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RADHAZ PROOF MAGNETIC COUPLING

9 MAY 1963

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND
ABSTRACT: A transformer can be constructed as a low frequency band pass filter by introducing a highly conductive and low permeability material between the primary and secondary windings of the transformer. Starting with Maxwell's equations, and assuming first a sinusoidal input signal, the output voltage is obtained as a function of frequency, along with an expression for the frequency where the maximum voltage occurs. Then with a step input, the voltage and power output characteristics are given as a function of time. Experimental data is presented confirming the predictions of both cases. At the end of the report two simple systems are proposed as possible communication links between the aircraft and weapon; one being a tuned circuit arrangement and the other a pulsing system. In both cases it is shown that RADAR will not affect the internal components of the weapon as long as the weapon is completely surrounded by a conducting shield.
This report is intended to show the effectiveness of a properly shielded transformer as a low frequency band pass filter, which could easily be adapted as a RADHAZ proof aircraft-weapon communication link. This project was conducted under Weptask No. RMMO-22-006-1.

R. E. Odening
Captain, USN
Commander

R. E. Grantham
By direction
ILLUSTRATIONS

FIGURE

1 (a) Shielded Magnetic Coupler
1 (b) Approximation to Semi-Infinite Plane
2. Magnetic Field Attenuation in Copper
3. Theoretical $V_{out}/V_{in}$ vs. Frequency as a Function of Shield Thickness
4. Input and Output Equivalent Circuits
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INTRODUCTION

1. The RADHAZ problem for free fall weapons has been attacked from various directions, where a multitude of partial solutions have been proposed. A complete solution to the problem is to completely encase the weapon in a conducting shield so that high frequency signals are not able to penetrate the skin. Now the problem arises as to how does one communicate with the weapon. The following report deals with a method of communication by propagating low frequency electromagnetic radiation through a conducting medium. The discussion is restricted to one dimensional magnetic field flow perpendicular to the conducting material. Essentially two cases are considered, a sinusoidal input and a step function input. It is shown theoretically and experimentally that energy from a low frequency magnetic field will penetrate the conductor and that energies from signals whose frequencies are above 10 KC will be attenuated to a negligible value, yielding a device which is a low frequency band pass filter.

THEORETICAL STUDY

2. The discussion in this section will be concerned with the transfer of magnetic energy through various media and boundaries. The problem can be briefly pointed out in Figure 1(a). A current \( i_1 \) flows in windings \( N_1 \), producing a magnetic field \( H_1 \), which is propagated through a new medium \( (x) \) as \( H_x \) and into the secondary core as \( H_2 \) and produces the current \( i_2 \) in the windings \( N_2 \). The evaluation of these parameters and their dependence upon the physics of the machine is best described through an interpretation of Maxwell's equations, i.e.,
\( \nabla \cdot \vec{B} = 0 \) 
\( \nabla \cdot \vec{E} = \rho / \varepsilon \) 
\( \nabla \times \vec{H} = \vec{I} + \varepsilon (\partial \vec{E} / \partial t) \) 
\( \nabla \times \vec{E} = -\partial \vec{B} / \partial t \)

where

- \( \vec{B} \) is the magnetic flux
- \( \vec{H} \) is the magnetic field
- \( \vec{I} \) is the true current density
- \( \vec{E} \) is the electric field intensity
- \( \rho \) is the charge density
- \( t \) is the time

The current density \( \vec{I} = \sigma \vec{E} \) where \( \sigma \) is the electrical conductivity tensor. If one takes the curl of equation (3) and (4) and assumes that the medium is a good conductor, so that \( \nabla \cdot \vec{B} = 0 \) and \( \nabla \cdot \vec{E} = 0 \), then a diffusion equation can be written for the quantities \( \vec{H}, \vec{E} \) and \( \vec{I} \), i.e.,

