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SOLAR RESEARCH NOTES

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GEOPHYSICS RESEARCH DIRECTORATE
AF CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
ON THE ENERGY BALANCE AND STRUCTURE
OF THE CHROMOSPHERE
by
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SOLAR RESEARCH NOTE NO. 23

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The conception of the heating of the chromosphere by compressional waves, originating in regions of turbulence connected with the convective zone of the Sun, is developed. It is shown that in the chromosphere a balance can be reached between the energy, produced as a result of the dissipation of shock waves, and the energy carried away by radiation. The shock waves provide for the appearance in the chromosphere of regions with different temperatures (a two-temperature chromosphere).

It is shown that the dissipation of magneto-hydrodynamic waves, because of losses due to friction, cannot provide sufficient energy.

The lowering of the chromosphere above spots is explained. The value of the density gradient in the chromosphere is discussed. An attempt is made to form a general idea on the chromosphere and on its generation.

The whole structure of the chromosphere is determined mainly by the value of the energy flux in acoustic noises, generated by turbulence produced in the convective zone of the Sun.
Recently much work was published devoted to the structure of the chromosphere and the possible ways of its heating. Many of them in some measure supplement and extend each other, and at the present time on the basis of this work one can attempt to put together for oneself several general ideas on the chromosphere. In the present work an attempt will be made to create such a scheme on the basis of ideas of the energy balance in the chromosphere. In 1948 Biermann [1] and Schwarzschild [2] put forward the supposition that the kinetic energy of granulation in some way (say, in the form of sound waves) reaches the corona and heats it to high temperatures. It appears that granulation corresponds to enormous quantities of energy and even small portions of it in a condition make up for the loss of radiation in the chromosphere and the corona through radiation. Schatzman [3] showed that when a granule rises turbulence must appear, giving rise, in its turn, to compressional waves, quickly being changed into shock waves by spreading upwards. On the basis of the theory of dissipation of shock waves, worked out by Brinkley and Kirkwood [4], was carried out the computation of the dissipation of shock waves and it was shown in particular, that in the lower corona the shock waves must dissipate entirely, since there the length of the free path becomes equal to the length of the wave (distance between fronts). On the other hand, the conductivity of the corona is so great, that equilibrium is established by a very small gradient of temperature over all layers, where there is the highest possible energy discharge. In 1955 Ueno and Kawanata [5] investigated in detail the process of formation of compressional waves - accoustical noise, capable of heating the chromosphere and the corona. They made use of the theory of aerodynamic generation of sound waves (noise) developed by Proudman [6] and Lighthill [7], and the model of convective zones, suggested by Vitense [8]. They concluded that the flux of energy upwards is approximately $10^4 - 10^5$ ergs/cm$^2$ per sec, and that this flux is proportional to the 6th power of the adopted value of turbulent velocity, so that the increase of the latter by one and one-half times will call forth an increase of greater than 25 times in the estimated magnitude of the flux. The turbulent velocity adopted in [5] is equal to 8% of the convection velocity, which is possibly underestimated by several percent (see [9]), as for example, a value of 11% of convection velocity gives a flow 10 - 15 times greater.

In 1956 Piddington [10] noted that the passage of sound waves in conditions of a highly conducting medium in the presence of magnetic fields must give rise to a spreading of magneto-hydrodynamic waves, the dissipation of which could explain the heating-up of the chromosphere. Piddington gave the formula for estimation of dissipation of magneto-hydrodynamic waves by means of so-called frictional loss.

In recent years more than once the consideration has been stated that in the chromosphere hot and cold regions must co-exist [11 - 14]. Not long ago Athay and Menzel [15], on the basis of observational material of the eclipse of 1952, constructed a model of such a two-temperature chromosphere. Athay and Thomas [16] theoretically showed the necessity for the existence of regions with different temperatures, thus, up to a certain
height regions with low temperatures (6300°) predominate, and above-regions with high temperatures (19000°).

Within each region the temperature changes little with height. The establishment of one or another temperature is conditioned by the balance between the escaping energy and the cooling mechanism; jumps in temperature are connected with the transition from cooling due to radiation of hydrogen to cooling by radiation of helium, which is effectively smaller for other temperatures.

Schklovsky and Kononovich [17], seeking to tie in the data of optical observations with observations in radio bands, given in [18], showed that the values for temperatures must be decreased to values of 6000° and 12000° accordingly. We considered the possibility of formation of regions with different temperatures due to accoustical waves for an atmosphere of pure hydrogen.

