CHARTS OF SPHERE STAGNATION HEAT-TRANSFER RATE IN AIR AND NITROGEN AT HIGH TEMPERATURES

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ABSTRACT

Fay and Riddell's theory for laminar heat transfer at the stagnation point is applied to the sphere in equilibrium air and nitrogen. Charts are presented of the heat-transfer rate as a function of stagnation temperature and pressure or free-stream velocity and stagnation pressure in the pressure range between 0.1 and 10 atm and in the temperature range from 2000 to 15,000°K. The effects of Lewis number are considered. Comparisons between Fay and Riddell's theory, the recent theory of Fay and Kemp, and shock-tube experiments are presented.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

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NOMENCLATURE

$c_p$ Specific heat at constant pressure, $\text{ft}^2/\text{sec}^2 - \text{K}$

$D$ Binary diffusion coefficient

$h$ Enthalpy, $\text{ft}^2/\text{sec}^2$

$h_D$ Dissociation energy, $\text{ft}^2/\text{sec}^2$

$h_E$ Reference enthalpy, $2.119 \times 10^8 \text{ft}^2/\text{sec}^2$

$K$ Constant (see Eq. (1))

$k$ Coefficient of thermal conductivity

$L_e$ Lewis number, $\rho c_p D/k$

$\frac{Nu}{\sqrt{Re}}$ Heat-transfer parameter based on wall conditions (see Eq. (4))

$p$ Pressure, atm

$q$ Heat-transfer rate, $\text{Btu/ft}^2\text{sec}$

$R$ Gas constant, $\text{ft}^2/\text{sec}^2 - \text{K}$

$r_n$ Sphere (nose) radius, in.

$S$ Entropy, $\text{ft}^2/\text{sec}^2 - \text{K}$

$T$ Temperature, $\text{K}$

$U_w$ Free-stream velocity, $\sqrt{2h_o} \text{ ft/sec}$

$u$ Velocity at the edge of the boundary layer, ft/sec

$x$ Surface distance from the stagnation point

$Z$ Compressibility factor

$\mu$ Dynamic viscosity, $\text{lb}_f\cdot\text{sec}/\text{ft}^2$

$\mu_r$ Reference viscosity (see Figs. 1 and 3)

$\rho$ Mass density in amagats where one amagat is the density of the perfect gas at one atmosphere pressure and 273.15$\text{K}$

SUBSCRIPTS

$a$ Reference condition

$o$ Stagnation condition downstream of normal shock

$w$ Evaluated at the wall condition
All logarithms are to the base 10

REFERENCE QUANTITIES

\[ T_a = 273.15^\circ K \]
\[ p_a = 1 \text{ atm} = 2116.22 \text{ lbf/ft}^2 \]

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<td>( 2.423609 \times 10^{-3} )</td>
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<td>( Z )</td>
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<td>( R ), \text{ft}^2/\text{sec}^\circ\text{K}</td>
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1.0 INTRODUCTION

Perhaps the most important and certainly one of the most investigated aerodynamic bodies is the sphere. For both drag and heat-transfer investigations, spheres and spherically nosed bodies have been extensively investigated throughout the history of aerodynamics. Since the sphere is so basic and amenable to theoretical and experimental investigations, it has been almost universally used as the nose of axisymmetric bodies. One can, in fact, use certain sphere data to predict results for other shapes. For these reasons heat-transfer rates at the stagnation point in equilibrium air and nitrogen are useful for wind tunnel and free-flight investigations.

Charts, developed by the von Kármán Gas Dynamics Facility (VKF), Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), are presented for the heat-transfer rate at the stagnation point of a sphere in air and nitrogen at chemical equilibrium. The calculations are based upon the modified formula of Fay and Riddell (Ref. 1) which is

\[ \dot{q} \sqrt{\frac{T}{g}} = K \left( \rho_w \mu_w \right)^{0.04} \left( \rho_o \mu_o \right)^{0.14} \left( h_o - h_w \right) \left( p_o / p_o \right)^{0.24} \times \]

\[ \left\{ 1 + (Le - 1) \left[ \left( h_{D,o} - h_{D,w} \right) / \left( h_o - h_w \right) \right] \right\}^{0.48} \]

Equation (1) implies a Newtonian velocity gradient at the stagnation point. The adequacy of Newtonian theory has been shown for hypersonic freestream conditions.

\*For \( T_w \sim 300^\circ K \) the two expressions in braces give almost identical results. However, for wall temperatures where dissociation may take place, Brokaw's formula should be more accurate.

