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WARHEAD SIZE DETERMINATION

by

F. Bisby

SUMMARY

A hypothesis is suggested for guidance in weapon design, that the best warhead size for the attack of a specified target is the threshold size required for a direct hit to produce a kill. It is based upon the fact that this threshold size is one or more orders smaller than that required to cater for near-misses and that this economy in warhead size can be exploited to increase the chance of getting a direct hit, either by increasing delivery accuracy or by deploying more weapons, thereby leading to greater weapon effectiveness. Multi-weapon deployment modes are discussed, leading in particular to the further principle that for weapons which have to be delivered in one attack opportunity, e.g. unguided air-to-surface weapons, the most efficient attack is with a spaced salvo of threshold weapons.

It is not claimed that the hypothesis is universally true. Whether it leads to a more effective weapon in any specific case will always require evaluation. Furthermore, it may be more applicable to air-to-surface weapons, mainly referred to as examples in this paper, than to other weapon types, such as anti-aircraft weapons, for which the warhead is a less dominant component.

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1 INTRODUCTION

1.1 Warhead optimisation

The question of warhead size optimisation constantly recurs in weapon design. Given the necessary functional relationships between the relevant parameters, in particular between warhead size on the one hand and lethality, accuracy and number of weapons deployable on the other, it should be possible to derive an optimum warhead size by the standard mathematical optimisation process. The difficulty lies in writing down these relationships in a form simple enough to be tractable and yet adequately realistic. An attempt at this type of solution is made in Appendix A, more in the way of an example to show that this approach is not a profitable one and can in fact lead to nonsensical answers, the reason being that the relationships are inadequate, particularly in one important respect which will emerge later.

An alternative approach is to take a particular situation and to evaluate the kill chances for a range of sizes of warhead to see which gives the best result. An example of this approach is given in Ref. 1. It can give valuable guidance for the particular situation investigated but it has the drawback that its solutions are usually determined by the particular constraints specified and, therefore, that it does not give any insight into the general problem, such as would lead to more universally applicable principles.

1.2 A hypothesis on warhead size

A hypothesis is put forward in this paper to provide guidance in the choice of warhead size for any given weapon design. Briefly the hypothesis is that it does not pay to cater for near-misses; more formally that the best warhead size is the minimum size required for a direct hit on the target to cause the required damage.

The hypothesis is not necessarily true in all cases and it will always be a matter for evaluation as to whether the cost of achieving a direct hit outweighs the advantage accruing from it. Furthermore, it may be more relevant to one type of weapon than another, for instance to air-to-surface weapons rather than to surface-to-air weapons, since the warhead is a more dominant missile component in the former. Nevertheless, this paper will indicate that the relationship between lethal range and warhead size against most targets is such that the 'threshold' size for a direct hit is an order or several orders smaller than that required for near-misses; that this advantage in size can be
exploited to increase the chance of getting a direct hit, either by increasing the number of weapons deployed or by increasing the delivery accuracy; and that this generally results in an overall gain in weapon effectiveness for a given weapon penalty in either weight or cost. This is particularly so for short-range unguided air-to-surface weapons for which the principle can be extended as follows: that the most efficient attack is with a spaced salvo of weapons whose warheads are of threshold size for direct hits.

2 THE TARGET VULNERABILITY CURVE

2.1 General form

The crux of the argument in this paper is that there is a peculiarity about target vulnerability which immediately points the way to the optimum warhead size. This peculiarity is best described by reference to the target vulnerability curve in Fig. 1, where the warhead size required for a 'kill', i.e. to cause some specified damage to the target, is plotted against distance from target centre to weapon impact point. No scales are put on the diagram because these will depend upon the particular target and type of warhead involved and the diagram is intended to depict a general form of relationship between critical warhead size and miss-distance which applies to any target, although a modification may be required for ship targets (see section 2.5 below).

For strikes on the target itself, henceforward referred to as 'direct-hits', a certain warhead size is required for a kill which is broadly constant wherever the hit occurs; but as soon as the impact point moves outside the target, into what is henceforward referred to as the region of 'near-misses', there is a big jump in the required warhead size, by about an order of magnitude, and as the miss-distance increases further the required warhead size rises rapidly, usually following some power law, typically the square or the cube of the miss-distance.

