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RADAR OBSERVATIONS OF THE PLANET VENUS IN THE SOVIET UNION
IN APRIL, 1961
(Sct. Rpt. of Institute of Radio Eng. and Elecfr., Moscow, 1961,
Translated from Russian)

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PREFACE

The results of radar observation of the planet Venus in the Soviet Union in April, 1961, are given in this report. The description and principle of operation of the equipment are set forth and the results of processing the reflected signals which were received are given. The value determined for the astronomical unit and the accuracy of measuring it are set forth. An analysis of the spectrum of the reflected signal is given.
This is a translation of a Technical Report of the Institute of Radio Engineering and Electronics of the Soviet Academy of Sciences in Moscow, dated 1961. A copy of this report was supplied us through the kindness of Academician Kotel'nikov, and we are distributing this translation because of its interest to American radar astronomers.
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INTRODUCTION

The Institute of Radio Engineering and Electronics of the Academy of Sciences of the USSR, with a group of other organizations, built special equipment and conducted radar observations of the planet Venus, in April, 1961.

The aim of this experiment was to improve the precision of the astronomical unit (the mean distance from the Earth to the Sun), to evaluate the rotation period of Venus, and to get data concerning its surface.

Until recently, the astronomical unit and the distances in the solar system determined by it were known only with an error of 1 part in $10^3$. Such accuracy is completely insufficient for calculating the trajectories of interplanetary space ships. The error of determining the distance to the planet Venus, for example, amounted to several multiples of its diameter.

The radar observations of the planet Venus conducted in the Soviet Union permitted us to decrease the errors in determining the astronomical unit by approximately 50 times.

These observations allow us to evaluate the width of the spectrum of the signal reflected from the planet Venus, and to determine the mean coefficient of reflection of its surface as well.

A preliminary partial analysis of the reflected signals was conducted during the reception. The analyzing device was connected directly to the output of the receiver, parallel to the magnetic tape recording. In this way we detected the reflected signals in a wide spectrum (several hundred cps) with the help of an analyzer with filters of a 60 cps width. The astronomical unit was first determined from this wide spectrum and the period of rotation was evaluated. The results of this analysis were published in "Pravda" and "Izvestia" on May 12, 1961.

The astronomical unit set forth in these reports was not determined precisely from observation, owing to the periodicity of the signals which were used. The difference was discovered on the basis of data existing at that time concerning the astronomical unit. These data were received during the course of many years by different researchers by means of astronomy and radioastronomy but, as is now clear,
they were in error. Therefore, the value of the astronomical unit reported in those newspapers was also in error.

Complete analysis of the signals reflected from Venus was made after conclusion of all radar observations from the magnetic recordings. Runs during which all the devices functioned normally were selected for this analysis. Monitoring of equipment operation was done from photooscillograms of the control functions of the equipment in time, as well as from the logs kept by the operators.

At first, with the help of an analyzer with 60 cps-wide filters a band of the spectrum, 600 cps in width, was studied. After data appeared on the discovery of a narrow spectrum in the radar signal from Venus in the USA in 1961, an analyzer with 4 cps-wide filters was made. With the help of these analyzers it was discovered that it is possible to represent the signals reflected from Venus in the form of two components: one wide-band, corresponding to a spectral width of more than 400 cps; and the other narrow-band, corresponding to a width of less than 4 cps.

The discordance in the determination of the astronomical unit caused by the periodicity of the sounding signals was eliminated by two independent methods: by the Doppler shift of the frequency of the reflected signal and by the constancy of the value of the astronomical unit being determined for all the days of the observations.

The results of the present work were reported by V. A. Kotel'nikov at:

1. A plenary meeting of the annual session of the A. S. Popov Society in Moscow, on June 23, 1961.


In the following report, the results of processing radar observations of the planet Venus for April 18, 19, 20, 21, 23, 24, 25, and 26 of 1961 are set forth. During this period, 200 separate observation runs were conducted and recorded on magnetic tape.
1. The Radar Equipment

For radar observation of the planet Venus in the Soviet Union in April, 1961, the frequency of the transmitter of the cosmic radar unit was about 700 mcps. The power density of the field amounted to 250 megawatts per steradian, which gave 15 watts on the surface of Venus. The polarization of the waves being transmitted was circular. For reception, the antenna had linear polarization.

The signal being transmitted had the form of rectangular impulses with a length of 128 or 64 milliseconds separated by the same intervals. On some days, a pulse of the same length but at another frequency occurred rather than blank intervals. Calculated corrections for the Doppler shift, which is caused by the change of the distance between the Earth and Venus and by the rotation of the Earth, were introduced into the frequency of the signal and modulation in the transmission. The frequencies of the transmitter and its modulation as well as the heterodynes of the receiver were set by a precision quartz generator with a stability of more than $10^{-9}$.

Transmission was conducted at intervals during the period of the passage of the signal from the Earth to Venus and back (about 5 minutes). Then, with the help of a mechanical commutator, the antenna was connected to the receiver and the polarization of the antenna was changed into a linear one, and the receiver was activated.

A simplified diagram of the transmitter is shown in Figure 1. In this diagram MO is the master oscillator; DC is the device introducing the correction for the Doppler frequency shift; FM is the frequency multiplier; T is the transmitter; D is the frequency divider, the signal from which is controlled by the switch $K_2$ which modulates the signal. The transmission is connected and disconnected by the switch $K_1$ actuated from the timer, which works with an accuracy up to 1 millisecond.

The incoming signals were received by a superheterodyne receiver with a parametric semiconductor amplifier. At the receiver output, the reflected signal should have had a frequency of about 700-750 cps (depending on the size of the astronomical unit) in the absence of rotation of Venus. This signal, along with noise, was recorded on magnetic tape in the band of 420-1020 cps. On the same tape a
Figure 1. Block Diagram of the Transmitting and Receiving Equipment
Figure 2. Time Diagram of the Analyzer Operation
sinusoid with a frequency of 2000 cps was recorded as well. This served as a scale for time, as well as for control and sustaining of the speed of movement of the magnetic tape during reproduction. The beginning of the wave recording corresponded precisely to the calculated moment of reception of the 5-minute series of reflected signals; this gave us the opportunity to determine how much the actual time of the passage of the signal to Venus and back differed from the calculated time.

The plan for the receiving part of the equipment is shown in Figure 1. In the figure, R is the receiver; FC is the frequency converter which places heterodyne waves in the receiver and waves of 2000 cps frequency on the magnetic recording M.

At first the signals from the magnetic tape were analyzed with the help of ten filters which had filter bands every 60 cps and covered the frequency band from 420 to 1020 cps. After each filter, we determined the differential energy,

\[ \Delta W_\tau = W' - W'' \]

where \( W' \) is the total energy of the wave at the output of the filter for the segments of time crosshatched once (see Figure 2), and \( W'' \) is the analogous energy for segments crosshatched twice.

In Figure 2, \( t_0 \) is the calculated instant of reception of the series of reflected signals (the moment of the beginning of the recording of the wave of 2000 cps); \( T \) is the period of modulation of the signal (256 or 128 milliseconds); \( \tau \) is the unknown lag established at random; \( I \) is the instantaneous intensity of the total of reflected signals and noise, \( 2 \) is the actual incoming reflected signal, and \( 3 \) is the 2000 cps waves.

When the lag \( \tau \) was such that the moment \( t_0 + \tau \) corresponded to the actual moment of reception of the series reflected signals (this case is shown in Figure 2), the energy \( W' \) was equal to the energy of the signal and noise and the energy \( W'' \) only to that of the noise. In such a case, the difference \( \Delta W_\tau \) was maximum (on the average) and corresponded to the energy of the reflected signals.

As a function of the delay \( \tau \), the differential energy of the signal has a sawtooth variation. For analysis, we made several tests with different \( \tau \), on the basis of which, taking into account the relation between the differential energy of the signal and the lag,
it is possible to determine the energy of the signal and its delay. To remove systematic errors, the modulation phase of the signals being transmitted was changed over an interval and the sign of the difference $\Delta W_\tau$ was changed correspondingly.

