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HYPERFINE INTERACTIONS IN THE GROUND STATE AND 21.7 KEV STATE OF Eu$^{151}$ IN EUROPIUM IRON GARNET

I. Nowik and S. Ofer
HYPERFINE INTERACTIONS IN THE GROUND STATE AND 21.7 KEV STATE OF Eu\textsuperscript{151} IN EUROPIUM IRON GARNET

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The hyperfine interaction of the ground state and 21.7 keV state of Eu$^{151}$ in europium iron garnet have been measured at 300°K, 81°K and 20°K using the technique of Mössbauer absorption. Values of (335 ± 50) koe, (570 ± 35) koe and (670 ± 100) koe were found for effective magnetic fields acting on the Eu nuclei in europium iron garnet at the temperatures of 300°K, 81°K and 20°K respectively. These values are in agreement with theoretical calculations of the non-diagonal matrix elements associated with the admixture of states produced by the exchange interaction. The value of the quadrupole interaction $eQ_{0}q_{eff}$ of the ground state of Eu$^{151}$ in europium iron garnet was found to be (-240 ± 100)μc/sec at 81°K. The results yield an assignment of a spin of 7/2 for the 21.7 keV level and a value of $\mu = (2.54 ± 0.15)$n.m. for the magnetic moment of this level.
Introduction

The hyperfine Zeeman splitting in the recoil-free absorption spectrum of the 21.7 kev γ-rays from Eu$^{151}$ ($T = 1.34 \pm 0.1 \times 10^{-8}$ sec$^1$) in Eu$^{151}$ situated in europium iron garnet (EuIG) have been observed at 200K, 810K and 3000K, using the technique of recoil free absorption. This is possible since the sublattice of the rare earth ions in EuIG displays a spontaneous magnetization below the Curie temperature (about 550°C), which is correlated with the existence of an effective magnetic field at the Eu nuclei$^2$. The spontaneous magnetization of the Eu sublattice is temperature dependent and this implies that there will be a corresponding dependence of the effective fields acting on the Eu nuclei on temperature.

The ionic ground state of Eu$^{3+}$ is $^7F_0$, which is diamagnetic. Wolf and Van Vleck$^3$ have pointed out, however, that the presence of nondiagonal matrix elements connected with the admixture of the excited $^7F_1$ state into the $^7F_0$ ground state produced by the exchange interaction accounts essentially for the behaviour of the spontaneous magnetization of EuIG below its Curie temperature. Using this approach, Wolf and Van Vleck succeeded in calculating the temperature dependence of the spontaneous magnetization of EuIG below its Curie temperature. Their results are in good agreement with the experimental data$^4$. In a recent paper, Gilat and Nowik$^5$, using the
same approach, have calculated the corresponding non-diagonal
matrix elements determining the hyperfine interaction on the
Eu nucleus in EuIG, and have in this way estimated, as a function
of temperature, the magnitude of the effective magnetic field
($H_{\text{eff}}$) and the part of the effective electric field gradient
($q_{\text{eff}}$) associated with the orientation of the orbital wave
functions produced by the exchange interaction through the spin
orbit coupling. Their calculated values of $H_{\text{eff}}$ and of $q_{\text{eff}}/h$
at $0^\circ\text{K}$ were $7 \times 10^5$ oe and $-550$ Mc/sec per barn respectively.
One of the main purposes of the present work was to measure
$H_{\text{eff}}$ and $q_{\text{eff}}$ acting on Eu$^{3+}$ ion in EuIG as a function of tempera-
ture and to compare the results to the theoretical predictions.
A very good agreement between the experimental and theoretical
values of $H_{\text{eff}}$ was found.

The spin of the ground state of Eu$^{151}$ is $5/2$. The value
of its magnetic moment is $(3.419 \pm 0.004)$ n.m.\(^{(6)}\) and of its
quadrupole moment is $(0.95 \pm 0.10)$ barn\(^{(7)}\). The 21.7 kev
transition is $M_1$. From the point of view of nuclear information,
concerning the 21.7 kev level, the results presented in this
article yield an assignment of a spin of $7/2$ for this state, and
a value of $\mu = (2.54 \pm 0.15)$ n.m. for the magnetic moment for
this state. The value of the magnetic moment is interpreted
in terms of two possible models.
Shirley et al.\textsuperscript{(8)} already observed the Mössbauer effect in the 21.7 keV transition of Eu\textsuperscript{151}, using oxide sources and absorbers. The absorption patterns obtained by Shirley et al. were single lines of approximately natural widths.

