Thermally-Activated Deformation of the Body Centered Cubic Metals at Low Temperatures

5 MARCH 1963

Prepared by H. CONRAD and W. MAYS
Materials Sciences Laboratory

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

LITERATURE DIVISION • AEROSPACE CORPORATION
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THERMALLY-ACTIVATED DEFORMATION OF THE BODY CENTERED CUBIC METALS AT LOW TEMPERATURES

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ABSTRACT

The activation energy $H$, activation volume $v^*$, and frequency factor $v$ for the deformation of Group VA (V, Cb, Ta) and Group VIA (Cr, Mo, W) metals were determined as a function of the thermal component of the stress, $\tau^*$, from data in the literature on the interrelationship of stress, temperature, and strain rate. It was found that $H_0 (\tau^* = 0) \approx 0.1 \mu b^3$, where $\mu$ is the shear modulus and $b$ is the Burgers vector. Also, $v^* \approx 50 b^3$ at $\tau^* = 2 \text{ kg/mm}^2$, increasing rapidly to values in excess of $100 b^3$ at lower stresses and decreasing to $2 - 5 b^3$ at high stresses, and $v = 10^4 - 10^{11} \text{ sec}^{-1}$. The activation energy and volume as a function of stress were independent of structure, indicating that the rate-controlling mechanism for the deformation of body centered cubic metals is overcoming the Peierls-Nabarro force. The experimental data were in agreement with Seeger's model for the thermally-activated nucleation of kinks. The kink energy and the Peierls-Nabarro energy and stress derived from the data were $3 - 4 \times 10^{-2} \mu b^3$, $2 - 4 \times 10^{-3} \mu b^3$ and $0.6 - 1.5 \times 10^{-2} \mu$, respectively.
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I. INTRODUCTION

A distinguishing characteristic of the body centered cubic (b.c.c.) transition metals, compared to the close packed metals (close packed hexagonal, c.p.h., and face centered cubic, f.c.c.), is the much stronger temperature dependence of the yield stress of the b.c.c. metals at low temperatures (below approximately 0.20 $T_m$ - see Fig. 1 and Ref. 1). The ductile-to-brittle transition in the b.c.c. metals is related to this strong temperature dependence. Consequently, it is technologically important and scientifically interesting to identify the rate-controlling deformation mechanism responsible for this strong temperature dependence.

Three dislocation mechanisms have been proposed as rate-controlling during the deformation of the b.c.c. metals at low temperatures and, hence, the mechanism responsible for the strong temperature dependence of the yield and flow stresses. These mechanisms include

a) overcoming the Peierls-Nabarro stress (Refs. 2-5),

b) nonconservative motion of jogs in screw dislocations (Ref. 6),

c) overcoming interstitial atom precipitates (Ref. 7).

Previous research on iron (Refs. 2-5, 8) indicates that the rate-controlling mechanism for the deformation of this metal at low temperatures is overcoming the Peierls-Nabarro stress. Some evidence that this mechanism is also the controlling one in the Group VA (V, Nb, Ta) and VIA (Cr, Mo, W) b.c.c. metals is provided by results presented in a report by the authors (Ref. 9). The results showed that the temperature dependence of the yield and flow stresses of these metals exhibits a behavior similar to that of iron and that the temperature dependence of the yield and flow stresses of all of these b.c.c. metals (with impurity content greater than 0.02 wt %) could be correlated on one plot. In this report data from the literature on
the interrelationship between stress, temperature, and strain rate for the Group VA and Group VIA metals are analyzed to provide additional information on the rate-controlling mechanism in the b.c.c. metals at low temperatures. The approach is similar to that previously employed for iron (Ref. 4), where parameters such as the activation energy, $H$, the activation volume, $v^*$, and the frequency factor, $v$, were determined from available experimental data and compared with predictions of various dislocation models.
II. PROCEDURE

