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UNDERWATER SHOCK WAVES FORMED BY EXPLODING WIRES

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ABSTRACT
A study has been made on underwater shock waves produced by exploding wires. Details on experimental equipment and procedure are given for both circuit and pressure measurements. The influence of capacitance, inductance, and gap length on maximum shock pressure $P_m$ is discussed. Experimental results correlating initial capacitor voltage (between 10 and 30 kv) with $P_m$ are given. An increase of voltage and gap length along with a decrease in inductance all tend to increase $P_m$. By using the empirical $P_m$ vs voltage curve and extrapolating to zero $P_m$, a voltage is obtained which corresponds closely to wire sublimation energy. Reproducibility, expressed quantitatively by standard deviation, is within 2 percent. The ease of $P_m$ variation, along with reproducibility, indicate the suitability of this method of shock formation as a research tool.

PROBLEM STATUS
This is an interim report; work on this project is continuing.

AUTHORIZATION
NRL Problem E01-01
Projects RR 010-04-41-5950, SR 007-12-01-0800,
and RAESOR-035/652-1/F012-11-001

UNDERWATER SHOCK WAVES FORMED
BY EXPLODING WIRES

INTRODUCTION

An underwater shock wave is produced between a pair of submerged electrodes by the rapid discharge of a capacitor through an exploding wire. The resulting plasma also dissipated radiant and thermal energy. This report deals only with acoustic or shock wave energy, the predominant energy conversion effect (1a, 2).

Chemical explosives and underwater sparks can also be used to generate shock waves. Explosives lack good reproducibility and, for a given explosive, shock wave variables are more difficult to vary since the chemical reaction times are fixed. While underwater sparks allow easier control of shock wave variables, reproducibility is poor (1b). Exploding wires, however, offer both advantages – reproducibility and convenient shock wave variation.

The purpose of this report is to discuss and present experimental work which correlates initial capacitor circuit parameters with the maximum shock wave pressure. Shock wave reproducibility is also discussed. Reproducibility and ease of variation indicate the suitability of this method of shock formation as a research tool, that is, in observing the effect of a shock wave on a test specimen, whether it be concerned with biology or the plastic deformation of metals.

BASIC CIRCUIT

The basic circuit of the capacitor method of exploding wires is shown in Fig. 1. When \( S_1 \) is closed, \( E \) can be used to charge \( C \) through \( R_1 \) to the required voltage. The voltage is indicated at \( V_0 \). Breakdown, or closing of \( S_2 \), then allows the capacitor to discharge through \( S_2 \) and \( G \).

The wire at \( G \) undergoes rapid heating and expansion. After vaporization, the resulting ionized fluid is termed a plasma. Shock wave development depends upon plasma (and

![Fig. 1 - Basic circuit of the capacitor method for exploding wires](image-url)
generated gas bubble) expansion. The inertia of the surrounding water resists accelerated movement of the plasma wall, resulting in the development of high pressure. This positive pressure disturbance propagates radially outward and eventually steepens into a shock front or wave.

Figure 2 shows a photograph of the discharge circuit. Four coaxial cables are used in parallel to minimize circuit resistance and inductance. To avoid reflections and shock wave distortions, a nondirectional type of geometry was used at G. Wires were held by clamp-type electrodes which had tungsten inserts (Fig. 3). This arrangement is similar to that used by Bennett (3) for vacuum studies of exploding wires. For purposes of insulation and waterproofing, the cable-electrode junction was potted in a high-dielectric epoxy resin.

Figure 4 is a block diagram indicating the general layout of the discharge circuit, along with pressure and current diagnostic equipment.

DISCHARGE CIRCUIT MEASUREMENTS

The following discharge circuit parameters can be used to control shock wave variables:

- C - capacitance
- \( V_0 \) - voltage to which the capacitor is initially charged
- L - total circuit inductance
- R - total circuit resistance, which includes the average resistance of the spark and wire gaps.

The average resistance of the discharge circuit is influenced by the initial resistance \( R_0 \) which can be discussed in terms of wire material, diameter, and length, along with electrode gap length.

It should also be mentioned that, beside the circuit parameters, the physical and chemical properties of the liquid media influence shock variables (4a). The electrode-gage geometry also affects the measured shock pressures (5).