**Experimental Details**

The source used was Gd\textsuperscript{151} in the form of Gd\textsubscript{2}O\textsubscript{3}. It was produced by a (p, n) reaction on enriched Eu\textsuperscript{151} oxide. The bulk of europium was removed by a Na amalgam reduction\textsuperscript{(9)} and the activity then precipitated with NH\textsubscript{4}OH using 5 mg Nd\textsubscript{2}O\textsubscript{3} carrier. The precipitate was then ignited at 800°C. The absorber used was 40 mg/cm\textsuperscript{2} EuIG containing enriched Eu\textsuperscript{151} of 91\% isotopic abundance. The 21.7 keV radiation was detected by a 1 mm thick NaI(Tl) scintillation counter with a 0.001" aluminum cover. The absorption as a function of relative velocity between source and absorber was recorded automatically on a multichannel pulse height analyzer as already described\textsuperscript{(10)}. A scale of velocities was established by using as a frequent calibration the absorption spectrum of α-Fe\textsubscript{2}O\textsubscript{3} which has been previously investigated\textsuperscript{(11)}. It turned out that the spectra obtained with the EuIG extended to velocities of about 6 cm/sec. On the other hand, the α-Fe\textsubscript{2}O\textsubscript{3} spectrum extends to about 1 cm/sec. In order that the calibration should cover the required range on the pulse height analyzer the A.C. voltage applied to the loudspeaker coil during the
calibration run was a suitable fraction of the voltage used in the EuIG measurement. The ratio which determines the conversion factor between the two scales of voltage was measured accurately using a helipot potentiometer. The procedure assumes linearity of the loudspeaker which was confirmed separately in the relevant range. A block diagram of the electronics is shown in Fig.1.

The measurements at $81^\circ$K were carried out using the cryostat shown in Fig. 2. The cryostat is suitable for work at a continuous range of temperatures between $81^\circ$K and room temperature. In the present experiment the heating coils were not used and the measurements were made with both the source and absorber at $81^\circ$K. Very thin rubber was used to close the hole between the lucite rod which held the source and the cryostat in order to prevent freezing of moisture in the cryostat which would impede the motion.

For the measurements at $20^\circ$K, the EuIG absorber was placed in a liquid hydrogen cryostat. The source was at room temperature. The radiation reached the absorber and detector through thin mylar windows.

**Experimental Results**

In order to measure the width of the recoilless emission line, measurements with Eu$_2$O$_3$ absorbers were carried out. Our results are in full agreement with those obtained by Shirley.
The spectrum obtained with oxide source and absorber at 810K is shown in Fig. 3. The measurements of the absorption by EuIG at 810K were carried out in two velocity ranges (Figs. 4 and 5). Six resolved absorption lines are seen. The spin of the ground state of Eu$^{151}$ is 5/2. As the 21.7 kev transition is of M1 character, the possible spin assignments for the 21.7 kev level are 3/2, 5/2 or 7/2. The Zeeman effect splits the ground and 21.7 kev level into magnetic sub-levels. The selection rules for a pure M1 transition permit 12, 16 or 18 transitions between the sublevels, corresponding to values of 3/2, 5/2 or 7/2 for the spin of the 21.7 kev level respectively. The fact that only six resolved absorption lines are obtained shows that the ratio of the gyromagnetic ratios of the 21.7 kev state and ground state is such that many of the possible absorption lines overlap. A preliminary analysis of the experimental results showed that the general shape of the absorption spectrum is consistent with either one of the following three assumptions:

(a) The spin of the 21.7 kev level is 3/2 and the ratio of the g-factors of the excited state and ground state respectively is about 1.5.

(b) The spin is 7/2 and the ratio of g-factors is about 2/3.

