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The selection of varactor diodes for a 2x multiplier

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This report presents a simplified procedure for the selection of varactor diodes in frequency multiplier design. Graphs are presented that enable the circuit designer to select the proper diode to fulfill the design objective.
THE SELECTION OF VARACTOR DIODES FOR A 2X MULTIPLIER

INTRODUCTION

The manufacturers of varactor diodes thus far have not followed any consistent standards for the classification of its electrical characteristics. Some manufacturers specify data when the diode is biased at voltage breakdown, some at 4 volts and still more at 6 volts. With this maze of information it is difficult to resolve the data into parameters that determine the input power and efficiency at a given operating frequency. It is certainly necessary to know this information when selecting a varactor for a particular application.

It is hoped that this report will make it easier for a circuit designer to select a varactor from some of the presently available commercial data which would satisfy requirements of input power and efficiency. Some of the diodes we have used are graphically presented with coordinates of input power vs. frequency. If desired, other diodes may also be added by the user to keep the graphs up to date. In all cases, presently available diodes with highest optimum input power and efficiency have been considered.

This report is separated into three sections. Section one contains the equations for graded junction (1/3 law) and abrupt Junction (1/2 law) varactor doublers. The second section explains the method of measuring varactors and includes an impedance plot of a diode taken at random. The information is extracted from the impedance plot and transferred into desired parameters. The third section contains graphs of some available diodes. Different commercially available diode types are plotted with efficiencies placed along "the diode line". Also included is an example indicating how an individual point on a "diode line" is obtained for each manufacturer's diode.
I. VARACTOR EQUATIONS

It is not the intention here to derive equations or give background material. For this material reference is made to a recently published book entitled "Varactor Applications", authored by R. Rafuse and Paul Penfield, Jr. This reference book includes work by Bliss L. Diamond and Marshall Greenspan and has derivations which are extremely helpful when designing frequency multiplier circuits.

In determining the varactor law the model of a series resistor and capacity is transformed into a shunt elastance (S) and conductance (G). This transformation into parallel components aids in the derivations. The elastance (S) is related to capacity by the equation:

\[ S = \frac{1}{C} = \frac{dv}{dq} \]  
Eq. 1

For the case of an abrupt junction diode the following equation is used:

\[ S = S_{\text{max}} \left( \frac{V_o + \theta}{V_b + \theta} \right)^{1/2} \quad \text{or} \quad C_j = C_{\text{min}} \left( \frac{V_b + \theta}{V_o + \theta} \right)^{1/2} \]  
Eq. 2

The same units are used to characterize both graded and abrupt junction diodes.

- \( C_j \) = junction capacity at \( V_o \).
- \( V_o \) = voltage across diode junction.
- \( V_b \) = voltage breakdown.
- \( \theta \) = contact potential of diode, which varies depending upon the materials used in the junction.

For the case of a graded junction diode the following equation is used,

\[ S = S_{\text{max}} \left( \frac{V_o + \theta}{V_b + \theta} \right)^{1/3} \quad \text{or} \quad C_j = C_{\text{min}} \left( \frac{V_b + \theta}{V_o + \theta} \right)^{1/3} \]  
Eq. 3

The graphs in this report are all plotted using either a 1/3 law in the case of Microwave Associates, Microstate, and Sylvania diodes or a 1/2 law.
for Pacific Semiconductors, Inc. diodes. Most of the manufacturers will concede that the diodes do not have uniform laws either from one to another or from one voltage point to another so "diode lines" will not have pinpoint accuracy. Usually if the manufacturer claims a 1/3 law diode the actual law will be closer to 1/3.6. When this occurs the resultant "diode line" will be shifted downward toward high power and less efficiency.