2.2 Physical explanation

There is a simple explanation of these features of the target vulnerability curve. The total chemical energy of a charge detonated internally, as well as the kinetic energy of the missile which delivered it, can be applied as a disruptive force to the target structure and contents, whereas for external explosions only that fraction of the explosive energy directed at the target can be so applied. By simple geometry this fraction is inversely
proportional to the square of the miss-distance and furthermore the energy is attenuated by the medium through which it passes, so that the proportion of the explosive energy eventually applied to the target decreases rapidly with miss-distance.

There are two additional factors which accentuate the discrepancy between the critical warhead sizes for internal and external explosions. The first is that energy applied asymmetrically from outside can often be absorbed by lateral movement of the whole or part of the target structure, thus reducing structural disruption. The second is that certain lethality-improving devices, such as shaped-charges, squash-heads and incendiary material, require contact with the target to be effective and can therefore only be used with direct hitting warheads (of course relatively large shaped-charges can be effective at relatively large stand-offs and may not therefore require direct-hitting but do require aligning in the target direction).

2.3 Examples

The foregoing explanation, based as it is on physical laws, is sufficient by itself to justify the acceptance of Fig. 1 as the fundamental form of the target vulnerability curve. Nevertheless, the provision of even one specific example of a well-established complete curve has been found well-nigh impossible, simply because the necessary target vulnerability data is not available. Apart from the aircraft vulnerability curve in Fig. 7, therefore, supporting evidence can only be given in the form of occasional spot points and a few such examples are given in Appendix B.

2.4 The difficulties of mathematical optimisation

If the target vulnerability curve of Fig. 1 is accepted — and henceforward in this paper it will be tacitly assumed that this is so — then the discontinuity at the edge of the target provides an immediate explanation of the difficulty of determining an optimum warhead size by a mathematical optimisation procedure such as that attempted in Appendix A. The power law assumed in Appendix A, in fact, only represents the near-miss portion of the vulnerability curve and ignores the direct-hit portion and the discontinuity at the edge of the target, the very features, it is argued here, which provide the clue to the true optimum warhead size. Such an over-simplified representation then leads to some absurd conclusions, such as that derived in Appendix A that the optimum weapon for the attack of heavy structural targets (dams, tunnels, bridges) is a cluster
of small bomblets each weighing a few pounds; because it assumes that the power law representing the near-miss portion of the vulnerability curve holds for all values of warhead size whereas, in fact, the power law breaks down for values below the threshold size and this threshold size can be quite large (order of 1000 lb) for these structural targets.

2.5 Modification for ship target

The possibility was mentioned in section 2.4 of the vulnerability curve needing modification for ship targets. A ship target differs from land or air targets in that a relatively small hole in a particular part, namely below water level, will allow flooding and thereby result in a higher category of damage than is warranted by the purely structural damage to the ship. An extreme example is the submarine, where the flooding caused by a hole in the pressure hull an inch or two in diameter may be beyond the capacity of the pumps and therefore constitute Category A damage. A full description of ship target vulnerability, in fact, requires two curves for the near-miss portion; one for air-bursts, which will have the same characteristics as that shown in Fig. 1; and another for underwater bursts, starting at a lower level than the air-burst curve for bursts in contact with, or very near to, the ship's plating and then increasing as the square of the distance (according to the well-known 'shock factor' for underwater bursts) - the starting point may in some cases be below the level for internal detonations.

3 THE EXPLOITATION OF THE THRESHOLD WARHEAD SIZE

3.1 Methods of exploitation

The minimum warhead size required to give a kill for a direct-hit on the target (or some specific part of the target) will be referred to as the 'threshold size' and it will be assumed that it is at least an order, and may be several orders, smaller than any warhead designed to have a near-miss capability. To establish the 'threshold hypothesis' enunciated in section 1.2 requires an exploitation of this advantage in warhead size so that, in comparison with the aforementioned near-miss weapon, an increase is achieved in the chance of getting a direct-hit and on balance thereupon an increase in overall weapon lethality in a specified set of attack conditions. There are two methods of exploitation:

(1) by increasing weapon delivery accuracy, and

(2) by increasing the number of weapons deployed.
The author's interest is mainly in air-to-surface weapons and for that reason the following discussion is mainly concerned with the application of the threshold hypothesis to these weapons, although there appears to be no reason why it should not apply to other weapons. For unguided air-to-surface weapons, such as bombs, rockets and guns, exploitation by numbers is the only possible method since the delivery accuracy of such weapons is largely determined by the aircraft's navigation/attack system and is therefore practically independent of missile size. The important penalty with airborne unguided weapons is weight, weapon costs being relatively small compared with aircraft and sortie costs, and valid comparisons of different weapon sizes can, therefore, be made on the basis of a given aircraft payload. Both exploitation methods are possible, however, for guided weapons (although physical factors such as glint and system noise may put a limit on the achievable accuracy) and furthermore comparisons of warhead sizes may have to be made on a cost as well as a weight basis. The two types of weapon therefore require separate consideration in this context, but before going on to do this in the next two sections it is convenient to consider the problem, common to any weapon exploiting by numbers, of the most efficient weapon deployment mode for a multi-weapon attack.

3.2 Multi-weapon deployment modes

There are in fact three distinct weapon deployment modes,

(1) Independent aiming – in which each weapon is aimed separately and independently at the target.

(2) Delivering a spaced salvo – in which the weapons are deliberately spaced in a pattern whose centre is aimed at the target.

(3) Delivering a random salvo – in which the weapons are all delivered with the same aim and weapon dispersion is relied upon to give a spread of impact points, generally in a bivariate normal distribution.

The derivation of the kill chance for these three modes is discussed in Appendix A and Fig. 2 plots kill chance against the parameter \( \left( \frac{N R^2}{S^2} \right) \) for each mode.
where $N$ is the number of weapons to be deployed,

$R$ is the radius of the target, assumed circular, and

$S$ is the standard deviation of the delivery error whose distribution is assumed to be unbiased, circular and bivariate normal.

For this idealised case it is shown in Appendix A that independent aiming and the ideal spaced salvo modes yield the same result and therefore are represented by the same curve in Fig.2. Furthermore, comparison with the curve for the random salvo mode shows that the two former modes are more efficient than the latter.

4 APPLICATION TO UNGUIDED AIR-TO-SURFACE WEAPONS

Independent aiming is not generally acceptable for a multi-weapon air-to-ground attack from a fixed wing aircraft since it would entail either repeated passes by one aircraft or a single pass by a number of aircraft and in either case multiple exposure of aircraft to enemy defences. In these circumstances, the result enunciated in the last paragraph is clearly an important one since it means that a strike aircraft can, by using the spaced salvo, deliver its full weapon quota for one target in one pass and with maximum efficiency with respect to weapon deployment mode. Thus the general hypothesis can be particularised as follows: that for weapons which must be delivered in one attack opportunity - in particular unguided air-to-surface weapons - the most efficient attack is with a spaced salvo of weapons whose warheads are of the threshold size for direct hits.

The most common instance of the spaced salvo is, of course, the "stick of bombs" which, with optimum spacing, is the ideal spaced salvo in one dimension only, namely along-track, and is well-suited to the beam attack of long targets, such as ships and bridges, where the across-track errors are absorbed by the target length. The British cluster weapon is an attempt to achieve the ideal spaced salvo in both dimensions and indeed is an example - probably the only one to-date - of the intentional application of the principle just enunciated.

5 APPLICATION TO GUIDED WEAPONS

Both methods of exploitation mentioned in section 3.1 are possible with a guided weapon, the first in the obvious way,

(1) by matching the guidance system to a single-shot attack with a weapon having a single warhead of threshold size, i.e. relying on high accuracy to
achieve a high chance of a direct hit with a single warhead; in some instances it may be that the limit on achievable accuracy precludes this solution, e.g. for S.A.G.W. where target glint and system noise tend to put a lower limit on the achievable miss distance;

the second in two ways,

(2) by deploying a number of cheaper and less accurate weapons than that of (1) above, each with a warhead of threshold size, and relying to some extent on numbers to achieve a direct hit - in this case the independent aiming mode is possible, or

(3) by using a cluster type warhead, the bomblets being of threshold size, on a larger missile than those of (1) or (2) above - in fact, this is the same solution as that for the unguided air-to-surface weapon, namely the use of a spaced salvo, but we must now define $S$ as the missile delivery error instead of the aircraft delivery error.