(c) The spin is 7/2 and the ratio of g-factors is about 0.5.
In order to decide which of the three assumptions fits the experimental results, the ratio of the areas corresponding to peaks "c" and "d" to the areas corresponding to peaks "b" and "e" was determined from Figs. 4 and 5. The ratio was found to be $1.05 \pm 0.20$. The theoretical ratio corresponding to assumption (a), (b) and (c) are 1.6, 2.3 and 1 respectively. Thus, the present experimental results indicate that the spin of the 21.7 kev level is $7/2$ and that the ratio of the $g$-factors of the 21.7 kev level and ground level is about 0.5. Following this interpretation, the transitions between the sub-levels are classified in Table 1 according to their positions in the experimental absorption spectra. The calculated relative transition probabilities are also given in this Table. The absorption lines corresponding to the $+5/2 \rightarrow +3/2$ and the $-5/2 \rightarrow -3/2$ transitions were not detected experimentally because of their very low relative intensity.

In the final analysis of the results, a possible axially symmetric quadrupole interaction was taken into account and it was assumed that each level was split according to the following expression:

$$W = mg\mu_n H_{eff} + 1/4 eqQ(3m^2 - I(I + 1))/[I(2I - 1)]$$

where $g$ is the gyromagnetic ratio of the level and $Q$ is its...
quadrupole moment. In addition a possible shift \( AE \) between the centroid of the absorption lines and the emission line was taken into account. The results obtained for the splitting parameters at \( 81^0K \) from the analysis of the experimental peak positions are given in Table 2.

From these results we see that \( g_1/g_0 = 0.53 \pm 0.03 \) and \( Q_1/Q_0 = 1.3 \pm 0.5 \). The magnetic moment of the ground state was measured by Pichanick et al and found to be \((3.419 \pm 0.004)\text{n.m.}\) (6) and, therefore, our results show that the magnetic moment of the 21.7 kev level is \((2.54 \pm 0.15)\text{n.m.}\). The present results show that the value of \( H_{\text{eff}} \) at \( 81^0K \) is \((570 \pm 35) \times 10^3 \text{ oe}\).

The recoil-less resonance absorption spectrum by EuIG at \( 300^0K \) is shown in Fig. 6. The peaks "a", "b", "c", "d", "e" and "f" can be identified in this spectrum too, although they are not very well resolved. By comparing this spectrum to that obtained at \( 81^0K \), a value of \( 0.59 \pm 0.08 \) was obtained for \( \frac{H_{\text{eff}}(300^0K)}{H_{\text{eff}}(81^0K)} \). Thus, the value of \( H_{\text{eff}} \) at \( 300^0K \) is \((335 \pm 50) \times 10^3 \text{ oe}\).

The absorption spectrum obtained at \( 20^0K \) is shown in Fig. 7. The measurement was carried out in this case for a relatively narrow range of velocities, with the intention of measuring accurately the separation between the peaks "c" and "d". By comparing the splitting between peaks "c" and "d" at \( 20^0K \) to the corresponding splitting at \( 81^0K \), a value of
(1.18 ± 0.15) was obtained for $H_{\text{eff}}(20^\circ\text{K})/H_{\text{eff}}(81^\circ\text{K})$. Thus, the value of $H_{\text{eff}}$ at $20^\circ\text{K}$ was found to be $(670 ± 100) \times 10^3\text{oe}$.

Although a 3/2 spin assignment to the 21.7 kev level seems very improbable, an analysis of the results was also made assuming such an assignment. Then, a value of $\sim 3$ n.m. was obtained for the magnetic moment of the 21.7 kev level and the values derived for $H_{\text{eff}}$ were about 10% lower than those given before.

Discussion

In Table 3, a summary of the experimental values of $H_{\text{eff}}$ and $eQ_{\text{eff}}$, together with their theoretical values according to Ref. 5, is given. Fig. 8 also displays the results for $H_{\text{eff}}$ as a function of temperature. From the table and the figure it can be seen that the magnitude of $H_{\text{eff}}$ and its temperature dependence are in good agreement with the theoretical predictions. The absolute values are also very close to the theoretical results. On the other hand, one must take into account the following qualifications with respect to the theoretical calculations. (a) Possible effects of core polarization were neglected (the contribution to $H_{\text{eff}}$ due to core polarization was estimated roughly by Freeman and Watson to be $-9 \times 10^4S$ where $S$ is the spin(12)). This contribution is negative and therefore diminishes the total effective magnetic field. (b) The values of the exchange field used in these calculations are obtained in a very approximate way.
(c) There is still considerable doubt concerning the true value of \( \langle 1/r^3 \rangle \) to be used in the theoretical formula for the hyperfine interaction\(^{(13)}\).