The work was based on the following ideas. Beneath the photosphere, in the convective zone of the sun, part of the energy of the directed motion of convective elements is converted into energy of non-directed turbulent motion. In a turbulently moving medium chaotic scattered condensation and rarefaction continuously occur. The rise of the latter is accompanied by the formation of compressional waves, which, because of the chaotic character of this same turbulent motion, bear an irregular character, generating accoustical noise. These waves pass through the photosphere, virtually not being absorbed in it. The velocity of magneto-hydrodynamic waves in regions of formations of compressional waves, apparently, is considerably less than the speed of sound, so that the ejection of energy from turbulent zones, connected with convection, must occur on the whole because of sound waves. As waves spreading upwards become shock waves, gaps arise in them [21]. The velocity of magneto-hydrodynamic waves increases with height far faster than the velocity of sound waves, and at a certain height becomes equal to it. In the chromosphere the absorption of waves increases; the part of the flux of energy transferable by waves is converted into heat. One of the problems of the present work is to clear up by what method the greatest quantities of heat are discharged; by dissipation of sound waves because of viscosity, through dissipation combined with non-adiabatic compression of gas by passage of shock waves of weak intensity, or by dissipation of magneto-hydrodynamic waves arising due to diffusion of compressional waves. Discharge of energy due to Joule heat is very slight, I considered the loss of magneto-hydrodynamic waves as due to collisions. This loss was great only where the ionisation of gas was very slight.

The heating of the chromosphere, connected with passage of waves, competes with its cooling by radiation. The problem consists of clearing up whether by dissipation of waves in the chromosphere such quantities of energy can be discharged sufficient to cover the expenditure of energy in radiation. It is supposed that the flux of energy is continuous in time, and
At every moment of time the radiation is equal. For the answer to this question it is necessary, first of all, to ascertain what radiation causes cooling in one or another layer of the chromosphere. On the whole the calculations of the present work are concerned with the lower chromosphere.

Every section of the chromosphere is in a field of radiation extending from above and from below. For those sections for which the optical thickness is very great, the quantity of energy absorbed in a certain volume must be equal to the quantity of radiated energy. For example, the radiation in lines of the Lyman series in this sense cannot originate from the lower chromosphere, since the absorption coefficient for these lines is on the order of $10^{-15}$ cm$^2$, and even at heights where the concentration of non-ionised atoms of hydrogen is $10^{10}$ cm$^{-3}$ the optical thickness reaches a total of unity in less than 1 km. Leaving the chromosphere the flux of Lyman-$a$ radiation cools higher layers of the chromosphere, on the whole. In the lower chromosphere the cooling agent can only be Balmer radiation, since, starting with some not very great height, the chromosphere is practically transparent. We show, for example, that the chromosphere is transparent in the center of line Ha up to the height of 1000 km. For this it is sufficient to avail oneself of the formula (usual notation):

$$S_{\nu 0} = \frac{\nu^2}{m_e c^2} \frac{\lambda^2}{\Delta \lambda_D} f_k \Delta \lambda_D = \lambda \sqrt{\frac{2kT}{m}}$$

and

$$\tau = S_{\nu 0} \int N_2 dl,$$

Indeed, we assume that the temperature in lower layers of the chromosphere is roughly equal to 6000°, and that the quantity of atoms of hydrogen at the second level can be calculated by Boltzmann's formula (the optical thickness in Lyman-$a$ is very great - hence such an assumption seems quite acceptable). Then $N_2 = 3 \cdot 10^{-9} N_0$ and $S_{\nu 0}(\text{Ha})_{6000°} = 6 \cdot 10^{-13}$. According to data on density of the chromosphere, reduced in [25], we can estimate that a column of section 1 cm$^2$ above the level of 1000 km in the chromosphere contains less than $4 \cdot 10^{20}$ hydrogen atoms, so that

$$\tau_{1000 \text{ km}}(\text{Ha}) = 3 \cdot 10^{-7} \cdot 6 \cdot 10^{-13} \int_{1000 \text{ km}} N_0 dl < 1.8 \cdot 10^{-21} \cdot 4 \cdot 10^{20} < 1,$$

from which it follows that above 1000 km the chromosphere can be considered as transparent for radiation in Ha.

We have assumed that radiation in Ha gives a sufficiently good notion on the cooling of the layers considered in the chromosphere. This is, to be sure, a first approximation, but the data of the investigation permits us to introduce corrections to it, where required. **

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* One must keep in mind that only true absorption of Ha-quanta hinders the cooling of the chromosphere.