Assuming that yielding and flow are each controlled by a single thermally-activated deformation mechanism, we can write

\[ \dot{\gamma} = \rho b \dot{s} = \rho b s v^* \exp \left[ \frac{H(\tau^*)}{kT} \right] \]  

where \( \dot{\gamma} \) is the shear strain rate, \( \rho \) is the number of dislocations actively participating in the deformation, \( b \) is the Burgers vector, \( \dot{s} \) is the average dislocation velocity, and \( s \) is the product of the number of places where thermal activation can occur per unit length of dislocation and the area swept out per successful thermal fluctuation. Also, \( v^* \) is the frequency of vibration of a dislocation segment, \( H(\tau^*) \) is the activation energy which is a decreasing function of the effective shear stress, \( \tau^* \), given by the difference between the applied shear stress, \( \tau \), and the long range internal stress, \( \tau_L \), which, in turn, is proportional to the shear modulus, \( \mu \) (i.e., \( \tau^* = \tau - \tau_L \)). The symbol \( k \) is the Boltzmann constant and \( T \) is the temperature. The values of \( H \) and \( v^* = -\frac{dH}{d\tau^*} \) can then be obtained from the following relationships (Refs. 4, 8, 10):

\[ H = k \left[ \frac{\partial \ln \dot{\gamma}}{\partial (\tau^*/T)} \right] = kT^2 \left[ \frac{\partial \tau^*}{\partial \tau} \right] \left[ \frac{\partial \ln \dot{\gamma}}{\partial \tau} \right] \]  

\[ v^* = -\frac{dH}{d\tau^*} = kT \left[ \frac{\partial \ln \dot{\gamma}}{\partial \tau} \right] = -\frac{k \ln(\rho b s v^*/\dot{\gamma})}{(\partial \tau^*/\partial \gamma)} \]
The value of \( v = \rho b s v^* \) can be obtained from the relations

\[
H = kT \ln \left( \frac{v}{\gamma} \right)
\]

or

\[
-T \left( \frac{\partial \ln \gamma}{\partial T} \right)_T \left( \frac{\partial T^*}{\partial T} \right)_T = \ln \left( \frac{v}{\gamma} \right)
\]

i.e., from the slope of the plot of \( H \) versus \( T \) or the plot of \( (\partial T^*/\partial T)_\gamma \) versus \( 1/T(\partial \gamma/\partial \ln \gamma)_T \).† For polycrystalline b.c.c. metals a reasonable assumption is that \( \tau = 1/2 \sigma \) and \( \gamma = 0.7 \epsilon \), where \( \sigma \) is the tensile stress and \( \epsilon \) is the tensile strain. Also \( (\partial T^*/\partial T)_\gamma \) is approximated by \( (\partial\tau/\partial T)_\gamma \) because \( d\tau/dT \) is small compared to \( (\partial\tau^*/\partial T)_\gamma \).

The materials and testing conditions considered in this analysis are for the most part given in Table I of Ref. 9. The sources of additional data are listed in Table I of this report.

To determine the effect of stress on \( H \) and \( v^* \), comparisons between the various materials of a given metal must be made at the same value of \( \tau^* \). To obtain \( \tau^* \), the following procedure was employed: the slopes of the curves of \((\tau - \tau_{300})\) versus temperature given in Ref. 9 were determined graphically and their logarithms were plotted versus the temperature. These latter plots were then extrapolated to a value of \( [\partial T/\partial T]_\gamma = 5 \times 10^{-3} \) kg/mm² - °K, where \( 1/T \left[ \partial \gamma/\partial \mu \right]_T \approx \left( 1/\mu \right) \left( d\mu/dT \right) \) and hence \( \tau = \tau_0 \)
(i.e., \( \tau^* \approx 0 \)).

