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METHOD OF PREDICTING SATELLITE DECAY

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A METHOD OF PREDICTING SATELLITE DECAY

by

GEORGE B. FINDLEY

April 1961

Project 1772
Task 17721

DIRECTORATE OF AEROSPACE
AIR PROVING GROUND CENTER
AIR FORCE SYSTEMS COMMAND
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ABSTRACT

Equations are derived for predicting the decay day and decay revolution of a satellite. Only two parameters, obtained from the quadratic equation that predicts equatorial crossings, are used. Examples of predictions are given.
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1. INTRODUCTION

Equations are derived for predicting the decay day and decay revolution of a satellite. Only two parameters, obtained from a quadratic equation which predicts equatorial crossings, are used. It appears that a predicted decay day might be useful in predicting equatorial crossings. Some examples of predictions are given.

2. DERIVATION OF EQUATIONS

Let

\[ \dot{P}_N = F(T_N) \]  \hspace{1cm} (1)

where \( F(T_N) \) is a continuous function of time and \( \dot{P}_N \) is the time derivative of the nodal period at revolution number \( N \). Integration of (1), by the first law of the mean for integrals, yields

\[ E_{N_i} = T_{N_i} - T_{N_i} = P_{N_i} - P_{N_i} = P_{N_i} - P_{N_i}, \]  \hspace{1cm} (2)

where

\[ E_{N_i} = \text{exact number of days to decay from revolution number } N_i, \]

\( N_i = \text{revolution number}, \)

\( N_d = \text{revolution number at decay}, \)

\( T_{N_d} = \text{time at decay (in days)}, \)

\(^1\) Figures in parentheses refer to equations.
\( T_{N_i} \) = time at revolution \( N_i \) (in days),

\( P_{N_i} \) = nodal period at \( T_{N_i} \) (in days),

\( P_{N_d} \) = period at \( T_{N_d} \) (in days),

\( P_{N_j} \) = mean value of \( P_{N} \) during the interval \( E_{N_i} \), and

\( T_{N_d} \geq T_{N_j} \geq T_{N_i} \).

Set

\[
P_{N_j} = \frac{\dot{P}_{N_i} (P_{N_i} - P_{N_d})}{0.6 \left(1 + \alpha_{N_i}\right) \left(\frac{88}{1440}\right)}
\]

\( \dot{P}_{N_i} \) is the rate of change of the period at \( T_{N_i} \), and

\[
\alpha_{N_i} = \alpha_{N_i} (P_{N_i}, P_{N_d}, P_{N_d}).
\]

The constants in (3) tend to minimize the \( \alpha_{N_i} \) when \( P_{N_i} \) is greater than \( \frac{88}{1440} \) days.

Substitution of (3) into (2) gives

\[
E_{N_i} = 0.6 \left(1 + \alpha_{N_i}\right) \left(\frac{88}{1440}\right) \frac{\dot{P}_{N_i}}{P_{N_i}}
\]

For predicting the time of equatorial crossing, The National Space Surveillance Control Center (NSSCC), (L. G. Hanscom Field, Bedford, Massachusetts) uses the equation

\[
T_{N_i} + N_i = T_{N_i} + N P_{N_i} + N^2 C_{N_i} + N^3 D_{N_i},
\]
where $D_{N_i}$ is generally omitted until the last hundred or so revolutions of a satellite.

Define $C_{N_i}$ in (6) by

$$C_{N_i} = \frac{P_{N_i}}{2} \cdot \frac{\dot{P}_{N_i}}{}.$$  \hspace{1cm} (7)

Define $A_{N_i}$ by either

$$A_{N_i} = \frac{0.6 (P_{N_i} - \frac{88}{1440})}{-P_{N_i}}.$$ \hspace{1cm} (8)

or

$$A_{N_i} = \frac{0.3 P_{N_i} (P_{N_i} - \frac{88}{1440})}{-C_{N_i}}.$$ \hspace{1cm} (9)

Substitution of (8) into (5) gives

$$E_{N_i} = (1 + a_{N_i}) A_{N_i}.$$ \hspace{1cm} (10)

No assumed value of decay is contained in (10).

In attempting to obtain an empirical representation for $a_{N_i}$, through an analysis of data from 15 decayed satellites, the predictions indicated that $|a_{N_i}| < 1$ if the predictions were made when $P_{N_i} > \frac{88}{1440}$ days. When this condition is fulfilled, the accuracy of the predictions, given by (9) indicates that (9) is a good approximation to the exact solution (2) for those 15 satellites.

