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THE USE OF AN HF LUNAR REFLECTION CIRCUIT IN THE STUDY OF IONOSPHERIC ELECTRON DENSITY

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ABSTRACT

A high-power hf transmitter has been used to illuminate the moon at moonrise. The returned signals which have been observed display both the smooth-sphere scattering components and the rough-scatterer components reported previously by observers utilizing the vhf and uhf bands. It is believed that this study represents the first evidence of similar scattering phenomena observed at high frequencies.

During most observation periods, the initial appearance of the moon-reflected signal was later than would be expected from considerations of antenna beam structure and ionospheric refraction alone. The received moon echoes were subjected to a high degree of predetection bandwidth narrowing and were analyzed to yield information regarding the possible extent of polarization-rotation undergone by radio waves traversing the ionosphere. The rotation of the plane of polarization has been found, on each occasion, to have a distinct echo-fading frequency as the ray path rises through the ionosphere. In the present investigation, the fading period varied from 90 to 113 seconds. The striking periodicity in amplitude exhibited by the moon-echo pulses indicates that the further study of the polarization-rotation mechanism will provide a powerful tool for investigating the electron density in regions above the F-layer maximum.

It is planned to modify the present high-power hf facility in order to carry out measurements at two slightly separated frequencies, and thus to extract data which will permit the determination of total ionospheric electron density along the earth-moon path.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

AUTHORIZATION

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THE USE OF AN HF LUNAR REFLECTION CIRCUIT IN
THE STUDY OF IONOSPHERIC ELECTRON DENSITY

INTRODUCTION

The study of radio waves reflected from the moon has been of interest to ionospheric
dischetists and radio engineers for many years. Its possibilities were recognized, in part,
as early as 1927 when Gernsback (1) suggested that radio waves of short enough wave-
length could penetrate the Heaviside layer and would then be useful for long-range com-
munications using reflection from the moon. The first successful experiments of Bay (2)
(see Kerr and Shain, Ref. 3), in Hungarian, and of the well-known team of U.S. Army Signal
Corps scientists in the United States (4), both performed in the mid-1940's, added impetus
to the efforts of investigators in lunar radio techniques. The possibilities of using the
moon both as a reflector for long-distance point-to-point communication and as a tool
for exploring the heretofore unprobed regions of the ionosphere above the F-region max-
imum were recognized early. Grieg (5) discussed the use of the moon for long-range
radio communications and pointed out several shortcomings of a lunar reflection circuit,
including the bandwidth limitation imposed by the moon's effective depth as a reflector.
However, it was not until experiments were conducted by Kerr, Shain, and Higgins (3,6)
at a wavelength of 15 m that a phenomenon was detected which indicated the possibly
greater value of lunar radio reflections as a means of studying the ionosphere. Kerr's
moon echoes underwent an unexpected slow, deep fading, with periods varying from five
to twenty minutes. He attempted to explain the effect by hypothesizing the existence of
ionospheric irregularities whose random motions impart slight frequency shifts to wave
components passing through different regions, thus causing these components to interfere
with one another.

An explanation which appears to be more satisfactory, and upon which most recent
work has been based, was offered by Murray and Hargreaves (7) who attributed the slow
fading to polarization-rotation of the radio waves as they pass through the ionosphere in
the presence of the earth's magnetic field (the Faraday effect). The results of later
workers (8-13), operating on a diversity of frequencies, have borne out the Faraday effect
hypothesis of Murray and Hargreaves.

One of the factors which determines the amount of polarization-rotation undergone
by a radio wave traversing a region where the Faraday effect occurs is the electron
density in that region. Hence, if the total polarization-rotation of a lunar radio echo can
be measured, and if all other pertinent parameters are known, the total electron density
of an earth-moon path may be estimated. Secondly, with simultaneous ionospheric
sounding data to describe the ionosphere below the F-layer maximum, a great deal may
be learned about ionospheric electron densities at levels above the reach of the conven-
tional ionosonde.