The choice between these three ways must be determined by the task required of the weapon, but in general the third would appear to have the widest application and to be the easiest to apply. The first, for instance, may be too difficult and too costly, and in choosing between the second and third the economic advantage of guiding a number of warheads in one package rather than individually will certainly favour the third. However, the third way implies putting all one's eggs in one basket and the second way has the advantage of being less vulnerable to the hazards of guidance system unreliability and target defences.

6 JUSTIFICATION ON EFFECTIVENESS GROUNDS

6.1 General

So far the case for the threshold hypothesis has rested on the advantage in warhead size of a direct hit weapon over one designed to have near-miss capability, but for a proper justification it is necessary to show that this results in an overall advantage in effectiveness. It is argued in section 2.2 that the advantage in size arises from the fundamental laws of physics. The question of effectiveness, however, depends upon the quantitative balancing of the warhead advantage against the accompanying disadvantage of needing to get a direct hit and there are no fundamental laws governing the relationship between these two opposing tendencies and hence no way of demonstrating analytically that the threshold hypothesis is necessarily true. Its justification must therefore rest on comparative effectiveness evaluations in specific cases.
6.2 Evidence on unguided weapons

It must be admitted that evidence along these lines is very scanty and that that which is available is restricted to the case of the attack of close-support targets by short-range unguided weapons. This evidence is summed up in Figs. 3 and 4, taken from Ref. 2, and shows that a cluster weapon, designed on the threshold hypothesis, has a clear advantage in effectiveness per unit weight of weapon over 540 and 1000 lb bombs, despite the latter's near-miss capability.

Against the structural interdiction targets, such as bridges, dams, tunnels, ships and railways, the evidence is confusing. There have been comparisons of different sized bombs, e.g. a stick of 3 x 1000 lb bombs against a single 2500 lb, and in general the results indicate an advantage to the smaller calibre, although not very marked. However, these assessments are not quite relevant to the issue under consideration, since they are generally based on the assumption that the targets are only vulnerable to direct hits whatever the size of bomb, the differences in size only affecting the chance of achieving a certain damage level given a hit. In fact these targets are such that the normal run of H.E. weapons, say limited to 3000 lb, the maximum load on current strike aircraft pylons, can have little or no near-miss capability and the right hand side of the target vulnerability curve could therefore only be relevant to nuclear weapons. If consideration were extended to nuclear warheads then to be consistent the comparison should be between say a megaton near-miss warhead weighing the order of 1000 lb and whatever sized nuclear warhead would cause the same damage given a direct hit and this presumably would be a sub-kiloton warhead weighing only the order of ten pounds. Then the 'structural' target attacked by nuclear weapons would present the same picture as the 'light' target attacked by H.E. weapons and the threshold principle would lead to the conclusion that the most economical attack is with a cluster of sub-kiloton bomblets.

6.3 Evidence on guided weapons

The author is unaware of any relevant evidence, in the form of comparative effectiveness evaluations, to justify the threshold hypothesis when applied to guided weapons, although two recent British designs of guided weapons could be put forward as examples of high-accuracy direct-hitters with threshold sized warheads and impact fuses: namely the A.S.G.W. Martel (TV version) for the attack of interdiction targets such as ships and bridges and the S.A.G.W. Rapier for the attack of low-flying aircraft. The anti-radar version of Martel
on the other hand cannot achieve the required accuracy to give a high chance of a hit on its target, the radar antenna, and since the missile can incorporate a big enough warhead to have a considerable near-miss effect then this is used together with an appropriate proximity fuse. The threshold warhead size for direct hits on the antenna would presumably be much smaller and in this case the solution suggested by the threshold hypothesis would be either to have a cluster warhead on the existing missile deploying bomblets of threshold size by means of the proximity fuse or to fire a larger number of smaller missiles each carrying a single warhead of threshold size and an impact fuse. It is a matter for evaluation as to how these two solutions compare on a cost/effectiveness basis with the present design, but on the face of it the second solution would appear to be uneconomical and the first solution probably a strong competitor.

