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Yielding and Flow of the Body Centered Cubic Metals at Low Temperatures

7 MARCH 1963

Prepared by HANS CONRAD
Materials Sciences Laboratory

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

COLUMBUS C. LEE
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AEROSPACE CORPORATION
El Segundo, California

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The available experimental data on the mechanical behavior of the body centered cubic (b.c.c.) transition metals at low temperatures (< 0.25 T_m) are reviewed and analyzed to establish the rate-controlling mechanism responsible for the strong temperature and the strain-rate dependence of the yield and flow stresses. The activation energy, H, activation volume, v*, and frequency factor, ν, were determined as a function of the thermal component of the stress, τ*.

It was found that H_0 (τ* = 1 kg/mm^2) ≈ 0.1 μb^3, where μ is the shear modulus and b the Burgers vector; v* ≈ 50 b^3 at τ* = 2 kg/mm^2, increasing rapidly to values in excess of 100 b^3 at lower stresses and decreasing to 2-5 b^3 at high stresses (50-60 kg/mm^2); and ν = 10^-6 - 10^{12} sec^-1, the higher values of ν generally being associated with the purer materials. H and v*, as a function of stress, were independent of structure. This along with other observations indicates that the thermally-activated overcoming of the Peierls-Nabarro stress is the rate-controlling mechanism. The values of H_0 and the change in H with stress at low stresses are in agreement with those predicted by Seeger's model for the nucleation of kinks. The Peierls-Nabarro stress and kink energy derived from the experimental data are approximately 10^-2 μ and 4 x 10^-2 μb^3, respectively.

The experimental data suggest that the yield point in the b.c.c. metals is associated with the sudden multiplication of dislocations by the double cross-slip mechanism, which in turn is controlled by the motion of dislocations through the lattice. Stress-strain curves for mild steel calculated on the basis of this mechanism are in good agreement with the experimental curves.
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I. INTRODUCTION

A distinguishing feature of the body centered cubic (b.c.c.) metals, as compared to the close packed hexagonal (c.p.h.), and face centered cubic (f.c.c.) metals, is the strong effect of temperature and of strain rate on the yield stress at low temperatures \( T < 0.2 T_m \), where \( T \) is the test temperature and \( T_m \) is the melting temperature, Figs. 1 and 2. An understanding of this difference is of technological, as well as of scientific interest, because the ductile-to-brittle transition in the b.c.c. metals is related to the strong temperature and to the strong strain-rate dependence of the yield stress.

A number of thermally-activated dislocation mechanisms have been proposed to account for the strong temperature and for the strong strain-rate dependence of the yield stress of the b.c.c. metals. In chronological order, these are:

a) breaking away from an interstitial atmosphere (Refs. 1-4)
b) overcoming of the Peierls-Nabarro stress (Refs. 5-9)\(^\dagger\)
c) nonconservative motion of jogs (Refs. 11-13)
d) overcoming of the interstitial precipitates (Ref. 14)
e) cross-slip (Ref. 15)

There is some experimental support for each of these mechanisms, which makes it difficult to decide which one is actually rate-controlling. From a review of the experimental data for iron available at the time, Conrad (Ref. 6) concluded that the best overall agreement was for the thermally-activated overcoming of the Peierls-Nabarro stress. Subsequent research

\(^\dagger\)Heslop and Petch (Ref. 10) first suggested that the strong temperature dependence of the yield and flow stress in iron might be due to a high Peierls-Nabarro stress. However, they attributed the temperature dependence to a change in width of the dislocation with temperature rather than the contribution of thermal fluctuations to overcoming of the Peierls-Nabarro stress. Their suggestion does not account for the strong effect of strain rate on the yield stress.
on iron (Refs. 8, 16) supported this conclusion. Recently, Conrad and Hayes (Ref. 9) analyzed the available data for the Group VA (V, Nb, Ta) and Group VIA (Cr, Mo, W) metals and again concluded that the rate-controlling mechanism at low temperatures was overcoming the Peierls-Nabarro stress. A similar conclusion was reached by Basinski and Christian (Ref. 5) for iron and by Christian and Masters (Ref. 7) for the Group VA metals.

In the present paper, pertinent data for all the b.c.c. transition metals are reviewed (including some of the more recent data, Refs. 7, 13, 17-19), and the rate-controlling mechanism during low temperature deformation is re-examined.
II. EXPERIMENTAL DATA

A. EFFECT OF TEMPERATURE ON YIELD AND FLOW STRESSES

1. Microdeformation

Employing a strain sensitivity of \(4 \times 10^{-6}\) and a strain rate of \(\sim 10^{-3} \text{ sec}^{-1}\), Brown and Ekvall (Ref. 15) found that the stress-strain behavior of ultrapure (\(\sim 0.004 \text{ wt } \%\) interstitials) and impure (\(\sim 0.04 \text{ wt } \%\) interstitials) iron consisted of three distinct regions, Fig. 3.

\(\sigma_E \uparrow\) is the stress where a hysteresis loop is first observed upon loading and unloading and represents the first evidence of dislocation motion. \(\sigma_E\) was only observed after previous straining (\(\varepsilon = 10^{-4}\) to \(5 \times 10^{-2}\)).

\(\sigma_A\) is the stress at which a permanent strain first occurs, i.e., the lowest stress at which the hysteresis loop does not close. This is generally the same as the stress at which a deviation from linearity can be measured and is usually called the proportional limit.

\(\sigma_F\) is commonly called yield or flow stress.

Brown and Ekvall (Ref. 15) found that \(\sigma_E\) was independent of temperature (between \(300^\circ\) and \(780^\circ\)K) for the impure and pure irons. The variation of \(\sigma_A\) with temperature is shown in Fig. 4. \(\sigma_A\) for the locked state (undeformed impure iron) here exhibits about the same temperature dependence as that previously reported (Refs. 20, 21) for the lower yield stress (of annealed or strain-aged states) and the subsequent flow stress of various impure irons (\(C + N > 0.015 \text{ wt } \%\)) and steels. \(\uparrow\uparrow\) On the other hand, \(\sigma_A\) for the unlocked (deformed) state has a lower temperature dependence than for yielding or for

\(\uparrow\) Throughout the present paper \(\sigma\) will be used to designate tensile stress and \(\tau\) shear stress. It will be assumed that \(\tau = 1/2 \sigma\). Similarly, \(\dot{\varepsilon}\) is the tensile strain rate and \(\dot{\gamma}\) the shear strain rate; \(\dot{\gamma} = 0.7 \dot{\varepsilon}\).