Capacitance, measured with an impedance bridge, was held constant at 0.5 \( \mu \)F. Voltage, measured with an electrostatic voltmeter, was varied between 10 and 30 kilovolts. If in a rate-of-change-of-current oscillogram (Fig. 5) the period of oscillation in an under-damped circuit \( f = 1/(2\pi \sqrt{LC}) \) is measured, the inductance may be calculated from

\[
f = 1/(2\pi \sqrt{LC})
\]

The value obtained for the circuit used was \( 325 \times 10^{-9} \) henrys. Circuit resistance includes the constant resistance of the leads, along with the resistance of \( S_2 \) and \( G \) which both change with time. With the gap at \( G \) shorted out, the average resistance of the circuit (with \( V_0 \) at 20 kilovolts and an \( S_2 \) gap length of 5/16 inch) was 0.05 ohm. This value was calculated from a slightly damped oscillogram using the damping factor \( e^{-Rt/2L} \) during the time interval 2.55 to 5.2 \( \mu \)sec.

The current in the discharge circuit can be measured directly with shunts or by integrating the voltage induced (by current change) in a Rogowski coil (8) or pickup loop secondary circuit. The pickup loop method of current measurement has the advantage of not being connected to the discharge circuit. Low-inductance shunts, which give greater accuracy, are of two types: ribbon (1c) and coaxial (7). At NRL the pickup loop shown in Fig. 6 is used for capacitor-discharge current measurements (8).
Fig. 2 - Discharge circuit equipment

Fig. 3 - Clamp-type electrodes

Fig. 4 - Block diagram of discharge circuit diagnostic equipment
Fig. 5 - Rate-of-change-of-current oscillogram from a shorted gap capacitor discharge. Horizontal scale: 1 μsec/division. This trace was used to calculate the discharge circuit inductance.

Fig. 6 - Pickup loop circuit used in NRL capacitor-discharge current measurements

The pickup loop was positioned near the discharge circuit so that the induced voltage fell within the voltage sensitivity range of a Tektronix 545A oscilloscope. Regarding the discharge circuit as the primary and the pickup loop as the secondary, the expression

\[
\frac{dI_p}{dt} = \frac{V_{sec}}{M}, \quad \text{where} \quad M \text{ is the mutual inductance,} \quad (1)
\]

gives the rate of change of current in the discharge circuit. Integration of the rate of change of current gives the discharge current

\[
I_p = \frac{1}{M} \int_{0}^{t} V_s \, dt. \quad (2)
\]

An RC integrator (9), shown schematically,

\begin{align*}
V_o & \quad R_i \\
\text{R} & \quad \int \quad \text{C} \\
\text{C} & \quad V_f
\end{align*}

gives the primary current as

\[
I(t)_p = \left( R_i C_1 / M \right) V(t)_f \quad (3)
\]

where

\[
V_f = \frac{1}{C_1} \int_{0}^{t} V_o \, dt. \quad (4)
\]

The calibration of the pickup loop requires the determination of \( M \). This is straightforward if \( L \), \( R \), and \( C \) are known. Both \( L \) and \( C \) were accurately measured. The value of \( R \), computed from the damping factor, represents an average value of \( S_2 \) and...
the linear circuit resistance (with $G$ shorted out), and this value is assumed to be nearly constant over a time interval equal to the period. During this same time interval, a peak current is computed.

The value of current as a function of time in a linear, underdamped (or oscillating) LRC circuit (10) is

$$I(t) = \frac{V_0}{L \omega} e^{-Rt/2L} \sin \omega t$$

(5)

where

$$\omega = 2\pi f = \frac{1}{\sqrt{LC} + \frac{R^2}{4L^2}}.$$  

(6)

If $1/\sqrt{LC} >> R^2/4L^2$, and if $t_m$ is the time at a peak current where $\sin \omega t_m = \text{unity}$, then

$$I_{\text{max}} = \frac{V_0}{\sqrt{LC}} e^{-Rt_m/2L}.$$  

(7)

For $V_0 = 20$ kilovolts at $t_o = 3.2 \mu \text{sec}$ (Fig. 7), and with the values of $L, R, C$ as previously calculated,

$$I_{\text{max}} = 19,230 \text{ amps}.$$

Using Eq. (3) with the integrator values $R_i = 2000\Omega$, $C_i = 0.1 \mu \text{f}$, and scope voltage 0.04 volt recorded at $t_m = 3.2 \mu \text{sec}$,

$$M = \frac{R_i C_i V_f}{I_{\text{max}}} = \frac{(2 \times 10^{-4})(4 \times 10^{-2})}{1.923 \times 10^4} = 4.2 \times 10^{-10} \text{ henrys}$$

to within an accuracy of 10 percent.