When computing input power and efficiency for these diodes the following equations are used:

1. The equation for input power in the abrupt junction case as given by R. P. Rafuse in his Sc. D thesis "Parametric Applications of Semiconductor Capacitor Diodes" and B. L. Diamond in his M. S. thesis "Idler Circuits in Varactor Frequency Multipliers" is as follows:

\[ P_{\text{input}} = 8 m_1^2 \left( \frac{F_0}{F_c} \right)^2 X_2 P_{\text{norm}} \]  

Eq. 4

where \( m_1 = \) modulation ratio (when fully pumped = .2)

and

\[ \frac{F_0}{F_c} = \text{input frequency} \]

\[ \text{cutoff frequency} \]

and

\[ X_2 = \text{real part of the input impedance} \]

and

\[ P_{\text{norm}} = \frac{V_0^2}{R_s} \]  

(P_{\text{norm}} determines the power handling capacity of the diode)

A. \[ P_{\text{input}} = 8 m_1^2 \left( \frac{F_0}{F_c} \right)^2 X_2 P_{\text{norm}} \]  

Eq. 5

where

\[ m_1 = 4 \times 10^{-2} \]

and

\[ F_0 = \text{input frequency} \]
\[ F_c = \frac{\left( \frac{1}{c_{\min}} - \frac{1}{c_{\max}} \right)}{2\pi R_s} \quad \text{or} \quad \frac{S_{\max} - S_{\min}}{2\pi R_s} \quad \text{Eq. 6} \]

\[ R_s = \text{series resistance} \]

\[ X_2 = \sqrt{1 + \frac{1}{k} \left( \frac{F_c m_l}{F_o} \right)^2} \quad \text{Eq. 7} \]

\[ P_{\text{norm}} = \frac{(V_b + \theta)^2}{R_s} \quad \text{Eq. 8} \]

\[ V_b = \text{breakdown voltage} \]

\[ \theta = \text{diode contact potential} \]

B. Efficiency (\(\epsilon\)) is equal to

\[ \epsilon = \left( \frac{X_2 - 1}{X_2 + 1} \right) \]

or as stated in "Varactor Applications"

\[ \epsilon = e^{-9.95 \left( \frac{F_{\text{out}}}{F_c} \right)} \quad \text{Eq. 9} \]

2. For all graded junction varactor doublers an approximation of input power is used.

A. \[ P_{\text{input}} = 1.18 \times 10^{-2} P_{\text{norm}} \left( \frac{F_o}{F_c} \right) \quad \text{Eq. 10} \]

where

\[ P_{\text{norm}} = \frac{(V_b + \theta)^2}{R_s} \quad \text{Eq. 11} \]

\[ F_o = \text{Input frequency} \]

\[ F_c = \frac{S_{\max} - S_{\min}}{2\pi R_s} \quad \text{Eq. 12} \]

\[ R_s = \text{Series Resistance} \]

\[ V_b = \text{Voltage Breakdown} \]
B. Efficiency ($\epsilon$) as stated in "Varactor Applications" is:

$$\epsilon = e^{-13 \left( \frac{P_{\text{out}}}{P_c} \right)}$$

Eq. 13

II. THE MEASUREMENT OF DIODES

For "pill" varactor measurements the holder was a converted M x 59B triplate module. When measuring "pigtails" diodes the leads were soldered directly across a General Radio to type N connector.

There are two types of measurements that may be made on a varactor. One measurement may be made when the diode is actively functioning as a multiplier, divider, etc. This measurement consists of an input impedance measurement of the diode. Unfortunately the impedance in this case is not just the varactor but also includes the effects of all the associated circuitry.

The second measurement is made using a small signal to find the passive characteristics of the varactor. If the measured signal is small (Fig. 1) compared to the total voltage swing of the varactor the measured impedance may be considered to be at the bias voltage. The bias may be varied to measure impedances associated with other voltages. A plot of impedance vs. voltage may be made to determine the diodes passive characteristics. This is the measurement that is used in this report (Fig. 2).