\(\uparrow\uparrow\) Data on the other b.c.c. metals also indicate that the proportional limit of annealed impure (> 0.02 \text{ wt } \%\) interstitials) material has a temperature dependence similar to that for the subsequent yield and flow stresses (Refs. 9, 20).
subsequent flow. Additional evidence that the proportional limit, \( \sigma_A \), after prestraining has a weaker temperature dependence than the flow stress, \( \sigma_F \), of iron has been reported by Conrad and Frederick (Ref. 8), Fig. 5, and by Kitajima (Ref. 22).

2. Macrodeformation

The yield or flow stress, \( \tau \), of b.c.c. metals can be considered to consist of three components (Refs. 6, 10, 20, 21):

\[
\tau = \tau^*(T, \dot{\gamma}) + \tau_\mu + K d^{-1/2}
\]

(1)

\( \tau^* \) is the thermal component which depends on temperature, \( T \), and strain rate, \( \dot{\gamma} \), and is associated with thermally-assisted overcoming of short-range obstacles. \( \tau_\mu \), represents the athermal component associated with long-range stress fields, is independent of strain rate, and varies with temperature only through the temperature variation of the shear modulus, \( \mu \). \( K d^{-1/2} \) is the component representing the grain size effect. \( K \) is the slope of the plot of the yield or flow stress versus the reciprocal of the square root of the grain size, \( d \), and for annealed impure material it is relatively independent of temperature and strain rate when compared to \( \tau^* \) (Refs. 21, 23-26). When \( K \) is independent of grain size, \( K d^{-1/2} \) gives the Hall-Petch relation (Refs. 27, 28). However, there are indications (Ref. 29) that \( K \), for both the lower-yield stress and flow stress, is a function of the grain size, and therefore the effect of grain size is not given by the simple Hall-Petch equation.

In the present discussion, we are principally concerned with the effect of temperature on the thermal component, \( \tau^* \), and wish to separate it from the others. This is accomplished by subtracting the stress at a given
temperature, $T$, from that at some reference temperature $T^0$ (Refs. 6, 20), assuming that $K$ is relatively independent of temperature, i.e.,

$$T_T - T_{T^0} = \tau^*(T, \dot{\gamma}) - \tau^*(T^0, \dot{\gamma}) = \Delta \tau^*(T, \dot{\gamma})$$

(2)

Typical results on iron and tungsten are shown in Figs. 5 and 6. Similar plots have been developed previously for iron (Ref. 20) and the Group VA and VIA metals (Refs. 9, 20). $\tau^*$ as a function of temperature is then derived from such plots by taking $\tau^* = 0$ at the temperature $T_0$, obtained by extrapolating plots of log $(\partial \tau^*/\partial \tau)$ versus $T$ to a value of $\sim 5 \times 10^{-3}$ kg/(mm$^2$ - °K), where $[(1/\tau)(d\tau/dT)] = [(1/\mu)(d\mu/dT)]$ and hence $\tau \approx \tau^*$. ($\partial \tau^*/\partial T$ is obtained by graphical differentiation of average curves similar to those in Figs. 5 and 6.) Plots of $\tau^*$ for yielding versus temperature for a strain rate of $10^{-4}$ sec$^{-1}$ derived in this manner from the available experimental data are given in Figs. 7-9. It is seen here that for the Group VA metals the variation of $\tau^*$ with temperature is relatively independent of purity and grain size (i.e., whether the specimen is single or polycrystalline), while for the Group VIA metals and iron, $\tau^*$ as a function of temperature is clearly dependent on purity and grain size. For the Group VIA metals and iron, the upper limit of $\tau^*$ at a given temperature is for polycrystals with interstitial contents $\geq 0.02$ wt %; the lower limit is for single crystals and polycrystals with less than 0.005 wt % interstitials. Single crystals and polycrystals with intermediate impurity levels exhibited a variation of $\tau^*$ with temperature between these two limits. In general, for the impure materials, $\tau^*$ at a given temperature was less for single crystals than for polycrystals of the same impurity content. Figures 5 and 10 show that previous thermal treatment and deformation may also influence the temperature dependence of $\tau^*$.

Values of $T_0$ (to the nearest 50°K) for a strain rate of $10^{-4}$ sec$^{-1}$ are given in Table 1. For the Group VIA metals and iron, $T_0$ for the pure materials is less than that for the impure materials, whereas for the Group V metals
there is no significant effect of impurity content on $T_0$. This difference and the difference in the effect of impurity content on the temperature dependence of $\tau^*$ may be related to the higher solubility of interstitials in the Group VA metals compared to the Group VIA metals and iron.

Table 1. Values of $T_0$ and the Ratio of $T_0/T_m$ for Pure and Impure B.C.C. Metals

<table>
<thead>
<tr>
<th>Metal</th>
<th>$T_m, ^\circ K$</th>
<th>$T_0, ^\circ K$</th>
<th>$T_0/T_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pure</td>
<td>Impure</td>
</tr>
<tr>
<td>V</td>
<td>2137</td>
<td>---</td>
<td>500</td>
</tr>
<tr>
<td>Nb</td>
<td>2741</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Ta</td>
<td>3269</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Cr</td>
<td>2148</td>
<td>---</td>
<td>500</td>
</tr>
<tr>
<td>Mo</td>
<td>2883</td>
<td>450</td>
<td>700</td>
</tr>
<tr>
<td>W</td>
<td>3683</td>
<td>500</td>
<td>850</td>
</tr>
<tr>
<td>Fe</td>
<td>1810</td>
<td>300</td>
<td>350</td>
</tr>
</tbody>
</table>

Pure b.c.c. metals (< 0.005 wt % interstitials)
Impure b.c.c. metals (> 0.02 wt % interstitials)
Strain rate $10^{-4} \text{ sec}^{-1}$

From Table 1 it is seen that the ratio $T_0/T_m$ is $0.22 \pm 0.04$ for all impure polycrystalline b.c.c. metals. In a previous paper (Ref. 9) it was shown that the yield stresses of all the impure polycrystalline b.c.c. metals correlate rather well on a single curve when $\tau^*$ is plotted versus the parameter $(T - T_0)/T_m$.