** Radiation of the chromosphere in line Ca II - the strongest metallic line - has the same order of values as radiation in Ha (see for example 27, 33 ).
For the construction of a general model of the chromosphere there is no immediate necessity to work from precise values of density at given heights. It is important to have the correct relationship of the extent of ionization, kinetic temperature, etc. to density, and (to have) the approximate relationship of density to height. Only a later theory of the chromosphere will explain the value of the density gradient and its change in relation to height and time. Hence, later on we will study the relationship of various processes to density and not to height in the chromosphere.

By simple reasoning it is possible to show that, for the calculation of ionization and density of population of lower levels of hydrogen atoms, we can use the formula of Saha and Boltzmann. The fact is that according to nearly all models, the temperature in the lower chromosphere is on the order of 6000°, the bordering temperatures on the sun are ~4500°; and since, for ionizing radiation (continuum beyond the limits of the Lyman series) and for the first lines in the Lyman series giving rise to the excitation of atoms, the chromosphere possesses great optical thickness, that field of radiation is near to the value corresponding to the intensity of radiation of a black body with temperatures of 5000° - 6000°. Under these conditions ionization by electrical shock plays an insignificant part.

Further on we will need the value of the extent of ionization of hydrogen and its radiation in Ha. According to Saha's formula the value will be calculated

\[ y = \frac{n_p}{n_0} \quad \text{and} \quad x = \frac{n_p}{n_0 n_0} \]

\( n_p \) - number of protons, and \( n_0 \) - number of neutral hydrogen atoms in 1 cm\(^3\) for various concentrations from \( 5 \times 10^{13} \) to \( 5 \times 10^8 \) atoms/cm\(^3\) and for temperatures from 4000 to 7000°. It was assumed that at these heights \( n_0 = n_p \).

The results of the calculations are shown in Figure 1. According to the data radiation in Ha per 1 gr hydrogen was calculated:

\[ E = A_{nn'} n^2 n' \frac{1}{n_m} n \left( \frac{n}{(y+1)} \right) e^{-\frac{X n n'}{k T}} \]

which for convenience was presented in the form

\[ E = \frac{G}{y+1} e^{-B} \]

It was assumed that the number of atoms in \( s \) through \( m \) levels could be computed by Boltzmann's formula. The necessary constants are taken from [20]. The calculation was carried out for the same values of density and temperature as above. The results are shown in Table 1.

In order to compare what waves give greater output of energy by dissipation in the chromosphere we calculated and compared the coefficients of dissipation for magneto-hydrodynamic waves and for weak shock waves, and we calculated also the damping coefficient for compressional waves due to viscosity. The flux of energy in waves in [5] was assumed to equal

\[ \cdots \]
F = 10^4 ergs/cm^2 sec. This same value can be obtained if one proceeds from the data on the thermal radiation of the corona [2] and from the assumption that all of this energy originally is supplied by compressional waves (acoustical noise). Taking into account, however, that the flux of energy, as will be apparent further on, is several times weaker in the chromosphere, we assume the value F = 5 \cdot 10^4 ergs/cm^2 sec. According to independent calculations of Schatzmann [3] and Unno and Kawabata [5] one assumes that wave frequency \( \omega = 0.6 \ sec^{-1} \).

For the damping coefficient of magneto-hydrodynamic waves we used Piddington's [10] formula:

\[
K_{mh} = \frac{\omega^2 \tau_n \tau_i}{2 \nu H S (1 + z)^4},
\]

where \( S = 1 \); \( \omega = 0.6 \ sec^{-1} \); \( z = \frac{n_o}{n_p} \); \( V_H = \frac{H}{\sqrt{4\pi \rho}} \);

\[
\tau_n = \tau_i, \quad \tau_i = \frac{3}{4} \frac{1}{n_o V}, \quad \sigma = 2.3 \cdot 10^{16} \ cm^2,
\]

\( \tau = \sqrt{3} \ c, \ c = \sqrt{\frac{kT}{m_H}} \) (c - speed of sound, because of the great role of radiation, speed of sound is supposedly isothermic). \( \rho = N m_H \), and \( \mu = \frac{1}{1 + x} \).

In the usual designation \( H \) is taken as \( \sim 20 \) grams. Finally, taking into consideration (1) we have

\[
K_{mh} = \frac{1 - x}{(1 + x)^4} \frac{D}{x \xi \rho T^4} : D = 1.3 \cdot 10^{-14},
\]

\[ E, \ ergs/gr \ sec \]

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and the dissipation of energy per 1 gr

\[ E_{mh} = \frac{1 - x}{(1 + x)^2} \left( \frac{1}{x} \frac{DF}{\rho T} \right) \]  \hspace{1cm} (6)

where \( F \) is the flux of energy in waves, and \( \rho \) is the density of the substance.