There are large errors in the experimental value of \( eQ_0q_{\text{eff}} \). It is interesting, however, to note, that at least the negative sign of the observed quadrupole interaction is in agreement with the theoretical estimate. As pointed out in the introduction, the calculated quadrupole interaction assumes that the only contribution to the gradient arises from the presence of the exchange field. In Ref. 5 reasons are given to justify the assumption that other contributions are relatively small. Good agreement between experiment and theory for the value of \( eQ_0q_{\text{eff}} \) at 81°K is obtained if we assume a value of about \( R = 0.4 \) for the Sternheimer shielding correction factor.

The properties of the ground and first excited states of \( \text{Eu}^{151} \) are, in many respects, very similar to those of the neighbouring lower Z, odd proton even-neutron isotopes. For example, in \( \text{Pr}^{141} \) the spin of the ground state is \( 5/2 \) and of the first excited state \( 7/2 \). In \( \text{La}^{139} \) and \( \text{Cs}^{133} \), the spin of the ground state is \( 7/2 \) and of the first excited state is \( 5/2 \). The value of the magnetic moment of the ground state of \( \text{Eu}^{151} \) (3.4 n.m.) is close to that of \( \text{Pr}^{141} \) (3.9 n.m.)\(^{(14)}\) and the value found in the present work for the magnetic moment of the 21.7 kev level
of $\text{Eu}^{151}(2.76 \text{ n.m.})$ is very close to those of the 7/2 ground states of $\text{La}^{139}(2.76 \text{ n.m.})$, $\text{Cs}^{133}(2.56 \text{ n.m.})$ and $\text{Cs}^{135}(2.71 \text{ n.m.})$. In these cases mentioned, the 5/2 spin states are identified as $d_{5/2}$ shell model states and the 7/2 spin states as $g_{7/2}$ states with positive parities. One possibility therefore is to interpret analogously the ground state of $\text{Eu}^{151}$ as a $d_{5/2}$ level and the 21.7 kev level as a $g_{7/2}$ level.

On the other hand, the quadrupole moment of the ground state of $\text{Eu}^{151}$ is about 1 barn. This moment corresponds to a deformation factor $\delta$ of about 0.15. According to the unified model, the ground state of $\text{Eu}^{151}$ is probably a $[532 \ 5/2]$, negative parity Nilsson state. For such a state the calculated magnetic moment for this ground state is a function of its deformation and for $\delta = 0.15$ one obtains a value of about 3.1 n.m., which is not too far from the experimental value. But, there does not seem to be any individual Nilsson state of negative parity which is suitable for the 21.7 kev level. The $[523 \ 7/2-]$ state is too greatly separated from the $[532 \ 5/2-]$ state in addition the magnetic moment for the latter state corresponding to $\delta \sim 0.15$, is not in agreement with the experimental value of the magnetic moment of the 21.7 kev level. Likewise, there is no pure rotational state which could occur at an energy so low as 21.7 kev. With a deformation of $\delta \sim 0.15$, the lowest rotational state should be several hundred
kev above the ground state.

A possible conclusion is that Nilsson's collective model does not describe in a complete and satisfactory way the properties of nuclei with small deformations and the spins and parities of the ground and 21.7 kev levels of Eu$^{151}$ are 5/2+ and 7/2+ respectively.

Another possibility which has been suggested is that the 21.7 kev level is not a regular Nilsson level and not a regular collective state, but a combination of an individual particle state with some collective excitation, for example, a combination of a 2+ Sm$^{150}$ vibrational state as a core plus a proton with spin 5/2(15). We have calculated the magnetic moment of the 7/2 state, assuming this model, in which we couple vectorially the magnetic moment associated with a 2+ vibrational state with g factor 0.4, to a single particle 5/2 spin state of magnetic moment 3.4 n.m. (as in the ground state of Eu$^{151}$) and we obtain a value of 2.9 n.m. close indeed to the measured value of 2.54 n.m.

Acknowledgements

We would like to thank Professor S.G. Cohen for helpful discussions and A. Mustachi for help in chemical problems.
**TABLE 1**

Identification of Observed Transitions Between Nuclear Zeeman Levels.