The available power of the measuring signal incident to the diode was $10^{-5}$ watts. Considering a standing wave occurring in the slotted line of 100 to 1 when the line is open circuited the peak to peak voltage existing at the high impedance point is:

$$\text{Signal Voltage} \quad (V_s) = 2\sqrt{2PR}$$

Eq. 14

where  

- $P$ = power input of $10^{-5}$ watts  
- $R = \text{VSWR} \times \text{line impedance} = 5 \times 10^3$  
- $V_s = .628$ volts (peak to peak)
This value of measuring voltage will suffice for these measurements except when the varactor is biased at a value near the contact potential. When this occurs the validity of the measurement is in doubt.

An example will be given to show how input power and efficiency may be extracted from the measured data.

The varactor selected is a Pacific Semiconductor diode type PC-141. Figure 3 shows an impedance plot vs. voltage externally applied to the varactor for this high power, low frequency diode. The input measuring frequency used was 400 mc and the case capacity being only a small fraction of the value of junction capacity is therefore neglected. The data from the impedance plot (Fig. 3) is as follows:

| Point | $|X_c|$ | $X_c$ | $C$ | $|R|$ | $R$ | $V_b$ |
|-------|-------|------|-----|------|----|-------|
| 1     | 0.12  | 6Ω   | 65pf| 0.15Ω| 7.5Ω| -130 volts |
| 2     | 5     | 250Ω | 1.6pf|      |    |        |

Point 1 is the $C_{max}$ point. Point 2 is the $C_{min}$ point.

From equation 2 the diode law $\eta = 0.717$

The cutoff frequency $(F_c) = \frac{S_{max} - S_{min}}{2\pi R_s} = 12.9 K \mu$ Eq. 15

The input power required to drive the diode to its maximum elastance value at 400 mc is:

$$P_{input} = 8 m_2 \left(\frac{F_c}{F_c^0}\right)^2 X_2 P_{norm}$$

Eq. 16

where

$$P_{norm} = \left(\frac{V_b}{R_s}\right)^2 = 2.245 \times 10^3$$

Eq. 17

and

$$\left(\frac{F_c}{F_c^0}\right)^2 = 9.48 \times 10^{-4}$$

Eq. 18

and

$$X_2 = 3.38$$

Eq. 19

Therefore $P_{input} = 2.3$ watts.
The efficiency of this diode at 400 mc is, 

\[ \epsilon = e^{-9.95 \left( \frac{F_{\text{out}}}{F_c} \right)} \]  
Eq. 20

where

\[ \frac{F_{\text{out}}}{F_c} = 6.2 \times 10^{-2} \]  
Eq. 21

and

\[ \epsilon = 51\% \]

It might be noted here that when a good diode \((R_s = 0)\) is measured the points would appear at the outer periphery of the Smith chart. Conversely a high resistance has its points close to the center of the chart.

III. CALCULATION OF POINTS ON "THE DIODE LINE"

This section outlines the method involved in translating the data given by manufacturers into points which make up a "diode line" for a times 2 multiplier.

1. Microwave Associates

The following is an explanation of how the diode lines were plotted for Figure 5.

The catalogue indicates the following characteristics:

<table>
<thead>
<tr>
<th>Type</th>
<th>Voltage Breakdown ((V_b))</th>
<th>Junction Capacity at (V_b)</th>
<th>(R_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA 4338A</td>
<td>-120 volts</td>
<td>.1 - .2 pf</td>
<td>2Ω</td>
</tr>
</tbody>
</table>

The junction capacity is then averaged and an expression for cutoff frequency is obtained.