For the signal analysis, the system pictured in Figure 3 was used. Signals from the magnetic recorder $M$ entered 10 filters $F_1, F_2 \cdots F_{10}$ (tuned circuits) which had a 60 cps bandwidth, and from their output entered the unit $RC$. If the amplitude of voltage at the output of the filter was greater than a certain threshold, the device $RC$ filtered the impulses being fed into it from the divider $D$ on the recorders $C'$ and $C''$. If it was less than such a threshold it was not filtered. The number of impulses being fed into the $RC$ device was 1000 per second.

The number of impulses at the output of the device $RC$, determined the energy of the wave coming in through the filter.

The 2000 cps oscillations recorded on the tape were processed by the filter $F$ and thence entered the DC pulse counter, which counted the assigned number of periods corresponding to the lag $\tau$ and closed the switch $K$, through which the 2000 cps waves entered the divider $D$. This divider began to supply pulses to the device $RC$ and to the change-over switch $S$, which operated on a modulation period of 256 or 128 milliseconds. This change-over switch fed impulses in the segments of time crosshatched once (see Figure 2) into the recorders $C'$, which stored energy $W'$, and in those which are crosshatched twice into the recorders $C''$ which stored energy $W''$. The difference of the readings of these recorders gave the quantity $\Delta W_\tau$ at the output of each filter in a numerical form convenient for further processing.

To remove that part of the spectrum which contains the narrow-band component of the signal, we connected a rejector filter in front of the analyzer. This filter was adjustable to the mean frequency of the signal and introduced an attenuation of 5-11 decibels in the $\pm 6$ cps band from the mean frequency.

Subsequently, the magnetically recorded signals were also analyzed more closely in the 40 cps band.

For such an analysis we used the scheme shown in Figure 4. It differs from Figure 3 in the following way: the filters $F_1, \ldots F_{10}$ were of a ladder-type and electromechanical, each with a 4 cps band. The recorders $C_1, \ldots C_{10}$ were connected directly to the device $RC$, and the signal feeding into the filters was gated synchronously
Figure 3. Block Diagram of the Wide Filter Analyzer
Figure 4. Block Diagram of the Analyzer with Narrow Filters
to the filters with the period of signal modulation by the switch K. The last changes were made, taking into account that in the present case the non-stationary processing by the filters $F_1 \ldots F_{10}$ was commensurate with the period of modulation.

In the playback of the magnetic recordings the switch K was closed for the intervals of time crosshatched once in Figure 2 and was disconnected for the remaining time. Thus the recorders C took a reading of the energy $W'$. Subsequently the same tape was played back a second time, while the lag $\tau$ was increased by $T/2$. In this case, the switch K was closed for the intervals of time crosshatched twice, and the recorders took a reading of the quantity $W''$. After this the difference $\Delta W_{\tau} = W' - W''$ was taken.

If the moment of time $t_0 + \tau$, corresponded to the actual moment of reception of the series of reflected signals, then in the first case the switch was closed during reception of the signal and $W'$ was equal to the total energy of the signal and noise. In the second case the switch was closed during blank intervals and the signal did not enter the filters. Thus $W''$ was equal to the energy of the noise. The difference, $\Delta W_{\tau} = W' - W''$, with a lag such as $\tau$, was maximal and equal to the energy of the signal.

### 2. Analysis of the Narrow-Band Component of the Reflected Signal

The results of measuring the spectrum of the narrow-band component of the signal on individual days of observation from April 18 to 26 of 1961 are set forth in Figures 5 and 6. The analysis was conducted with a collection of filters with 4 cps bandwidths, each according to the block-diagram shown in Figure 4. Along the abscissa in the figures is plotted the frequency of the spectral components of the signal at the output of the receiving apparatus ($f_i$), and along the ordinate is plotted the relation of the mean power of the signal in the band of the filter (4 cps) to the power of the noise in a band of 1 cps:

$$b_{\tau}(f_i) = \frac{2\Delta W_{\tau}(f_i)}{T_C \cdot N_0} \quad (2)$$
Figure 5. The Energy of the Signal Accumulated by the Narrow-Band Filters ($\Delta f \phi = 4 \text{ cps}$) for Frequency Modulation of 4 pulses per second (mean for an interval of one day).
Figure 6. The energy of the Signal Accumulated by the Narrow-Band Filters ($\Delta f \varphi = 4$ cps) for Frequency Modulation of 8 pulses per second (mean for an interval in one day).
where $\Delta W_\tau (f_1)$ is the mean differential energy referred to the analyzer input for the run received at the lag $\tau$ in the filter with a mean frequency $f_1$. The lag $\tau$ was taken relative to the calculated time lag of the signal which was estimated from a value of the astronomical unit of 149,600,000 km. The measurement of the power was made at the lag $\tau = \tau_0 = 0$; $N_0$ is the spectral intensity of the noise (the power of the noise in a band of 1 cps) at the analyzer input; $T_C$ is the mean length of one run equal to approximately 300 seconds.

The mean-square error of the corresponding measurements is designated in the figures by a horizontal dotted line. The date of observation and the number of intervals ($N$) over which the measurement was made are shown on the left side of each graph.

In Figure 5, the spectrum of the narrow-band component is shown for the signal modulated at 4 pulses per second, and in Figure 6 for the signal modulated at 8 pps. The compensation for the Doppler frequency shift in these runs was made working from the value of 149,474,440 km as the astronomical unit. If the compensation for the Doppler frequency shift were complete, then the center of the spectrum of the narrow-band signal component could correspond to a frequency of 743 cps, which in Figure 5 is shown by vertical dotted lines. However, as is shown in these graphs, the mean frequency of the spectrum of the narrow-band signal component, because of the partial compensation of the Doppler shift, is less than the calculated one (743 cps) and decreases each day in proportion to the recession of Venus.

Above the abscissa on each graph is placed a scale of values for the astronomical unit calculated for the moment of transit of each day on the condition that the values of the mean frequency of the narrow-band signal component are those plotted on the abscissa. From these scales it is clear that the mean frequency of the signal being received corresponds approximately to a value of 149,600,000 km for the astronomical unit.

In Figure 7 the total spectrum of the narrow-band component is shown separately for the signal modulated at 4 pps and for a signal modulated at 8 pps on the condition that the compensation for the Doppler shift of frequency was made working from a value of 149,600,000 km for the astronomical unit. Along the ordinate in these graphs the
Figure 7. Energy of the Signal Accumulated by the Narrow-Band Filters (Δfφ = 4 cps) (mean for an interval of several days).
quantity $b_{r_0} (f_i)$ is also plotted, and along the abscissa the deviation of the frequency of the spectrum components of the signal ($\Delta f$) from the calculated value of the carrier frequency is plotted.

From the graphs that have been presented, it is clear that the width of the spectrum of the narrow-band signal component is determined fundamentally by the modulation of the signal (4 pps and 8 pps). From analysis of the spectra and estimation of the equipment stability, it is possible to draw the conclusion that the expansion of the spectrum width of the narrow-band signal component caused by the properties of the reflecting surface of Venus does not exceed 4 cps. Existing data do not permit us to make a more accurate estimation.

The values of all the power in the narrow-band signal component for individual days of operation are set forth in Figure 8. Along the abscissa in the figure the dates of the measurements are plotted, and along the ordinate is plotted the ratio of the total power of the narrow-band signal component of all the signal filters to the spectral intensity of noise:

\[ B_{r_0} = \sum_{i} b_{r_0} (f_i) \]

For a signal modulated by 4 pps (April 18-24) the summation was made in a 12 cps band every three signal filters; for a signal modulated at 8 pps it was made every 5 signal filters in a 20 cps band. The length of the vertical segments which passed through the points of the $B_{r_0}$ values is equal to twice the root-mean-square value of the error which occurred in the given measurements (not taking into account the systematic errors). As is clear from this figure, the power of the narrow-band signal component was approximately identical on all days.

The energy of the signals reflected from Venus was determined by comparing the energy detected by the equipment from the extraterrestrial discrete source Cassiopeia A, the intensity of which is well known. The mean coefficient of reflection of the surface of Venus was computed from the value of this energy. For the narrow-band component the reflected power which was received amounted to 8% of the power which would have been received in substituting for Venus a well conducting smooth sphere of the same size.
Figure 8. The Total of Energies Accumulated in Narrow-Band Filters in 12 cps Bands (April 18–21, 1961) and of 20 cps Bands (April 21–26, 1961) (mean for the interval).
Values of the astronomical unit were computed from the Doppler shift of the mean frequency of the spectrum of the narrow-band signal component in individual runs during the course of 8 days from April 18-26, 1961. These values are set forth in Figure 9. Along the abscissa in the drawing, the index number of the run is plotted, and along the ordinate the value of the astronomical unit is plotted. The length of the segments plotted in the figure corresponds to the uncertainty caused by the filter transmission band (4 cps). Beneath the abscissa, days of observation are designated, and above it the form of modulation is designated. Significant deviations of the middle points of the segments from the mean value, which have been observed particularly in recent days, are caused obviously by the inaccuracy introduced by the Doppler shift correction operator.

