UNCLASSIFIED

AD 402 728

Reproduced by the
DEFENSE DOCUMENTATION CENTER
FOR
SCIENTIFIC AND TECHNICAL INFORMATION
CAMERON STATION, ALEXANDRIA, VIRGINIA
NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
ANALYSIS OF CRITERION RELATIONSHIPS FOR LIQUIDS BOILING IN TUBES

IZV. VYSSH. UCHEBN. ZAVEDENII, ENERGETIKA, (5), 108-114, 1959, U.S.S.R.

By
I.I. Sagan'

TRANSLATED AND ISSUED
APRIL 1963
BY
TECHNICAL INFORMATION AND LIBRARY SERVICES
MINISTRY OF AVIATION
Analysis of criterion relationships for liquids boiling in tubes

I.I. Sagan'

The boiling process is a complex process which has as yet not been fully studied. Owing to the great complexity of the heat transfer process in boiling, there are as yet no firmly established criteria and consequently no universally accepted relationships which could be used for all cases of boiling liquids and solutions in the absence of experimental data.

For generalising experimental data relating to boiling a number of criterion equations have been put forward. The better known of these are the equations of S.S. Kutateladze (1), G.N. Kruzhilin (2), M.A. Kichigin - N.Y. Tobilevich (3) and V.I. Tolubinskii (4).

The equations of V.I. Tolubinskii and those of M.A. Kichigin - N.Y. Tobilevich are the result of vast experimental material relating to the heat transfer process in boiling water and various solutions in tubes. In the resolution of the Science Session of the Commission on High Pressure Steam Research of the Academy of Science of the USSR and the Institute of Thermoenergetics of the Academy of Science of the Ukrainian SSR, made in 1951, it was pointed out that the formulae of M.A. Kichigin-N.Y. Tobilevich and those of V.I. Tolubinskii have certain practical advantages in comparison with the others. It was mentioned that until further special research is undertaken, it was impossible to express any decisive preference for the one or the other system. In generalising our experimental data, therefore, relating to the boiling process in highly concentrated sugar solutions in tubes, we used only the criterial equations of M.A. Kichigin-N.Y. Tobilevich and V.I. Tolubinsky.

The M.A. Kichigin-N.Y. Tobilevich equation

$$ \text{Nu} = 3.25 \times 10^{-4} \text{Pe}^{0.6} \text{Ga}^{0.125} \text{Kp}^{0.7} $$

(1)

is recommended by authors for calculating evaporators and condensers.

The V.I. Tolubinskii equation

$$ \text{Nu} = 54 \text{Fr}^{0.6} \text{Pr}^{0.3} $$

(2)

is recommended by the authors for determining the heat transfer coefficient $\alpha_{q}$ for horizontal and vertical evaporators.

Up to recent times there were no experimental data available relating to the heat transfer in the boiling process of highly concentrated sugar solutions, in tubes of large diameters, with optimum levels of the boiling liquid. For these conditions the validity of the above mentioned criterion relationships (1) and (2) have not yet been proved. For this reason we have carried out a special experimental investigation into the heat transfer in highly concentrated boiling sugar solutions, using a single tube evaporator with natural circulation and interchangeable steel heating tubes of 159/150, 95/87 and 57/48 mm in diameter, installed at the Oxfen Sugar Factory. The content of dry substances was varied from 0 to 86%, with the pressure varying from 0.4 to 1 atm.abs and the heat flow $q$ varying from 5 to 90 thousand kcal/m².h., with a depth of immersion of the heating surface of up to 200%. In addition, the kinematic viscosity in the experiments varied from $0.295 \times 10^{-6}$ to $215 \times 10^{-6}$ m²/sec.
i.e. about 730 times and the Pr number varied from 1.72 to 1540 i.e. about 900 times.

In determining the relationship between the similarity criteria in heat transfer to boiling liquids, it is important to determine experimentally the effect of the separate physical properties of the boiling liquid on the heat transfer coefficient \( a_2 \). The difficulty, however, is that as one of the properties changes the other properties vary as well.