The cutoff frequency as defined in equation 5 is:

\[ F_c = \frac{S_{\text{max}} - S_{\text{min}}}{2\pi R_s} \]  
Eq. 22

where

\[ S_{\text{max}} = \frac{1}{C_{\text{min}}} \]  
Eq. 23
and \[ S_{\text{min}} = \frac{1}{C_{\text{max}}} \]  
\[ F_c = 41.5 \text{ Kmc} \]

Using equation 3 shows \[ C_{\text{max}} = 0.83 \text{ pf} \]

The input power using the low frequency asymptotic formula for a graded junction doubler at an input frequency of 100 mc is:

\[ P_{\text{input}} = 1.18 \times 10^{-2} \left( \frac{F_c}{F_o} \right) P_{\text{norm}} \]  
\[ \text{Eq. 25} \]

where \[ P_{\text{norm}} = \left( \frac{V_b}{R_s} \right)^2 = 6.86 \times 10^2 \]  
\[ \text{Eq. 26} \]

and \[ \frac{F_c}{F_o} = 2.41 \times 10^{-3} \]  
\[ \text{Eq. 27} \]

\[ P_{\text{input}} = 1.95 \times 10^{-2} \text{ watts} \]

This is point 1 of Figure 4. By changing \( F_o \) in the above equation from \( 10^8 \) cps to \( 10^9 \) cps the input power increases to \( 0.195 \) watts. This is point 2 of Figure 4. These two points are connected together to form "the diode line". Also shown on this graph in a solid line is actual input power capabilities of the diode. The difference in the two powers indicates the useful limit of the equations used in this report.

2. Pacific Semiconductors, Inc.

The following is an explanation of how a point on "the diode line" is obtained for Figure 6.

The catalogue indicates the following characteristics:

<table>
<thead>
<tr>
<th>Type</th>
<th>( V_b )</th>
<th>( C_{-4V} )</th>
<th>( Q_{-4} ) at 50 mc</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC-141</td>
<td>-100</td>
<td>6.5 pf</td>
<td>125</td>
</tr>
</tbody>
</table>
The series resistance \( R_s \) = \( \frac{1}{2\pi FC_{-4}Q_{-4}} \) = 3.9\( \Omega \) \hspace{2cm} \text{Eq. 28}

The capacity excursions are:
\[
C_{\text{max}} = C_{-4V} \times (4)^{1/2} = 13 \text{ pf} \hspace{2cm} \text{Eq. 29}
\]
and
\[
C_{\text{min}} = \frac{C_{\text{max}}}{(100)^{1/2}} = 1.3 \text{ pf} \hspace{2cm} \text{Eq. 30}
\]
The cutoff frequency \( F_c \) = \( \frac{S_{\text{max}} - S_{\text{min}}}{2\pi R_s} \) = 32 Kmc \hspace{2cm} \text{Eq. 31}

The input power for an abrupt junction doubler at 1 Kmc input frequency is:
\[
P_{\text{input}} = 8m_1^2 \left( \frac{F}{F_c} \right)^2 X_2 P_{\text{norm}}
\]
\[
P_{\text{input}} = 3.58 \text{ watts} \hspace{2cm} \text{Eq. 32}
\]

3. Microstate

The following is an explanation of how a point on "the diode line" is obtained for Figure 7. The catalogue indicates the following characteristics:

<table>
<thead>
<tr>
<th>Type</th>
<th>Junction capacity at ( V_b )</th>
<th>( V_b )</th>
<th>( F_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS 4540-C</td>
<td>0.2 - 0.4 pf</td>
<td>-120 V</td>
<td>60 Kmc</td>
</tr>
</tbody>
</table>

The capacity at \( V_b \) is averaged to be 0.3 pf
\[
C_{\text{max}} = C_{\text{min}} (120)^{1/2.6} = 1.89 \text{ pf} \hspace{2cm} \text{Eq. 33}
\]
The series resistance \( R_s \) = \( \frac{S_{\text{max}} - S_{\text{min}}}{F_c \cdot 2\pi} \) = 7.5\( \Omega \) \hspace{2cm} \text{Eq. 34}
The input power for a 1/3 law doubler at 1 Kmc input frequency is:
\[
P_{\text{input}} = 1.18 \times 10^{-2} \left( \frac{F}{F_c} \right) P_{\text{norm}}
\]
\[
P_{\text{input}} = 9 \text{ watts} \hspace{2cm} \text{Eq. 35}
\]
where

\[ P_{\text{norm}} = 1.93 \times 10^3 \]  
Eq. 36

and

\[ \frac{F_0}{F_c} = \frac{1}{60} \]  
Eq. 37

\[ P_{\text{input}} = .378 \text{ watts} \]

4. Sylvania Diodes

The following is an explanation of how a point on "the diode line" is obtained for Figure 8.