As a result of the averaging of the individual measurements set forth in Figure 9 and the analysis of the total spectrum for entire days of observation set forth in Figures 5, 6, and 7, the astronomical unit determined by this method can be estimated as $149,598,000 \text{ km} \pm 10,000 \text{ km}$.

3. Determination of the Astronomical Unit According to the Retardation of the Envelope of the Signal

Observations for the retardation of the envelope of the narrow-band signal component reflected from Venus permitted us to determine the size of the astronomical unit more accurately. This was ensured by the fact that the apparatus guaranteed a higher relative accuracy for measuring the distance in comparison with the relative accuracy of measuring the Doppler frequency shift. The determination for the retardation of the envelope was made for each run individually as well as on the average for entire days of accumulation according to the values of the differential energy with several lags $\tau$ (see Appendix 1).

The relationship of the differential energy of the narrow-band signal component to the size of the lag for the signal modulated at 8 pps. is shown in Figures 10 and 11. In Figure 10 the result of accumulation for 28 runs on April 21, 23, and 24, 1961, is shown for a signal with amplitude modulation, and in Figure 11 the result of
Figure 9. The Size of the Astronomical Unit obtained from the Doppler Frequency Shift at Separate Intervals
Figure 10. Relation of the Differential Energy to the Amount of Lag for 28 runs on April 21, 23, and 24, 1961
Figure 11. Relation of the Differential Energy to the Amount of Lag for 18 Intervals on April 25 and 26, 1961
accumulation for 18 runs on April 25 and 26, 1961, is shown for a signal with frequency shift modulation of 420 cps (in this only the signal having a higher frequency was used). Along the ordinate in the figures the relationship of the total differential energy in all the runs $B_n(\tau)$ to the corresponding root-mean-square error of measurement $\sigma_n$ for two cases of accumulation is plotted: for the mean signal filter (dash-dot line, $n = 1$) and for the total of the energy in five signal filters (unbroken line, $n = 5$). The value of the lag $\tau$, relative to the calculated time of signal retardation computed for a value of 149,600,000 km as the astronomical unit is plotted along the abscissa. The experimentally derived points were approximated by the theoretical curves calculated for a point reflection (see Appendix 1).

As is clear from the figures, with amplitude modulation (Figure 10) the maximum of the curve of $B_n(\tau)$ is displaced by $\Delta \tau_1 = -1.4$ milliseconds, and with frequency shift modulation (Figure 11) by $\Delta \tau_2 = -2$ milliseconds, which, converting into astronomical unit kms amounts to $\Delta A_1 = -700$ km and $\Delta A_2 = -1040$ km respectively.

The values for the astronomical unit derived from the measurement of the retardation of the envelope of the narrow-band signal component in the Earth-Venus-Earth path for individual intervals in the course of 8 days from April 18-26, 1961, are set forth in Figure 12, where the same designations are assumed as in Figure 9. The value of the root-mean-square error of measurement for one interval is shown in the figure by a dash-dot line, and the value of the resulting root-mean-square error of measurement determined from the dispersion in all the measurements is shown by a dotted line.

Averaged results for the determination of the astronomical unit from the retardation of the envelope on the basis of data given in Figure 12 are set forth in Table 1. The root-mean-square measurement errors specified only by the dispersion of values with individual measurements are also set forth.
### TABLE 1

Results of the Determination of the Astronomical Unit from a Measurement of the Retardation of the Narrow-band Signal Component Reflected from Venus

<table>
<thead>
<tr>
<th>No.</th>
<th>Form of modulation and days of observation</th>
<th>Number of runs</th>
<th>Mean value of the astronomical unit (km)</th>
<th>Root-mean-square error of measurement (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Amplitude modulation--4 pps, April 18, 19, 20, 21, 1961</td>
<td>41</td>
<td>149,599,470</td>
<td>630</td>
</tr>
<tr>
<td>2.</td>
<td>Amplitude modulation--8 pps, April 21, 23, 24, 1961</td>
<td>30</td>
<td>149,599,300</td>
<td>440</td>
</tr>
<tr>
<td>3.</td>
<td>Frequency modulation--8 cps, April 25, 26.</td>
<td>18</td>
<td>149,598,960</td>
<td>700</td>
</tr>
<tr>
<td>4.</td>
<td>Amplitude and frequency modulation of 8 cps together, April 21, 23, 24, 25, 26.</td>
<td>48</td>
<td>149,599,200</td>
<td>390</td>
</tr>
<tr>
<td>5.</td>
<td>All intervals of 8 cps and 4 cps together, April 18-26, 1961.</td>
<td>89</td>
<td>149,599,280</td>
<td>330</td>
</tr>
</tbody>
</table>

In computing the astronomical unit it was assumed that:

1. The speed of light is 299,792.5 km/sec.
2. The radius of Venus is 6100 km.
Figure 12. Size of the Astronomical Unit (From a Measurement of the Envelope Delay of the Narrow-Band Component of the Reflected Signal).
As is clear from Table 1, the mean value of the astronomical unit determined from the retardation of the envelope of the narrow-band signal component for the days from April 18 through April 26, 1961, was equal to 149,599,300 km.

The value of the astronomical unit determined from the retardation of the signal by the method employed in this work is not unambiguous. This is clear from the formula from which the computation of the astronomical unit was made:

\[ A = A_p + \frac{\alpha(\Delta \tau + nT)C}{2} \]  

(3)

Here: \( A_p \) is the value of the astronomical unit assumed in the calculation; \( \Delta \tau \) is the value for the signal retardation correction (caused by the discrepancy between the value for the astronomical unit assumed in the calculation and the real value) derived from experiments; \( C \) is the speed of light; \( \alpha \) is the factor representing the relationship of the calculated value of the astronomical unit to the size of the calculated distance from Earth to Venus at the moment of the measurements (determined from ephemerides); \( n = 0, 1, 2 \).

As follows from this formula, with \( T = 256 \) milliseconds the value of the astronomical unit can be greater or smaller by \((120-130) \) n thousand km, depending on \( \alpha \).

The accuracy of determining the astronomical unit from the Doppler shift (see Part 2) permits us to resolve this ambiguity with confidence and choose the quantity 149,599,300 km.

As is clear from Formula (3), the size of the astronomical unit with an incorrect choice of the offset will change from day to day because of the change in the size of \( \alpha \). Since \( \alpha \), during the period from April 18-26, changed by \( \Delta \alpha = 0.085 \), the size of the astronomical unit during this time, with an incorrect exposure of the difference, would have changed by \( \pm \Delta \alpha \frac{mT}{2} C = \pm 11,000 \) m. km, where \( m = 1, 2, \ldots \) As is clear from Figure 12, this does not occur.
4. Evaluation of the Error in Determining the Astronomical Unit

The root-mean-square error in the measurement of the astronomical unit determined from the dispersion of the results in individual measurements was found equal to 330 km (see Table 1). In this section the systematic errors are appended.

An error comes from an inexact calculation of the signal retardation in the transmitter and receiver circuitry. The root-mean-square value of this error can be taken equal to 0.7 milliseconds or, in the astronomical unit, to 340 km.

It is possible to evaluate the limits of the lack of knowledge of the speed of light by the quantity \( \pm 0.6 \) km/sec, [5], which upon conversion to astronomical unit terms gives a root-mean-square error of 100 km.

The following factors could also have had an effect on the size of the signal retardation and hence on the astronomical unit: inexact knowledge of the radius of Venus, additional retardation of the signal in the ionosphere of the Earth and Venus and in space due to the free electrons which are in it, and the inexact knowledge of the location of the reflecting elements on Venus. In our calculations, the radius of Venus was assumed to be 6100 km. If we grant a maximum error of \( \pm 200 \) km, this gives the additional root-mean-square error in the astronomical unit of 220 km.