The damping of sound waves due to viscosity may be calculated with the aid of the coefficient \( K_{vis} \) [21]:

\[ K_{vis} \approx \frac{\omega^2}{\rho c^3} \gamma \]  \hspace{1cm} (7)

where \( \gamma \) is the dynamic coefficient of viscosity. Since \( \gamma = \frac{1}{3} \sum n_i \bar{v}_i \lambda_i \), where \( \lambda_i \) is the average length of free path of \( i \)-particles, equal to \( 10^{16} \) for collisions with neutral atoms (\( N \) is the common number of particles in 1 \( \text{cm}^3 \)) and for collision of ions \( \lambda_i = \frac{10^{13}}{N} \), then

\[ \gamma = \frac{1}{\sqrt{3}} c \rho \left( \frac{(1-x)10^{16}}{N} + \frac{x \cdot 10^{13}}{N} \right). \]  \hspace{1cm} (8)

The lowest possible value \( \gamma = \frac{c}{\sqrt{3}} \frac{\rho}{N} 10^{13} \), which corresponds to \( x = 1 \), i.e., complete ionization. In the lower chromosphere, where in (8) we can ignore the second term, we have according to (7):

\[ K_{vis} = \frac{1 - x}{1 + x} \frac{G}{\rho T} \quad G = 5.3 \cdot 10^{-17}. \]  \hspace{1cm} (9)

\[ E_{vis} = \frac{1 - x}{1 + x} \frac{GF}{\rho Z T}. \]  \hspace{1cm} (10)

The expression for the damping coefficient for faint shock waves is easily obtained from the formula for changes in entropy at the front of weak shock waves [30]. Changes in entropy by passage of a front of a shock wave, calculated for 1 gr of matter works out as:

\[ \Delta S = \frac{1}{12} \left( \frac{\partial \Delta V}{\partial V} \right) (P_1 - P_2)^3. \]  \hspace{1cm} (11)

Together with the adiabatic curve of Poisson \( pV = \text{const} \), \( V_1 = \frac{1}{12} \gamma^2 \), this gives, for a weak shock wave, a heating by the passage of the front of

\[ h = T \Delta S = \frac{1}{12} \frac{v + 1}{v^2} \frac{V \Delta p^3}{p^2} \rho = \frac{1}{12} \frac{(v + 1) \gamma p \Delta V^3}{V^2}. \]  \hspace{1cm} (12)

In a 1 cm path of the wave \( dW = \rho h \), where \( W \) is the entire energy in the shock wave. The coefficient of absorption is the value \( K \) from the ratio

\[ K = - \frac{dW}{W} = - \frac{\rho h}{W} = - \frac{dF}{F}. \]  \hspace{1cm} (13)
We assume that
\[ W_0 = \rho_0 u^2 U t_0 = F_0 t_0. \] (14)

where \( U \) is the velocity of the front, \( u \) is the velocity of the substance behind the front, and \( t_0 \) is the time interval characterizing the steepness of the front (Fig. 6).

Considering that \( t_0 \) characterizes the quasi-period of the wave, \( U t_0 = \lambda \).

Taking into account that \( \frac{\partial \rho}{\partial t} = \frac{c^2}{V} \) where \( c \) is the speed of sound, we have
\[ h = \frac{y + 1}{12} \frac{c^2 \Delta V^3}{V^2}. \] (15)

From the term \( \rho (U - u) = \rho_0 U \) we compute \( \Delta V/V \) and finally, substituting (13) and (14) in (13) we obtain \( K_{sh} = \frac{y + 1}{12} \frac{u}{c^2 t_0} \), which for \( t_0 = 10 \) and \( y = 5/3 \) gives the expression
\[ K_{sh} = \frac{1}{45} \frac{u}{c^2} = \frac{1}{45} \frac{\rho_0 F h}{c^2 \rho_0 h}. \] (16)

where \( u \) is the velocity of the movement of matter in wave \( N \). An analogous expression is used in (3). We can write
\[ K_{sh} = \frac{1}{(1 + x)} \frac{M F h^2}{\rho_0 h} = \frac{M F h^2}{\rho_0 h}. \] (17)

whence
\[ E_{sh} = \frac{1}{1 + x} \frac{M F h^2}{\rho_0 h} \]

(18)

The flux of energy with height changes when
\[ F = \left( \sqrt{F_0 - \frac{M h^2}{2}} \right)^2 \] (19)