Values of $\tau^*_0$ obtained by extrapolating the curves of Figs. 7-9 to $0^\circ K$ are given in Table 2. Because of the rapid increase in $\tau^*$ at very low temperatures, there is some uncertainty associated with these values. However, if one plots the logarithm of $\tau^*$ versus temperature, an approximately linear region occurs at low temperatures (Fig. 11) allowing for an easier extrapolation. Extrapolation of this linear region to $0^\circ K$ gives values of $\tau^*_0$. 

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slightly higher than those based on the linear plots, Table 2. Both methods yield values of \( \tau_0^* \) of the order of \( 10^{-2} \mu \).

Table 2. Values of \( \tau_0^* \) Obtained by Extrapolating \( \tau^* \) to 0°K

<table>
<thead>
<tr>
<th>Metal</th>
<th>( \mu \times 10^3 ) kg/mm²</th>
<th>( \tau_0^* ), kg/mm²</th>
<th>( \tau_0^*(\text{Avg})/\mu ) x 10⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>5.2</td>
<td>60</td>
<td>62</td>
</tr>
<tr>
<td>Nb</td>
<td>4.0</td>
<td>58</td>
<td>61</td>
</tr>
<tr>
<td>Ta</td>
<td>7.0</td>
<td>60</td>
<td>69</td>
</tr>
<tr>
<td>Cr</td>
<td>11.4</td>
<td>79</td>
<td>83</td>
</tr>
<tr>
<td>Mo</td>
<td>12.7</td>
<td>78</td>
<td>85</td>
</tr>
<tr>
<td>W</td>
<td>15.7</td>
<td>100</td>
<td>122</td>
</tr>
<tr>
<td>Fe</td>
<td>7.4</td>
<td>49</td>
<td>58</td>
</tr>
</tbody>
</table>

\( \mu = 3/8 \ E \), where \( E \) is Young's modulus at 300°K (Tietz and Wilson, Ref. 30).

B. EFFECT OF STRAIN RATE ON THE YIELD AND FLOW STRESSES

In Fig. 2 the typical variation of the strain-rate parameter, \( \Delta \sigma/\Delta \ln i \) (\( \Delta \sigma \) is the incremental increase in yield or flow stress for an increase in strain rate from \( i_1 \) to \( i_2 \)), with temperature for the b.c.c. metals is shown.† It initially increases with decrease in temperature below \( T_0 \), goes through a maximum, and then decreases again, approaching zero as the temperature goes to absolute zero. It depends on structure (i.e., on impurities and on thermal and mechanical history) in a similar manner, as does the parameter \( \Delta \tau^*/\Delta T \); e.g., compare the effect of strain on \( \Delta \sigma/\Delta \ln i \) and on \( \Delta \tau^*/\Delta T \) for iron in Figs. 2 and 5.

†More detailed data on the variation of \( \Delta \sigma/\Delta \ln i \) with temperature are found in Refs. 5, 7, 8, and 13.
Another type of experiment which also gives the relationship between the yield stress and strain rate is the so-called delay-time test employed by Wood and Clark (Ref. 31) and Kraft and Sullivan (Ref. 32). In general, the parameter $\Delta \sigma / \Delta \ln t_d$, where $t_d$ is the delay time for yielding, obtained from such tests has a similar value and exhibits the same trends as does the parameter $\Delta \sigma / \Delta \ln \dot{i}$.

C. ACTIVATION ENERGY, ACTIVATION VOLUME, AND FREQUENCY FACTOR FOR DEFORMATION

1. General

It is now generally accepted that the deformation of metals may be thermally activated, and if a single mechanism is rate-controlling one can write for the shear strain rate $\dot{\gamma}$

$$\dot{\gamma} = \rho b \dot{s} = \rho b s v* \exp\left[-\frac{H(\tau, T)}{kT}\right]$$

where $\rho$ is the density of dislocations contributing to the deformation, $b$ the Burgers vector, $\dot{s}$ the average velocity of the dislocations, $s$ the product of the number of places where thermal activation can occur per unit length of dislocation and the area swept out per successful thermal fluctuation, $v*$ the frequency of vibration of the dislocation segment involved in the thermal activation, and $H$ the activation enthalpy (energy) which may be a function of the shear stress, $\tau$, and the temperature, $T$. For the b.c.c. metals, it has been established (Refs. 6, 8, 9, 16) that $H$ is primarily a function of the effective shear stress, $\tau^*$ (i.e., the thermal component of the yield or flow stress) given by the difference between the applied stress, $\tau$, and the long range internal stress $\tau_m$, i.e., $\tau^* = \tau - \tau_m$. Further, one can show that (Ref. 33)
\[ H = -k \left( \frac{\partial \ln \dot{\gamma}}{\partial 1/T} \right)_{T^*} \]  \hfill (4a)

\[ = -kT^2 \left( \frac{\partial \ln \dot{\gamma}}{\partial T} \right) T \left( \frac{\partial T^*}{\partial T} \right)_{\dot{\gamma}} \]  \hfill (4b)

Rearranging Eq. 3 and differentiating

\[ -\frac{dH}{dT^*} = kT \left( \frac{\partial \ln \dot{\gamma}}{\partial T} \right)_T \]  \hfill (5a)

\[ = \frac{k \ln (\dot{\gamma}/\nu)}{(\partial T^*/\partial T)_\dot{\gamma}} \]  \hfill (5b)

where \( \nu = \rho b s \nu^* \) and \( -dH/dT^* \) is defined as the activation volume, \( \nu^* \). The value of \( \nu \) can be obtained from the relations

\[ H = kT \ln \left( \frac{\nu}{\dot{\gamma}} \right) \]  \hfill (6a)

or

\[ -T \left( \frac{\partial \ln \dot{\gamma}}{\partial T} \right)_T \left( \frac{\partial T^*/\partial T}{\dot{\gamma}} \right)_T \ln \left( \frac{\nu}{\dot{\gamma}} \right) \]  \hfill (6b)

i.e., from the slope of a plot of \( H \) versus \( T \) or a plot of \( (\partial T^*/\partial T)_{\dot{\gamma}} \) versus \( (1/T)(\partial \dot{\gamma}/\partial \ln \dot{\gamma})_T \). If \( \nu \) is relatively independent of temperature and stress per se, the values of \( H, \nu^*, \) and \( \nu \) can then be derived from the relationships between stress, temperature, and strain rate obtained from the usual
mechanical tests. For polycrystalline b.c.c. metals (and also single crystals), a reasonable assumption is that \( \tau = \frac{1}{2} \sigma \), and \( \gamma = 0.7 \varepsilon \), where \( \sigma \) is the tensile stress, and \( \varepsilon \) is the tensile strain; also, \( \frac{\partial \tau / \partial T}{\gamma} \) is approximated by \( \frac{\partial \tau / \partial T}{\gamma} \), because \( \frac{d\varepsilon}{\partial T} \) is small compared to \( \frac{\partial \tau / \partial T}{\gamma} \).