The Earth's ionosphere gives a supplementary retardation of less than 0.01 milliseconds in a frequency of 700 megacycles. Space, if the concentration of electrons in it is taken as even 1000 electrons in a cubic centimeter, gives 0.02 milliseconds. Thus, if we assume that the ionosphere of Venus is approximately the same as ionosphere of Earth, the overall supplementary retardation will be less than 0.04 milliseconds, which can decrease the value of the astronomical unit by no more than 20 km.

Finally, the narrow component of the spectrum, from which the distance in the determination for the astronomical unit is measured, is already approximately two orders of magnitude wide. Hence, it follows (if we assume that the wide spectrum to be described in Section 5 arises from a reflection from the entire surface of Venus) that the radius of the reflecting spot responsible for the narrow component of the spectrum should be approximately two orders of magnitude less than the radius of Venus, i.e. not more than 100 km. With a smooth surface the points of such a spot will have a
difference of distance to the Earth not exceeding 1 km. Such difference can give an error in the astronomical unit of 3 km.

But, if we assume that the surface of Venus has the same character as the surface of the Moon, then the depth of the basic reflecting field should be of the order of 30 km. This gives an error in the determination of the astronomical unit of 45 km.

Thus the total root-mean-square error in the astronomical unit can be estimated by the quantity \( \sqrt{330^2 + 340^2 + 100^2 + 220^2} = 510 \) km. We should add to this the error of the ephemerides, which is estimated at 220 km. Then the full root-mean-square error for the determination of the astronomical unit is \( \sqrt{510^2 + 220^2} = 560 \) km. Thus, it is possible to assume the maximum error to be \( +3 \times 560 = +1700 \) km.

The results of the measurements for the astronomical unit by different methods are set forth in Table 2. In the table the values of the maximum (tripled root-mean-square) error of the measurements are given, including the errors for the ephemerides and the inexact value for the radius of Venus.

### TABLE 2

Astronomical Unit Measurement Results

<table>
<thead>
<tr>
<th>Method of Measurement</th>
<th>Astronomical Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the Doppler shift of the center of the narrow spectrum for April 18-21.</td>
<td>149,598,000 ± 10,000 km</td>
</tr>
<tr>
<td>From the retardation of the reflected signal for April 18-21 (the length of the pulses: 128 milliseconds).</td>
<td>149,599,500 ± 2,300 km</td>
</tr>
<tr>
<td>For April 21, 23, 24 (the length of the pulses: 64 milliseconds A. M.).</td>
<td>149,599,300 ± 2,000 km</td>
</tr>
<tr>
<td>For April 25, 26 (the length of the pulses: 64 milliseconds F. M.).</td>
<td>149,599,000 ± 2,700 km</td>
</tr>
<tr>
<td>Average from all the measurements of the signal retardation</td>
<td>149,599,300 ± 1,700 km</td>
</tr>
<tr>
<td>Author</td>
<td>Value of the Astronomical Unit and Error of Measurements</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>USSR IRE</td>
<td></td>
</tr>
<tr>
<td>JPL USA</td>
<td></td>
</tr>
<tr>
<td>LINCOLN USA</td>
<td></td>
</tr>
<tr>
<td>JODRELL ENGLAND</td>
<td></td>
</tr>
<tr>
<td>JODRELL ENGLAND</td>
<td></td>
</tr>
<tr>
<td>LINCOL, USA</td>
<td></td>
</tr>
<tr>
<td>PIONEER V, USA</td>
<td></td>
</tr>
<tr>
<td>BRAUER</td>
<td></td>
</tr>
<tr>
<td>RABE</td>
<td></td>
</tr>
<tr>
<td>ADAMS</td>
<td></td>
</tr>
<tr>
<td>SPENCER JONES</td>
<td></td>
</tr>
<tr>
<td>SPENCER JONES</td>
<td></td>
</tr>
<tr>
<td>AVERAGE OF 7 OBSERVATIONS</td>
<td></td>
</tr>
<tr>
<td>IAU</td>
<td></td>
</tr>
</tbody>
</table>

Figure 13. Results of the Determination for the Value of the Astronomical Unit by Radar and Astronomical Methods, in Thousands of km.
The following were the values for the astronomical unit obtained in other countries with the help of radar observations of Venus in 1961:

a) Jodrell Bank (England) \[ 1 \] 149,600,000 ± 5000 km;

b) Lincoln Laboratory of the Massachusetts Institute of Technology (USA) \[ 2 \] 149,597,700 ± 1500 km;

c) Jet Propulsion Laboratory of the California Institute of Technology (USA) \[ 3 \] 149,598,500 ± 500 km.

In distinction from our value in the estimation of the exact value, the errors caused by the inexactness of the ephemerides and the inexact value of the radius of Venus were not taken into account.

The results of the radar location, and also of basic astronomical determinations for the astronomical unit are set forth in Figure 13. In this figure the method of measurement and authors of these measurements is shown. The shaded rectangles in the figure correspond to the maximum errors of the measurements according to the estimation of the authors themselves.

As is shown in the figure, all the radar observations of Venus in 1961 give very close values for the astronomical unit. The previously announced values for the astronomical unit received in 1958 in the USA and in 1959 in England were approximately 130,000 km smaller (noted in the figure by brackets).

5. **Analysis of the Wide-band Component of the Reflected Signal**

In Figures 14 and 15 the results of measurement for the differential energy of the wide-band component (mean for the interval) for various channels are set forth. In these measurements, the filters in the channels were single pole circuits with a 60 cps bandwidth (3 db points) the tuned center frequencies of the adjacent filters differed by 60 cps. The portion of the spectrum at the input of the analyzer containing the narrow-band component was removed by a band elimination filter, which effected an attenuation of 5-11 decibels in a ± 6 cps band. Along the abscissa in the graphs the center frequencies of the filters are plotted, and along the ordinate the values of the quantity \( b_\tau (f_i) \) (see Equation 2) are plotted. The results are set forth for two values of the lag(\( \tau \)) which differ by one fourth of the repetition period of the transmission: \( \tau = 16 \) milliseconds
and $\tau = 80$ milliseconds for intervals with a repetition frequency of 4 cps and $\tau = 16$ milliseconds and $\tau = 48$ milliseconds for intervals with repetition frequency of 8 cps. The arrows over the graphs show the position of the mean frequency of the narrow-band component of the reflected signal.

The lags enumerated above were selected for the following considerations. With two lags differing by one fourth of the repetition period of the transmission, for any distance to the reflecting point with one of the values of the lag, the differential energy of the reflected signal is close to the maximum possible.*

Let us note that a differential energy of positive sign will correspond to a lag of 16 milliseconds, assuming a reflection from the point of the surface of Venus nearest to us; a differential energy of negative sign will correspond to lags of 48 milliseconds (for intervals with a repetition frequency of 8 cps) and 80 milliseconds (for intervals with a repetition frequency of 4 cps). The differential energies of the noise corresponding to the lags which differ by one-fourth of the repetition period of the transmission are independent.

For an estimation of the values of the quantity $b_{\tau}(f_1)$ for one day we made from the runs a simple averaging of the measurement results for the differential energy in the corresponding filters. Such a grouping is close to optimal if we assume that the retardation of the reflected signal, as well as the intensity of the signal and of the noise in each filter during the day change insignificantly.

In determining the mean results of the measurements for several days ($b_{\tau}(f_1)$, see the lower graphs in Figures 14 and 15), we took into account the value of intensity of the noise at the analyzer output and the number of runs for each day.

In Figures 14 and 15, the level corresponding to the root-mean-square dispersion of the estimation of the quantity $b_{\tau}(f_1)$ from the influence of the receiver noise in the given number of intervals (N) is plotted by a dotted line.