Lunar radio reflection experiments are being conducted by the Naval Research
Laboratory (NRL), with the use of a high-power hf transmitter operating at 26.6 Mc. A
high-gain array, with an extremely low angle of takeoff, is utilized for both transmission
and reception, and received signals are subjected to extensive predetection bandwidth
narrowing, involving a filter having an equivalent bandwidth of a fraction of a cycle per
second. Data have been taken on several occasions in the spring of 1962, and the results
of preliminary analysis are presented in the section titled "Data and Analysis." In the
next section appears a brief sketch of the theoretical basis upon which the experiments
were performed.
THEORETICAL CONSIDERATIONS

Ionospheric Effects

Neglecting absorption and refraction, as is acceptable for many purposes at 27 Mc in the nighttime ionosphere, the principal effect of the ionosphere upon an hf wave passing through it is a change in polarization due to the magneto-ionic effect. A linearly polarized wave traversing the ionosphere is propagated as though composed of two elliptically polarized components with opposite directions of rotation, each rotating with a different phase velocity. For a high enough signal frequency (considerably greater than the ionospheric electron collision frequency, gyrofrequency, and plasma frequency) and appropriate orientation of the ray path with respect to the earth's magnetic field, the two components are circularly polarized, and their resultant is linearly polarized. Because the components possess slightly different phase velocities, however, the plane of polarization of the total signal rotates as the wave traverses the ionosphere. Signals directed moonward from NRL at 26.6 Mc satisfy the requirements for such rotational behavior, which, coupled with the geomagnetic orientation of a Washington-moon path, permits use of the quasi-longitudinal approximation to the Appleton-Hartree relation.

The angular displacement of each of the two component wave vectors along its propagation path is given by

\[ \theta_i = \frac{\mathbf{k} x}{v_i} \]

where \( \theta_i \) is the angular transmission frequency, \( v_i \) indicates a component's phase velocity, and \( x \) denotes its distance of travel away from the source. Considering an incremental advance \( dx \) along the ray path, the difference in phase between the two components becomes

\[ d\theta = \frac{1}{2} \left( \frac{d\theta_1}{v_1} - \frac{d\theta_2}{v_2} \right) dx \]

Then, since the index of refraction \( n_i \) is equal to \( c v_i \),

\[ d\theta = \frac{1}{2c} \left( n_1 - n_2 \right) dx \]

For wave frequencies satisfying the conditions delineated above, the Appleton-Hartree equation for the index of refraction may be approximated by

\[ n^2 = 1 - \frac{1}{\left( \frac{\omega_p^2}{2n^2} \sin^2 \alpha \right) \left( \frac{\omega_p^2}{2n^2} \left[ \frac{\omega_p^2 \sin^2 \alpha}{2n^2} \right] + \cos^2 \alpha \right)^\alpha} \]

(see Ref. 14), where \( \omega_p \) is the angular plasma frequency, \( \omega_m \) is the angular gyrofrequency, and \( \alpha \) is the angle between the propagation direction and the earth's magnetic field. For the quasi-longitudinal case the Appleton-Hartree relation becomes

\[ n^2 = 1 - \frac{1}{2} \left( \frac{\omega_p}{\omega} \right)^2 \left( 1 + \frac{\omega_m}{\omega} \cos \alpha \right) \]
and

\[ \eta_1 \cdot \eta_2 = \left( \frac{e^2}{2\epsilon} \right) \left( \frac{\omega}{c} \right)^2 \cos \beta. \]

then,

\[ d\phi = \frac{\omega}{2c} \left( \frac{e}{\epsilon} \right)^2 \cos \beta. \]

Also,

\[ \eta_2 \cdot B \cdot e \cdot m \cdot \frac{e^2}{2\epsilon} \cdot 2 \cdot 4 \cdot 2N \cdot m, \]

where \( B \) is magnetic field strength, \( N \) is electron density, \( e \) is the electronic charge, and \( m \) is the electron mass. Then

\[ d\phi = \frac{2 \cdot e^2 \cdot B \cdot N}{(m \cdot c)^2} \cos \beta \]

which may be integrated along the propagation path to give the total polarization-rotation of the radio wave.