Values \( E_{mh} \), \( E_{vis} \) and \( E_{sh} \) can be represented conveniently for the calculation in the form \( (F = 5 \cdot 10^8 \text{ ergs/cm}^2 \text{ sec}) \):
\[ E_{mh} = \frac{1 - x}{x} \frac{2.82 \cdot 10^{26}}{\frac{T h}{N h} \left( \frac{1}{1 + x} \right)^h} ; \] (20)
\[ E_{vis} = \frac{1 - x}{1 + x} \frac{9.4 \cdot 10^{35}}{T N^{2h}} ; \] (21)
\[ E_{sh} = \frac{5.5 \cdot 10^{31}}{N h^{\frac{2}{3}}} \left( \frac{1}{1 + x} \right)^h. \] (22)

Values of \( E \), computed by this formula, for various densities and temperatures are given in Table 2.

For comparison of the importance of various mechanisms of dissipation
of energy. Coefficients $K_{mh}$, $K_{vis}$ and $K_{sh}$ were computed on the supposition that $F = 10^4 \text{ ergs/cm}^2\text{sec}$ (Fig. 2), for various values of $N$, temperature $T = 6000^\circ$ and ionization as determined according to Figure 1. In Figure 2 it is apparent that dissipation of magneto-hydrodynamic waves can play an appreciable role only in the very bottom of the chromosphere, but that due to great density, the discharge of energy per $1 \text{ gr}$, and consequently the heating, is very small. We have no basis to assume that hydrogen remains almost totally non-ionized up to a considerable height. In addition, one must keep in mind that, as B. E. Stepanov has pointed out, the value $K_{mh}$ obtained according to Piddington's formula is much higher. If, aside from the value $K$, the exit of energy depends further on the correlation of the velocity of shock and magneto-hydrodynamic waves, then the discharge of energy in the lower chromosphere $E_{mh}$ stays less than $E_{sh}$ since $V_H/C$ is small.

In the chromosphere $K_{sh}$ and $K_{vis}$ are on the order of $10^{-6} \text{ cm}^{-1}$; this means that throughout the chromosphere the flux of energy changes in all by several times, i.e., formulas (6), (10) and (18) can be used.

The balance of energy in the chromosphere must be provided for by the equality of energy emitted by dissipation of wave $E$ and the energy carried off by radiation $G$.

### Table 2

<table>
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<th>$E_{vis}$, ergs/gr-sec</th>
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In Figure 3 we have plotted the dependence of $H_\alpha$ on the value $N$ - the concentration of hydrogen atoms and ions in $1 \text{ cm}^3$ for various possible temperature values, and also the dependence of these same parameters for quantities of energy emitted by dissipation of shock waves. Curve E hardly changes with temperature, but for each value of $N$ it intersects with the curve for $\delta$, of some corresponding temperature.

From the drawing Figure 3 one can determine at what temperature at a given height (for a given density) it is possible to assume a balance of energy. From this drawing it is apparent that, starting with some height, whatever the value of temperature of the hydrogen radiation, no condition provides a balance between emitted energy and energy carried off. According to [16] this indicates that, for a given flux of energy, the radiation of hydrogen cannot provide a balance, and that here a leap in temperature must occur, to that value at which some other mechanism of cooling, - for example, the radiation of ionized helium (temperature on the order of 12000 - 19000$^\circ$K) - enters into operation. For a nearly invariable flux of energy, starting at some height, the temperature in the chromosphere must increase sharply. In other words, this can be expressed: above some level in the chromosphere hot regions must predominate; below some level, cold regions must predominate.

It is necessary to note that in hot regions the optical thickness of lines of the Lyman series remains great to a considerable height [23]. With the aid of drawing Figure 3 we can determine the dependence between the concentration of hydrogen atoms and temperature for lower portions of the chromosphere. This dependence is given in Figure 4. The data of Figure 4 can be used (with the aid of Figure 1) in order to find the dependence of the degree of ionization of hydrogen $x$ on the concentration. This dependence is given in Figure 1 - thick line. It is apparent that the path of ionization with density is rather abrupt. The results obtained agree well with the results of Wooley and Allen [24] who also obtained an abrupt path of ionization, although half ionization was obtained for the same value and same temperature as ours.