The effect of viscosity on the heat transfer coefficient can be determined from the experimental data, obtained in the course of the investigation. The properties relating to the vapour phase \((\gamma'', \nu'')\) hardly vary. The physical constants \( \sigma, \lambda, c \) and \( \gamma \) for sugar solutions vary comparatively little. The main factor affecting the heat exchange process is the viscosity, which varies within wide limits (table 1) and, in fact, the viscosity is the basic factor determining the quantitative value of the non-dimensional criterion of the physical parameters i.e. Prandtl's number Pr.

### Table 1  Some physical constants at p = 0.4 atm abs.

<table>
<thead>
<tr>
<th>Name of constant</th>
<th>Units</th>
<th>Concentration in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Surface tension</td>
<td>kg/m</td>
<td>0.63x10^{-2}</td>
</tr>
<tr>
<td>Coefficient of thermal</td>
<td>kcal/m</td>
<td>0.521</td>
</tr>
<tr>
<td>conductivity</td>
<td>m/min/degree</td>
<td></td>
</tr>
<tr>
<td>Specific gravity</td>
<td>kg/m³</td>
<td>972</td>
</tr>
<tr>
<td>Thermal capacity</td>
<td>kcal/kg/degree</td>
<td>1.0</td>
</tr>
<tr>
<td>Coefficient of kinematic</td>
<td>m²/sec</td>
<td>0.376x10^{-6}</td>
</tr>
<tr>
<td>viscosity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prandtl number</td>
<td></td>
<td>2.2</td>
</tr>
</tbody>
</table>

On the basis of the then existing experimental data it was only possible to make an approximate assessment of the effect of viscosity on the heat transfer coefficient \( a_2 \) in a boiling liquid and, therefore, also only an approximate evaluation of the \( \gamma'' \) power index in the M.A. Kichigin - N.Y. Tolubovich equation or the Pr index in the V.I. Tolubinski equation, since the experiments were carried out in a narrow range of viscosity changes. For instance, in V.I. Tolubinski's experiments the Pr number varied only from 1.7 to 100 and in N.Y. Tolubovich's experiments it varied from 0.7 to 170.

Comparing the experimental results of different authors, we observe different values for the power index of the Pr number. In the case of convective heat exchange in a non-boiling liquid the Pr index, in calculation formulae, is of the order of 0.33 to 0.43(5). In criterial equations relating to the heat transfer in boiling the Pr power index varies from + 0.21 [L.S. Sterman (6)] to - 0.5 [G.N. Krushilin (7)].

The relationship of the heat transfer coefficient \( a_2 \) to the coefficient of kinematic viscosity \( \nu \) is shown in fig. 1. In this figure the results of our experiments are compared with the data of N.Y. Tolubovich (3), according to which \( a_2 \sim \nu^{0.27} \). In our case \( a_2 \sim \nu^{-0.18} \) at atmospheric pressure and \( a_2 \sim \nu^{-0.18} \) at a pressure of 0.4 atm abs. According to V.I. Tolubinski (4)
\( a_2 \sim v^{-0.3} \), whereas according to V.D. Popov (8) \( a_2 \sim v^{0.23} \) and according to A.F. Sorokin (9) \( a_2 \sim v^{0.4} \) to \(-0.5\).

**Fig. 1. Relationship of the heat transfer coefficient \( a_2 \) to the viscosity coefficient \( v \).**

1 - the author's experiments, \( p = 0.4 \text{ atm.abs.}, q = 40 \times 10^3 \text{kcal/m}^2\text{h} \).
2 - N.Y. Tobilevich's experiments, \( p = 1.0 \text{ atm.abs.}, q = 10 \times 10^3 \text{kcal/m}^2\text{h} \).
3 - the author's experiments, \( p = 1.0 \text{ atm.abs.}, q = 10 \times 10^3 \text{kcal/m}^2\text{h} \).

Analysing the curves of fig. 1, we can conclude that in the transfer of heat to boiling sugar solutions at a pressure \( p < 1 \text{ atm.abs.} \), the ratio of \( a_2 \) to \( v \) decreases slightly.