2. Activation Energy, H

The variation of H with \( \tau^* \) obtained from the available experimental data is shown in Figs. 12-18. Identical results were obtained from the use of either Eq. (4a) or (4b), supporting the validity of the assumptions inherent in these equations. The curves drawn in these figures represent the author's interpretation of the variation of H with \( \tau^* \) indicated by the data points. However, there is some doubt as to whether a change in curvature actually occurs at the low stress, and for the most part the data suggest equally well a rather rapid increase in H as \( \tau^* \) approaches zero. Of particular significance in Figs. 12-18 is that, within the scatter of the data, H as a function of \( \tau^* \) is independent of the yielding or flow phenomena considered (microcreep, delay time, proportional limit, upper yield stress, flow, and dislocation velocity) and of the structure (impurity content, grain size, and previous thermal or mechanical history) for a given metal. In Fig. 19, it is seen that \( H_0 \), the value of H at \( \tau^* = 1 \text{ kg/mm}^2 \), is approximately equal to 0.1 \( \mu \text{b}^3 \), when comparing the various b.c.c. metals.

A number of investigators (Refs. 2, 6, 34-36) have reported that the activation energy for yielding in iron decreases in a linear manner with the logarithm of the total applied stress, \( \tau \). For comparison, plots of H versus log \( \tau^* \) are given in Fig. 20 for the various impure polycrystalline metals.

---

†The values of H, \( \gamma^* \), and the product \( s\gamma^* \) can also be obtained from measurements of the effect of stress and temperature on dislocation velocity by replacing the strain rate, \( i \), in Eqs. 4-6 with the velocity, \( \dot{\gamma} \).

†The shear modulus values were derived from the relation \( \mu = \frac{3}{8} E \), where \( E \) is the Young's modulus at \( T_0 \) (Ref. 30).
b.c.c. metals. It is seen here that for such plots there appears to be two linear regions, one at very low stresses and the other at high stresses, with a transition region in the vicinity of $\tau^* = 1-5 \text{ kg/mm}^2$. The slope in the high stress region is about 10 times that in the low stress region. Extrapolation of the straight lines at high stresses to $H = 0$ gives values of $\tau^*$ in reasonable agreement with those obtained by the other two methods, Table 2.

3. Activation Volume, $v^*$

Typical variation of $v^*$ with $\tau^*$ is shown for Ta, W, and Fe in Figs. 21-23. There was agreement between values obtained from Eq. (5a) and Eq. (5b) and from graphical differentiation of the curves of Figs. 12-18, except at the lowest values of $\tau^*$.†† It is shown in Fig. 24 that the activation volume as a function of stress is similar for all the b.c.c. metals. The values given here were taken from average curves, such as those drawn in Figs. 21-23. It is seen from Fig. 24 that, for all of the b.c.c. metals, $v^*$ is about $50 \text{ b}^3$ at $\tau^* = 2 \text{ kg/mm}^2$, increasing rapidly to values in excess of $100 \text{ b}^3$ at lower values of stress and decreasing with stress to values as low as $2-5 \text{ b}^3$. Again, as for $H$, $v^*$ as a function of stress is independent of the yielding or flow phenomena considered and of the structure, i.e., of mechanical and thermal history.

4. Frequency Factor, $v$

Typical proportionality between $H$ and temperature obtained for the b.c.c. metals is shown in Fig. 25. Plots of the average curves of $H$ vs $T$ for all the impure polycrystalline b.c.c. metals are given in Fig. 26. Average values of $v$ derived from such plots for pure ($< 0.005 \text{ wt }\%$) and impure ($> 0.02 \text{ wt }\%$) materials are given in Table 3.

†The values of $H$ and $\tau^*$ plotted in Fig. 20 were taken from the average curves drawn in Figs. 12-18.

††This disagreement will be discussed in a subsequent section.
Table 3. The Frequency Factor, $v$, for the B.C.C. Metals Obtained from Yield or Flow Stress Measurements

<table>
<thead>
<tr>
<th>Metal</th>
<th>Authors</th>
<th>$v$, sec$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pure Single or Polycrystals</td>
</tr>
<tr>
<td>V</td>
<td>Conrad, Present</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Christian and Masters (Ref. 7)</td>
<td>---</td>
</tr>
<tr>
<td>Nb</td>
<td>Conrad, Present</td>
<td>10$^6$</td>
</tr>
<tr>
<td></td>
<td>Christian and Masters (Ref. 7)</td>
<td>---</td>
</tr>
<tr>
<td>Ta</td>
<td>Conrad, Present</td>
<td>10$^8$</td>
</tr>
<tr>
<td></td>
<td>Chambers (Ref. 39)</td>
<td>10$^7$</td>
</tr>
<tr>
<td></td>
<td>Christian and Masters (Ref. 7)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Mordike (Ref. 13)</td>
<td>10$^9$</td>
</tr>
<tr>
<td>Cr</td>
<td>Conrad, Present</td>
<td>---</td>
</tr>
<tr>
<td>Mo</td>
<td>Conrad, Present</td>
<td>10$^{11}$</td>
</tr>
<tr>
<td>W</td>
<td>Conrad, Present</td>
<td>10$^{11}$</td>
</tr>
<tr>
<td>Fe</td>
<td>Conrad, Present</td>
<td>10$^{11}$</td>
</tr>
<tr>
<td></td>
<td>Basinski and Christian (Ref. 5)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Conrad (Ref. 6)</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Lean, Plateau, and Crussard (Ref. 38)</td>
<td>---</td>
</tr>
</tbody>
</table>

Additional evidence of the proportionality between $H$ and temperature is provided by the variation of the ductile-to-brittle transition temperature in the b.c.c. metals with strain rate. Generally, a straight line is obtained when the logarithm of the strain rate is plotted versus the reciprocal of the transition temperature (Refs. 37, 38), suggesting a rate equation of the form $i = Ae^{-H/kT}$. Taking the logarithm of both sides of this equation and rearranging, one obtains $H = kT \ln A/i$ (Fig. 27), which
agrees with Eq. 6 when \( A = \nu \). This is consistent with the analysis of Eqs. 3-6, if the transition from ductile-to-brittle behavior occurs at a constant stress. The values of \( \nu \) derived from the effect of strain rate on the ductile-to-brittle transition are given in Table 4. They are in reasonable agreement with those obtained from the yield and flow stress measurements listed in Table 3, indicating that the ductile-to-brittle transition temperature is determined by the dynamic motion of dislocations, as has been proposed by Cottrell (Ref. 40) and Petch (Ref. 24).