We can make an attempt to explain the existence of hot regions below the level where this transition takes place by the natural decrease of density with height. The superimposing of waves, the fluctuation of flux of energy can lead to an increase in the flux of energy in some regions. Assuming that at a given level the gas pressure is a fixed value, we must assume that value $N$ will change inversely proportionally to $T$, since $p = (1+x)NkT$. Hence according to formula (22) the discharge of energy must be proportional to $T^{1/4}$. For each value of density there is a value of temperature close to which the slope of the curves for $\delta$ becomes less than 1/4. This means that, for this density, by a sudden increase in temperature a stated critical value is reached, then the increase of temperature will continue until a new mechanism of cooling, providing sufficiently effective rejection of heat, will be effective. In other words, for a given density, as soon as that value of temperature is reached where radiation depends on temperature to an extent less than 1/4, a jump in temperature must occur at a level lower than that.
requiring a decrease of density with height for a similar case. Sudden
increases in the flux of energy in any region can be very important. The
decrease of the flux leads to the reverse formation of cold regions. Of
course, a change in flux or a sudden change in temperature can result in the
formation of hot regions only in the cases where these changes are sizable
enough so that they cannot be compensated for by the change in hydrogen
radiation. Thus, the same hot regions prove to be dynamic formations,
the lifetime of which is not very great.

The thinning of the observed chromosphere above spots, mentioned by
Mistel [11] and Krat [31] can be explained also. Really, if, in regions of
magnetic fields, total pressure equal to the sum of magnetic $H^2/8\pi$ and
gaseous (flux) $NkT$ pressure must be equal to gaseous pressure from the
surface, and the temperature in the region of the magnetic field (for example
above a spot) and outside of it are the same, then equilibrium is possible
only when the density in the regions with magnetic fields is less than in
adjacent regions. And the decrease of density, as was shown above, leads
to an increase in the output of energy and consequently to the formation of
hot regions at comparatively lower heights. Consequently we propose to
calculate the possible contrast between hot and cold regions which must be
observed in Hα and compare this with the observed contrast between dark
and bright details of the chromospheric layers of unperturbed regions on
the solar disk. The question of reflection of acoustic waves in the chromo-
sphere is studied in [3]. One can consider that acoustic waves experience
reflection only at the edges of a region of a jump in temperature. In the case
considered, the flux changes only by a factor of two, not changing on the
order of magnitude, since according to Schatzmann the flux of energy changes
inversely proportionally to the square of the speed of sound, i.e., propor-
tionally to the ratio of temperature. For magneto-hydrodynamic waves the
reflection in the chromosphere is insignificant [10].

In the same lower portions of the chromosphere the temperature,
apparently, is less than 5000°. The energy radiated in these regions can
be discharged by the already-mentioned dissipation of magneto-hydrodynamic
waves, and is yielded because of heat conductivity. Continuous emission
emitted from these layers arises chiefly because of the formation of
negative hydrogen ions. It is necessary to keep in mind that part of the
radiation in the continuous spectrum can be explained simply by scattering
of photospheric radiation, i.e., not all the radiation emitted from these
layers results in cooling them.

For upper, hot, and completely ionized layers of the chromosphere
the comparison of $E_{sh}$ with the calculation for radiation of ionized helium,
made in [16] shows that because of compression waves a sufficient quantity
of energy is emitted.

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*The formation of hot or cold regions can be connected with the fluctuation
of density at a given height.
One can attempt to estimate roughly at what height break-off points appear in sound waves, and at what height these break-off points become so great that it is impossible to consider them faint.

A profile of a sound wave is shown on the upper portion of Figure 7. Point B moves in a stationary system of coordinates with the speed of sound wave c, and point A moves with speed \( u + c \). The velocity of point A relative to point B is equal to \( u \). Roughly, we can consider that the break-off point occurs when the situation is fulfilled that is represented in the lower portion of the figure, i.e., when A advances relative to point B by a distance \( \lambda/4 \). At this time, equal to say, \( t \), the wave advances a distance \( L \). Thus

\[
t = \frac{L}{c} = \frac{\lambda/4}{u}; \quad \lambda = ct_0; \quad u = \sqrt{\frac{p}{\rho c}}.
\]

Consequently,

\[
L = t_0 c \frac{\rho k}{4 F} ^{\frac{1}{2}} = 2.5 \frac{k \rho T \theta}{m_H \rho k} N \frac{\lambda}{4}.
\]

Assuming \( T = 5 \cdot 10^3 \) and \( F = 10^4 \) we obtain \( L = 5 N \frac{\lambda}{4} \). Thus, in the same lower portion of the chromosphere where \( N = 10^{14} \), \( L = 5 \cdot 10^7 \), i.e., for the whole length of a 500 km path the sound wave crosses the shock wave.

The formula for shock waves of weak intensity cannot be used if \( u > c \). If \( \frac{u}{c} \) becomes several times greater than unity, the shock wave must be considered as strong; it, as is known, fades more sharply than a shock wave of weak intensity.