![Oscillogram for calculation of the mutual inductance](image)

**Fig. 7** - Oscillogram for calculation of the mutual inductance. Vertical scale: 12,000 amp/division; horizontal scale: 1$\mu$sec/division. Capacitor discharge voltage $V_o = 20 \text{ kv}$.

**PRESSURE MEASUREMENTS**

Gage Geometry and Circuit

Shock waves were generated in a stainless steel tank (4 ft high and 4 ft in diameter) filled with fresh water. A gage holder positioned on the bottom of the tank oriented a 1/4-in.-diam tourmaline gage * in an edge-on (gage axis parallel to wire axis) position (Fig. 8) at a distance of 6 inches from the exploding wire. All pressure measurements

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*Crysal Research Incorporated, Cambridge, Mass.*
were taken with this geometry held constant. The gage housing (Fig. 9) includes a copper tube for electromagnetic shielding and wax for waterproofing and prevention of gage oscillations.

When a shock wave strikes the gage, a charge is produced across the total circuit capacitance. For short coaxial cable lengths, the voltage measured across this capacitance is directly proportional to the absolute value of the shock wave pressure (11a).

\[
V_s(t) = \frac{Q(t)}{C} = C_g V_g(t)/C = K_A P(t)/C
\]

where

- \( V_s \) = oscilloscope recorded voltage as a function of time
- \( P \) = pressure (psi) as a function of time
- \( K_A \) = crystal constant (\( \mu \text{coul}/\text{psi} \))

"Compound C-276, manufactured by Zophar Mills, Brooklyn, N.Y."
\[ C = \text{sum of cable } C_s \text{, and terminating } C_t \text{ capacitance (nsf)} \]
\[ C_e = \text{equivalent electrostatic capacitance of gage} \]
\[ R_k = \text{leakage resistance of crystal} \]
\[ R_s = \text{oscilloscope impedance} \ (= 10^6 \text{ ohms}) \]
\[ Q = \text{charge (coul) as a function of time} \]
\[ V_e(t) = \text{emf produced by gage as a function of time} \]

The shock variables recorded include the maximum pressure \( P_m \) and the corresponding pressure decay \( P(t) \).

Experimental Difficulties

Only \( P_m \) measurements could be taken uninfluenced by gage oscillations along with electrostatic, gage hysteresis, and other distortions. Multiple peaks observed on some pressure traces (Fig. 10) might be the result of nonlinear interference of pressure fronts from different parts of the wire (11b). Even for the \( P_m \) measurements, absolute values differed by 15 percent between tests run some time apart, and between tests run with different gages and wax coatings. When a test was completed over a short time interval and with the same diagnostic setup, the data was found to be consistent although absolute values still differed. Therefore, rather than compute absolute values of pressure, reduced or normalized values of pressure (i.e., generated voltage) are usually given.

![Fig. 10 - Multiple pressure peaks, possibly associated with nonlinear interference of pressure fronts from different parts of the exploding wire. A 0.5-in.-long, 5-mil-diam nichrome wire was used at a gage-to-wire distance of 6-in. Vertical scale: 220 psi/division; horizontal scale: 20 \( \mu \text{sec/division} \).](image)

Other problem areas in measuring shock variables included (a) oscilloscope baseline drift, (b) lowering of leakage resistance by water seepage between the gage-to-cable connections, and (c) ground loops. These and other problems are discussed by A. Caggiano et al. (4b).

When the capacitor is discharged, the rate of change of current induces a voltage in the gage circuit at the gage-to-cable connection (Fig. 9, left). This signal presents no problem, however, since it decays before the shock voltage is generated and is actually used to trigger the scope sweep.

More detailed information on shock measurements using tourmaline gages may be found in the literature (12).
In capacitor discharge applications, $V_0$ is the most convenient of the initial parameters ($V_0$, $L$, $R_0$, and $C$) to vary. Experiments were performed at NRL to correlate $P_m$ and $V_0$.

At a particular $V_0$, $P_m$ was observed for a number (between one and five) of discharges. Over the voltage range 10-30 kv (25-225 joules), $R_0$ was held constant using 0.5-in. long, 5-mil-diam nichrome wire. At 30 kv the average peak piezoelectric voltage generated by four separate shock waves was set equal to unity. This average peak voltage corresponded to $P_m \approx 480$ psi. At lower values of $V_0$, the average $P_m$ was expressed as some decimal fraction. Typical pressure pulses at 20 and 30 kv are compared in Fig. 11. Notice that the 30-kv pressure pulse has a greater area (corresponding to greater acoustic energy) under the curve. Figure 12 shows the corresponding current trace for the 20-kv discharge. Comparing Fig. 12 with Fig. 13 (the current trace of the exploding wire in air) it is seen that greater damping is achieved with an underwater discharge.