Table 4. Values of \( \nu \) Derived from the Effect of Strain Rate on the Ductile-to-Brittle Transition Temperature (Refs. 37 and 38)

<table>
<thead>
<tr>
<th>Metal</th>
<th>( \nu, \text{sec}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>( 10^{10} )</td>
</tr>
<tr>
<td>Mo</td>
<td>( 10^{8} - 10^{12} )</td>
</tr>
<tr>
<td>W</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>Fe</td>
<td>( 10^{8} - 10^{12} )</td>
</tr>
</tbody>
</table>
A. RATE-CONTROLLING MECHANISM

The fact that the various relationships of Eqs. 3-6 gave the same values of $H$, $v^*$, and $v$ indicates that the postulated assumptions are valid, at least to a first approximation. Specifically, this supports the contention that during the low temperature deformation ($< 2.0 \, T_m$) of the b.c.c. transition metals, a single dislocation mechanism is rate-controlling and that $v$ is relatively independent of stress and temperature per se. Furthermore, the fact that identical values of $H$ and $v^*$ were obtained for all yielding and flow phenomena (and the ductile-to-brittle transition) indicates that the same dislocation mechanism is controlling in all cases and that this is associated with the motion of dislocations through the lattice, as distinct from a generation mechanism, such as breaking away from an interstitial atmosphere. Finally, the fact that $H$ and $v^*$ as a function of stress were independent of structure (i.e., thermal and mechanical history) strongly suggests that the rate-controlling mechanism is overcoming the inherent resistance of the lattice, i.e., overcoming the Peierls-Nabarro stress. Further support for the Peierls-Nabarro mechanism is that dislocations in the b.c.c metals are often observed to lie along the close-packed directions (Ref. 41-43). A summary of the experimental evidence negating the other mechanisms mentioned in the Introduction is given in Table 5.

A possible thermally-activated mechanism for overcoming the Peierls-Nabarro stress (energy) is that mechanism originally proposed by Seeger (Ref. 44) to explain the Bordoni peak in f.c.c. metals, which is shown in Fig. 28. It involves the formation of a pair of kinks in a dislocation line lying in a close-packed direction by the combined action of thermal fluctuations and the applied stress, and the subsequent lateral propagation of the kinks along the dislocation line, resulting in the forward motion of the
Table 5. Evidence Against Specific Mechanisms Proposed as Rate-Controlling During the Low Temperature Deformation of the B.C.C. Metals

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Contrary Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaking-away from an interstitial atmosphere</td>
<td>All yielding and flow phenomena exhibit the same values of $H$ and $v^*$.</td>
</tr>
<tr>
<td></td>
<td>$H$ and $v^*$ are independent of interstitial content.</td>
</tr>
<tr>
<td></td>
<td>$H$ and $v^*$ are the same for the mobility of dislocations as determined by etch pits as for initial yielding.</td>
</tr>
<tr>
<td>Nonconservative motion of jogs</td>
<td>$H$ and $v^*$ are independent of structure.</td>
</tr>
<tr>
<td></td>
<td>$H$ and $v^*$ for yielding and flow are the same as for the mobility of edge dislocations, which do not move nonconservatively.</td>
</tr>
<tr>
<td>Overcoming interstitial precipitates</td>
<td>$H$ and $v^*$ are independent of impurity content.</td>
</tr>
<tr>
<td></td>
<td>$H$ and $v^*$ are independent of the quantity of interstitial precipitates.</td>
</tr>
<tr>
<td></td>
<td>$v^*$ decreases with increase in quantity of precipitates.</td>
</tr>
<tr>
<td>Cross-slip</td>
<td>$H$ and $v^*$ for yielding and flow are the same as for the mobility of edge dislocations, which cannot cross-slip.</td>
</tr>
</tbody>
</table>

dislocation. Seeger (Ref. 44) calculated the activation energy for this process at low stresses to be

\[ H = H_K \left[ 1 + \frac{1}{4} \log \left( \frac{16\tau_0^p}{\pi \tau^*} \right) \right] \tag{7} \]

where \( H_K \) is the energy of a single kink, and \( \tau_0^p \) is the Peierls-Nabarro stress at 0°K. Furthermore, Seeger gives

\[ H_K = 2a \left( \frac{2E_0ab\tau_0^p}{\pi} \right)^{1/2} \tag{8} \]

and

\[ H_{PN} = \frac{\tau_0^p ab^2}{2\pi} \tag{9a} \]

\[ \frac{H_{PN}^2 \pi^2 b}{16a^2 E_0} \tag{9b} \]

where \( 2H_{PN} \) is the Peierls-Nabarro energy per atomic length, \( a \) is the distance between close-packed rows, \( b \) is the Burgers vector, and \( E_0 \) is the line energy of a dislocation. Taking the average value of \( \tau^* \)(from Table 2) for \( T_p^o \) and taking \( E_0 = 1/2 \mu b^2 \), one obtains from Eqs. (7) and (8), \( H_0 \) (\( \tau^* = 1 \text{ kg/mm}^2 \)) \( \approx 0.1 \mu b^3 \), which is in good agreement with measured values of \( H_0 \), Table 6. Furthermore, from Eqs. (7) and (8), \( H_K = 3-4 \times 10^{-2} \mu b^3 \)

\[ \dagger \text{The thermal component of the stress } \tau^* \text{ has been substituted for the total stress } \tau \text{ in Seeger's equation.} \]
and from Eq. 9, \( H_{PN} = 1.2 \times 10^{-3} \mu b^2 \), Table 6. Taking the derivative of Eq. 7 with respect to \( \tau^* \) gives

\[
\frac{dH}{d\tau^*} = v^* = \frac{1}{4} \frac{H_K}{\tau^*}
\]  

Values of \( H_K \) derived from the values of \( v^* \) at \( \tau^* = 1 \text{ kg/mm}^2 \) are also given in Table 6 and are in agreement with those obtained from Eqs. (7) and (8). For comparison, values of \( H_K \) derived from the slope of the plots of \( H \) vs \( \log \tau^* \) (Fig. 20) at low stresses (< 1 kg/mm\(^2\)) are approximately 1/6 to 1/3 those calculated using Eqs. (7) and (8), while those derived from the slope at high stresses (> 10 kg/mm\(^2\)) are about 3 to 4 times larger. Agreement occurs in the intermediate stress range \( (\tau^* = 1-5 \text{ kg/mm}^2) \), where the plots show curvature.