†The values of \( H, v^* \), and the product \( s v^* \) can also be obtained from measurements of the effect of stress and temperature on dislocation velocity by replacing the strain rate \( \dot{\gamma} \) in Eqs. (2-5) with the velocity \( \dot{\mu} \).
Table 1. Materials and Testing Conditions (Additional to Those)

<table>
<thead>
<tr>
<th>Author</th>
<th>Material and Condition</th>
<th>Interstitial Content (wt %)</th>
<th>Grain Size mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>C</td>
<td>O</td>
</tr>
<tr>
<td>Sargent and Johnson, Ref. 18</td>
<td>Columbium</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>E. B. rod</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sherwood, Ref. 19</td>
<td>Tantalum</td>
<td>0.0015</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>E. B. rod</td>
<td>0.0010</td>
<td>0.0100</td>
</tr>
<tr>
<td>Mordike, Ref. 20</td>
<td>E. B. rod</td>
<td>0.0001</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0002</td>
<td>0.0040</td>
</tr>
<tr>
<td>Armstrong, Ref. 21</td>
<td>Molybdenum</td>
<td>0.007</td>
<td>0.002</td>
</tr>
<tr>
<td>Hendrickson, et al., Ref. 22</td>
<td>P. M. rod</td>
<td>0.033</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>A. C. rod</td>
<td>0.006</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>P. M. rod</td>
<td>0.007</td>
<td>0.0023</td>
</tr>
<tr>
<td>Weinstein, et al., Ref. 23</td>
<td>A. C. rod</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>Rose, et al., Ref. 24</td>
<td>Tungsten</td>
<td>0.004</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>E. B. rod</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*E. B. = Electron-beam melted
P. M. = Powder Metallurgy
A. M. = Arc Melted

**T = Tension
T_{td} = Tension - delay time
C = Compression
Tor = Torsion
Table 1. Materials and Testing Conditions (Additional to Those Listed in Reference 9)

<table>
<thead>
<tr>
<th>Material and Testion</th>
<th>Interstitial Content (wt %)</th>
<th>Grain Size mm</th>
<th>Strain Rate sec(^{-1})</th>
<th>Type of Test**</th>
<th>(\tau_{300})† kg/mm(^2)</th>
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</thead>
<tbody>
<tr>
<td>Nbium</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3. rod</td>
<td>-</td>
<td>0.017</td>
<td>10(^{-6}) - 10(^{-2})</td>
<td>T</td>
<td>6</td>
</tr>
<tr>
<td>tantalum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. rod</td>
<td>0.0015</td>
<td>0.0010</td>
<td>0.0015</td>
<td>0.0001</td>
<td>0.5 - 0.8</td>
</tr>
<tr>
<td>3. rod</td>
<td>0.0050</td>
<td>0.0100</td>
<td>0.0200</td>
<td>0.0010</td>
<td>10(^{-6}) - 10(^{-2})</td>
</tr>
<tr>
<td>Bdenum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M. rod</td>
<td>0.007</td>
<td>0.002</td>
<td>0.012</td>
<td>-</td>
<td>0.032</td>
</tr>
<tr>
<td>C. rod</td>
<td>0.033</td>
<td>0.003</td>
<td>0.0002</td>
<td>-</td>
<td>4 \times 10(^{-5}) - 4 \times 10(^{-2})</td>
</tr>
<tr>
<td>M. rod</td>
<td>0.006</td>
<td>0.0019</td>
<td>0.011</td>
<td>-</td>
<td>10(^{-2}) - 10(^{3}) sec</td>
</tr>
<tr>
<td>3. rod</td>
<td>0.007</td>
<td>0.0023</td>
<td>0.013</td>
<td>-</td>
<td>10(^{-2}) - 10(^{3}) sec</td>
</tr>
<tr>
<td>C. rod</td>
<td>0.033</td>
<td>&lt;0.0005</td>
<td>&lt;0.0003</td>
<td>-</td>
<td>5 \times 10(^{-2}) - 12.5</td>
</tr>
<tr>
<td>magsten</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. rod</td>
<td>0.004</td>
<td>-</td>
<td>0.007</td>
<td>-</td>
<td>4 \times 10(^{-6}) - 4 \times 10(^{-2})</td>
</tr>
</tbody>
</table>