Assume for a given satellite the $P_{N_i}$ and $C_{N_i}$ in (11) are such that predicted equatorial crossings are in good agreement with observations.
Substitution of these values of \( P_{N_i} \) and \( C_{N_i} \) into (9) gives the days to decay from \( T_{N_i} \). The decay day \( T_{N_d} \) is then

\[
T_{N_d} = T_{N_i} + A_{N_i} .
\] (12)

The \( T_{N_d} \) in (12) can be used to determine \( C_{N_k} \) at some later epoch \( T_{N_k} \). From the NSSCC bulletins we obtain \( T_{N_k} \) and \( P_{N_k} \). Since \( T_{N_d} \) is now known, we have

\[
A_{N_k} = T_{N_d} - T_{N_k} ,
\] (13)

and

\[
C_{N_k} = \frac{0.3 \, P_{N_k} \left( \frac{P_{N_k} - 0.88}{1440} \right)}{T_{N_k} - T_{N_d}} .
\] (14)

Appendix II gives an application of this technique for determining the \( C_{N_k} \) to be used on later bulletins.

3. THE DECAY REVOLUTION

In order to predict the approximate decay revolution number \( N_{d'} \), use (12) to obtain \( T_{N_d} \). Then let \( N_q \) be the revolutions from \( N_i \) to \( N_d \) and solve (15) for \( N_q \).

\[
T_{N_d} = T_{N_i} + N_q = T_{N_i} + N_q \, P_{N_i} + N_q^2 \, C_{N_i} .
\] (15)
For a better approximation of \( N_d \), especially when predicting far in advance of decay, the above method should be modified in the following manner. We first derive an expression for \( D_{N_i} \) in terms of \( P_{N_i} \) and \( C_{N_i} \). This derivation requires that the \( P_{N_d} \) in (17) be assigned the value \( \frac{88}{1440} \) days. When this is done, the approximate solution (9) is exactly the same as that obtained if (1) is replaced by

\[
\hat{p}_N = J \left( P_N - P_{N_d} \right)^{-2/3},
\]

(17)

where \( J \) is a constant. From (17) we obtain

\[
\hat{P}_{N_i} = -\frac{2}{3} \left( P_{N_i} - \frac{88}{1440} \right)^{-1} \left( P_{N_i} \right)^{2/3},
\]

(18)

Define \( D_{N_i} \) in (6) by

\[
D_{N_i} = \frac{P_{N_i}^2 \cdot P_{N_i}}{P_{N_i} - P_{N_d}^2}.
\]

(19)

Substitution of (7) and (18) into (19) gives

\[
D_{N_i} = -\frac{4}{9} C_{N_i} \left( P_{N_i} - \frac{88}{1440} \right)^{-1}.
\]

(20)

We substitute \( D_{N_i} \) into (21) and solve for \( N_q \).

\[
T_{N_d} = T_{N_q} + N_i = T_{N_i} + N_q P_{N_i} + N_q^2 C_{N_i} + N_q^3 D_{N_i}.
\]

(21)
This $N_q$ when substituted into (16) will in general give a better approximation of $N_d$ than will the $N_q$ obtained from (15). To illustrate the last statement we use 1959 Epsilon-2 as an example. The first two NSSCC bulletins for this satellite gives for epoch revolution number 2,

\begin{align*}
T_2 &= 46.08599 \\
P_2 &= 7.241102 \times 10^{-2} \\
C_2 &= -6.795 \times 10^{-7}.
\end{align*}

(22)

Substitution of (22) into (9) gives

\begin{align*}
A_2 &= 361.25278 \text{ days.}
\end{align*}

Equation (15) becomes

\begin{align*}
361.25278 = N_q P_2 + N_q C_2
\end{align*}

(23)

From (23) we obtain $N_q = 5247$, and from (22) and (20) we obtain

\begin{align*}
D_2 &= -1.816 \times 10^{-11}.
\end{align*}

Substitution of $D_2$ into (21) gives $N_q = 5289$. Hence the first method gives $N_d = 5249$ and the second method gives $N_d = 5291$. Since the decay revolution was approximately 5313.5, it is quite apparent that the introduction of $D_2$ gives a better prediction of $N_d$. It should be pointed out that (20) may also serve a useful purpose in determining the values of $D_{N_1}$ to use in (6) when predicting equatorial crossings during the last hundred or so revolutions of a satellite.
4. REMARKS

Some of the variations in the predicted decay day, (obtained by using $T_{N_1}$, $P_{N_1}$, and $C_{N_1}$ from NSSCC bulletins), are due to the fact that bulletins must be issued even though insufficient or no observations are available on which to determine good values of $T_{N_1}$, $P_{N_1}$, and $C_{N_1}$. Since the predicted decay day depends only on these values, the best available values should be used. Such values are obtained by fitting (11) to the observations. An example of this is the prediction of decay for 1958 Epsilon in Appendix I.
APPENDIX I - EXAMPLES OF DECAY PREDICTIONS