The good agreement between the values of \( H_o \) and \( H_K \) obtained from the various relationships, Eqs. (7)-(10) indicates rather strongly that the nucleation of kinks is the rate-controlling mechanism during low temperature deformation of the b.c.c. metals. Although the derived values of \( \tau^O_p \), \( H_K \) and \( H_{PN} \) are somewhat higher than those usually given for close-packed metals (Ref. 45), they are in accord with those calculated using the original Peierls-Nabarro equations (Ref. 46) and the more recent calculations of Kuhlmann-Wilsdorf (Ref. 47) and Hobart and Celli (Ref. 48). The good agreement between the values of \( H_o \) and \( H_K \) obtained using only experimental data and those obtained using \( E_0 = 1/2 \mu b^2 \) indicates that the line energy in the b.c.c. metals is very nearly 1/2 \( \mu b^2 \).

According to Seeger (Ref. 44) the width, \( w \), of a kink is given by

\[
w = \frac{1}{2} a \left( \frac{E_0 b}{H_{PN}} \right)^{1/2}
\]  

\[
(11)
\]
Table 6. Values of $H_o$, $H_K$, and $H_{PN}$ Derived from the Experimental Data

<table>
<thead>
<tr>
<th>Metal</th>
<th>$b$ (\text{Å})</th>
<th>$\mu b^3$ (\text{ergs x 10^{-12}})</th>
<th>$H_o (\tau^* = 1 \text{ kg/mm}^2) / \mu b^3$</th>
<th>$H_K / \mu b^3$</th>
<th>$H_{PN} / \mu b^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>2.63</td>
<td>9.4</td>
<td>9.5</td>
<td>3.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Nb</td>
<td>2.86</td>
<td>9.5</td>
<td>10.7</td>
<td>3.8</td>
<td>2.4</td>
</tr>
<tr>
<td>Ta</td>
<td>2.86</td>
<td>16.6</td>
<td>8.9</td>
<td>3.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Cr</td>
<td>2.50</td>
<td>17.6</td>
<td>7.8</td>
<td>3.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Mo</td>
<td>2.73</td>
<td>25.5</td>
<td>7.6</td>
<td>3.3</td>
<td>1.1</td>
</tr>
<tr>
<td>W</td>
<td>2.73</td>
<td>31.5</td>
<td>8.4</td>
<td>3.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Fe</td>
<td>2.48</td>
<td>11.2</td>
<td>7.8</td>
<td>3.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

$\mu = 3/8E$, where $E$ is Young's modulus at 300$^\circ$K (Tietz and Wilson, Ref. 30).
and the critical separation, \( f^* \), of the kinks during thermal activation is

\[
f^* = \frac{w}{\pi} \log \left( \frac{32H_{PN}}{\tau^* b^3} \right)
\]

(12a)

\[
\frac{H - H_K}{2H_{PN}} = \frac{H_{PN}}{2H_{PN}}
\]

(12b)

The values of \( w \) and of \( f^*(at \tau = 1 \text{ kg/mm}^2) \) obtained from Eqs. (11), (12a), and (12b) using the average values of \( H_K \), \( H_{PN} \), and \( H (\tau^* = 1 \text{ kg/mm}^2) \) from Table 6 and taking \( E_0 = 1/2 \mu b^2 \), are given in Table 7. It is here seen that \( w = 7-10 \text{ b} \) and \( f^*(\tau^* = 1 \text{ kg/mm}^2) = 12-22 \text{ b} \), which are quite reasonable.

Again, of significance is the good agreement between values of \( f^* \) from Eqs. (12a) and (12b).

Table 7. Values of \( w \) and \( f^* \) Derived from the Experimental Data

<table>
<thead>
<tr>
<th>Center</th>
<th>w/b</th>
<th>( f^*/b )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Eq. (12a)</td>
</tr>
<tr>
<td>V</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Nb</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Ta</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>Cr</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>Mo</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>W</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>Fe</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>
For the Peierls-Nabarro mechanism, the frequency factor can be given by

\[ \nu = \rho b \left( \frac{1}{\ell^*} \right) (Lb) \left( \frac{b}{\ell^*} \nu_d \right) \]  

(13)

where \( \rho \) is the density of dislocations participating in the deformation, \( b \) the Burgers vector, \( \ell^* \) the length of dislocation segment involved in the thermal activation, \( L \) the maximum lateral spread of the kinks, and \( \nu_d \) the Debye frequency. The first term within the brackets represents the number of places per unit dislocation length where thermal fluctuations may nucleate a loop of length \( \ell^* \); the second term is the area of the slip plane swept out per successful thermal fluctuation; the third term is the frequency of vibration of a segment of length \( \ell^* \). Taking \( \rho = 10^9 \text{ cm}^{-2}, \ell^* = 10 b, L = 10^{-4} \text{ cm} \) and \( \nu_d = 10^{13} \text{ sec}^{-1} \), one obtains \( \nu = 10^8 \text{ sec}^{-1} \), which is in agreement with that observed experimentally for many materials, Tables 3 and 4.

Since \( H \) and \( \nu^* \) as a function of \( \tau^* \) were independent of structure, the effect of impurities, precipitates, grain size, dislocations, and other aspects of previous thermal or mechanical history on the temperature dependence of the yield or flow stress is then due to a change in the frequency factor, \( \nu \), i.e., in the number of dislocations, \( \rho \), participating in the deformation or in the lateral distance, \( L \), a kink can move before encountering an obstacle. In this regard, Conrad and Frederick (Ref. 8) investigated the effect of straining and of interstitial precipitates in iron on the temperature dependence of \( \tau^* \). Some of their results are given in Fig. 3, which shows that a weaker temperature dependence results from straining and from the presence of precipitates. In Figs. 29 and 30 (from Ref. 8) it is shown that the weaker temperature dependence is associated with a larger value of \( \nu \), given by the slope of the plot of \( H \) vs temperature. From the relation \( \nu = \rho b v^* \) and taking the value of \( v^* \) derived from the dislocation velocity measurements of Stein and Low (Ref 49), they obtained values of \( \rho \), and their increase with strain (Table 8) in agreement with dislocation densities determined by Keh
and Weissman (Ref. 42) by thin-film electron microscopy, indicating that the increase in $v$ was due primarily to an increase in $p$. They further concluded from their results that precipitates represented good sources for dislocations, which is in agreement with observations of Leslie (Ref. 50) and Van Torne and Thomas (Ref. 51).