Taking into account the results obtained by plotting the relationship \( a_2 = f(v) \), we carried out a comparative approximation correction of the M.A. Kichigin - N.Y. Tobilevich and the V.I. Tolubinskii criterial equations, based on our experimental data. In doing so, we did not necessarily aim at obtaining a relationship valid only for working conditions specific to vacuum condensers and evaporators employed in the sugar industry, although it would be possible to do that. What we had in mind is to maintain or rather widen the sphere of applicability of these generalised relationships to a wider range of values, embracing a number of liquids with differing physical properties and boiling in a wider range of pressure variations.

The corrected or refined criterion relationship of M.A. Kichigin - N.Y. Tobilevich \( \frac{\text{Nu}}{\alpha_0 0.09 K_p 0.77} = f(\text{Pe}) \), based on our experimental data, is shown in fig. 2. Although the tested sugar solutions differ considerably in their physical properties, nevertheless the experimental points appear to be grouped quite satisfactorily along the mean criterion line with a scatter of \( \pm 15\% \). If the same data are generalised to the M.A. Kichigin - N.Y. Tobilevich criterion system without altering the power indices, then the scatter of the points relative to the mean line exceeds \( \pm 49 \) and \(-28\% \).

**Fig. 2. Generalisation of the author's experimental data with the corrected M.A. Kichigin - N.Y. Tobilevich criterion relationship**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Sugar content %</th>
<th>( P ) atm.abs.</th>
<th>Outside diameter d mm</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0</td>
<td>1.0</td>
<td>48</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>1.0</td>
<td>87</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>0.4</td>
<td>87</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>Sugar solution</td>
<td>60</td>
<td>1.0</td>
<td>48</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>1.0</td>
<td>87</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>1.0</td>
<td>150</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>0.4</td>
<td>48</td>
<td>( \Phi )</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>0.4</td>
<td>87</td>
<td>( \Phi )</td>
</tr>
</tbody>
</table>
With a view to confirming the validity of the corrected M.A. Kichigin - N.Y. Tobilevich equation for a wider range of liquids, extensive experimental material was used drawn from different authors. The generalisation of these experimental results, relating to boiling liquids widely differing in their physical properties, is shown in the same coordinates 

\[ \frac{\text{Nu}}{0.09 \text{Kp}^{0.77}} = f(\text{Pe}) \] in fig. 3. The scatter of the experimental points relative to the mean line is within the limits of ±15%. The result of generalising all the above experimental data, including our own, can be expressed by a corrected, non-dimensional equation of the following type

\[ \text{Nu} = 2.71 \times 10^{-4} \text{Pe}^{0.6} \text{ca}^{0.09} \text{Kp}^{0.77} \]

which, after converting, will take the following form

\[ \alpha_2 = 2.71 \times 10^{-4} \frac{\lambda \alpha_c \rho}{\text{Pe}} \frac{y}{n}^{0.6} \text{c}^{0.6} \text{p}^{0.6} \text{c}^{0.77} \text{Kp}^{0.77} \]

In this formula, in view of the narrow range of pressure variations and since \( \gamma > \gamma' \), it was assumed that \( \gamma - \gamma' = \gamma \), in order to simplify the calculations.

**Fig. 3.** Generalisation of the experimental results of other authors with the corrected M.A. Kichigin - N.Y. Tobilevich criterion relationship