Under normal ionospheric conditions and an assumed parabolic dependence of ionospheric electron density upon height, typical values of total polarization-rotation through the ionosphere for a 27-Mc signal vary from 40 to 80 rotations. An earthbound observer, then, would be unable to determine the exact number of such rotations undergone by a radio wave reflected from the moon. To resolve the ambiguity, however, use may be made of the dependence of phase upon operating frequency. Utilization of two slightly different frequencies separated by a few kilocycles will make it possible to extract the exact number of rotations experienced by a wave traveling the earth-moon path. From the final expression for phase, as given above,

\[ \phi = \left( \frac{2 \cdot e^2}{m^2 \cdot c^2 \cdot 2} \right) \left( \frac{1}{\epsilon} \right) \int_{path} BN \cos \beta \ dx \]

\[ (1/\epsilon) \psi (B, N, \beta), \]

and

\[ d\phi = (-2 \cdot 2 \cdot 3) \psi (B, N, \beta) \ d\lambda. \]

\[ (-2 \cdot 2 \cdot 3). \]

Taking a small difference in frequency \((d \lambda, \Delta \omega)\) and considering \(d\phi = (d\phi/d\lambda) \Delta \lambda\), to allow for observation over a period of time \(\Delta t\),

\[ = \left( \frac{2}{\epsilon} \right) \left( \frac{1}{\epsilon} \right) \left( \frac{1}{\epsilon} \right). \]
The quantity \( \frac{d\phi}{dt} \) may be extracted from the fading rate of either component (\( \Delta \omega \) must be chosen small enough so that the extent of polarization-rotation of the two components differs by less than \( \pi \) radians; for such a situation both fading rates are approximately the same), and \( \Delta t \) may be taken from the difference in time between each component's passage through a prescribed stage in the fading cycle (e.g., a maximum or minimum).

System Expectations

Considerations of signal-to-noise ratio and available received power are of importance in moon-reflection studies for their use in investigating such subjects as moon reflectivity and effective echo area, ionospheric absorption, and dispersion. The radar equation for a spherical target contained wholly within the transmitting and receiving antenna beams may be written

\[
\frac{P_r}{P_t} = C \left( \frac{G_t A_r \Delta (\pi R^2)}{16(\pi L^2)^2} \right)
\]

where the following notation is followed:

- \( P_t \) = transmitted power
- \( P_r \) = received power
- \( G_t \) = gain of transmitting antenna
- \( A_r \) = effective collecting area of receiving antenna
- \( \rho \) = reflection coefficient of target
- \( D \) = directivity of target
- \( R \) = radius of target
- \( L \) = distance to target
- \( C \) = factor to account for pulse length narrower than moon radius.

Details of the equipment used in the experiment cannot be specified - military equipment has been modified and used as a high-power, coherent, MTI radar for the purposes of the moon study.

The moon's radius is taken as \( 1.74 \times 10^6 \) m, the earth-moon distance is considered to be \( 3.84 \times 10^8 \) m. If the moon's reflectivity is considered to be 0.1, as suggested by Browne, et al. (8), and if its directivity is estimated to be unity, as suggested by Winter (16), Daniels (17), Evans and Pettengill (18), and Siegel (private communication), the radar equation as given above yields for the system an expected peak received power of approximately \( 3.75 \times 10^{-14} \) watts. If the cosmic noise temperature at 10 m is considered to be \( 10^4 \) K (19) and a noise bandwidth of 4 kc is considered representative of the bandwidth of the stage at which signal level measurements are made (considerable predetection bandwidth narrowing is utilized later in the analysis process as well), the noise power becomes

\[
P_n = k T_n A f = 2 \times 10^{-15} \text{ watts}.
\]
where \( k \) is Boltzmann's constant, \( T \) is the cosmic noise temperature, \( n \) is the noise figure of the receiver, and \( \Delta f \) is the energy bandwidth of the receiver. Thus the effective signal-to-noise ratio expected of a moon-reflected signal at the receiver output terminals is approximately 13 db.

Refractive effects are expected to be pronounced at the low takeoff angles to which the 27-Mc energy is subjected for the purposes of the lunar echo investigation. Bailey (20) discusses a method by which the angle through which a radio signal is refracted in traversing the ionosphere may be approximated with the use of ordinary vertical-incidence ionospheric data. Rough calculations based on Bailey’s suggestions indicate that a not-abnormal nighttime ionosphere could cause as much as a 3-degree refraction of 27-Mc radio signals launched at low angles (e.g., 3 to 6 degrees).