Table 8. Effect of Strain on the Dislocation Density Participating in the Plastic Flow of Iron Determined from the Frequency Factor $v$ (Ref. 8)

<table>
<thead>
<tr>
<th>Material</th>
<th>Strain</th>
<th>$v$, Sec$^{-1}$</th>
<th>$p$, cm$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum Melted Electrolytic Iron, Water Quenched from 920°C</td>
<td>$1 \times 10^{-3}$</td>
<td>$2.1 \times 10^7$</td>
<td>$8.4 \times 10^8$</td>
</tr>
<tr>
<td></td>
<td>$5 \times 10^{-2}$</td>
<td>$8.2 \times 10^7$</td>
<td>$3.3 \times 10^9$</td>
</tr>
<tr>
<td>Ferrovac, Decarburized</td>
<td>$2 \times 10^{-2}$</td>
<td>$3.8 \times 10^6$</td>
<td>$1.5 \times 10^8$</td>
</tr>
<tr>
<td></td>
<td>$10 \times 10^{-2}$</td>
<td>$1.7 \times 10^8$</td>
<td>$6.6 \times 10^9$</td>
</tr>
<tr>
<td></td>
<td>$20 \times 10^{-2}$</td>
<td>$5.3 \times 10^9$</td>
<td>$2.1 \times 10^{11}$</td>
</tr>
<tr>
<td>Ferrovac, Annealed</td>
<td>$&gt; 5 \times 10^{-2}$</td>
<td>$5.3 \times 10^9$</td>
<td>$2.1 \times 10^{11}$</td>
</tr>
</tbody>
</table>

From Table 3 it is seen that the weaker temperature dependence of $T^*$ for single crystals or pure polycrystals, as compared to impure polycrystals in the Group VIA metals and iron, is associated with a frequency factor that is larger by 3 to 5 orders of magnitude. These larger values of $v$ cannot be due entirely to a greater dislocation density, $p$, because this would require unreasonably large values for $p$. Rather, it appears that this difference in $v$ is primarily due to larger values of $L$ for the pure as compared to the impure materials, suggesting that interstitial atoms or precipitates influence the extent to which the kinks can spread before encountering an obstacle. Besides acting as obstacles to kink motion, the interstitial atoms or precipitates may induce cross-slip, which in turn limits the dislocation loop length.
on the slip plane. Of interest in this regard are the observations of Schadler and Low (Ref. 19), who report that under some conditions dislocations in tungsten crystals can move long distances without multiplying, in agreement with the high values of \( v \) given in Table 3 for single crystals of tungsten.

All of the above supports overcoming the Peierls-Nabarro stress by thermally-activated nucleation of kinks as the rate-controlling mechanism in the b.c.c. metals at low temperatures. However, explanation is needed for the fact that in the vicinity of \( T_0 \) (i.e., \( \tau^* = 0 \)) \( H \), for the most part, does not increase as rapidly with decrease in stress (or increase in temperature) as is expected from the values of \( v^* \) or the straight line portion of the \( H \) versus temperature curves at lower temperatures. As indicated earlier, the scatter in the data allows for a more rapid increase in \( H \) than is indicated by the curves drawn in Figs. 12-18. The low values of \( H \) for stresses only slightly greater than zero may then simply reflect the difficulty in defining \( \tau^* = 0 \) exactly. Also, \( \nu \) may actually decrease with increase in temperature (or decrease in stress).† On the other hand, a different mechanism may become rate-controlling in the very low stress range. Additional work is needed to resolve this problem.

Finally, one needs to explain the much smaller temperature dependence of the proportional limit after straining and the fact that \( \sigma_E \), as defined by Brown and Ekvall (Ref. 15), is independent of temperature. Also, in recent investigations on the determination of \( H \) as a function of stress in Ta by creep tests, Chambers (Ref. 39) found a spectrum of activation energies for very small strain rates \((10^{-6} - 10^{-11} \text{ sec}^{-1})\) rather than a single

†The change in \( \nu \) may be the result of straining at different temperature (or stresses) rather than the effect of stress or temperature per se.
activation energy. † There are two possible explanations for these various effects:

First, these phenomena represent the motion of those specific dislocations located in the most favorable internal stress fields, and the applied stress primarily gives direction to the motion of these dislocations and does not contribute significantly to the thermally-activated process.

Second, another less difficult mechanism is rate-controlling at the very low stress levels, for example the lateral motion of kinks, as proposed by Brailsford (Ref. 52), or the lateral motion of jogs as suggested by Chambers. ††

In both cases the easier motion would soon die out, and to obtain gross macroscopic flow the more difficult mechanism of nucleating kinks would become rate-controlling. The rapid strain hardening associated with the early part of the stress-strain curve would then be an exhaustion hardening rather than an interaction hardening, which occurs subsequently during macroflow. Here, also, additional research is needed to resolve this question.

B. YIELD POINT AND WORK HARDENING

The fact that K in Eq. (1) is relatively independent of temperature and that the activation energy and activation volume as a function of stress are the same for all deformation phenomena suggests that the yield point in the b. c. c. metals is due to the thermally-assisted unpinning of dislocations from their interstitial atmosphere, as proposed by Cottrell (Ref. 1), but results from the sudden multiplication of dislocations by the double cross-slip mechanism of Koehler (Ref. 53) and Orowan (Ref. 54), as proposed by Johnson and Gilman (Ref. 55) for LiF. In this latter model, the multiplication of dislocations is controlled by their motion through the lattice, which agrees

†As an upper limit, Chambers (Ref. 39) reported an H versus stress relationship in agreement with that given in Fig. 14.

††R. C. Chambers, private communication.
with the experimental facts. As mentioned previously (Ref. 16), three factors favor the occurrence of such a yield point in the b.c.c. metals:

First, there exists only a small number of dislocations which can contribute to the plastic flow, due to the pegging of the available dislocations by interstitial precipitates (as distinct from pinning by an interstitial atmosphere).