\( \nabla^2 \vec{H} = \sigma \mu (\partial \vec{H} / \partial t) \) 
\( \nabla^2 \vec{E} = \sigma \mu (\partial \vec{E} / \partial t) \) 
\( \nabla^2 \vec{I} = \sigma \mu (\partial \vec{I} / \partial t) \)

where

- \( \mu \) is the magnetic permeability
3. First let us consider the case where the current flowing in windings \( N_1 \) and the magnetic field produced by the current are sinusoidal as \( e^{i\omega t} \). Equation (5) then becomes

\[
\nabla^2 \mathbf{H} = j\omega \mu \mathbf{H} \tag{8}
\]

The case in general may be approximated by Figure 1(b), where the magnetic field \( \mathbf{H} \) is directed in the \( x \) direction only, producing a current \( i \) in the copper media in the \( z \) direction only. Equation (7) is written as

\[
\frac{\partial^2 i}{\partial x^2} = j\omega \mu i_z = \tau^2 i_z \tag{9}
\]

where

\[
\tau^2 = j\omega \mu
\]

Hence

\[
\tau = (1 + j) \sqrt{\frac{\mu_0}{2}} = \frac{1 + j}{\delta} \tag{10}
\]

where

\[
\delta = \sqrt{\frac{2}{\omega \mu_0}} \tag{11}
\]

Which is called the skin depth, i.e., that penetration where the induced current and associated magnetic field \( H_x \) is reduced to \( 1/e \) of its value at the surface \( x = 0 \).

4. Equation (8) in our case can be written in the same form as equation (9), i.e.,
\[ \frac{\partial^2 H_x}{\partial x^2} = j\mu \sigma H_x \]  

(12)

When the appropriate boundary conditions are applied the solution of equation (12) is written as

\[ H_x = H_0 e^{- (1 + j)x/\delta} \]  

(13)

where \( H_0 \) is \( H_x \) when \( x = 0 \).

Which describes the complex magnetic field in the conductor as is shown in Figure 2.

5. By coupling equation (13) with Faraday's induction law, i.e.,

\[ V = \mu \int_A^{\partial} H_x \, dA \]  

(14)

The voltage developed in the secondary windings \( N_2 \) is then

\[ V_2 = j\omega AN_2H_0 e^{- (1 + j)x/\delta} \]  

(15)

whose absolute value is

\[ |V_2| = \omega AN_2\mu H_0 e^{-x/\delta} \]  

(16)

6. Assuming at this point that \( H_0 \) is constant with frequency than a plot of \( V_2 \) versus \( \omega \) will show a peak at

\[ \omega_c = \frac{8}{x^2 \mu \sigma} \]  

(17)

as is shown in Figure 3 for various values of \( x \). In practice, however, \( H_0 \) is not a constant with \( \mu \) since it is a direct
NOLTR 63-55

function of \( i_1 \). At very low frequencies \( i_1 \) is not a severe function of \( \omega \), but at frequencies above 100 cycles per second \( i_1 \) is definitely affected by \( \omega \) since

\[
i_1 = \frac{V_0}{R_1 + j \omega L_sl1} e^{j\omega t} \tag{18}
\]

where the equivalent circuit is shown in Figure 4(a). The resistance \( R_1 \) is more than the dc resistance of the windings \( N_1 \). The conducting sheet \( x \) acts as a single turn load to the primary and is essentially resistive, although it is frequency depended. Hence at frequencies above about 100 cycles per second \( H_0 \) is written as

\[
H_0 = \frac{N_1}{2\pi r} \times \frac{V_0}{R_1 + j \omega L_sl1} e^{j\omega t} \tag{19}
\]

where \( r \) is the radius of the solenoid.