![Fig. 11](image1) ![Fig. 12](image2) ![Fig. 13](image3)

---

**Fig. 11** - Comparison of underwater $P(t)$ traces for (a) 20-kv (100 joules) discharge and (b) 30-kv (225 joules) discharge. For both discharges $R_0$ was kept constant by using a 0.5-in.-long, 5-mil-diam nichrome wire at a gage-to-wire distance of 6 in. Vertical scale: 110 psi/division; horizontal scale: 20 $\mu$sec/division.

**Fig. 12** - Current oscillogram of the 20-kv underwater discharge shown in Fig. 10 (a). Vertical scale: 10,000 amp/division; horizontal scale: 1 $\mu$sec/division.

**Fig. 13** - Current oscillogram of exploding wire in air. Wire and initial circuit conditions as for Fig. 12. Vertical scale: 10,000 amp/division; horizontal scale: 1 $\mu$sec/division.
Figure 14 shows a curve through the experimental points; the curve can be expressed empirically beyond 11 kv by $P_m \approx 0.417 \cdot V_0^{0.198/V_0^{1/2}} - 0.52$. Beyond 20 kv (100 joules), the curve appears linear. This part of the curve agrees with underwater spark work by Caulfield (13a) and Gardner (14a) which indicated that $P_m$ increased linearly with $V_0$ over the voltage ranges 6 to 12 kv and 12 to 25 kv.

### Sublimation Energy

The curve in Fig. 14 becomes more nonlinear between 20 and 10 kv when the energy required to vaporize the wire (sublimation energy) becomes a larger percentage of $(1/2)CV_0^2$. Sublimation energies for various materials are given in Table 1. In general, the less energy required for wire sublimation, the more energy available for shock formation. At 10 kv, as shown in Fig. 14, lead had the greatest $P_m$ value, followed by aluminum, nichrome, and copper, respectively. When sublimation energy is small compared to $(1/2)CV_0^2$, critically damped discharges will produce a greater $P_m$ than an oscillating discharge. At 10 kv (25 joules), however, the lead wire produced the larger $P_m$ in spite of the fact that the discharge was underdamped compared to the aluminum discharge (Fig. 15). The high frequency “hash” may be attributed to the spark gap switch (15). At 300 joules on the other hand, 10-mil copper, 1-mil tungsten, 10-mil silver, 5-mil lead, and 5-mil nichrome all produced identical oscillatory current traces and identical values of $P_m$. In other words, if initial wire resistance influenced $P_m$ more than the sublimation energy, a distinct difference in $P_m$ should have been noticed.

Pressures recorded by Caggiano (4c) using different wire materials in a one-inch gap at 10 kv (12,000 joules) are shown in Table 2. The pressures listed, which were produced by 62-mil wires, had the greatest range and were chosen to minimize the influence of experimental scatter. The order of decreasing $P_m$ corresponds to the increase in sublimation energies.
Dotted line A of Fig. 14 represents the type of curve one obtains when the sublimation energy becomes a large percentage of \((\frac{1}{2})CV_0^2\) (44). It is even possible to produce a pressure pulse when the sublimation energy is greater than \((\frac{1}{2})CV_0^2\). This occurs because wire expansion can take place without requiring all of the wire mass to be vaporized. Solid, liquid, and vapor states may exist simultaneously in such an exploding wire (16).

Dotted line B of Fig. 14 represents an extrapolation to zero psi. The calculated value of \((\frac{1}{2})CV_0^2\) using the zero psi voltage is within 10 percent of the sublimation energy of nichrome. With refinements, such an extrapolation technique might provide a new method for measuring sublimation energies of conducting materials which could be made into wire.

![5-mil aluminum wire](image1)

![5-mil lead wire](image2)

FIG. 15 - Comparison of 10-kv (25 joules) discharge current for identical aluminum and lead wires. In this case the underdamped discharge produced a greater \(P_m\). Vertical scale: 5000 amp/division; horizontal scale: 1 ms/division.