The antenna used in the moon-echo studies is an electronically steerable array which may be slewed over a 60-degree arc in azimuth centered at an approximate bearing of 080 degrees from true North. During each month, the bearing of the moon at moonrise from NRL varies from the southern extreme of the antenna’s steering range to slightly north of its center. The vertical pattern of the antenna contains lobes at 3, 9, and 15 degrees of elevation, providing nearly an hour of moon coverage during each observation period (interrupted by antenna nulls at 6 and 12 degrees) at the moon’s northernmost excursions, and progressively less time for occasions when the moon rises further to the south.

Ionospheric absorption was expected to cause serious attenuation of the 27-Mc signals under daylight propagation conditions, so observations were restricted to periods when the full transionospheric portion of the path was dark. The combination of antenna constraints and path requirements restricted observations to a few moonrises each month (as few as two days during equinoctial months, and as many as seven days during the extremes of the year).

DATA AND ANALYSIS

Although equipment difficulties have plagued the operators throughout the experiment, useful moon-reflected signals were received on several occasions in the spring of 1962. Figure 1 shows an oscillogram of a lunar-echo signal made at a receiver i-f level during the first observation period (Jan. 23, 1962). Periods of interest in the photograph are denoted by lower case letters below the illustration. Period (a) represents an interval in which the receiver was gated off during transmission of a pulse. Because the pulse repetition period of the equipment used for the study was small compared to the transit time of a moon echo, the interval between the transmitted pulse (a) and the appearance of the moon return (b) is of no significance for present purposes. Period (b) represents a component of the moon echo which may be attributed to specular reflection from the center of the moon’s face. Period (c) represents a slowly decaying component, attributed to diffuse scatter, which persists for approximately 4.5 msec before being obscured by noise. The occurrence of these two types of scattering phenomena of rf energy by the moon has been observed repeatedly at frequencies in the vhf and uhf bands (21,22), but this is believed to be the first evidence of similar behavior in the hf region.

Echo pulses have varied in total length from one msec to over 4 msec, while the transmitted pulse length has generally remained close to 1/2 msec. It should be emphasized, however, that the echo pulse lengths observed correspond solely to the periods during which received energy exceeded the ambient noise level – an appreciable component of cosmic noise affects receivers operating in the 10-m band.

During most observation periods, the initial appearance of the moon-reflected signal was later than would be expected from considerations of antenna beam structure and ionospheric refraction. On Feb. 20, for which some data are illustrated below, the
The earliest lunar reflection was obtained a full half hour after optical moonrise, and 16 minutes after the moon passed through the geometric projection of the antenna's principal lobe. On the other hand, echoes which were easily confirmed by time delay measurement to be bona fide moon reflections have been detected well before moonrise. On Mar. 21, although no data were recorded at the time, observers were able to discern verifiable lunar echoes 10 minutes before optical moonrise. The total refraction experienced by these radar signals can be estimated to be in excess of 3.5 degrees.

Signal strengths have upon occasion approached the expected values; during one period on June 28, shortly before ionospheric sunrise, a received power level of $2.25 \times 10^{-14}$ w was detected, although for only a few seconds. This value is within 1.5 db of the value which would be expected from consideration of the radar equation and indicates that ionospheric absorption was quite low at the time of the measurement. Amplitude-versus-time presentations of the data obtained during three of the observation periods are presented in Figs. 2 to 4.

It is evident from Figs. 2 to 4 that a remarkably clear fading structure may be extracted from each set of data. Although evidence of the existence of higher frequency fading components is present in each figure, a consideration of the deepest minima will yield a reasonably consistent fading period in each case. For example, if in Fig. 2 the minima at approximately 2025:12, 2028:30, 2028:00, and 2029:45 are considered to represent the most significant fades, a mean period of 90 seconds results. A similar consideration of the minima in Fig. 3 at about 1945:30, 1947:00, 1948:50, 1950:30, 1952:00, 1953:50, 1955:30, 1957:00, 1959:00, and 2001:00 yields a mean fading period of 106 seconds. It is of interest in Fig. 3 that a strong fading component with approximately one-third the gross 108-sec period is also present. The data in Fig. 4 display a mean fading period of about 106 seconds.
Fig. 2 - Amplitude of moon echoes received Feb. 20, 1962

Fig. 3 - Amplitude of moon echoes received Mar. 21, 1962
The method of signal processing to which the moon echo data were subjected precludes the computation of a pulse-to-pulse autocorrelation function, and, hence, precise information regarding the scattering law obeyed by hf moon echoes cannot be extracted from the data. The calculation of an autocorrelogram for periods greater than the system sampling period (two seconds) is valuable in segregating the fading periods evident in Figs. 2 to 4.