The catalogue indicates the following characteristics:

<table>
<thead>
<tr>
<th>Type</th>
<th>( V_b )</th>
<th>Cutoff frequency at -6V</th>
<th>Junction capacity ((C_j)) at -6V</th>
</tr>
</thead>
<tbody>
<tr>
<td>D 4253 D</td>
<td>-60</td>
<td>60 Kmc</td>
<td>2 - 4 pf</td>
</tr>
</tbody>
</table>

\[ R_s = \frac{1}{2\pi F_c C_j} = .88 \]  
Eq. 38

\[ C_{\text{max}} = C_{-6V} \left( \frac{1}{6} \right)^{1/3} = 5.46 \text{ pf} \]  
Eq. 39

\[ C_{\text{min}} = \frac{C_{\text{max}}}{(60)^{1/3}} = 1.39 \text{ pf} \]  
Eq. 40

The cutoff frequency \((F_c)\) is:

\[ F_c = 97 \text{ Kmc} \]  
Eq. 41

The input power for a graded junction doubler at 1 Kmc input frequency is:

\[ P_{\text{input}} = 1.18 \times 10^{-2} \left( \frac{F_0}{F_c} \right) P_{\text{norm}} \]  
Eq. 42

when

\[ P_{\text{norm}} = 4.08 \times 10^3 \]  
Eq. 43

and

\[ \frac{F_0}{F_c} = \frac{1}{97} \]  
Eq. 44

\[ P_{\text{input}} = .495 \text{ watts} \]
V. CONCLUSION

The graphs enclosed provide a handy tool for a design engineer to use when selecting a varactor diode for doubler work. They were prepared using existing formulas for optimum conversion efficiency. The accuracy of the diode lines are dependent upon the ability of the manufacturer to control the characteristics of the diodes. An important point is that input powers are computed for the optimum conversion efficiency case, not for the maximum power output case. Many experimenters have reported successfully overdriving the diodes by as much as a factor of four times the optimum input power without appreciable degradation in efficiency. One reason for this is the inaccurate law of the diode, while another reason is that the diffusion capacity effects begin to appear as the input frequency approaches 1 Kmc.

Usually when a diode is purchased the parameters that are received are slightly different than specified in catalogues. When this happens the diode lines may be displaced. If the series resistance is different than specified, there is little change in "diode line", only the efficiency suffers. If the $C_{\text{min}}$ varies from catalogue specifications, the input power varies accordingly.

\[
\text{Actual input power } (P_a) = \frac{P_s C_a}{C_c} \quad \text{Eq. 45}
\]

where

$P_s = \text{Power read from graph.}$

and

$C_c = \text{Capacity listed in catalogue.}$

and

$C_a = \text{Actual capacity of diode.}$

In spite of variations in device characteristics, the design parameters are easily obtained from the graphs.
Bibliography


Figure 1

Varactor Diode Curve Showing Small Measuring Signal
Figure 2
Block Diagram of Varactor Diode Measurements
Figure 3
Impedance Plot of PC-141 Varactor Diode Taken at 500 Mcps
Figure 4

Graph Showing the Usability of the Low Frequency
Asymptotic Formula for Graded Junction X2 Multipliers
Figure 5
Microwave Associates Diodes Input Power vs. Input Frequency When Used in a 2X Multiplier
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