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DETECTION
and
DESIGNATION

Atmospheric Property Approximation

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MONOGRAPH NR. 2

ATMOSPHERIC PROPERTY APPROXIMATION

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I. INTRODUCTION

II. ATMOSPHERIC DENSITY APPROXIMATION

III. ATMOSPHERIC PRESSURE APPROXIMATION

IV. DEFINITION OF SYMBOLS

V. REFERENCES
This monograph is concerned with two properties of the earth's atmosphere which are pertinent to the description of the phenomenon associated with the reentry of a hypersonic body. These two properties, density and pressure, are functions of the geometric altitude and vary in a manner which can be approximated by exponential relations. Specific relations for the properties are given for altitudes up to 350 Kft.

Since the altitude regime was considered in several increments and the most recent data was utilized i.e., The 1959 Model NDC Atmosphere, the approximations provide a reasonably accurate determination of the atmospheric property values. In addition, the relations are in a form amenable to incorporation into expressions of motion and aerodynamic heating parameters.
In the initial simplified analysis of the motion and heating characteristics of bodies reentering at hypersonic velocities\(^{(1)}\), Allen and Eggers approximated the density property of the earth's atmosphere. By using the data then available\(^{(2,3)}\), a relation of the following form was derived:

\[
\rho = \rho_c \cdot e^{-\frac{h}{\eta}}
\]

(1)

where for 20 Kft \(< h \leq 180\) Kft

\[
\rho_c = 3.4 \times 10^{-3} \text{ slugs/ft}^3
\]

\[\eta = 22 \text{ Kft}\]

The above density relation is plotted in Figure 1 along with the most recent atmospheric data\(^{(4)}\). It can be seen that the agreement in the lower altitude regime is excellent and, even throughout the entire range, it provides an average value of the density.

Recent interest in higher altitudes, however, has necessitated a more accurate approximation for use in the analysis of reentry phenomena. A relation of the above form is to be preferred; accordingly, the 1959 ARDC Model atmosphere has been approximated by three specific relations, applicable in respective altitude regions where the parameters \(\rho_c\) and \(\eta\) assume the following values:

a. Region I, Figs. 2, 3

50 Kft \(< h \leq 135\) Kft

\[
\rho_c = 3.6 \times 10^{-3}
\]

\[\eta = 21.44\]

b. Region II, Figs. 2, 4

135 Kft \(< h \leq 240\) Kft

\[
\rho_c = 1.07 \times 10^{-3}
\]

\[\eta = 26.48\]
The approximations, it is hoped, will be useful in obtaining rapid estimations of the atmospheric density.

**ATMOSPHERIC PRESSURE APPROXIMATION:**

An attempt, similar to the previous section regarding density, for the best estimate of atmospheric pressure is presented here. The same reference, ARDC 1959, has been used and the two lower altitude regions are the same as those used in the density case. However, it seemed necessary to write separate pressure equations in the altitude range of 240 Kft to 300 Kft. Region three covers 240 Kft to 300 Kft and region four covers 300 Kft to 350 Kft.

The general equation for atmospheric pressure is as follows:

\[ p = k e^{-m h} \]  

(2)

where \( k \) is 1lb/ft²

\( m \) is Kft⁻¹

The four specific relations, applicable in respective altitude regions, are given below where the parameters \( k \) and \( m \) assume the following values:

a. Region I, Figs. 1, 2

\[ 25 \text{ Kft} < h < 135 \text{ Kft} \]

\[ k = 3.3376 \times 10^3 \]

\[ m = 4.528 \times 10^{-2} \]

b. Region II, Figs. 1, 3

\[ 135 \text{ Kft} < h < 240 \text{ Kft} \]

\[ k = 1.3186 \times 10^3 \]

\[ m = 4.076 \times 10^{-2} \]
c. Region III, Figs. 1, 4

\[ 240 \text{ Kft} < h < 300 \text{ Kft} \]
\[ k = 1.15920 \times 10^5 \]
\[ m = 5.94 \times 10^{-2} \]

d. Region IV, Figs. 1, 5

\[ 300 \text{ Kft} < h < 350 \text{ Kft} \]
\[ k = 1.24717 \times 10^4 \]
\[ m = 5.196 \times 10^{-2} \]

Of course, the equations are not meant to give exact results. However, it is hoped that they represent approximations close enough to be useful as rapid calculations of the atmospheric pressure within the prescribed altitudes.


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<thead>
<tr>
<th>SYMBOL</th>
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<tr>
<td>h</td>
<td>Altitude</td>
<td>ft</td>
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<tr>
<td>p</td>
<td>Pressure</td>
<td>lbf/ft²</td>
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