** T = Tension

T\(\text{td}\) = Tension - delay time

C = Compression

Tor = Torsion

†\(\tau_{300}\) = Yield stress at 300\(^{0}\)K and \(\dot{\epsilon} = 10^{-4}\) sec\(^{-1}\)
The temperature at which this relationship first occurs is defined as $T_0$. The value of $\tau^0$ at a given temperature was then taken as

$$
\tau^0 = [ (\tau_T - \tau_{300}) + (\tau_{300} - \tau_{T_0}) ]
$$

For comparisons with previous work on iron (Ref. 4) $\tau^0(Fe) = [(\tau - \tau^0) + 2 \text{ kg/mm}^2]$. For iron tested at a strain rate of $10^{-4} \text{ sec}^{-1}$, $\tau^0 = 0$ at $T \geq 350^0 \text{ K}$. 
III. RESULTS

A. ACTIVATION VOLUME, \( v^* \)

The variation of \( v^* \) with stress for the Group VA and VIA metals is shown in Figs. 2-7. It is shown that for all of these metals \( v^* \) is approximately 50 \( b^3 \) at \( \tau^* = 2 \text{ kg/mm}^2 \), increasing rapidly to values in excess of 100 \( b^3 \) at lower stresses and decreasing with stress to values as low as 2 - 5 \( b^3 \).

The similarity among the various b.c.c. metals is especially evident in Fig. 8, where the activation volumes taken from the curves of Figs. 2-7 and those obtained for iron (Ref. 4) are plotted on one graph. Figures 2-7 also show that for a given metal \( v^* \) as a function of stress is identical for the various yield or flow stresses and is independent of impurity content and grain size (including single crystals), as was previously found for iron (Refs. 4, 5, 8).

B. ACTIVATION ENERGY, \( H \)

The variation of \( H \) with stress is shown in Figs. 9-14. The curves were obtained from the relationship \( H = -v^* kT[(\delta \tau^*)/(\delta T)]_T \), using for \( v^* \) values from the curves of Figs. 2-7, and for \([[(\delta \tau^*)/(\delta T)]_T] \) the slope of the \((\tau_T - \tau_{300}) \) versus temperature curves given in Ref. 9. The plotted data points were obtained with the use of the equation \( H = -k[[(\delta 1n\gamma)/(\delta 1/T)]_T] \).

It is shown that the values and stress dependence of \( H \) for a given metal are independent of the yield or flow stresses and of impurity content and grain size, as was the case for \( v^* \). Figure 15 shows that the value of the activation energy for \( \tau^* = 0 \), defined as \( H_0 \) in Figs. 9-14, is approximately equal to 0.1 \( \mu b^3 \). The shear modulus values in Fig. 15 are for the temperature \( T_0 \) and were derived from the relation \( \mu = 3/8 E \), where \( E \) is the Young's modulus taken from Ref. 11. The values of \( \mu, b, \) and \( \mu b^3 \) are given in Table 2.

Plots of \( H \) versus \( T \) are given in Fig. 16. These plots were obtained by substituting temperature for \( \tau^* \) in Figs. 9-14. The values of \( v \) obtained from the slopes of the initial straight line portions of the curves of Fig. 16
are given in Table 3. This table shows that values of $v$ for the yield stress of impure polycrystalline metals vary between $10^4$ to $10^{11}$ sec$^{-1}$.

Table 2. Values of Shear Modulus $\mu$ at Temperature $T_0$, Burgers Vector $b$, and the Product $\mu b^3$ for the B.C.C. Metals

<table>
<thead>
<tr>
<th>Metal</th>
<th>$T_0 (i = 10^{-4} \text{sec}^{-1})$ $^0K$</th>
<th>$\mu(T_0)$ $10^{11} \text{dynes/cm}^2$</th>
<th>$b$ $\AA$</th>
<th>$\mu b^3$ $10^{-12} \text{ergs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>450</td>
<td>4.8</td>
<td>2.63</td>
<td>8.7</td>
</tr>
<tr>
<td>Cb</td>
<td>500</td>
<td>4.0</td>
<td>2.86</td>
<td>9.3</td>
</tr>
<tr>
<td>Ta</td>
<td>600</td>
<td>6.6</td>
<td>2.86</td>
<td>15.4</td>
</tr>
<tr>
<td>Cr</td>
<td>500</td>
<td>11.1</td>
<td>2.50</td>
<td>17.3</td>
</tr>
<tr>
<td>Mo</td>
<td>650</td>
<td>11.9</td>
<td>2.73</td>
<td>24.2</td>
</tr>
<tr>
<td>W</td>
<td>950</td>
<td>14.5</td>
<td>2.73</td>
<td>29.4</td>
</tr>
<tr>
<td>Fe</td>
<td>350</td>
<td>7.4</td>
<td>2.48</td>
<td>11.2</td>
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</tbody>
</table>