<table>
<thead>
<tr>
<th>Author</th>
<th>Liquid</th>
<th>Sugar content %</th>
<th>P atm.abs.</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.Y. Tobilevich</td>
<td>NaCl solution</td>
<td>20</td>
<td>1.0</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Water</td>
<td>0</td>
<td>1.0</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Sugar solution</td>
<td>25</td>
<td>1.0</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>1.0</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47</td>
<td>1.0</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>1.0</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>65</td>
<td>0.2</td>
<td>(7)</td>
</tr>
<tr>
<td>Y.I. Tolubinskii</td>
<td>Water</td>
<td>0</td>
<td>0.7</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0.4</td>
<td>(9)</td>
</tr>
<tr>
<td>N.Y. Tobilevich</td>
<td>Sugar solution</td>
<td>25</td>
<td>1.46</td>
<td>(10)</td>
</tr>
<tr>
<td>N.G. Yampol'skii</td>
<td></td>
<td>65</td>
<td>0.24</td>
<td>(11)</td>
</tr>
<tr>
<td>A.G. Bendor'</td>
<td>NaOH solution</td>
<td>5</td>
<td>1.0</td>
<td>(12)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>1.0</td>
<td>(13)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.62</td>
<td>(14)</td>
</tr>
<tr>
<td>R.Y. Ladiev</td>
<td>NH₄NO₃ solution</td>
<td>10</td>
<td>1.0</td>
<td>(15)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>1.0</td>
<td>(16)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>1.0</td>
<td>(17)</td>
</tr>
</tbody>
</table>

It follows from the above that the corrected M.A. Kichigin - N.Y. Tobilevich criterion formula satisfactorily generalises the experimental data relating to a large group of substances and is valid for rather a wide range of variations of the basic factors affecting the heat transfer process in boiling liquids.
The corrected or refined criterion system shows rather less scatter of the experimental points, relative to the concentration, than the previous one. However, for other liquids it cannot be considered reliable and would require additional verification for conditions not applying to the generalised experiments.

The V.I. Tolubinskii criterion equation has attracted considerable interest owing to its simple form. Fig. 4 shows the generalisation of the author's experimental data with the V.I. Tolubinskii corrected criterion relationship $\text{NuPr}^{0.2} = f(K)$. The experimental points are distributed along the mean line with a scatter of ±20%. If the same data are generalised to the criterion system with the Pr power index taken as 0.3, then the deviation of the points from the mean line will exceed +56 and -36%. Generalisation of experimental results of other authors to the V.I. Tolubinskii refined criterion system, has shown that the deviation of the points from the mean line amounts to -17 and +30%.

**Fig. 4.** Generalisation of the author's experimental data with the corrected V.I. Tolubinskii criterion relationship

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Sugar content %</th>
<th>$\nu$ atm.abs.</th>
<th>$d$ mm</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0</td>
<td>1.0</td>
<td>87</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>1.0</td>
<td>48</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>0.4</td>
<td>87</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>Sugar solution</td>
<td>60</td>
<td>1.0</td>
<td>150</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>1.0</td>
<td>87</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>0.4</td>
<td>150</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>0.4</td>
<td>87</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>60</td>
<td>0.4</td>
<td>48</td>
<td>$\textcircled{0}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>75</td>
<td>1.0</td>
<td>150</td>
<td>$\textcircled{0}$</td>
</tr>
</tbody>
</table>

The result of generalising all the experimental data can be expressed by a corrected non-dimensional equation of the following type

$$\text{NuPr}^{0.2} = 41.3K^{0.5}$$

or

$$a_2 = 1.4 \frac{1.2 \nu^{0.3} \gamma^{0.06}}{n^{0.6} Y^{0.66} \nu^{0.2} \delta^{0.5} \rho^{0.2}} q^{0.6} \left( \frac{\text{kal}}{m^2 \cdot h \cdot \text{degree}} \right)$$

When comparing the generalised results of the two criterion systems, we see that a greater deviation of the experimental points from the mean line is obtained when using V.I. Tolubinskii's criterion equation. The greater deviation of the experimental data, from the line corresponding to this criterion equation, is probably due to an inaccurate evaluation of $d\gamma$, since experimental information relating to the value of this term for different liquids and pressures is as yet somewhat scanty. Both the author and V.I. Tolubinskii have used values of $d\gamma$ relating to water at pressures of 1 to 5 atm.abs., carbon tetrachloride, sodium chloride solutions and glycerine at atmospheric pressure. As far as other liquids are concerned, there are as yet no experimental data available for them relating to the $d\gamma$ value for atmospheric or any other pressures. In order to develop further, therefore, the given criterion system, it will be necessary to carry out special
preliminary experiments for ascertaining the true values of $d_0U$ for various liquids and pressures.