<table>
<thead>
<tr>
<th>Peak Observed</th>
<th>Transition Between Zeeman Substates</th>
<th>Relative Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+5/2 → +5/2</td>
<td>0.28</td>
</tr>
<tr>
<td>&quot;a&quot;</td>
<td>+3/2 → +1/2</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>+5/2 → +7/2</td>
<td>1.0</td>
</tr>
<tr>
<td>&quot;b&quot;</td>
<td>+3/2 → +3/2</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>+1/2 → -1/2</td>
<td>0.28</td>
</tr>
<tr>
<td>&quot;c&quot;</td>
<td>+3/2 → +5/2</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>+1/2 → +1/2</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>-1/2 → -3/2</td>
<td>0.48</td>
</tr>
<tr>
<td>&quot;d&quot;</td>
<td>+1/2 → +3/2</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>-1/2 → -1/2</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>-3/2 → -5/2</td>
<td>0.71</td>
</tr>
<tr>
<td>&quot;e&quot;</td>
<td>-5/2 → -7/2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>-3/2 → -3/2</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>-1/2 → +1/2</td>
<td>0.28</td>
</tr>
<tr>
<td>&quot;f&quot;</td>
<td>-3/2 → -1/2</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>-5/2 → -5/2</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>-5/2 → +3/2</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>-5/2 → -3/2</td>
<td>0.04</td>
</tr>
</tbody>
</table>
**TABLE 2**

Results for the Hupertfine Interaction and Chemical Shift Observed in EuIG at 81°K. \( g_o \) and \( Q_o \) correspond to the ground state of Eu\(^{151}\). \( g_\perp \) and \( Q_\perp \) correspond to the 21.7 keV level.

<table>
<thead>
<tr>
<th>( \Delta E/h ) (Mc/s)</th>
<th>( g_o H^\text{eff}/h ) (Mc/s)</th>
<th>( g_\perp H^\text{eff}/h ) (Mc/s)</th>
<th>( eQ_o q^\text{eff}/h ) (Mc/s)</th>
<th>( eQ_\perp q^\text{eff}/h ) (Mc/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17 ± 25</td>
<td>595 ± 35</td>
<td>315 ± 35</td>
<td>-240 ± 100</td>
<td>-320 ± 100</td>
</tr>
</tbody>
</table>

**TABLE 3**

Summary of Experimental and Theoretical Values

<table>
<thead>
<tr>
<th>Temperature (°K)</th>
<th>( H^\text{eff}(\text{exp.}) ) (koe)</th>
<th>( H^\text{eff}(\text{theory}) ) (koe)</th>
<th>( eQ_o q^\text{eff}(\text{exp.}) ) (Mc/s)</th>
<th>( eQ_o q^\text{eff}(\text{theory}) ) (Mc/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>335 ± 50</td>
<td>320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>570 ± 35</td>
<td>665</td>
<td>-240 ± 100</td>
<td>-480</td>
</tr>
<tr>
<td>20</td>
<td>670 ± 100</td>
<td>700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES

Fig. 1  Block diagram of the electronics used for the automatic measurement of the recoil-less absorption as a function of relative velocity between source and absorber.
Fig. 2 Experimental arrangement for carrying out recoil-less absorption measurement with source and absorber at liquid nitrogen temperature.
Fig. 3 The absorption by $\text{Eu}_2\text{O}_3$ at 810 K of the 21.7 keV $\gamma$-ray emitted from an oxide source of Gd$^{151}$ at 810 K, as a function of relative velocity between source and absorber.
Fig. 4  The absorption by europium iron garnet at 81°X of the 21.7 kev γ-ray emitted in the decay of $^{151}$Gd, as a function of relative velocity between source and absorber. (Range of velocities -6 cm/sec to +8 cm/sec).
Fig. 5 The absorption by europium iron garnet at 81°C of the 21.7 kev γ-ray emitted in the decay of \(^{151}\text{Gd}\), as a function of relative velocity between source and absorber. (Range of velocities -4 cm/sec to +4 cm/sec).
Fig. 6 The absorption by europium iron garnet at 300°K of the 21.7 kev γ-ray emitted in the decay of \( \text{Gd}^{151} \), as a function of relative velocity between source and absorber.
Fig. 7  The absorption by europium iron garnet at 20⁰K of the 21.7 kev γ-ray emitted in the decay of Gd¹⁵¹, as a function of relative velocity between source and absorber.
Fig. 8 Solid line represents the theoretical temperature dependence of $H_{\text{eff}}$ action of the Eu nuclei in EuIg. Points represent the experimental values of $H_{\text{eff}}$ obtained in the present work.
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