---

* [The author appears to mean that if one of the lags coincides with that of a point target, the output $b_{\tau}$ will be greater than for other values of lag $\tau$. Then the output for a second lag displaced by one-quarter of a repetition period from the first lag will register noise only.]
Figure 14. Differential Energy Accumulated by the Wide-Band Filters ($\Delta f = 60$ cps) from the Measurements of April 18-21 (mean for the interval)
Figure 15. Differential Energy Accumulated by the Wide-Band Filters 
\((\Delta f_\varphi = 60 \text{ cps})\) from the Measurements of April 23-24 
(mean for the interval)
In Figures 16a and b estimates are given for various days of the mean differential energy for a wide-band component of the reflected signal in the field of frequencies overlapped by ten filters (approximately 600 cps), with values of the lag differing by one-fourth of the repetition period of the dispatchings. Along the abscissa the date of the measurements are plotted, and along the ordinate the values of the quantity $B_\tau$ are plotted.

$$B_\tau = \sum_{i=1}^{10} b_\tau (f_i)$$

In Figure 16a the results corresponding to $\tau = 16$ milliseconds are given; in Figure 16b the results corresponding to April 18-21, received with $\tau = 80$ milliseconds (the repetition period $T = 256$ milliseconds) and the results corresponding to April 21, 23, and 24, received with $\tau = 48$ milliseconds ($T = 128$ milliseconds) are given. The length of the vertical segments is equal to twice the root-mean-square deviation of the quantity $B_\tau$, which is caused by the receiver noise. It should be noted that in plotting the graph we did not take into account the changes in the equivalent noise temperature of the receiver at different intervals. This however, did not exceed $\pm 35\%$ and consequently we could not determine an observed fluctuation of the level of the wide-band component of the reflected signal.

In estimating the noise intensity at the analyzer output (the root-mean-square value of the dispersion of the quantity $b_\tau (f_i)$ from one interval $\sigma_{b_\tau}$) there arose differences in the results of the measurements at two adjacent intervals, from which the dispersion of the measurement caused by the noises was estimated. In certain cases a similar operation was performed on the results of the measurements for the differential energy accumulated separately in the even and odd repetition periods of one and the same run. In this case, it is possible to consider the noises practically independent, since the product of an effective width of the filtration circuit (90 cps) and the duration of the repetition period (1/4 or 1/8 seconds) is substantially greater than unity. Let us observe that such an estimate of the noise intensity in the presence of an irregular signal will be greater than the corresponding estimate gotten from analogous measurements with only one noise; but with a signal that is weak and slowly changing from
Figure 16. Total of the Differential Energies Accumulated by the Wide-Band Filters in the 600 cps Band (mean for the interval)
period to period, or correspondingly from run to run, the difference in the estimates will not be substantial.

The results of the estimation, which was conducted in such manner for every day of the measurements, for the root-mean-square value of the noise at the output of one channel of the analyzer with calculations of $\sigma_b$ for one run are set forth in Table 3. The channels were assumed to be identical and independent.

<table>
<thead>
<tr>
<th>Date</th>
<th>Number of sets of independent measurements</th>
<th>$\sigma_b T \left[ \frac{1}{\text{sec}} \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 18, 1961</td>
<td>120</td>
<td>0.8</td>
</tr>
<tr>
<td>April 19, 1961</td>
<td>135</td>
<td>0.9</td>
</tr>
<tr>
<td>April 20, 1961</td>
<td>165</td>
<td>0.8</td>
</tr>
<tr>
<td>April 21, 1961</td>
<td>45</td>
<td>0.9</td>
</tr>
<tr>
<td>April 23, 1961</td>
<td>560</td>
<td>0.8</td>
</tr>
<tr>
<td>April 24, 1961</td>
<td>260</td>
<td>0.9</td>
</tr>
</tbody>
</table>

* The bandwidth of the circuit $\Delta f_{0.7} = 60$ cps.

In Figures 17, 18 and 19 values are given for the distances obtained from signals in various filters with a 60 cps band in the elimination of the narrow-band component. Along the abscissa are plotted the resonant frequencies of the filters, and along the ordinate are plotted the distances gotten from the retardation of the signal envelope in the corresponding filters (the beginning of the record corresponds to the distance to the nearest point of the planet Venus, which is determined from the retardation of the narrow-band component of the signal).

In the figures, proceeding from the frequency of the sequences of the transmissions, the integral lengths of the simple distance determinations are laid out. The dotted line indicates the circle corresponding to the contour of the planet Venus, on the assumption that the maximum width of the spectrum of the reflected signal determined by the rotation of the planet is 400 cps.
Figure 17. Distances Received from Signals in Different Filters (without narrow-band component) from Measurements Conducted April 18-21.
Figure 18. Distances Received from Signals in Different Filters (without narrow-band component) from Measurements Conducted April 23-24
Figure 19. Distances Received from Signals in Different Filters (without narrow-band component) from Measurements Conducted April 18.
For each filter, the estimate of the distance was made on the basis of the results for the measurement of the differential energy accumulated by the filter in the runs with a transmitter sequence frequency of 4 cps (April 18-21, Figure 17) and of 8 cps (April 23-24, Figure 18). Separately in Figure 19 the results gotten on the basis of measurements conducted on April 18 are set forth. On this day, the wide-band component was particularly intense.

In determining the distance, we proceeded from the hypothesis that the signal corresponding to the band of each filter was reflected from a "point" target.

6. Possible Sources for Appearance of the Wide-Band Component for the Reflected Signal

Let us examine the possible sources for appearance of the wide-band component for the reflected signal:

1. Widening of the spectrum due to the Doppler components caused by the rotation of Venus.
2. Widening of the spectrum due to the reflection from some formations which are moving in the vicinity of the surface of Venus.
3. Widening of the spectrum due to reflections from formations in space.
4. The same from formations in the vicinity of Earth (in the ionosphere, etc.).
5. Widening of the spectrum due to the equipment.
6. Random realization of noise being taken for the reflected signal.

An estimate of the probability that receiver noise and interference coming from the antenna were mistaken for the reflected wide-band signal is set forth in various ways in Appendix 2. These evaluations gave the following results: $4 \cdot 10^{-4}$; $1.5 \cdot 10^{-2}$; $2.0 \cdot 10^{-2}$. In that appendix we have excluded the possibility of a substantial influence by the narrow-band signal component on the result obtained.

For continuous emission control we used a special receiver, to the input of which the signal from the transmitter was constantly supplied. At the output of this receiver the transmitted frequency was converted to audio frequencies and was monitored.
aurally and on the oscillograph screens. This check permits us to establish that in the transmitter there was no spurious modulation capable of explaining the appearance of the observed wide-band component.

Operation of the receiving channel was tested in the following manner. At the input of the parametric amplifier through the directional coupler from a special "simulator" a correspondingly weakened test signal was supplied. From the output of the receiver, the signal of the simulator was supplied to the analyzer and was separated just as it is done in operation. In this test, widening of the spectrum line of the signal was not observed.

Since the receiver was switched on approximately half a minute after the conclusion of the transmission, no signal reflections from formations close to the Earth (for example, from the ionosphere) could effect the receiver.

The appearance of the wide-band component due to the reflections from formations in space likewise is not very probable. To receive the signal registered by the analyzer from them, it is necessary that they move with approximately the same speed with respect to both Earth and Venus. Otherwise the Doppler shift, which reached 30 kcps in the signal reflected from Venus, would be different, and these signals would not pass through the receiver. In addition, Figures 17, 18 and 19 show that the distance to the points of reflection causing the wide-band component lie in a field close to the surface of Venus. If the reflections originated from random formations in space, then we should expect that these distances would be more evenly distributed in the intervals of ambiguity shown in Figures 17, 18 and 19.

Thus, reasons 1 and 2 remain.

In such a case the reflecting points, in order to give a spread of the radial lines for + 200 cps should have radial speeds of ± 40 meters/second with respect to the center of Venus. The following pictures which explain the observed phenomena are conceivable.

A. The wide-band component is formed as a result of the reflection of the signal from the entire surface of Venus as well as a result of the Doppler shift, which is caused by its rotation. The narrow-band component is caused by the reflection from that portion of the surface of Venus which is nearest to us (the highlight).
Since the widening of the lines of the spectrum in the narrow-band component of the signal is at least 100 times less than in the wide-band component, it should be assumed that the "highlight" has dimensions less than 1/100 of the diameter of Venus. This can hold true if the surface of Venus is significantly more even than the surface of the Moon, on which half of the intensity of the radar signals is reflected from a central "spot" with a diameter equal to 1/10 of the diameter of the Moon.

According to this hypothesis for the spread of lines of ± 200 cps, the period of rotation of Venus should be about 10 days, if its axis of rotation is perpendicular to the direction of the Earth and the whole surface is a reflecting one. If the axis of rotation is 60° to the direction of the Earth (in accordance with the hypothesis of Kuiper [6]), then the period is decreased to 9 days. If we have not recorded the entire spectrum and it is actually wider than 400 cps, then the period of rotation should be still smaller.