Manuscript received June 1963.
2.0 THERMODYNAMIC AND TRANSPORT PROPERTIES

2.1 AIR

Thermodynamic properties were obtained from the data of Hilsenrath, Klein, and Woolley (Ref. 3) for air. These data were fit with thermodynamic surfaces by Lewis and Burgess (Ref. 4). For the range of the present calculations (0.1 ≤ \( p \) ≤ 10, 2000 ≤ \( T \) ≤ 15,000°K), the maximum errors were

\[
\begin{array}{cccc}
\text{Percent Error} & \rho & h/R & Z & T \\
2.42 & 1.96 & 0.75 & 2.24 \\
\end{array}
\]

The viscosity data of Hansen (Ref. 5) were used, and these data are shown on Fig. 1. The reference viscosity was

\[
\mu_r = \frac{3.0485 \times 10^{-4} \sqrt{T} \left[ 1 + (112/T) \right]}{1 + (112/T) \left[ 1 + (112/T) \right]^{-1}}
\]

The Lewis number was correlated by Cohen (Ref. 6) as a function of an enthalpy ratio and pressure. His correlation function, shown in Fig. 2, is independent of the pressure within 8 percent for 0.1 ≤ \( p \) ≤ 10. The error is negligible at \( p = 0.1 \) and increases as \( p \) increases, becoming a maximum at \( p = 10 \).

If one assumes that high temperature air consists only of oxygen and nitrogen molecules and atoms, and that the dissociation of oxygen is complete before the nitrogen dissociation begins, then

\[
h_D = \begin{cases}
1.830923 \times 10^8 (Z_o - 1), & 1.0 - Z \leq 1.21153 \\
3.87295 \times 10^{-7} + 3.512878 \times 10^{8} (Z_o - 1.21153), & Z > 1.21153
\end{cases}
\]

For the present calculations, \( T_w = 300°K \) was assumed; thus, \( h_D, w = 0 \).

2.2 NITROGEN

The thermodynamic properties of equilibrium nitrogen were obtained from empirical surface fits of the data of Hilsenrath (Ref. 7) by Lewis and Burgess (Ref. 4). The maximum errors in the surface fits were

\[
\begin{array}{cccc}
\text{Percent Error} & \rho & h/R & Z & T \\
4.74^* & 3.62 & 0.86 & 5.09^* \\
\end{array}
\]

*The maximum errors for \( T \leq 12,000°K \) were 2.49 percent in \( \rho \) and 2.42 percent in \( T \).
The viscosity data for nitrogen were taken from Ahtye and Peng (Ref. 8) and are shown plotted in Fig. 3.

3.0 RESULTS AND DISCUSSION

The stagnation heat-transfer rate for a sphere in equilibrium air with \( Le = 1 \) is shown in Fig. 4, where \( Z \) at \( p = 0.1 \) is also shown. The bends in the heat-transfer curve at the same pressure are easily correlated with changes in \( Z(T; p = 0.1) \).

The Lewis number effect is shown in Fig. 5, where, for comparison, the \( Le = 1 \) results for three pressures are also shown. At the highest temperatures, the non-unity Lewis number effect on the heat-transfer rate is about 12 percent.

The stagnation heat-transfer in equilibrium nitrogen (\( Le = 1 \)) is shown in Fig. 6.

A comparison of air and nitrogen with \( Le = 1 \) yields some interesting results. For definiteness, consider the pressure level \( p_o = 0.1 \). The air and nitrogen results are almost identical up to \( T_o = 2500^\circ K \). Between 2500 and 6000\(^\circ \)K, the heat-transfer rate in air is substantially higher (as much as 50 percent) than that in nitrogen. In this region the oxygen and nitrogen are dissociating. The oxygen dissociation is complete (\( Z = 1.209 \)) at about 4200\(^\circ \)K at which temperature only about 1 percent of the nitrogen is dissociated.* At 6000\(^\circ \)K about 33 percent of the nitrogen is dissociated. This is indicated by an examination of both the air and nitrogen results. At about 6000\(^\circ \)K the heat-transfer rates are again almost equal. From 6000 to about 11,000\(^\circ \)K, the heat-transfer rate in nitrogen is slightly higher (\( \leq 10 \) percent) than the air results. Between about 11,000 and 12,500\(^\circ \)K the results are once again almost identical. Above 12,500\(^\circ \)K there is another departure of results where the heat-transfer rate in nitrogen is again about 10 percent higher than in air. These comments generally apply at higher pressures; however, the temperatures quoted above will be changed as the pressure increases.

A comparison of the heat-transfer rates in air and nitrogen at \( Le = 1 \) as a function of stagnation enthalpy or free-stream velocity indicates almost identical results for all temperatures 2000 \( \leq T \leq 15,000 \)\(^\circ \)K and pressures \( p = 0.1 \) to 10. This is in agreement with the assumption of Fay and Kemp (Ref. 9).

---

*This tends to substantiate the earlier assumption that the oxygen dissociation is complete before the nitrogen dissociation begins.
A correlation of the heat-transfer rate in terms of the parameter \( \frac{q}{\sqrt{\rho} \sqrt{P_o}} \) as a function of the free-stream velocity is shown in Fig. 7. The results in Figs. 4 and 6 for air and nitrogen are now reduced to a single curve. The solid line is accurate within \( \pm 5 \) percent for both air and nitrogen in the pressure range \( 0.1 \leq p \leq 10 \). The dashed curve represents an accuracy of \( \pm 10 \) percent; however, it should be noted that in this latter region the Lewis number effects would presumably be greater than \( 10 \) percent (see Fig. 5).