1958 EPSILON (EXPLORER IV)

Equatorial crossing times for the first 937 revolutions of this satellite are contained in "Interim Definitive Orbit for the Satellite 1958 Epsilon IV" NASA TN D-410. From this reference we obtain for epoch revolution 100, $T_{100} = 3.267095$ August 1958. The equation

$$T_N + 100 = T_{100} + NP_{100} + N^2C_{100}$$

predicts $T_{200}$ with zero error and $T_{300}$ with an error of 14 seconds, when $P_{100} = 7.63746 \times 10^{-2}$ and $C_{100} = -7.84 \times 10^{-7}$. Substitution of these values into (9) gives $A_{100} = 446.1$ (days to decay from $T_{100}$). The decay day was approximately 23.2 October 1959, so the error in $A_{100}$ is less than 0.4 days.

The following predictions were all obtained by using the $T_{N_i}$, $P_{N_i}$, and $C_{N_i}$ given on NSSCC bulletins issued at epoch revolution number $N_i$.

1958 DELTA (Sputnik III)

<table>
<thead>
<tr>
<th>$N_i$</th>
<th>$A_{N_i}$</th>
<th>$T_{N_d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7400</td>
<td>171.68</td>
<td>463.17</td>
</tr>
<tr>
<td>7600</td>
<td>147.66</td>
<td>452.52</td>
</tr>
<tr>
<td>8100</td>
<td>131.07</td>
<td>469.12</td>
</tr>
<tr>
<td>8250</td>
<td>115.09</td>
<td>463.03</td>
</tr>
</tbody>
</table>
The average value of $T_{N_d}$ is 462.2. The actual decay of the satellite occurred approximately at 462.3.

1959 ZETA (Discoverer VI)

<table>
<thead>
<tr>
<th>$N_i$</th>
<th>$A_{N_i}$</th>
<th>$T_{N_d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>51.81</td>
<td>285.57</td>
</tr>
<tr>
<td>80</td>
<td>59.11</td>
<td>296.18</td>
</tr>
<tr>
<td>130</td>
<td>57.05</td>
<td>297.41</td>
</tr>
<tr>
<td>190</td>
<td>55.31</td>
<td>299.61</td>
</tr>
<tr>
<td>340</td>
<td>46.65</td>
<td>300.75</td>
</tr>
<tr>
<td>500</td>
<td>27.54</td>
<td>292.00</td>
</tr>
<tr>
<td>610</td>
<td>17.54</td>
<td>289.06</td>
</tr>
<tr>
<td>700</td>
<td>13.84</td>
<td>291.09</td>
</tr>
<tr>
<td>720</td>
<td>14.01</td>
<td>291.39</td>
</tr>
</tbody>
</table>

The average value of $T_{N_d}$ is 293.67. The last NSSCC bulletin estimated the decay occurred about 293.75.
1959 KAPPA (Discoverer VII)

<table>
<thead>
<tr>
<th>$N_i$</th>
<th>$A_{N_i}$</th>
<th>$T_{N_d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>22.73</td>
<td>335.54</td>
</tr>
<tr>
<td>30</td>
<td>11.79</td>
<td>325.59</td>
</tr>
<tr>
<td>50</td>
<td>15.59</td>
<td>330.69</td>
</tr>
<tr>
<td>110</td>
<td>12.19</td>
<td>331.84</td>
</tr>
<tr>
<td>200</td>
<td>5.84</td>
<td>330.61</td>
</tr>
</tbody>
</table>

The average value of $T_{N_d}$ is 330.85. The last NSSCC bulletin estimated that decay occurred shortly after 330.78.

1959 EPSILON-2 (Capsule of Discoverer V)

In Appendix II we find that the first two bulletins predict $A_2 = 361.25$ and $T_{N_d} = 407.34$. Since these bulletins gave accurate predictions on equatorial crossings, assume that the decay day is $(407 \pm 30)$. The average value of $T_{N_d}$ obtained from the first 12 bulletins, which predict within the above limits, is 408.33. All twelve predictions were made over 225 days before decay. The actual decay was $T_{N_d} = 408.77$. 
APPENDIX II - EXAMPLE OF PREDICTED DECAY DAY USED TO PREDICT EQUATORIAL CROSSINGS