B. The reflecting properties of Venus are approximately the same as those of the Moon. Therefore, the narrow-band component of the reflected signal should correspond to the reflection from a spot having 1/10 the radius of Venus. In this case, taking into account that this component according to our data is smaller than 4 cps, we get a period of rotation of more than 100 days.

In this alternative, it is not possible to explain the wide-band component by the reflection from the surface of the planet and we should assume that it occurred as a result of the reflection from formations moving with radial speeds up to ± 40 meters/second or even more rapidly, for example, from strongly ionized streams. However, for this the ionization in these streams should be much greater (approximately 1000 times) than in the ionosphere of the Earth.

In the radar observations of Venus in 1961 in the USA at MIT and Cal Tech, only the narrow-band component of the reflected signals was recorded. The parameters of this component which were measured there do not contradict the data that we received. In this work the wide-band component of the signal was not detected. In the radar location of Venus in England at Jodrell Bank, the spectrum was not measured.

On the basis of the measurements for the narrow-band component of the spectrum, considering the surface of Venus to be similar to the surface of the Moon, the period of rotation of Venus in the USA was estimated at 200-600 days.
At present there is not yet sufficient material from the radar experiments conducted here and abroad to determine the true picture, and it is necessary to conduct additional experiments.

CONCLUSION

Full processing of all the material on radar observations of the planet Venus which were conducted in 1961 in the USSR gave the following results:

1. The astronomical unit equals 149,599,300 km with a maximum error of $\pm$ 2000 km.

2. In the reflection from Venus the lines of the spectrum of the signal were widened. The equipment permitted us to separate the narrow-band and the wide-band parts of the signal. The resolved narrow-band part of the signal which was reflected from Venus corresponded to the widening of the line of less than 4 cps and its intensity changed practically none during all the days of observation.

3. The coefficient of reflection of Venus, determined from the narrow-band component, is equal to $8\%$ (with respect to an ideal conducting sphere of the same dimensions).

4. The spectrum of signals reflected from Venus which was received in 1961 in the USA agrees with the spectrum of the narrow-band component of the reflected signal which we received. The wide-band component of the signal was not recorded in the USA.

5. On the basis of the spectra that we received for the signals reflected from Venus, it is not possible at the present time to make a reliable estimate of the rotation period of Venus.

6. The results of the measurement of the astronomical unit which were gotten in the USSR are in close agreement with the results gotten in the USA and England in 1961.

7. A comparison of the results of the radar location of Venus in 1961 with information about the radar location of Venus in past years (1958 in the USA and 1959 in
England) permits us to draw the conclusion that this information was erroneous and that in these experiments random realizations of noise were taken for reflected signals.

For the first time radar location of the planet Venus was successfully conducted in the USSR, the USA, and England in 1961.
Procedure for Conducting an Analysis of the Narrow-Band Signal Component

The analysis of the narrow-band signal component was conducted according to the following procedure:

1. First we conducted a preliminary determination of the astronomical unit from runs for April 19, 1961, by two methods: from the Doppler shift and from the retardation of the envelope of the narrow-band signal component. The value of the astronomical unit in both methods was close to 149,600,000 km, and thus the accuracy of determination from the Doppler shift permitted us to resolve the ambiguity which was gotten in determining the retardation due to signal periodicity. The value of the astronomical unit which was gotten differed significantly from the value 149,474,440 km, from which the initial introduction of the correction for the Doppler shift and the retardation of the reflected signal was made. Such a large difference between the astronomical unit established in the beginning calculations and the true value leads to a variable undercompensation for the Doppler shift and for the retardation of the signal recorded on the tape. Such compensation lessens the possibility of a prolonged signal accumulation in the filters with a constant adjustment with a constant lag. Therefore a new program was made for additional correction of the Doppler shift and retardation in the analysis of all the intervals selected for processing, proceeding from the value of the astronomical unit \( A = 149,600,000 \) km.

The correction for the Doppler shift \( \Delta f \) and the retardation \( \Delta \tau \) was calculated from the formulae:

\[
\Delta f = \Delta A \cdot \frac{f'_{\text{center}}}{A'} \quad (5)
\]

and

\[
\Delta \tau = \Delta A \cdot \frac{r'}{A'} \quad (6)
\]
where: $\Delta A = A - A'$ is the difference of the values of the astronomical units, and $A' = 149,474,440 \text{ km}$; $\tau'$ is the beginning calculated value of the retardation, calculated with $A'$; $f_{\text{center}}$ is the beginning Doppler shift of the frequency caused only by the speed of the relative shift of the centers of the masses of Venus and Earth.

2. Then an analysis was made of all the runs for spectrum determination for the narrow-band signal component with a zero lag ($\tau = 0$) using a correction for retardation and Doppler shift from the new program with $A = 149,600,000 \text{ km}$. Analysis of the spectrum was made for each interval individually as well as for the results of an accumulation for the entire day. From the results of the analysis, the intensity and width of the spectrum of the narrow-band component of the reflected signal were estimated. In addition, an estimation of the size of the astronomical unit from the Doppler shift of the spectrum frequency was conducted.

3. After analysis of the spectrum, an attempt was made to make the astronomical unit more precise from the retardation of the envelope of the narrow-band signal component. To shorten the time of processing, an additional selection of runs from the results of the spectrum analysis was made. The retardation was determined only from those runs in which the total of the differential energy of the signal filters with a neutral lag exceeded the normal deviation by more than twice.

For the signal modulated by 4 cps keying ($T = 256 \text{ milliseconds}$), the determination of the retardation was made from the values of the differential energy ($\Delta W_\tau$) in three signal filters with two values of the lag ($\tau_1 = 16 \text{ milliseconds}$; $\tau_2 = 112 \text{ milliseconds}$), shifted for $3/8$ of the period of the keying. From these values the shift ($\Delta \tau$) was calculated for the maximum value of the differential energy $\Delta W_\tau$, with respect to the neutral lag ($\tau = 0$).

For the signal modulated by 8 cps keying ($T = 128 \text{ milliseconds}$), the retardation calculation was made from the values of the differential energy $\Delta W_\tau$ in five signal filters with the following lag values: $\tau_1 = 0$; $\tau_2 = 8 \text{ milliseconds}$; $\tau_3 = 16 \text{ milliseconds}$; $\tau_4 = 48 \text{ milliseconds}$; $\tau_5 = 56 \text{ milliseconds}$.

The theoretically calculated function $\Delta W(\tau)$, which was computed for the point reflection, was entered into the experimentally determined values of $\Delta W_\tau$ in the best
Figure 20. Theoretical Dependence of the Differential Energy on the Size of the Lag (reflection from a point)
way possible, and the shift of its maximum with respect to the neutral lag \( \tau = 0 \) was found. The theoretically calculated dependence of the differential energy on the lag for the signal reflected from a point reflector is set forth in Figure 20. Along the abscissa the relationship of the lag \( \tau \) to the period of the keying \( T \) is plotted, and along the ordinate the relationship of the differential energy of the carrier and of two lateral frequencies to the maximum value of their total energy is plotted.

Curve "1" is computed only for the carrier frequency. Its dependence is expressed by the formula:

\[
\frac{B_1(\tau)}{B_\Sigma(0)} = \frac{\pi^2}{\pi^2 + 8} \left[ 1 - 4 \frac{\tau}{T_0} \right]
\]

Curve "2" is computed for the total of the first two sidebands, and its dependence is expressed by the formula:

\[
\frac{B_2(\tau)}{B_\Sigma(0)} = \frac{8}{\pi^2 + 8} \left\{ \sin^2 \left[ \frac{\pi}{2} \left( 1 - 2 \frac{\tau}{T_0} \right) \right] - \sin^2 \left[ \pi \frac{\tau}{T_0} \right] \right\}
\]

Curve "3" is computed for the total of the carrier and the two sidebands

\[
\frac{B_\Sigma(\tau)}{B_\Sigma(0)} = \frac{B_1(\tau) + B_2(\tau)}{B_\Sigma(0)}
\]

\( B_\Sigma(0) \) is the maximum value of the differential energy for the total of the carrier and the two sidebands when \( \tau = 0 \).