The stagnation point heat-transfer rate in nitrogen based upon the Fay and Riddell theory was compared with the recent theory of Fay and Kemp and the shock-tube measurements of Rose and Stankevics (Ref. 10), and the results are shown in Fig. 8. In the present application of Fay and Riddell's theory in nitrogen for non-unity Lewis number, it was assumed that \( L_e = 0.6 \) (constant) as was done by Fay and Kemp. Also, here it was assumed that

\[
h_D, e = \begin{cases} 
3.619269 \times 10^4 (Z_0 - 1), & \text{if } Z \leq 2 \\
1.0743546 \times 10^4 (Z_0 - 2) + 3.619269 \times 10^4, & \text{if } Z > 2 
\end{cases}
\]

where the dissociation energy of nitrogen was 9.756 electron volts per molecule and the ionization energy was 14.48 electron volts per atom.

As previously noted by Rose and Stankevics (Ref. 10), the scatter in the experimental data was such that no theory could be judged superior. However, they indicated that their measurements clearly refuted the theory of Scala (Ref. 11) and the measurements reported by Warren (Ref. 12).

One interesting result is noted from the present calculation. If one simply multiplies the present Fay and Riddell results for Lewis number unity by \( 1.15 \), the results agree with Fay and Kemp's theory (equilibrium nitrogen, \( L_e = 0.6 \)) within about \( 5 \) percent for all velocities in \( 20,000 \leq U \leq 55,000 \) ft/sec. It should be noted that the Fay and Kemp theory includes the effects of ionization which are neglected in the Fay and Riddell theory, and the ionization becomes significant at the higher velocities or temperatures (see, e.g., Fig. 4). Because of the present state of experimental data and the neglect here of radiation, an empirical modification of Fay and Riddell's theory may suffice for engineering estimates of the convective heat transfer. The modified formula becomes

\[
\frac{Nu}{\sqrt{Re}} = \frac{(\frac{-\tau_w}{c_{p_w} x})(\frac{\rho_w u x}{\mu_w})^{1/4}}{k_w (h_o - h_w)(\frac{\rho_o \mu_o}{\rho_w \mu_w})^{0.8}} = 0.77 \left(\frac{\rho_o \mu_o}{\rho_w \mu_w}\right)^{0.8}
\]

(4)
4.0 CONCLUDING REMARKS

Fay and Riddell's theory has been used to compute the heat transfer to the stagnation point on a sphere. Air and nitrogen in chemical equilibrium were used in the calculations. Based upon these calculations the following results were obtained:

1. The effect of non-unity Lewis number was investigated in the air calculations. For the range of temperatures and pressures considered, the Lewis number effect was not greater than 15 percent.

2. There are rather large ranges of temperature and pressure over which the heat-transfer rates in air and nitrogen are essentially the same. However, oxygen dissociation strongly affects the heat-transfer rate in air between 2500 and 6000°K. However, the heat-transfer rates as a function of stagnation enthalpy and pressure in air and nitrogen (Le = 1) are almost identical (within 5 percent) in the range 2000 ≤ T ≤ 15,000°K.

3. Comparison of the Fay and Riddell theory (including only dissociation effects) with recent theories (such as that of Fay and Kemp which includes ionization effects) and shock-tube experimental measurements indicates that a simple empirical modification to the Fay and Riddell formula is adequate for engineering estimates.

REFERENCES


Fig. 1 Viscosity Ratio for Air

\[
\frac{\mu}{\mu_r} = 3.0485 \times 10^{-8} \sqrt{T} \left(1 + \frac{112}{T}\right)^{-1}
\]

\( r_0 \text{ atm} \)

0
1
10
0.1
1
1.2
1.4
1.6
0
2
4
6
8
10
12
14
16

\( T \times 10^{-3}, ^{0}\text{K} \)
\[ \mu_r = 2.9511 \times 10^{-8} \sqrt{T} \left(1 + \frac{112}{T}\right)^{-1} \]

Fig. 3 Viscosity Ratio for Nitrogen
Fig. 4 Concluded
Fig. 7 Stagnation Heat-Transfer Parameter in Air or Nitrogen

\[ q \sqrt{\frac{r_n}{\rho_0}} = 7.467 \times 10^{-7} U_\infty^{2.14}, \quad 6700 \leq U_\infty \leq 35,000 \]

- \( T_w = 300^\circ K \)
- \( Le = 1 \)
- \( 0.1 \leq \rho_0 \leq 10 \)
Fig. 8 Summary of Various Data and Theories for Stagnation Point Heat Transfer in Partially Ionized Air after Rose and Stankevics (Ref. 10)
### Arnold Engineering Development Center
Arnold Air Force Station, Tennessee


**Unclassified Report**

**Fay and Riddell's theory for laminar heat transfer at the stagnation point is applied to the sphere in equilibrium air and nitrogen. Charts are presented of the heat-transfer rate as a function of stagnation temperature and pressure or free-stream velocity and stagnation pressure in the pressure range between 0.1 and 10 atm and in the temperature range from 2000 to 15,000K. The effects of Lewis number are considered. Comparisons between Fay and Riddell's theory, the recent theory of Fay and Kemp, and shock-tube experiments are presented.**

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