7 CONCLUSIONS

To sum up: the threshold hypothesis is put forward in the belief that it is a good working guide. There is very little supporting evidence, but its validity really rests upon,

(1) the fact, itself based upon the fundamental laws of physics, that the threshold warhead size is much smaller than that required to cater for near-misses, and

(2) the opportunity thereby offered to exploit this advantage in warhead size to increase the chance of a direct hit either by increasing delivering accuracy or by increasing the number of weapons deployed.

Since it cannot be shown to be universally true, the application of the threshold hypothesis to any specific set of circumstances must always be a matter for evaluation. Nevertheless it is suggested that in formulating proposals for any new weapon it would be worthwhile considering whether a weapon designed on the threshold hypothesis provides the best solution.

ACKNOWLEDGEMENT

The author gratefully acknowledges the assistance given by Mr. G.T.J. Pullan during the preparation of this report and in supplying the aircraft vulnerability data in Appendix B and in Fig. 7.
Appendix A

MATHEMATICAL WARHEAD OPTIMISATION

A.1 To find an optimum warhead weight by means of the standard mathematical procedure requires weapon effectiveness to be expressed as a function of warhead weight. Consider the idealised case of a point target attacked by a weapon whose lethal range is $R$ (or the identical case of a target of radius $R$ attacked by a weapon with a warhead of threshold size) and a delivery error distribution which is circular, bivariate normal and unbiased with standard deviation $S$. Then the kill chance for a single shot is

$$P_{KSS} = 1 - \exp\left(-\frac{R^2}{2S^2}\right)$$

(1)

and if $N$ such weapons are independently aimed at the target then

$$P_{KN} = 1 - \left(1 - P_{KSS}\right)^N$$

$$= 1 - \exp\left(-\frac{NR^2}{2S^2}\right)$$

(2)

Thus $N$ weapons of lethal radius $R$ independently aimed at a target are exactly equivalent in kill chance to a single weapon of lethal radius $\frac{R}{N^2}$. The ideal spaced salvo will be so designed that the lethal areas of the $N$ weapons are compactly grouped with no overlap and no gaps, to give a total lethal area of $NR^2$, showing that the ideal spaced salvo and independent aiming yield the same result. The random salvo is not so easily dealt with because an additional parameter must be introduced, namely the weapon dispersion within the salvo, say $\sigma$. However, it can be shown that for given values of $N$, $R$ and $S$ there is an optimum value of $\sigma$ and if it is assumed for the ideal case that $\sigma$ has been optimised then $P_K$ is a monotonic increasing function of $\frac{NR^2}{S^2}$ as plotted in Fig.2, where is also plotted the $P_{KSS}$ for the independent aiming and ideal spaced salvo weapon deployment modes as given by expression (2) above. (To avoid misunderstanding regarding the claimed equivalence of the spaced salvo and independent aiming the foregoing should be qualified by pointing out that the criterion of effectiveness is the
chance of at least one hit on a vulnerable area and that, for instance, the spaced salvo would be inferior to independent aiming if the criterion were the total cumulative damage to the target, or the expected number of hits on the target. Furthermore, the spaced salvo is assumed above to be the ideal one with no overlap and no gaps between the lethal areas achieved by its components; this is well nigh impossible to achieve in practice due to irregularity of weapon spacing and the usually inappropriate shape of lethal areas. Thus, where it is possible, and other things being equal, independent aiming should always be preferred to the spaced salvo.)

A.2 The specific case will now be considered of the unguided air-to-surface weapon, for which \( S \) is largely determined by the aircraft's weapon delivery system and can therefore be regarded as independent of warhead weight. Thus the optimisation process need only concern itself with the expression \( NR^2 \) and the requirement therefore is to express \( N \) and \( R \) as functions of the warhead weight \( W \).

A.3 The simplest relationship for \( N \) is to make it inversely proportional to \( W \), but this ignores the packaging problem, i.e. the fact that there are certain weight overheads which are largely independent of \( W \) and which therefore result in a relationship of the form,

\[
N = \frac{A}{B + W}
\]  

where \( A \) and \( B \) are empirical constants, \( A \) being some overall weight restriction such as the payload of an aircraft and \( B \) the weight overhead for each weapon.

