22,500 rpm for nylon bearings with a pressure angle of 90°. However, the rise from zero up to 22,500 rpm should be identical to that indicated on Figure 5 for a pressure angle of 45°. This would indicate that the rise time for nylon and steel ball bearings is almost the same (i.e., $\approx 0.3$ sec for 250-knot air speed) even though the steady state angular velocities are different.

16. The vane with steel ball bearings continues to spin at a constant angular velocity (cut-off speed) probably until bearing failure. However, the vane with nylon bearings tends to slow down somewhat with time after reaching its cut-off speed. By the time $t = 0.5$ sec, for the case of
Figure 5, the speed had dropped 700 rpm from the peak value of 27,600 rpm. This drop continued as a function of time; however, the data is not now available to present quantitatively.

17. Figure 6 presents the steady state angular velocity for nylon ball, steel ball, and teflon 0-ring bearings (all unlubricated) as a function of air speed. Obviously the steep slopes of the two ball bearing curves present a greater velocity difference to use for air speed discrimination. It should be noted that the data and calculations for steel ball bearings use a constant pressure angle of 90° while the nylon ball bearings seem to range from a pressure angle of 45° to 90°.

CONCLUSIONS

18. The assumption of viscous friction in combination with a cut-off velocity determined by gyroscopic action in the ball bearing gives a good representation of the motion of the vane (see Figures 2 and 4).

19. The vane spin-up characteristics using nylon or steel ball bearings are very similar (see Figure 5).

20. The ball bearing gyroscopic effect is present in nylon ball bearings as well as in steel ball bearings.

21. A similar analysis would hold for other rotating assemblies mounted with thrust ball bearings.

22. The ball bearing gyroscopic effect is experienced as a step function and for a given configuration is only a function of axial load (i.e., air speed) and consequently assists in air speed discrimination.

23. The vane steady state angular velocities using nylon or steel ball bearings are of the same order of magnitude for similar air velocities up to about 250 knots (see Figure 6).

24. The vane steady state angular velocities using teflon 0-ring bearings are about a factor of three less than those when either nylon or steel ball bearings are used (see Figure 6).
25. The cut-off angular velocity due to the ball bearing gyroscopic effect can be fairly accurately calculated for the nylon ball bearing if the pressure angle is known accurately.

26. However, if the bicycle-type race is used there may be a significant uncertainty in the exact value of the pressure angle. Consequently, the bicycle-type race is very poor for this application in speed discrimination (see Figure 4).

27. When ball bearings are used as thrust or combination thrust-radial bearings for high-speed applications where uniformity of velocity history is desired, plane surfaces may be used as races to eliminate the uncertainty in the pressure angle introduced by concave surfaces.

J. E. HIGHDALE

D. L. BLANCHARD
APPENDIX

NUMERICAL EXAMPLE

The following data are for one run in the wind tunnel tests of 17 August 1961. The bearings had steel balls, and the wind speed was 200 knots.

\[ I = 0.004832 \text{ lb-in.}^2 \]
\[ r = \frac{3}{16} \text{ in.} \]
\[ \tau_1 = 0.04 \text{ sec} \]
\[ t_1 = 0.24 \text{ sec} \]
\[ \dot{\phi}_1 = 3250 \text{ rpm} \]
\[ \dot{\phi}_2 = 17,350 \text{ rpm} \]

Substituting into equations (9) and (10) gives

\[ f(\dot{\phi}) = 0.8127 - e + 0.187 e \]
\[ f'(\dot{\phi}) = 1.552 e - 1.744 e \]

Now using equation (8) to find a first value of \( \beta \) gives

\[ \dot{\beta}_1 = 0.03 \text{ in.-lb/sec} \]
\[ f(\dot{\beta}_1) = -0.0002 \]
\[ f'(\dot{\beta}_1) = 0.1629 \text{ sec/in.-lb} \]

To find a better approximation for \( \beta \) use equation (11).

\[ \dot{\beta}_2 = 0.03 - \frac{0.0002}{0.1629} = 0.0312 \text{ in.-lb/sec} \]
\[ f(\dot{\beta}_2) = 0 \]

Using \( \dot{\beta} = 0.0312 \text{ in.-lb/sec}, \dot{\phi}_2 = 17,350 \text{ rpm}, \)

and \( t_2 = 0.24 \text{ sec} \) in equation (7) gives the following

equation for \( \ddot{\phi} \) as a function of \( t \).

\[ \ddot{\phi} = 68,850 (1 - e^{-1.2107t}) \]
Since $\dot{\phi} = 3250$ rpm at $t = 0.04$ sec was used to find $\beta$, one may check the validity of the value of $\beta$ calculated above by using $t = 0.04$ sec and solving for $\dot{\phi}$.

$$\dot{\phi} = 68,850 \left(1 - e^{-0.04843}\right) = 3253 \text{ rpm}$$

The variation between the given and calculated values of $\dot{\phi}$ is less than 0.1% which is well within the accuracy of an observation; thus the value of $\beta$ is sufficiently precise.

To determine the maximum velocity one must calculate the axial force on the vane assembly in a 200-knot airstream.

$$F = \rho A v^2$$

$$F = (0.002375) (0.008305) (338)^2 = 2.25 \text{ lb}$$

Using this in equation (20) with

$\alpha = \pi/2$

$Z = 9 \text{ balls}$

$D = 3/8 \text{ in.}$

$d = 1/16 \text{ in.}$

$gives$

$n = 22,800 \text{ rpm}$

This is only 150 rpm greater than that determined in the wind tunnel tests (22,650 rpm). This indicates that the coefficient of sliding friction is very close to the assumed value of 0.02.
FIG. 1 FUZE MK 258 AS MODIFIED FOR TESTING SHOWING BEARING LOCATIONS
STEADY STATE $\omega = 68,850$ RPM

FROM EQUATION OF MOTION

EQUATION (7)

CALCULATED GYROSCOPIC CUT-OFF VELOCITY $\omega = 22,800$ RPM

EXPERIMENTALLY OBSERVED STEADY STATE VELOCITY $\omega = 22,650$ RPM

O WIND TUNNEL DATA

FIG. 2 VANE ANGULAR VELOCITY AS A FUNCTION OF TIME FOR STEEL BALL BEARINGS AND AN AIR SPEED OF 200 KNOTS
Fig. 3 Vane angular velocity as a function of time for steel ball bearings and air speed of 150, 200, 250 and 525 knots.
FIG. 4 OBSERVED AND CALCULATED STEADY STATE VANE SPEED AS A FUNCTION OF AIR SPEED WITH NYLON BALL BEARINGS IN THE MODIFIED FUZE MK 258.
Fig. 5 Angular vane velocity as a function of time for both steel and nylon ball bearings in the modified fuze MK 258 for an air speed of 250 knots.
FIG. 6 STEADY STATE VANE SPEED AS A FUNCTION OF AIR SPEED FOR BOTH TEFLOM O-RING AND NYLON BALL BEARINGS IN THE MODIFIED FUZE MK 258
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