The values of the retardation of the envelope in the individual intervals were averaged for the individual forms of modulation (for 2-3 days) according to the formula:

\[
\overline{\Delta \tau} = \sum_{b=1}^{N} K_b \cdot \Delta \tau_b
\]

where \( N \) is the number of intervals from which the averaging is made;

\[
K_b = \frac{(\Delta W_{\tau, b})_{max}}{\sum_{1}^{N} (\Delta W_{\tau, b})_{max}}
\]

is the weight coefficient of the run;
and \((\Delta W_{\tau, l})^{\text{max}}\) is the maximum value of the differential energy in the signal filters.

From the values of the retardation \(\Delta \tau\) which were found, the size of the astronomical unit was made more precise according to the formula:

\[
\Delta A = \Delta \tau \cdot \frac{A}{\tau_A}
\]

(11)

where \(A = 149,600,000\) km, and \(\tau_A\) = the retardation of the signal in the Earth-Venus-Earth path, computed with \(A = 149,600,000\) km.

The values of the astronomical unit received as a result of the averaging are set forth in Section 3.

The size of the astronomical unit gotten from the various forms of modulation were averaged according to the rules of the method of least squares with allowance for their weight. Thus the following formulas were applied:

a) For finding the mean value;

\[
\bar{A} = K_1 \bar{A}_1 + K_2 \bar{A}_2
\]

(12)

b) For finding the root-mean-square error of the measurement;

\[
\sigma_{\text{meas.}} = K_1 \frac{N_1}{N_1 + N_2} \left( \sigma_1^2 + \frac{\Delta A_1^2}{N_1} \right) + K_2 \frac{N_2}{N_1 + N_2} \left( \sigma_2^2 + \frac{\Delta A_2^2}{N_2} \right)
\]

(13)

In these formulae the following substitutions were made:

\[
K_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}; \quad K_2 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}
\]

(14)

\(\bar{A}_1\) and \(\bar{A}_2\) are the mean values of the astronomical unit gotten as a result of averaging of the two individual groups of measurements; \(N_1\) and \(N_2\) are the number of runs, as a result of the averaging of which \(\bar{A}_1\) and \(\bar{A}_2\) are obtained; \(\sigma_1\) and \(\sigma_2\) are the root-mean-square errors of the measurement of \(\bar{A}_1\) and \(\bar{A}_2\),

\[
\Delta A_1 = \bar{A}_1 - \bar{A}; \quad \Delta A_2 = \bar{A}_2 - \bar{A}.
\]

(15)
Appendix 2

Estimation of the Probability for a Null Hypothesis

Let us examine the probability that the results of the measurements for the wide-band echo-signal component were caused by the noise of the receiver, antenna, and so on. This hypothesis is usually called "null", and the probability corresponding to it is usually called "the probability of a null hypothesis" ($P(0)$).

To answer the question which has been raised let us examine the quantity $S_\tau$.

$$S_\tau = \frac{B_\tau - \bar{B}_\tau + \tau/4}{\sqrt{2} \sigma_{B_\tau}}$$

(16)

where $\bar{B}_\tau$ is the quantity analogous to the quantity $B_\tau$, see Expression (4), but gotten from all the measurements with the rejection filter (see Figure 16); $\sigma_{B_\tau}$ is the root-mean-square value dispersion of the quantity $\bar{B}_\tau$ caused by the noise.

Denoting the value which was obtained for the quantity $S_\tau$ by $J_\tau$ on the basis of Figure 16 we have $J_\tau = 4.8$ ($\tau = 16$ milliseconds). In the absence of the reflected signal the quantity $\bar{S}_\tau$ is a Gaussian random quantity with zero mean and a dispersion equal to unity. Designating the probability of exceeding the value $J_\tau$ by the random value $\bar{S}_\tau$ in the absence of the signal by $P_{\bar{S}_\tau}(0)$, then on the basis of what we have stated above we have:

$$P_{\bar{S}_\tau}(0) = \Phi (J_\tau) \approx 10^{-6} \quad (J_\tau = 4.8)$$

(Here $\Phi(J_\tau)$ denotes the error function.

$$\Phi (J_\tau) = \frac{1}{\sqrt{2\pi}} \int_{J_\tau}^{\infty} e^{-x^2/2} dx.$$ )

Such a small probability of the neutral hypothesis ($\approx 10^{-6}$) means in practical terms that we should discard this hypothesis.
It is possible, however, to advance the hypothesis that for the result which was obtained we are indebted not to the presence of the wide-band component, but to the insufficient rejection of the narrow-band component of the echo-signal. The insertion of the narrow-band component into the quantity \( J_{\tau} \) was estimated; it did not exceed 30%.

However, for reliability it is expedient to conduct an analogous estimation, drawing from only those filters where the narrow-band component is in practice completely absent. Thus, the processing of the measurement results from 8 filters (except the two central ones: the fifth and the sixth) on the basis of Figures 14 and 15 gives:

\[
J'_{\tau} \approx 3.4^2 (\tau = 16 \text{ milliseconds}) \quad \text{and} \quad P_{J'_{\tau}} \approx 4 \cdot 10^{-4}.
\]

(The prime indicates that the result was received from 8 filters).

Insertion of the narrow-band component into \( J'_{\tau} \) does not exceed 5% in this case.

The result which was obtained gives exceedingly reliable evidence about the presence of the wide-band component of the echo-signal.

Let us examine still another criterion for estimating the accuracy of the neutral hypothesis: the criterion of signs. This criterion consists of the following: For \( n \) measurements one determines the number of measurements \( (K) \) in which the result had a "necessary" sign (previously determined). The probability of getting in at least \( K \) out of \( n \) measurements the "necessary" sign in the neutral hypothesis is \( P^{(0)} (K, n) \). Such is the quantity on the basis of which this hypothesis can be discarded. The value of such an examination lies in the fact that with it knowledge of the corresponding root-mean-square values which, generally speaking, are always known with an error is not demanded.

As the quantity being measured, let us examine a) \( \overline{B}_{\tau} \) and b) \( \overline{S}_{\tau} \), which are being determined for each day from the measurements with the rejector filter.

The results of the measurements of the quantity \( \overline{B}_{\tau} \) and \( \overline{S}_{\tau} \) (we have designated them by \( \overline{B}_{\tau} \) and \( \overline{J}_{\tau} \) respectively) are entered in the tables. In Table 4 the results
TABLE 4

Results of Processing from 10 Filters

<table>
<thead>
<tr>
<th>Date</th>
<th>Repetition Frequency of the transmission</th>
<th>$\beta_T$</th>
<th>$\beta_{T+T/4}$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-18-1961</td>
<td>4 cps</td>
<td>3.8</td>
<td>-0.3</td>
<td>2.9</td>
</tr>
<tr>
<td>4-19-1961</td>
<td>4 cps</td>
<td>0.4</td>
<td>-2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>4-20-1961</td>
<td>4 cps</td>
<td>0.5</td>
<td>-1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>4-21-1961</td>
<td>4 cps</td>
<td>1.3</td>
<td>-0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>4-23-1961</td>
<td>8 cps</td>
<td>2.0</td>
<td>-1.4</td>
<td>2.4</td>
</tr>
<tr>
<td>4-24-1961</td>
<td>8 cps</td>
<td>1.7</td>
<td>-2.2</td>
<td>2.8</td>
</tr>
</tbody>
</table>

TABLE 5

Results of Processing from 3 Filters

(Except the two central figures: the fifth and the sixth)

<table>
<thead>
<tr>
<th>Date</th>
<th>Repetition Frequency of the transmission</th>
<th>$\beta'_T$</th>
<th>$\beta'_{T+T/4}$</th>
<th>$e'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-18-1961</td>
<td>4 cps</td>
<td>3.3</td>
<td>-0.7</td>
<td>2.8</td>
</tr>
<tr>
<td>4-19-1961</td>
<td>4 cps</td>
<td>-0.4</td>
<td>-1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>4-20-1961</td>
<td>4 cps</td>
<td>0.9</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>4-21-1961</td>
<td>4 cps</td>
<td>1.5</td>
<td>-0.1</td>
<td>1.1</td>
</tr>
<tr>
<td>4-23-1961</td>
<td>8 cps</td>
<td>2.1</td>
<td>-1.1</td>
<td>2.3</td>
</tr>
<tr>
<td>4-24-1961</td>
<td>8 cps</td>
<td>1.2</td>
<td>-1.3</td>
<td>1.8</td>
</tr>
</tbody>
</table>
of processing from all ten filters are set forth, and in Table 5 the results from eight filters (except the two central ones) are set forth.