A.4 Empirical data suggests a relationship for \( R \) of the form

\[
R = CW^n
\]

and it is possible to justify this theoretically for some damage criteria. Combining (3) and (4) gives the effectiveness parameter \( NR^2 \) in terms of \( W \) namely,

\[
NR^2 = A0^2 \left( \frac{W^{2n}}{B + W} \right)
\]

from which the following criteria can be derived:
when $n > \frac{1}{2}$, the kill chance increases monotonically with $W$ and the optimum warhead weight is therefore the largest possible, namely
$$W_{\text{OPT}} = (A - B)$$ and then $n = 1$,
when $n < \frac{1}{2}$, equating to zero the derivative of $5$ with respect to $W$ gives,
$$W_{\text{OPT}} = \left(\frac{2n}{1 - 2n}\right)B$$ (6)

For instance a common value of $n$ is $\frac{1}{3}$ giving a value for $W_{\text{OPT}}$ of $2B$. $B$ is normally a very small fraction of $A$ (e.g. for the 600 lb cluster weapon containing 147 x 2 lb bomblet, $B = 1.6$ lb) so that gives a $W_{\text{OPT}}$ of a few pounds weight.

A.5 The procedure outlined above therefore leads to the result that the best way to attack a target is either with a single large weapon or with a large number of small weapons according to whether the value of $n$ in equation (4) is greater or smaller than $\frac{1}{2}$. This result would be credible if it meant that the single large weapon solution applied to the massive structural type target and the small multi-weapon solution applied to the light battlefield target, i.e. if the value of $n$ for these targets were respectively greater and less than $\frac{1}{2}$. But, in fact, this is not so, as a glance at the data in Ref.5 will show, and in particular the structural targets tend to have a value of $n$ of $\frac{1}{3}$ leading to the nonsensical answer, on the above analysis, that the best way of attacking these targets is with a cluster of small bomblets. The reason why the above analysis leads to such a false conclusion is discussed in the main text, section 2.4, but in essence it stems from the fact that the power law of equation (4) breaks down for values of $W$ below a certain 'threshold value' and this threshold value can be very large, say of the order of 1000 lb, for major structural targets.
Appendix B

TARGET VULNERABILITY DATA

B.1 Tank target

B.1.1 Direct hits

Fig. 5 plots the conditional kill chance for a direct hit on a typical tank target as a function of charge diameter for a shaped charge projectile. This curve typifies a complication to the direct-hit portion of the target vulnerability curve of Fig. 1 which applies to most targets, namely that some parts of the target are more vulnerable than others and that the percentage of target area vulnerable to a direct hit is a function of warhead size. This does not, of course, invalidate the curve of Fig. 1 nor the ensuing arguments given in the main text, it merely means that we can choose various levels of this direct-hit portion for different conditional kill probabilities and the choice still remains open to choose a level corresponding to 100% conditional kill chances. Fig. 5 suggests that a tank target can be killed by a warhead whose H.E. weight is in the range \( \frac{1}{2} \) to 5 lb (2 inch to 6 inch charge diameter) corresponding to conditional kill chances from 25% to 70%.

B.1.2 Near misses

Fig. 6 plots estimates from various sources of lethal range as a function of H.E. weight for near-misses against the same tank target, indicating for instance that at 20 ft from the centre of the tank the critical H.E. weight for a kill is in the range 100-1000 lb according to source and damage level and that it increases as the cube of the miss distance.

B.2 Aircraft Target

B.2.1 Direct hits

For internal detonations within an aircraft on the ground the H.E. weight varies between 1 \( \frac{1}{2} \) lb and 3 lb over a substantial portion of the target. For hits involving detonation on the skin of the target trials, results available suggest that the amount of H.E. required can vary from 3 \( \frac{1}{2} \) lb to 5 lb on a fighter type and from 5 lb to 20 lb on bombers, depending on the part of the target struck. This is shown diagramatically in Fig. 7 where the fighter fuselage radius is arbitrarily taken as 2\( \frac{1}{2} \) ft and the bomber fuselage as 4\( \frac{1}{2} \) ft radius.
B.2.2 Near-misses

Trials in which aircraft were damaged by external blast have been intensively analysed by Boulton\textsuperscript{6}, who deduced that for major structural damage the lethal range measured from the centre line of the nearest part of the aircraft structure was proportional to the square root of the weight of explosive. The factor of proportionality varied with the type of target, even the part of the target struck and also to some extent with the angle of impingement. Fig. 7 illustrates the variation of lethal distance and warhead weight for external blast warheads against hard and soft parts of the aircraft.