Table 3. Values of the Frequency Factor $v$ for Impure B.C.C. Metals (<99.98 wt %) from Fig. 16

<table>
<thead>
<tr>
<th>Metal</th>
<th>$v$ (sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>Cb</td>
<td>$10^4$</td>
</tr>
<tr>
<td>Ta</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Cr</td>
<td>$10^9$</td>
</tr>
<tr>
<td>Mo</td>
<td>$10^7$</td>
</tr>
<tr>
<td>W</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Fe</td>
<td>$10^8$</td>
</tr>
</tbody>
</table>
IV. DISCUSSION AND CONCLUSIONS

The current results obtained for the Group VA and VIA b.c.c. metals are in accord with the previous results obtained for iron. The variation of $H$ and $\nu^*$ with stress is independent of structure (i.e., impurity content and distribution, dislocation structure, and grain size), supporting the earlier conclusion (Refs. 3-5, 8, 9) that the rate-controlling mechanism responsible for the strong temperature dependence of the yield and flow stress of b.c.c. metals is overcoming the Peierls-Nabarro stress.† A possible thermally-activated mechanism for overcoming the Peierls-Nabarro stress is that proposed by Seeger (Ref. 13) for the Bordoni peak in f.c.c. metals which is shown in Fig. 17. The mechanism involves the formation of a pair of kinks in a dislocation line lying in a close-packed direction by the combined action of thermal fluctuations and the applied stress. Seeger's analysis of this mechanism for the activation energy at low stresses gives

$$H = H_K \left[ 1 + \frac{1}{2} \log \left( \frac{16 \tau^0_p}{\pi \tau^*} \right) \right]$$

(7)

where $H_K$ is the energy of a single kink, and $\tau^0_p$ is the Peierls-Nabarro stress at 0°C. He further gives

$$H_K = \frac{2a}{\pi} \left( \frac{2E_0 a b \tau^0_p}{\pi} \right)^{1/2}$$

(8)

†Subsequent to the writing of this manuscript, the authors became aware of results by Christian and Masters (Ref. 12) on V, Nb, and Ta, which also support the overcoming of the Peierls-Nabarro stress as rate-controlling mechanism at low temperatures.

††The thermal component of the stress $\tau^*$ has been substituted for the total stress $\tau$ in Seeger's equation.
and

\[ H_{PN} = \frac{\tau_0 ab^2}{2\pi} \]  \hspace{1cm} (9a)

\[ = \frac{H_K s^2 b}{16a^2 E_0} \]  \hspace{1cm} (9b)

where \( 2H_{PN} \) is the Peierls-Nabarro energy per atomic length, \( a \) is the distance between close-packed rows, \( b \) is the Burgers vector, and \( E_0 \) is the line energy of a dislocation. The derivative of Eq. (7) with respect to \( \tau^0 \) gives

\[ \nu^0 = -\frac{dH}{d\tau^0} = \frac{H_K}{4\tau^0} \]  \hspace{1cm} (10)

The values of \( H_K \) obtained from the experimental values of \( \nu^0 \) at \( \tau^0 = 1 \text{ kg/mm}^2 \) are approximately \( 4 \times 10^{-2} \mu \text{b} \) and are given in Table 4.