7. The magnitude of \( V_2 \) in this case is then

\[
|V_2| = K_1 \frac{\omega e^{-x/\delta}}{\sqrt{\omega^2 L_sl1^2 + R_1^2}} \tag{20}
\]

where

\[
K_1 = \frac{N_1 N_2 \mu_{Fe} AV_1}{2\pi r}
\]

\( \mu_{Fe} \) representing the permeability of iron. The maximum frequency here is given by

\[
\omega_c = \frac{8}{x^2 \mu} - f(L, R) \tag{21}
\]

where \( f(L, R) \) varies with the core geometry turns ratio and the material and thickness of the conducting material.
8. To investigate energy transfer from a step input function we begin again with Figure 4. When switch $S$ is close the current flowing in windings $N_1$, is

$$i_1 = i_0 \left(1 - e^{-\left(\frac{R_1}{L_{sl}}\right)t}\right)$$  \hspace{1cm} (22)

where $i_0 = \frac{V_o}{R_1}$, and the associated magnetic field produced is

$$H_1 = H_0 \left(1 - e^{-\left(\frac{R_1}{L_{sl}}\right)t}\right)$$  \hspace{1cm} (23)

where

$$H_0 = \frac{N_1}{2\pi r} i_0 = \frac{N_1}{2\pi r} \frac{V_o}{R_1}$$

A solution of equation (12) after applying the appropriate boundary condition, is in this case

$$H_x = H_0 e^{-x/\delta_s} \left(1 - e^{-\left(\frac{R_1}{L_{sl}}\right)t}\right)$$  \hspace{1cm} (24)

where $\delta_s$ is

$$\delta_s = \left[ \frac{1 - e^{-\left(\frac{R_1}{L_{sl}}\right)t}}{\sigma \mu \left(\frac{R_1}{L_{sl}}\right) e^{-\left(\frac{R_1}{L_{sl}}\right)t}} \right]^{1/2}$$  \hspace{1cm} (25)

The skin depth ($\delta_s$) from a step input is therefore dependent on time as well as the electrical conductivity $\sigma$, the permeability $\mu$ and the circuit parameters $R_1$ and $L_{sl}$. The voltage $V_2$ developed across $N_2$ is then
\[ V_2(t) = N_2 \frac{\partial}{\partial t} \int_0^A H_x \cdot dA \]

\[ = \frac{N_1 N_2 A_{\mu \varepsilon}}{2 \pi r} \frac{V_0}{L_{s1}} e^{-x/\delta_s} \left( e^{-\left(\frac{R_1}{L_{s1}}\right)t} \right) + \frac{x}{2 \delta_s} \]  

9. The energy \( W \) delivered to the load resistor \( R_L \) is given by (c.f. Figure 4(b))

\[ W = i_2(t) R_L T \]

where \( T = \) the duration of \( V_2(t) \) and

\[ i_2(t) = \frac{N_1 N_2 A_{\mu \varepsilon}}{2 \pi r} \frac{V_0}{L_{s1}} \frac{e^{-x/b_s}}{R_2 + R_L} \left\{ \begin{array}{c} \frac{x}{2 \delta_s} \\ 1 - e^{-\left((R_2 + R_L)/L_{s2}\right)t} \\ \frac{R_1 L_{s1} L_{s2}}{L_{s1}(R_2 + R_L)} = R_1 L_{s2} \\ (e^{-(R_1/L_{s1})t} - e^{-\left((R_2 + R_L)/L_{s2}\right)t}) \end{array} \right\} \]

The energy delivered to a load is then dependent on the size, geometry and turns ratio of the transformer and highly dependent on the thickness and physical properties of the conducting shield. The dependence on \( L_{s1} \) and \( L_{s2} \) is in turn essentially constant with \( x \), as will be shown in the next section.
PARAMETER EFFECTS AND EXPERIMENTAL RESULTS

10. In the preceding section we developed both voltage and current equations, which describe the energy propagation process from one medium to another, when the input signal is sinusoidal and when it is a step function. In this section we hope to confirm these expressions, at least in part, by some experimental results that have been obtained.

11. Let us take the sinusoidal case first where only a limited amount of experimental work was performed. In this case voltage measurements were made across a secondary load of 0.75 ohms as a function of the driving frequency \( \omega/2\pi \).