In this appendix we illustrate how the predicted decay day can be used to obtain the $C_{N_1}$ terms for later bulletins. We use 1959 Epsilon-2 (Capsule of Discoverer V) as an example. The first revolution of this satellite was arbitrarily chosen as the first revolution on 15 February 1960. The first and second bulletins issued by NSSCC gave good predictions over an interval of 173 revolutions. Both bulletins used $T_2 = 46.08599$, $P_2 = 7.241102 \times 10^{-2}$, and $C_2 = -6.795 \times 10^{-7}$. From this data we compute $A_2 = 361.25298$ and $T_{Nd} = 407.33897$. From bulletin #3, we obtain $T_{140} = 56.06614$ and $P_{140} = 7.22269 \times 10^{-2}$. Then

$$C_{140} = \frac{0.3 \; P_{140} \; (P_{140} - 88)}{T_{140} - T_{Nd}}$$

$$= -6.86 \times 10^{-7}.$$ 

We use (11) which becomes

$$T_{200} = T_{140} + 60 \; P_{140} + 3600 \; C_{140},$$

to predict $T_{200}$. On bulletin #5, $T_{200} = 71.19978$. The error in the prediction is 21 seconds.

On bulletin #33, $T_{3000} = 256.91272$ and $P_{3000} = 6.8334 \times 10^{-2}$. 

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We use the above $T_d$ and repeat the procedure to predict the epoch $T_{3100}$ on bulletin #35, $T_{3100} = 263.73626$. The error in the prediction is 2 seconds.

On bulletin #48, $T_{4100} = 330.76048$ and $P_{4100} = 6.65805 \times 10^{-2}$.

We repeat the above procedure to predict the epoch $T_{4200} = 337.32994$ on bulletin #50. The error in the prediction is 24 seconds.

An application of the above to four satellites still in orbit indicates that this technique may have some value.
APPENDIX III - DECAY PREDICTION PROGRAM WITH EXAMPLES

In this appendix we discuss a program for obtaining three predictions of $T_{Nd}$ from one epoch $T_{Ni}$. $B_{Ni}$ is the $T_{Nd}$ predicted by the $T_{Ni}$, $P_{Ni}$, $C_{Ni}$ given on the bulletin. $\overline{B_{Ni}}$ is the $T_{Nd}$ predicted if $C_{Ni}$ is changed to $C_{Ni+1}$ in order that (11) will predict $T_{Ni+1}$ with zero error. $\overline{B_{Ni}}$ is the $T_{Nd}$ predicted if $C_{Ni}$ is changed to $C_{Ni}$ in order that (11) will predict $T_{Ni+2}$ with zero error.

Let $P_{Ni+j}$, $T_{Ni+j}$, $C_{Ni+j}$, $(j = 0, 1, 2)$, be the elements on three successive bulletins.

Compute

$$A_{Ni} = \frac{3 P_{Ni} (P_{Ni} - \frac{88}{1440})}{-C_{Ni}} = -G_{Ni} (C_{Ni})^{-1},$$

$$B_{Ni} = T_{Ni} + A_{Ni},$$

$$\overline{C_{Ni}} = \frac{T_{Ni} + 1 - T_{Ni} - (N_{i} + 1 - N_{i}) P_{Ni}}{(N_{i} + 1 - N_{i})^2},$$

$$\overline{A_{Ni}} = -G_{Ni} (C_{Ni})^{-1},$$

$$\overline{B_{Ni}} = T_{Ni} + \overline{A_{Ni}},$$

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The equations were programmed for the IBM 709 computer. The printout gives the above eight quantities and also the orbital elements contained on the bulletins. The program starts with \( i = 1 \), where \( N_1 \) is the epoch revolution on the first bulletin, and then repeats the procedure for all values of \( i \).

It was believed that the program might provide \( B \) and \( \overline{B} \) values which gave a more accurate predicted decay day than did the \( B \) values.

Additional calculations are necessary before the program can be evaluated.

Some predictions calculated by this program are given below.

1959 LAMBDA (Discoverer VIII)

\[
\begin{align*}
A_{780} &= 39.8 & B_{780} &= 54.3 \\
\overline{A}_{780} &= 53.8 & \overline{B}_{780} &= 68.3 \\
A_{780} &= 51.7 & B_{780} &= 66.2
\end{align*}
\]

Decay day was approximately 68.0.
1959 EPSILON-1 (Discoverer V)

\[ A_9 = 29.1 \]
\[ \bar{A}_9 = 45.5 \]
\[ A_9 = 44.7 \]
\[ B_9 = 255.4 \]
\[ \bar{B}_9 = 271.8 \]
\[ B_9 = 271.1 \]

Decay day was between 271 and 272.

1958 EPSILON (Explorer IV)

\[ A_{4170} = 150.0 \]
\[ \bar{A}_{4170} = 141.1 \]
\[ A_{4170} = 150.6 \]
\[ B_{4170} = 297.4 \]
\[ \bar{B}_{4170} = 288.5 \]
\[ B_{4170} = 298.0 \]

Decay day was approximately 296.2.