Substituting this value into Eq. (8) and using \( E_0 = \frac{1}{2} \mu b^2 \), one derives \( \tau^0 \approx 10^{-2}\mu \). For further comparisons it is necessary to know the experimental value of \( \tau^0_0 \), i.e., the value of \( \tau^0 \) at \( 0^0 \text{K} \). An estimate of \( \tau^0_0 \) can be obtained by extrapolating the curves of \( \tau_T - \tau_{T_0} \) versus temperature (given in Ref. 9) to \( 0^0 \text{K} \). The extrapolated values, \( \tau^0_0 \), given in Table 4, are of the order of \( 10^{-2}\mu \) and are in agreement with the calculated values. Using the values of \( \tau^0_0 \) for \( \tau^0 \) in Eqs. (7, 8, and 9), one obtains \( H_K, H(\tau^0 = 1 \text{ kg/mm}^2) \) and \( H_{PN} \) equal to \( 3-4 \times 10^{-2} \mu \text{b}^3, 7-14 \times 10^{-2} \mu \text{b}^3 \) and \( 1-2 \times 10^{-3} \mu \text{b}^3 \), respectively.

(See Table 4.) Of special significance is the good agreement between the calculated values of \( H(\tau^0 = 1 \text{ kg/mm}^2) \) and the experimental values taken from Figs. 9-14. The agreement supports Seeger's model for the nucleation of kinks as the rate-controlling mechanism during the deformation of the b.c.c. metals at low temperatures.
Table 4. Values of $H(\tau^* = 1 \text{ kg/mm}^2)$, $H_K$, and $H_{PN}$

<table>
<thead>
<tr>
<th>Metal</th>
<th>$\tau_0^*$</th>
<th>$\tau_0^*/\mu$</th>
<th>$H_K/\mu b^3$</th>
<th>$H(\tau^*)$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>kg/mm$^2$</td>
<td>x10$^{-2}$</td>
<td>Eq. (7) x10$^{-2}$</td>
<td>Eq. (8) x10$^{-2}$</td>
</tr>
<tr>
<td>V</td>
<td>60</td>
<td>1.25</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Cb</td>
<td>60</td>
<td>1.50</td>
<td>3.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Ta</td>
<td>65</td>
<td>0.99</td>
<td>4.2</td>
<td>3.6</td>
</tr>
<tr>
<td>Cr</td>
<td>75</td>
<td>0.68</td>
<td>3.7</td>
<td>3.0</td>
</tr>
<tr>
<td>Mo</td>
<td>75</td>
<td>0.63</td>
<td>3.6</td>
<td>2.9</td>
</tr>
<tr>
<td>W</td>
<td>95</td>
<td>0.65</td>
<td>3.2</td>
<td>2.9</td>
</tr>
<tr>
<td>Fe</td>
<td>45</td>
<td>0.61</td>
<td>3.5</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Values of $H(r^* = 1 \text{ kg/mm}^2)$, $H_K$, and $H_{PN}$ Derived from Experimental Data

<table>
<thead>
<tr>
<th>$H_K/\mu b^3$</th>
<th>$H(r^* = 1 \text{ kg/mm}^2)/\mu b^3$</th>
<th>$H_{PN}/\mu b^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (8) $x10^{-2}$</td>
<td>Eq. (10) $x10^{-2}$</td>
<td>Avg. $x10^{-2}$</td>
</tr>
<tr>
<td>4.0</td>
<td>4.2</td>
<td>4.1</td>
</tr>
<tr>
<td>4.4</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td>3.6</td>
<td>5.5</td>
<td>4.4</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
<td>3.8</td>
</tr>
<tr>
<td>2.9</td>
<td>4.0</td>
<td>3.5</td>
</tr>
<tr>
<td>2.9</td>
<td>4.1</td>
<td>3.4</td>
</tr>
<tr>
<td>2.8</td>
<td>3.8</td>
<td>3.4</td>
</tr>
</tbody>
</table>
The above values of $\tau_0^*, H_K$, and $H_{PN}$ are somewhat higher than those usually given for close-packed metals, but they agree with those calculated using the original Peierls-Nabarro equations (Ref. 14) and the more recent calculations of Kuhlmann-Wilsdorf (Ref. 15).