<table>
<thead>
<tr>
<th>Material</th>
<th>(k\text{(joules/in.-mils}^2))</th>
<th>Resistivity (ohmmeter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>0.137</td>
<td>4.8 (\times) 10^-8</td>
</tr>
<tr>
<td>Pb</td>
<td>0.138</td>
<td>22.0 (\times) 10^-8</td>
</tr>
<tr>
<td>Ag</td>
<td>0.418</td>
<td>2.83</td>
</tr>
<tr>
<td>Al</td>
<td>0.357</td>
<td>1.63</td>
</tr>
<tr>
<td>Nichrome</td>
<td>0.572</td>
<td>100</td>
</tr>
<tr>
<td>Ti</td>
<td>0.589</td>
<td>3.2</td>
</tr>
<tr>
<td>Cu</td>
<td>0.612</td>
<td>1.72</td>
</tr>
<tr>
<td>W</td>
<td>1.130</td>
<td>5.51</td>
</tr>
</tbody>
</table>

* S.E. = \(kLD^2\) where S.E. is in joules, L is length in inches, and D is diameter in mils.

Table 2

<table>
<thead>
<tr>
<th>Sublimation Energy (% of (1/2)CV₀²)</th>
<th>Material</th>
<th>Pm (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Mg</td>
<td>1500</td>
</tr>
<tr>
<td>10</td>
<td>Al</td>
<td>880</td>
</tr>
<tr>
<td>16</td>
<td>Ti</td>
<td>660</td>
</tr>
<tr>
<td>20</td>
<td>Cu</td>
<td>480</td>
</tr>
<tr>
<td>35</td>
<td>W</td>
<td>400</td>
</tr>
</tbody>
</table>

Conditions for Greatest Pm

Experimental results from sparks and exploding wires will be used to qualitatively state initial (t₀) parameter conditions which will tend to produce the greatest Pm.

Voltage

As indicated in Fig. 14 and in the literature, for V₀ greater than a certain minimum value an increase in V₀ produces an approximately linear increase in Pm. This approximate linearity applies only to wires for which sublimation energy is small compared to (1/2)CV₀². The value of V₀ is limited by the capacitor ratings, circuit insulation, and corona losses. The last two limitations present little difficulty under 30 kv.

Inductance

Spark work by Bailitis (17) and Caulfield (13a), along with exploding wire test by Caggiano, et al. (4e), indicate that an increase in Pm requires a decrease in L. Sufficient agreement between the above experiments allows the following statement: Pm α 1/√L.

The most effective way to reduce inductance is to parallel a number of the low-inductance coaxial cables which are used to connect the underwater gap to the capacitor bank.

Capacitance

Changes in the capacitance C apparently have little influence on Pm. Spark work by Gardner (14b) indicates that beyond 150 joules, capacitance changes from 3 to 16.3 μF had little, if any, effect on Pm. Caulfield (13b) reports that while capacitance was varied from 8 to 500 μF, Pm remained nearly constant. The increase in C naturally increased (1/2)CV₀², but the additional energy went into the shock wave, as an increase in shock wave duration or area under the curve, not as an increase in Pm.

Resistance

Since the resistance of an exploding wire is an unknown function of time, linear LRC analysis cannot be used to indicate the initial resistance R₀ which will give an optimum Pm for a given L, C, and V₀. The initial resistance R₀ can be controlled by gap or wire length, along with wire size and material. By choosing smaller diameter wires, sublimation energy can be held to a minimum and wire material becomes unimportant. Varying R₀ then becomes a matter of changing the gap length.
Optimum $P_m$ will be obtained when the discharge is approximately critically damped. Upon observing the current, if the discharge is oscillatory, that is, if $1/LC > R/4L$, an increase in gap length will increase damping and $P_m$.

Studies indicate that longer gaps produce greater $P_m$ (14c, 4f, 18). Figure 16 compares $P_m$ from 5-mil nichrome with 0.5- and 2.5-in. gap lengths. For sparks, the gap length is limited by breakdown potential. Wire length limitations, aside from sublimation effects, have not been determined as yet.

To provide maximum transfer of energy to the exploding wire rather than to the spark switch and joule heating of the leads, circuit resistance should be kept as low as possible, that is,

$$R_{sw} + R_{leads} < R_G.$$

Fig. 16 - Variation of the peak pressure $P_m$ as the 5-mil nichrome wire length is increased from (a) 0.5 in. to (b) 2.5 in. The discharge voltage $V_0$ is 34.5 kv (300 joules). Vertical scale: (a) 110 psi/division, (b) 220 psi/division; horizontal scale: 20 usec/division for both cases.