The value \( u = c \) is attained for density \( \rho \), determined from the ratio

\[
\frac{u}{c} = \left( \frac{F}{\rho c^3} \right)^{\frac{1}{2}} = 1, \quad \text{i.e., when}\;
\frac{F}{c^3} \quad \text{and} \quad N = \frac{5 \cdot 10^4 m}{k \theta T} \frac{k}{10^3}.
\]

For higher layers of the chromosphere \( T = 2 \cdot 10^4 \) so that the break-off point becomes large at \( N = 10^{10} \), i.e., the above leap in temperature.

Since at heights where \( N = 10^{10} \) the density in the chromosphere changes very slowly with height, then \( u \) must be several times greater than \( c \) at rather great heights. Hence the examination carried out in the present work can be shown to be correct for great heights. It is possible that magnetic fields also retard the formation of strong break-off points.

We investigate the question of the dependence of density on height. For layers with constant temperature gradients the density must be, in the case of \( T = 5000^\circ \) and a non-ionized medium (lower chromosphere),

\[
\rho_1 = \frac{L_1}{RT} = \frac{L_1}{RT(1+\alpha)} \approx 6.56 \cdot 10^{-8}, \quad (23)
\]
and for the case $T = 19000^\circ$ and full ionization $\beta_2 = 0.87 \cdot 10^{-8}$ cm$^{-1}$.

In connection with the fact that with the increase of height the relative quantity of hot regions is increased, one must note the smooth transition of gradient values of density $\beta$ from large values to smaller ones. In Figure 5 is shown the observed trend of densities according to the data of Van de Hulst [25] drawn through the gradients of $6.56 \cdot 10^{-8}$ and $0.57 \cdot 10^{-8}$. For cold regions of the chromosphere the theoretical dependence of density on height (dotted line on Figure 5) is constructed with the aid of Figure 4 and numerical integration of the equation of hydrostatic equilibrium:

$$h_2 - h_1 = \frac{R}{g} (1 + x) \int_{N_1}^{N_2} \frac{T(N)}{N} dN. \quad (24)$$

From the initial point, the value $N = 10^{13.4}$ at height $h = 1000$ km is assumed [25]. As should have been expected, this dependence falls more steeply than the observed curve, which fact is connected, as shown above, with the increase of relative quantity of hot regions with height.

It is necessary to take into account a possible supporting of the chromosphere conditioned by sound waves.

In this case the equation of hydrostatic equilibrium has the form

$$dp + d(\rho u^2) = \rho g dh, \quad (25)$$

where $u = \left( \frac{F}{\rho c} \right)^{1/2}$ is the speed of a substance in a sound wave. Spectroscopically this value is the same as the turbulent velocity; giving rise to broadening of lines. The question of turbulence in the chromosphere has been investigated by A. B. Saverny [28] and, with regard to sound waves, by Unno and Kawabata [5]. One should note only that the turbulent velocity observed by us spectroscopically is not equal to $u$ (if $u$ is the speed of a substance immediately in front of the wave), but is the averaged value, averaged according to time and direction, i.e., it must be several times less than $u$ [3]. The value $u$ increases sharply with height because of a decrease in density $\rho$ and reaches $10^6$ cm/sec.

The second term of the right-hand part of equation (25) can be written as

$$d\left( \frac{F}{c^2} \right) = \frac{c dF - F dc}{c^2} \quad (26)$$

Then

$$\frac{dp}{dh} + \frac{1}{c} \frac{dF}{dh} \frac{F}{c^2} \frac{dc}{dh} = \rho g. \quad (27)$$

We equate the separate terms of the left-hand part of this equation:
\[
\frac{F}{c^2} \frac{dc}{dh} \approx 5 \cdot 10^4 \frac{10^4}{10^8} \sim 5 \cdot 10^{-12}, \quad \frac{1}{c} \frac{dF}{dh} = \frac{K}{c} = \frac{10^{-9}}{10^6} \sim 10^{-14},
\]

and \[\frac{dp}{dh} = \frac{2kTdT}{dh} = 2kTN\beta,\]
i.e., at any place in the lower chromosphere \(dp/dh > 10^{10}\). Consequently, the damping of sound waves in the lower chromosphere does not give rise to appreciable change of density gradients. In the upper portion of the chromosphere the supporting by sound waves is important, apparently, especially in the regions of jumps in temperature (term \(-dc/dh\)).

The question of the spreading of shock waves in the presence of magnetic fields, and of their dissipation in the presence of magnetic fields must be considered separately. Estimates made show, however, that the presence of magnetic fields in non-perturbed chromosphere does not change in an essential way the results obtained in the given work.