The normalized autocorrelation function

\[
\hat{c}(\tau) = \frac{\sum_{i=0}^{n-1} [f(i) - \bar{f}] [f(i+m) - \bar{f}]}{\sum_{i=0}^{n-1} [f(i) - \bar{f}]^2}
\]

where \( f(i) \) represents the amplitude of the i-th data point, \( \bar{f} \) represents the mean amplitude of the total sample of \( n+1 \) points, and \( m \) represents the lag between data points to be correlated, was computed for the three sets of data in Figs. 2 to 4 with the aid of a General Precision LGP-30 digital computer. The results, plotted in Figs. 5 to 7, indicate the presence in each case of a strong fading period varying from 90 seconds (in Fig. 5) to 113 seconds (in Fig. 7). The quality of the autocorrelograms in Figs. 5 and 7 is degraded by the brevity of the sample represented in each illustration; this brevity amplifies the sensitivity of the function to anomalous variations in signal level. Furthermore, in the latter case a spurious response of the analysis system, varying in period from 20 seconds to one minute, was discovered to have been affecting the recorded signal level; this response thus appears in addition to the fade-rate data in the autocorrelogram in Fig. 7.

The difference in Faraday fading period between the sample illustrated in Figs. 2 and 5 (90 seconds) and the other two samples (approximately 110 seconds) indicates that during the former period the moon signal path was subject to a higher rate of electron density variation than in the other two periods. An explanation for this circumstance cannot readily be derived from ionospheric conditions and geometrical considerations, however. During both of the observation periods on Feb. 20 and on Mar. 21, for which the fading periods approximated 90 seconds and 104 seconds, respectively, moonrise occurred during the early evening hours when ionospheric electron density normally changes quite rapidly. On Feb. 20 the data were taken 2.5 hours after sunset, when the moon was at an altitude of 22 degrees. On Mar. 21 the data were taken 1.5 hours after sunset, when the moon was at an altitude of only 15 degrees. Both of these circumstances would indicate that the fading period experienced by the moon signal on Feb. 20 would be longer than that exhibited by the data taken on Mar. 21, for the rate of change in the amount of ionosphere
Fig. 5 - Autocorrelogram of moon echo data from Feb. 20, 1962

Fig. 6 - Autocorrelogram of moon echo data from Mar. 21, 1962

Fig. 7 - Autocorrelogram of moon echo data from May 31, 1962
subtended by the antenna beam is greater at low takeoff angles, and the rate of change of ionospheric electron density is greatest immediately after sunset. Vertical incidence ionograms, presently on order from the Central Radio Propagation Laboratory, should help in clarifying this situation. That the data taken on May 31 would exhibit the longest fading period (113 seconds) is consistent with the normally slowly changing ionospheric conditions in the hours immediately preceding sunrise, during which time the measurements for the date were made.

CONCLUSIONS

The striking periodicity in amplitude exhibited by the moon-echo pulses discussed above indicates that the polarization-rotation mechanism provides a powerful tool for exploring the ionosphere. The technique described will permit the acquisition of valuable information regarding electron density above the F-layer maximum of ionization. It is planned to modify the present high-power hf facility in order to carry out measurements at two slightly separated frequencies, and thus to extract data which will permit the determination of total ionospheric electron density along the earth-moon path. It is further expected that more accurate measurements of received signal level may be made, and that the resultant data will clarify some of the uncertainties surrounding the factors mentioned in the section "System Expectations" as contributing to path losses. It is also proposed to attempt an investigation of echo shape, with the intention of acquiring more precise information on the type of roughness encountered by a radio signal upon reflection from the moon. It is hoped, finally, that a vertical-firing antenna, presently under construction at NRL's high-power hf transmitter facility, may be exploited to extract additional information from the earth-moon path.

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