Geometry

An expression by Gardner (5)

$$P_m \approx \frac{P_0 (2a_0)^{0.6}}{(2D_1)^{1.15}},$$

where

- $P_0$ = peak plasma wall pressure
- $a_0$ = initial plasma radius
- $G$ = gap length
- $D_1$ = perpendicular distance between wire and gage,

indicates that larger gap lengths and shorter gage-to-wire distances ($D_1$) allow greater measured $P_m$ at a distance $D_1$.

Reproducibility

Reproducibility may be defined as the degree of closeness obtained when a given quantity is measured repeatedly. The standard deviation $\sigma$ will be used to express reproducibility:
\[ \sigma = \left( \frac{1}{n-1} \sum_{i=1}^{n} \left| P_m - P_{m_1} \right|^2 \right)^{1/2} \]

where

- \( P_m \) = mean or average value of \( P_m \)
- \( P_{m_1} \) = a particular value of \( P_m \)

The standard deviation for two series of discharges with a one-in. gap using 10-mil copper wire at 20 kv (100 joules) are listed below.

<table>
<thead>
<tr>
<th>Series</th>
<th>No. of Discharges</th>
<th>Standard Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>2.01</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Both series were taken with the same parameter values. Extremely fine copper residue suspended in the water may have changed water conductivity enough to allow further scatter than one would receive with aluminum or nichrome wire. When using fresh water, the lack of electrode insulation normally presents no problem compared to spark gaps, which require watertight insulation for the ungrounded electrode. Pressure oscillograms (Fig. 17) obtained from 5-mil aluminum wire (using the oscilloscope delay sweep) show two consecutive discharges.

Empirically, the important conditions for reproducibility are (a) to control initial capacitor voltage, and (b) to maintain a fixed length of exploded wire.

Work by Gilstein (19) indicates that standard deviations for bubble pulse characteristics are less than 3 percent.

**SUMMARY**

The essential results may be summarized as follows:

1. Reproducibility of \( P_m \) values can be controlled to within 2 percent.

2. Sublimation energies of exploding wires (0.5-in. long, 5-mil diam) were found to within 10 percent by extrapolating the \( P_m \) versus \( V_0 \) curve to zero \( P_m \).

3. When the sublimation energy is small compared to \((1/2)CV_0^2\), the wire material and initial wire resistance (using straight wire) has no effect on \( P_m \).

4. Experiments at NRL and elsewhere indicate that the circuit parameters (when sublimation energy effects can be neglected) affect \( P_m \) as follows.
a. A voltage increase causes a near-linear increase in $P_m$

b. An inductance increase causes $P_m$ to decrease according to $P_m \sim 1/\sqrt{L}$

c. Capacitance has little effect on $P_m$

d. Initial gap resistance $R_0$ (to obtain the greatest $P_m$ for a given $V_0$, $L$, and $C$) should be set so that:

\[
(1) \quad R_{\text{switch}} + R_{\text{leads}} << R_0
\]

\[
(2) \quad \text{Gap length is as long as possible}
\]

e. Geometry — larger gap lengths and shorter gage-to-wire distances allow a greater measured $P_m$

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REFERENCES

   (a) pp. 117-118
   (b) p. 48
   (c) pp. 18-22


   (a) pp. 42-4'i
   (b) p. 39
   (c) p. 78
   (d) p. 66
   (e) p. 88
   (f) p. 75


   (a) pp. 178 and 188
   (b) p. 255

   (a) See Fig. 10a
   (b) See Fig. 10b

   (a) Section 2, p. 19
   (b) Section 2, p. 20
   (c) Section 2, p. 22


A study has been made on underwater shock waves produced by exploding wires. Details on experimental equipment and procedure are given for both circuit and pressure measurements. The influence of capacitance, inductance, and gap length on maximum shock pressure \( P_m \) is discussed. Experimental results correlating initial capacitor voltage (between 10 and 30 kV) with \( P_m \) are given. An increase of voltage and gap length along with a decrease in inductance all tend to increase \( P_m \). By using the empirical \( P_m \) vs voltage curve and extrapolating to zero \( P_m \), a voltage is obtained which...
corresponds closely to wire sublimation energy. Reproducibility, expressed quantitatively by standard deviation, is within 2 percent. The ease of $P_0$ variation, along with reproducibility, indicate the suitability of this method of shock formation as a research tool.