According to Seeger (Ref. 13), the width of $w$ of a kink is given by

$$w = \frac{1}{2} a \left( \frac{E_0 b}{H_{PN}} \right)$$  \hspace{1cm} (11)

and the critical separation of the kinks, $l^*$, during thermal activation is

$$l^* = \frac{w}{\pi} \log \left( \frac{32H_{PN}}{\tau^* b^3} \right)$$ \hspace{1cm} (12a)

$$= \frac{(H - H_K)b}{2H_{PN}}$$ \hspace{1cm} (12b)

The values of $w$ and of $l^*$ at $\tau^* = 1 \text{ kg/mm}^2$ obtained from Eqs. (11, 12a, and 12b) using the average values of $H_K$, $H_{PN}$, and $H(\tau^* = 1 \text{ kg/mm}^2)$ from Table 4 and taking $E_0 = 1/2\mu b^3$ are given in Table 5. It is shown that $w = 7-10 b$ and $l^*(\tau^* = 1 \text{ kg/mm}^2) = 13-22 b$. Again, good agreement is obtained for $l^*$ calculated from Eqs. (12a and 12b).

For the Peierls-Nabarro mechanism the frequency factor is given by

$$\nu = \rho b \left( \frac{1}{l^*} \right) \left( \frac{Lb}{l^*} \right) \left( \frac{b}{l^*} \right) \nu_d$$ \hspace{1cm} (13)

where $\rho$ is the density of the dislocations participating in the deformation, $l^*$ is the length of dislocation segment involved in the thermal activation, $L$ is the maximum lateral spread of the kinks, and $\nu_d$ is the Debye frequency.
Table 5. Values of $w$ and $I^*$ Derived from Experimental Data

<table>
<thead>
<tr>
<th>Metal</th>
<th>$w/b$</th>
<th>$I^<em>/b(\tau^</em> = 1 \text{ Hg/mm}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Eq. (12a)</td>
</tr>
<tr>
<td>V</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Cb</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>Ta</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Cr</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>Mo</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>W</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Fe</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

The term $(1/I^*)$ represents the number of places per unit dislocation length where thermal fluctuations may nucleate a loop of length $I^*$; the term $(Lb)$ is the area of the slip plane swept out per successful thermal fluctuation; and the term $(\nu db/I^*)$ is the frequency of vibration of a segment of length $I^*$. The effect of impurities, strain, and grain size (e.g., single crystals as compared to polycrystals) on the temperature dependence of the yield or flow stress should then result from an effect of structure on the frequency factor $\nu$, i.e., on the number of dislocations participating in the deformation, $\rho$, or in the lateral spread of the kinks, $L$. This relationship was found to exist for iron where sufficient data were available for the necessary analysis (Ref. 8). If this relationship is true, then the stress at 0 K should be a constant for a given metal independent of structure and those materials which exhibit a weaker temperature dependence of the yield or flow stress at higher temperatures should show a more rapid increase of these stresses at very low temperatures. That this phenomenon actually occurs is evident from the data of Lawley, et al. (Ref. 16) given in Ref. 9.

Explanation is needed for the fact that the activation volume $v^* = -dH/d\tau^*$ increases very rapidly as $\tau^*$ approaches zero, while the slope of the curve
of H versus \( \tau^* \) generally shows no such rapid increase. The large values of \( v^* \) indicate that the activation energy curves in Figs. 9-14 should increase rapidly as \( \tau^* \) approaches zero. In fact such a behavior is expected as \( \tau^* \) becomes zero and an athermal condition prevails. The low values of H for stresses near zero might then simply reflect the difficulty in exactly defining \( \tau^* = 0 \). Furthermore, the fact that the plots of H versus T in Fig. 16 depart from a straight line as T approaches \( T_0 \) (i.e., as \( \tau^* \) becomes zero)\(^\dagger\) indicates that \( v \) changes with temperature or stress as \( \tau^* \) approaches zero.\(^\dagger\dagger\) On the other hand, perhaps a different mechanism with approximately the same activation energy, but with a much larger activation volume, becomes rate-controlling near \( T_0 \). A positive choice between the above alternatives cannot be made on the basis of the available experimental data.