Second, the dislocation density contributing to the plastic flow increases very rapidly with strain. This is inherent in the double cross-slip mechanism for multiplication and has been observed experimentally in iron by Keh and Weissman (Ref. 42) and in molybdenum by Benson.†

Third, the change in stress for a given change in dislocation velocity, \( \frac{\partial \sigma}{\theta \ln \dot{\varepsilon}} \) or \( \frac{\partial \sigma}{\theta \ln \dot{\varepsilon}} \), is relatively large, Fig. 2.

To check the proposed interpretation of the yield point, stress-strain curves for mild steel were calculated (Ref. 16) using Eqs. (1) and (3) and the available information on the activation energy as a function of stress (Fig. 18), the value of \( s/v^* \) derived from etch pit measurements in silicon-iron (Ref. 16), the increase in dislocation density with strain (Ref. 42), and the increase in flow stress associated with the increase in dislocation density (Ref. 42). The good agreement between the calculated and experimental curves is shown in Fig. 31. Since only plastic strain was considered, the upper yield point was taken as the stress at a plastic strain of \( 10^{-4} \), which is approximately the observed pre-yield microstrain in iron and steel.

†R. Benson, private communication.
Fig. 1. Initial Yield Stress of Various Polycrystalline Metals vs Ratio of $T/T_m$
Fig. 2. Effect of Strain Rate on the Yield and Flow Stresses of F.C.C. and B.C.C. Metals
Fig. 3. Idealized Stress-Strain Curves for an Ultrapure Iron at Two Temperatures (from Brown and Ekvall, Ref. 15)
Fig. 4. Temperature Dependence of $\sigma_A$ for Various Materials and States of Locking (from Brown and Ekvall, Ref. 15)
Fig. 5. Effect of Temperature on the Initial Yield Stress, the Proportional Limit After Straining, and the Flow Stress of Electrolytic Iron
Fig. 7. Variation of $\tau^*$ with Temperature for Pure (<0.005 wt % Interstitials and Impure (>0.02 wt % Interstitials)
Group VA Metals, Strain Rate $10^{-4}$ sec$^{-1}$
Fig. 8. Variation of $\tau^*$ with Temperature for Pure (<0.005 wt % Interstitials and Impure (>0.02 wt % Interstitials) Group VIA Metals, Strain Rate $10^{-4}$ sec$^{-1}$

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Fig. 10. Temperature Dependence of Reversible Flow Stress in Iron (Data from Basinski and Christian, Ref. 5)
Fig. 11. Log $\tau^0$ vs Temperature for Impure Polycrystalline B.C.C. Metals
Fig. 12. Effect of Stress on the Activation Energy for the Deformation of Vanadium

Fig. 13. Effect of Stress on the Activation Energy for the Deformation of Niobium
Fig. 14. Effect of Stress on the Activation Energy for the Deformation of Tantalum

Cr

Fig. 15. Effect of Stress on the Activation Energy for the Deformation of Chromium
Fig. 16. Effect of Stress on the Activation Energy for the Deformation of Molybdenum
Fig. 17. Effect of Stress on the Activation Energy for the Deformation of Tungsten
Fig. 18. Effect of Stress on the Activation Energy for Yielding and Flow in Iron and Steel
Fig. 19. Relationship Between $H_0 \ (\tau^* = 1 \text{ kg/mm}^2)$ and $\mu b^3$ for the B.C.C. Metals
Fig. 20. Activation Energy vs Log $\tau^*$ for the B.C.C. Metals
Fig. 21. Effect of Stress on the Activation Volume for the Deformation of Tantalum
Fig. 22. Effect of Stress on the Activation Volume for the Deformation of Tungsten
Fig. 23. Effect of Stress on the Activation Volume for Yielding and Flow in Iron and Steel
Fig. 24. Effect of Stress on the Activation Volume for Deformation of the B.C.C. Transition Metals
Fig. 25. Variation of Activation Energy with Temperature for Ingot Iron and Mild Steel
Fig. 26. Variation of Activation Energy with Temperature for Impure Polycrystalline B. C. C. Metals
Fig. 27. Variation of the Activation Energy for Brittle Fracture with Temperature (from Lean et al., Ref. 38)
Fig. 28. Seeger's Model for Thermally-Activated Overcoming of the Peierls-Nabarro Energy
Fig. 29. Variation of Activation Energy with Temperature for Electrolytic Iron, Water Quenched from 920°C
Fig. 30. Variation of Activation Energy with Temperature for Ferrovac Iron
Fig. 31. Comparison of Calculated and Experimental Stress-Strain Curves for Mild Steel
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The available experimental data on the mechanical behavior of the body centered cubic (b.c.c.) transition metals at low temperatures (<0.25 T_m) are reviewed and analyzed to establish the rate-controlling mechanism responsible for the strong temperature and the strain-rate dependence of the yield and flow stresses. The activation energy, H; activation volume, V_a; and frequency factor, v, were determined as a function of the thermal component of the stress, \( \sigma^\theta \). It was found that \( H = (10^3 \text{ kg/mm}^2 \text{ s}^0.1 \mu_b) \), where \( \mu \) is the shear modulus and \( b \) is the Burgers vector; \( v = 50 \text{ b}^2 \text{ at } \sigma^\theta = 2 \text{ kg/mm}^2 \), decreasing rapidly to values in excess of 100 \text{ b}^2 at lower stresses and decreasing to 2-5 \text{ b}^2 at high stresses (50-60 kg/mm\(^2\)); and \( v = 10^5 - 10^6 \text{ sec}^{-1} \), the higher (over).
values of $\nu$ generally being associated with the purer materials. $H$ and $\nu^*$, as a function of stress, were independent of structure. This along with other observations indicates that the thermally-activated overcoming of the Peierls-Nabarro stress is the rate-controlling mechanism. The values of $H_0$ and the change in $H$ with stress at low stresses are in agreement with those predicted by Seeger's model for the nucleation of kinks. The Peierls-Nabarro stress and kink energy derived from the experimental data are approximately $10^{-14}$ and $4 \times 10^{-2}$ $\mu B^2$, respectively. The experimental data suggest that the yield point in the b.c.c. metals is associated with the sudden multiplication of dislocations by the double cross-slip mechanism, which in turn is controlled by the motion of dislocations through the lattice. Stress-strain curves for mild steel calculated on the basis of this mechanism are in good agreement with the experimental curves.

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