A plot of the output to input voltage ratio versus the driving frequency \( \omega/2\pi \) is given in Figure 5. The physical properties of the device are shown in the figure. Over the frequency range of 1 KC to 200 KC the variation of \( V_{out}/V_{in} \) ranges from \( 10^{-1} \) to \( 2 \times 10^{-5} \) indicating a very definite filtering action. The corresponding curves shown in Figure 3 show that the curve should have peaked at about 20 KC, but a closer look at the equation (22) indicates a term dependent upon the resistance and inductance of the circuit which is subtracted from the term \( 8/x^2\omega \) which compensates for the variation in the two curves.

12. The bulk of the experimental work was performed where the input was a step function. The information desired was the energy transmitted through the conducting shield to the resistive load connected across the secondary windings. Figure 6 shows plots of the energy delivered to the resistive load \( R_L \) as a function of the shielding thickness \( x \) and the core area cross section. The physical parameters of the materials are given in the figure. The input voltage in all cases was 28 volts. The energies transferred range from 1000 to 800,000 ergs, which are adequate for firing explosive devices. The theoretical curves and experimental data
(points) on the curve fit very well. The values of the leakage inductance looking from the secondary $L_{s2}$ was constant with the turns ratio $N_2/N_1$ and both the primary and secondary leakage inductances $L_{s1}$ and $L_{s2}$ were constant with the shielding thickness $x$.

13. All the discussions of this report have been restricted to electrical energy transfer. It is also possible to transmit electrical energy by doing mechanical work, such as separating two sections of a magnetic core. This type of investigation has been conducted at the Naval Ordnance Laboratory by Mr. Roland Schlie.

14. Figures 7 and 8 are possible communication systems, using the aforementioned principles as the communication link. Figure 7 shows a system which uses a sinusoidal input to drive different tuned circuits. Each tuned circuit operating at a different frequency would be a selection for the pilot. Prior to separation or bomb release the core is energized with a dc current, and as the bomb is released a signal on the secondary windings arms the weapon. Figure 8 shows a system which incorporates a series of step input voltages to select the arming and fuzing sequences desired. The second transformer is energized prior to separation and the pulse generated at separation arms the weapon. Both systems would have to be armed deliberately by the pilot so that an accidental release or a desired jettison would leave the weapon in a safe condition.

CONCLUSIONS

15. A transformer whose primary and secondary windings are wound on iron cores will exhibit the behavior of a low frequency band pass filter, if the iron cores are separated by a thin conducting sheet. This has been shown theoretically and experimentally. For a copper sheet 0.040
inches thick the attenuation of the magnetic field at 100 KC is greater than 2 orders of magnitude, and at one megacycle it is attenuated by seven orders of magnitude. The associated electric field is attenuated some 8 orders of magnitude below that of the magnetic field. If then a weapon is completely encased in a conducting material the internal components of the weapon will not be affected by radar. The split transformer, however, provides a method of communication which, while not very efficient, has been demonstrated to be quite feasible from an energy transfer, reliability and size viewpoint.
FIG. 1 (A) SHIELDED MAGNETIC COUPLER

(B) APPROXIMATION TO SEMI INFINITE PLANE
\[ |H_X| = H_0 e^{-\chi/\delta} \]

\[ \delta = \sqrt{\frac{\pi \mu_0 \sigma_c}{\chi}} \text{ SKIN DEPTH} \]

**FIG. 2 MAGNETIC FIELD ATTENUATION IN COPPER**
FIG. 3 THEORETICAL $\frac{V_{\text{OUT}}}{V_{\text{IN}}}$ VERSUS FREQUENCY AS A FUNCTION OF SHIELD THICKNESS
FIG. 4 INPUT AND OUTPUT EQUIVALENT CIRCUITS
FIG. 5 \( \frac{V_{OUT}}{V_{IN}} \) VS FREQUENCY OF EXPERIMENTAL MODEL

- \( R_L = 0.75 \Omega \)
- \( A = 0.063 \text{ in}^2 \)
- \( Y = 0.04 \text{ in}^2 \)
- \( \frac{N_2}{N_1} = 10 \)
FIG. 6 TRANSMITTED ENERGY VS SHIELD THICKNESS
FIG. 8 PLANE TO WEAPON COMMUNICATION BY PULSED SIGNAL
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