In the absence of the signal, $\bar{B}_r$ and $\bar{S}_r$ are Gaussian random quantities with zero means, and consequently in the absence of the signal they can have positive or negative signs with equal probability. It is thus easy to show that $\bar{B}_r$ and $\bar{B}_{r+T/4}$ are independent. For the reflecting surface corresponding to the surface of the planet Venus with the selected size of the lag ($\tau = 16$ milliseconds), in the absence of noises we would have:

$$\bar{B} > 0, \quad \bar{B}_{r+T/4} < 0 \quad \text{and} \quad \bar{S}_r > 0.$$  

Introducing the indices $\bar{B}$ and $\bar{S}$ for the full number of corresponding tests ($n_{\bar{B}}$, $n_{\bar{S}}$) and for the number of tests which have given the "necessary" sign ($K_{\bar{B}}$, $K_{\bar{S}}$), we have:

On the basis of Table 4

$$n_{\bar{B}} = 12 \quad n_{\bar{S}} = 6$$

$$K_{\bar{B}} = 12 \quad K_{\bar{S}} = 6$$

$$P_{\bar{B}}^{(0)} = \left(\frac{1}{2}\right)^{12} = 3 \cdot 10^{-4} \quad P_{\bar{S}}^{(0)} = \left(\frac{1}{2}\right)^{6} = 1.5 \cdot 10^{-2}$$

On the basis of Table 5

$$n_{\bar{B}} = 12 \quad n_{\bar{S}} = 6$$

$$K_{\bar{B}} = 10 \quad K_{\bar{S}} = 6$$

$$P_{\bar{B}}^{(0)} = \left(\frac{1}{2}\right)^{12} \left[\frac{12}{10! \cdot 2!} + \frac{12}{11! \cdot 1!} + 1\right] = 2.0 \cdot 10^{-2} \quad P_{\bar{S}}^{(0)} = \left(\frac{1}{2}\right)^{6} = 1.5 \cdot 10^{-2}$$

Thus the estimates of the null hypothesis probability gotten on the basis of the criterion of signs also point very reliably to the existence of the wide-band component of the echo signal.
APPENDIX 3

Calculations of Corrections for the Doppler Frequency Shift and Signal Retardation

1. Requirements for the Calculations. Operational Formulae

Preliminary calculations retardation $\tau$ of the reflected signal taking into account the time of the signal emission were made according to the formula

$$\tau(t) = \frac{2D(t)}{C} \left[ 1 + \frac{3\dot{D}(t)}{2C} \right],$$  \(17\)

where $\tau(t)$ is the time of the echo-signal retardation, with respect to the moment of emission of the signal $t$;

$D(t)$ is the distance between the observer and the nearest point of the surface of the planet Venus at moment $t$;

$\dot{D}(t)$ is the radial speed of the observed point with respect to the nearest point on the surface of the planet Venus at moment $t$ ($\dot{D} > 0$ in the recession of Venus);

$C$ is the speed of light.

The calculations were conducted with an accuracy of up to 1 millisecond.

The frequency of the emitted signal ($f_{\text{rec}}$) varied in time according to the law,

$$f_{\text{trans}} = f_0 + \Delta f(t)$$  \(18\)

where $f_0$ is the constant tuned frequency of the receiver;

$\Delta f(t)$ is the correction compensating for the Doppler shift. The values of $\Delta f(t)$ were computed earlier from the condition that the frequency of the reflected signal remained constant during the whole period of observation and equalled $f_0$. In the computation of $\Delta f(t)$ it was assumed that the frequency of the reflected signal ($f_{\text{refl}}$) is determined according to the formula

$$f_{\text{refl}}(t_3) = f_{\text{trans}}(t_1) \frac{C - \dot{D}(t_3; t_2)}{C + D(t_1; t_2)}$$  \(19\)
where $t_1, t_2, \text{and } t_3$ are the moments of the time of emission, reflection, and reception of the signal, respectively; and $\dot{D}(t_i; t_j)$ is the radial speed of the observer at the moment $t_i$ with respect to the nearest point of the surface of the planet Venus at the moment $t_j$.

Calculation of the relativistic effect leads to the appearance in the right side of equation (19) of the factor

$$\sqrt{\frac{C - V^2_1}{C - V^2_3}},$$

where $V^2_1$ and $V^2_3$ are the moduli of the vector of the full velocity of the observer at moments $t_1$ and $t_3$ respectively, with respect to the nearest point of the planet Venus taken at the moment $t_2$. Calculations showed that in the specific case under examination, the relativistic correction did not exceed $10^{-2}$ cps. Since the accuracy of the introduction of the correction $\Delta f$ did not exceed 1 cps; no calculation for the relativistic effect was made.

Simultaneous solution of equations (18) and (19) gives the following expression for frequency correction.

$$\Delta f = \frac{f_0}{C} \left[ \dot{D}(t_1; t_2) + \dot{D}(t_3; t_2) \right] \left[ 1 + \frac{\dot{D}(t_1; t_2)}{C} \right]$$

which was assumed for the calculations. In the final result, the correction value which was gotten was divided by the coefficient $K$, taking into account that the value of the correction being introduced before emission was multiplied in the apparatus by the coefficient $K$.

The values of the frequency correction $\Delta f' = \frac{\Delta f}{K}$ were computed with an accuracy up to 1 cps. The results of the computations were given in the form of a table for values of $\Delta f'$ with one step for each cps, and with a designation of Moscow time corresponding to each of the values of $\Delta f'$.

2. **Source Data for the Calculations**

For the computation of the values $D$ and $\dot{D}$ we used tables of the positions and speeds of the center of the masses of the Earth-Moon system and of the center of the mass of Venus in rectangular heliocentric equatorial system of coordinates.
In computing the position and speed of the observer with respect to the center of the masses of the Earth-Moon system, we used a table of positions and speeds of the Moon in rectangular heliocentric equatorial system of coordinates.

The tables referred to were compiled on the basis of the analytical theory of Newcomb taking into account the corrections for elements of the orbits of Venus (according to the data of Duncombe [7]) and of the Earth-Moon system (according to the data of Morgan, confirmed in the ITA of the USSR Academy of Sciences from observations over the period 1925-1953).

In computing the target designation for the guiding of the antenna and also for the signal retardation and the correction of the frequency $\Delta f'$, we used the following values of basic constants (not taken into account by the tables of ephemerides):

- Astronomical unit of distance: $149,474,440$ km
- Speed of light in a vacuum: $299,792.5$ km/sec.
- Radius of the planet Venus: $6,100$ km.
- Equatorial radius of the Earth (according to Krasovskii): $6,378.245$ km.
- Ratio of the masses of the Earth and the Moon: $81.53$

3. Accuracy of the Ephemeride Computations

An analysis of the errors in computing the position of the center of the mass of the planet Venus and the center of the mass of the Earth-Moon system (without calculating the errors due to the inexact knowledge of the astronomical unit in km) showed that the maximum values of these errors are determined by the following table.

**TABLE 6**

<table>
<thead>
<tr>
<th></th>
<th>Radial Component</th>
<th>Tangential Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venus</td>
<td>146 km</td>
<td>500 km</td>
</tr>
<tr>
<td>Earth-Moon</td>
<td>93 km</td>
<td>213 km</td>
</tr>
</tbody>
</table>

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The accuracy of tying the position of the observer to the center of the masses of the Earth-Moon system is at least one order of magnitude greater; and therefore, it is possible to disregard the errors in determining this position. The errors in computing the speeds (also without calculating the inexact knowledge of the speed in astronomical units), according to the approximate estimations, are such that they lead to errors of less than 1.5 cps in the computation of the Doppler frequency.

Taking into account all that has been stated above, the maximum error of the computed value of the distance from the observer to the center of the mass of the planet Venus was, for example, on April 21, equal to ~ 200 km.
Figure 21. Distance from the Earth to Venus on the Days of Observations (April, 1961)
Figure 22. Radial Speed of Venus with Respect to the Earth on the Days of Observations (April, 1961)
BIBLIOGRAPHY