Conclusions

1. A study was made of the heat exchange process in boiling highly concentrated sugar solutions with natural circulation and interchangeable heating tubes of 159/150, 95/87 and 57/48 mm in diameter, the sugar content being 0.6, 0.75, 0.82 and 0.86%, the pressure varying from 0.4 to 1.0 atm.abs. with a depth of immersion of the heating surface of up to 200%.

2. The experiments, carried out for a sufficiently wide range of variations of kinematic viscosities $\nu$ (from $0.295 \times 10^{-6}$ to $215 \times 10^{-6}$ m$^2$/sec), allowed of ascertaining the effect of the viscosity on the heat transfer coefficient $\alpha_2$ and of determining the values of the $Ga$ and $Pr$ power indices in the M.A. Kichigin-N.Y. Tobilovich and in the V.I. Tolubinskii critical systems.

3. The new experimental data made it possible to obtain a corrected criterion equation for the M.A. Kichigin-N.Y. Tobilovich system

$$Nu = 2.74 \times 10^{-4} Pe^{0.6} Ga^{0.09} Kp^{0.77}$$

and a corrected criterion formula for the V.I. Tolubinskii system

$$Nu Pr^{0.2} = 41.3 K^{0.5}$$

and to extend the applicable range of these formulae for Prandtl numbers ranging from 0.7 to 1540.

4. Since the M.A. Kichigin-N.Y. Tobilovich corrected criterion system yields a smaller error in generalising experimental data relating to a large group of experimental substances and is valid in a wide variational range of factors affecting the heat transfer in boiling liquids, its application should be given preference, at least at the present stage of progress, in the study of the heat exchange processes in boiling.

Bibliography


4. V.I. Tolubinskii. Determination of the heat transfer coefficient from the wall to the liquid in horizontal and vertical evaporators. Tr.in-ta teploenergetiki ANUSSR (Trans.of the Institute of Thermoenergetics, Academy of Science of the Ukrainian SSR), (5), 1952.

5. V.P. Popov. Determination of the heat transfer coefficient in preheaters. Tr.KTIPP. (Trans.of the KTIPP), (7), 1948.


Рис. 1. Зависимость коэффициента теплоотдачи $e$ от коэффициента
кинематической вязкости $\nu$:
1 — опыт автора, $p = 0,3$ атм, $q = 10^{-3}$ ккал/м²-час;
2 — Н. Ю. Тобелевича, $p = 2,0$ атм, $q = 10^{-3}$ ккал/м²-час;
3 — автора, $p = 1,5$ атм, $q = 10^{-3}$ ккал/м²-час.

Рис. 2. Обобщение опытных данных автора в уточненной критериальной зависимости
М. А. Кичингина — Н. Ю. Тобелевича

Рис. 3. Обобщение опытных данных других исследователей в уточненной
критериальной зависимости М. А. Кичингина — Н. Ю. Тобелевича
Рис. 4. Обобщение опытных данных автора в уточненной критериальной зависимости В. И. Толубинского
**MINISTRY OF AVIATION**

TIL 1(a)  
1
TIL 1(a) Transl.  
1
R.A.E. LIBY. FARNDOROUGH  
2

**OTHER GOVT. DEPTS.**

PAT. OFF. LIBY.  
1
BRIT. MUS. LIBY.  
1
J.I.E./D.S.I. (Min. of Def.)  
1  
Attn. Miss Amoss

SCI. MUS. LIBY.  
1
D.S.I.R./H.L.L. Boston Spa  
1
35  
Prof. Spalding  
6

**OVERSEAS**

**U.S.A.**

N.A.S.A. Bethesda  
25
J.S.R.F.  
18
LIBY. OF CONGRESS  
1  (thro' B.D.S. Washington)

**CANADA**

DEF. RES. MEMBER/ CANAD JT. STAFF  
1
S.L.O./C.S.L. OFFICE  
3

**AUSTRALIA**

DEPT OF SUPPLY AUST. HOUSE  
2
R.A.A.F.  
1

**FRANCE**

O.N.E.R.A. (Service de Documentation)  
1
S.D.I.T. (Ministere de L'Air)  
1