Because the activation energy and activation volume and their stress dependence are the same for all deformation phenomena (i.e., microcreep, proportional limit, lower yield stress, upper yield stress, flow stress) for the Group VA and VIA b.c.c. metals and for iron it is concluded that the mechanism of yielding in these metals is similar to that previously postulated for iron (Refs. 4, 8); i.e., the upper and lower yield stresses are caused by the sudden multiplication of dislocations by the double cross-slip mechanism rather than by the thermally-activated tearing of dislocations from their interstitial atmosphere. In this model the three phenomena, microcreep, yielding, and flow, represent the movement of free dislocations and the rate-controlling mechanism is the thermally-activated overcoming of the Peierls-Nabarro stress.

\(^\dagger\) An exception to this was found from the data of Bazinski and Christian (Ref. 3) on decarburized iron. For this material H increased rapidly in the region of \( \tau^* = 0 \) (Ref. 5).

\(^\dagger\dagger\) The change in \( v \) with temperature (or stress) in the vicinity of \( T_0 \) may reflect differences in structure resulting from straining at different temperatures, rather than an effect of temperature or stress per se, as suggested by Guard (Ref. 17).
Figure 1. Initial Yield Stress of Various Polycrystalline Metals Versus Ratio of $T/T_m$
Figure 2. Effect of Stress on the Activation Volume for the Deformation of Vanadium
Figure 3. Effect of Stress on the Activation Volume for the Deformation of Columbium
Figure 4. Effect of Stress on the Activation Volume for the Deformation of Tantalum
Figure 5. Effect of Stress on the Activation Volume for the Deformation of Chromium
Figure 6. Effect of Stress on the Activation Volume for the Deformation of Molybdenum
Figure 7. Effect of Stress on the Activation Volume for the Deformation of Tungsten
Figure 8. Comparison of the Activation Volume for Deformation of the Various B.C.C. Metals
Figure 9. Effect of Stress on the Activation Energy for the Deformation of Vanadium
Figure 10. Effect of Stress on the Activation Energy for the Deformation of Columbium
Figure 11. Effect of Stress on the Activation Energy for the Deformation of Tantalum
Figure 12. Effect of Stress on the Activation Energy for the Deformation of Chromium

\[ H = -\nu^* T \left( \frac{\partial \tau^*}{\partial T} \right) \gamma \]

Cr
POLYCRYSTALS (<99.98%)
○ L.Y.-MARCKINKOWSKI
AND LIPSITT
Figure 14. Effect of Stress on the Activation Energy for the Deformation of Tungsten

\[ W \]

POLYCRYSTALS (< 99.98 %)
- Y.S. (0.2 %) - UNION CARBIDE

\[ H = -v^* T \left( \frac{\partial \tau^*}{\partial T} \right) \gamma \]

SINGLE CRYSTALS (~99.995 %)
- P.L. - ROSE et al.
Figure 15. Relationship between $H_0$ and $\mu b^3$

$H_0 \approx 0.1 \mu b^3$
Figure 16. Variation of Activation Energy $H$ with Temperature for Impure, Polycrystalline B.C.C. Metals

$\epsilon = 10^{-4}$ SEC$^{-1}$

PURITY: $<99.98$ wt %

POLYCRYSTALLINE

$T_0$ ($\tau^* = 0$)
ACKNOWLEDGEMENT

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Aerospace Corporation, El Segundo, California.


(Report TDR-169(3240-11)TN-7; SSD-TDR-63-46)

(Contract AF 04(695)-169) Unclassified report

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activation energy and volume as a function of stress were independent of structure, indicating that the rate-controlling mechanism for the deformation of body-centered-cubic metals is overcoming the Peierls-Nabarro force. The experimental data were in agreement with Seeger’s model for the thermally-activated nucleation of kinks. The kink energy and the Peierls-Nabarro energy and stress derived from the data were $3 - 4 \times 10^{-4} \mu b^2$, $2 - 4 \times 10^{-3} \mu b^3$ and $0.6 - 1.5 \times 10^{-2} \mu$, respectively.

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