Some corroboration of the assumption made in the present work can be obtained if we compare the observed output of energy in Ha [27, 29] with that which must be observed according to our calculations. For example, we obtained, at a height of 1500 km, the value \(T \approx 5200^\circ\), and emission in Ha, by calculation for 1 gr of substance, is (Table 1):

\[
\xi = 2 \cdot 10^9 \text{ ergs/gr sec. (28)}
\]

Absolute photometry of chromospheric lines, done by Vyasanitsin [27, 29] gives, for full emission in Ha at height \(h = 1500\) km, the values \(lg \xi(h) = 15.5\) and gradient \(\beta \cdot 10^{-8}\).

Solving Abell's equation [26] we can obtain emission of 1 cm\(^3\) at a height 1500 km as:

\[
j(h) = \frac{E(h) \beta^3}{\sqrt{2\pi R_0}} \approx 5 \cdot 10^{-3} \text{ ergs/cm}^3\text{sec},
\]

which, in conversion to 1 gr (the concentration of hydrogen atoms at this height is \(N = 12.0\) given according to Figure 5), gives

\[
E \approx 3 \cdot 10^9 \text{ ergs/gr sec. (29)}
\]

Value (29) is found to be in quite good agreement with value (28).

Summarizing all we have said before, and omitting details, we can draw up the following schematic presentation of the structure of the chromosphere and of the causes of its formation. We visualise the sun, possessing neither convective zones, nor chromosphere, nor corona. If, at some moment, convection begins, then inevitably the chromosphere and the corona must emerge, which is precisely what is, indeed, observed. Indeed, in the upper portions of the convection zones, as a result of dispersion of convective
cells, turbulent motion begins, which in its turn gives rise to the formation of compressions and rarefactions, and consequently to generation of sound waves. In the photospheric layers these waves do not experience dissipation, and are spread upwards. Upon rising, the waves are changed very quickly into weak shock waves. As soon as the density becomes sufficiently small the waves dissipate and the energy discharged provides an increase in temperature that, in its turn, decreases the density gradient. The warming-up and decrease of density gradient will proceed until an instability occurs in the temperature and density at which is achieved a balance between the energy discharged and the energy carried off. Such a balance is provided at various heights by various emission mechanisms. In the lower layers there is the radiation of hydrogen; in the upper layers there is the radiation of ionized helium; and finally, in the corona there are the radiation of highly-ionized atoms of various metals, the re-combinative radiation of hydrogen and ionized helium, and "evaporation" of particles from the corona[32]. The greatest discharge of energy takes place in the corona, where, in connection with the decrease in density and rise in temperature, the free path of particles becomes equal to the length of the sound wave (the distance between fronts for shock waves). At this height the compression waves dissipate completely, and in the higher layers the temperature of the corona is maintained only because of heat conductivity. Thus the corona is heated not by the chromosphere, as some authors consider, but directly.* Furthermore, a part of the energy discharged in the corona passes into the upper chromosphere.

If the idea stated here is correct then all of the structure of the chromosphere and corona is conditioned, on the whole, by the value of only one parameter - the value of the flux of energy carried by sound (shock) waves. The details of the structure depend, of course, on the frequency of waves, on the value of the intensity of waves, etc. The value of flux of energy, F, is used here, and values obtained for the chromosphere are, of course, only tentative. Available data on the mechanisms of cooling of the chromosphere, of the coefficients of absorption, of the flux of energy, etc., can be used for the creation of a general schematic representation of the chromosphere, but a model of the chromosphere can be constructed only on the basis of direct observations.

In conclusion I express my thanks to corresponding member of the Academy of Science, USSR, A. B. Severny for his useful discussions of the manuscript.

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Bibliography

Captions for Figures

Fig. 1. Ionization of hydrogen at various temperatures and concentrations, according to Saha's formula. The dark line shows the assumed path of ionization in the chromosphere.

Fig. 2. Comparison of coefficients of dissipation.

Fig. 3. Comparison of energy carried off by emission of hydrogen in Hα (curve C) and dissipation of energy by shock waves (curve E).

Fig. 4. Relation of temperature and concentration of hydrogen in cold regions of the lower chromosphere.

Fig. 5. Dependence of density on height in the chromosphere. Continuous line - data of Van de Hulst [25], dotted line - calculations of the present work for cold regions of the chromosphere. Drawn straight to gradients corresponding to $\beta = 6.56$ and $0.57$.

Fig. 6. Form of shock wave. $t_0$ - time interval between passage of fronts.

Fig. 7. Propagation of sound waves in shock waves.