The lethal range of a well designed continuous rod can be taken to be of the same order as the maximum hoop radius and for a given weight this depends on the cross section of metal. Fig. 7 shows broadly the variation of warhead weight and lethal radius of the continuous rod against aircraft targets. There are believed to be practical difficulties in making an effective rod warhead below the minimum weight shown in Fig. 7. Trials results show that the structural damage by a rod depends on the nature of the target structure at the section struck, but very broadly they confirm the indication in Fig. 7 that a continuous rod warhead of a given weight will have a longer damage radius than a blast warhead. However, it must be remembered that rod warheads are not 'isotropic' warheads such as blast warheads are and they involve a matching problem of burst point relative to target.

B.3 BRIDGE TARGET

B.3.1 Direct hits

Fig. 8 plots conditional kill chance for hits on a single-lane all-truss bridge as a function of H.E. weight, indicating a threshold H.E. weight of the order of 1000 lb.

B.3.2 Near-misses

For data on near-miss effects by blast against bridges we have to go to nuclear sized charge weights and Ref. 7 gives a curve for a truss bridge of 150-250 ft span indicating that to cause 50% probability of severe damage requires at a distance of 300 ft, the lowest point on the curve, an equivalent H.E. weight for blast of 400 000 lb and increasing as the cube of the distance. The same reference suggests that ground shock will require even bigger charges, e.g. $10^6$ lb at 300 ft, scaling again as the cube of the distance. It should be noted that 'near'-misses is a misnomer for the nuclear data quoted above since
the scaling law will break down for miss-distances small enough to be of interest for non-nuclear weapons. This is due to the fact that the quoted scaling law, based as it is on peak over-pressure, does not take into account the variation of blast duration with charge weight. It is not possible therefore to fill in the bridge vulnerability curve for miss-distances between zero and the 300 ft mentioned above, since the necessary data from a relevant range of H.E. explosions does not exist.

B.4 Ship targets

B.4.1 Direct hits

Fig. 9 suggests that the threshold size of M.C. warhead to cause Category B damage with a direct hit on a destroyer is of the order of 1000 lb, i.e. about 500 lb of H.E. detonated internally.

B.4.2 Near-misses

Again for air-burst near-miss data we have to go to nuclear weapons and Ref. 7 gives a curve showing a lethal range of about 900 ft for a 1 kiloton air-burst. 1 kiloton is roughly equivalent in blast to $10^6$ lb of H.E. so the above-mentioned curve suggests a lethal range for nuclear weapons of 900 ft where C is the equivalent H.E. charge weight in lb. For the reasons given in section B.3.2 this relationship cannot be used for air-burst miss-distances of interest to H.E. weapons but for instance at 300 ft miss-distance it would give a critical H.E. weight of 40,000 lb. For under-water burst the near-miss portion of the vulnerability curve can be based on the well-known shock factor $C^2/R$ where C is the charge weight and R is the lethal range. This factor varies from 0.2 to 2 according to the target type and the damage category being considered, but whatever the value it means that on this portion of the vulnerability curve C increases as $R^2$. However, for the reasons discussed in section 2.5 of the main text, the curve may start in the region of, or even lower than, the threshold size for internal detonations.
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<th>No.</th>
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Fig. 1 Required weapon size as a function of miss distance.
Fig. 2 Variation of $P_k$ with $\frac{NR^2}{S^2}$ for different weapon delivery modes

- Independent aiming and ideal spaced salvo
- Random salvo (with optimised weapon dispn)
Fig. 3 Comparison of cluster bombs and bombs against a tank
Fig. 4 Comparison of cluster bombs and bombs against a parked aircraft
Fig. 5 Tank target: direct hits:
Conditional kill chance v charge dia
Fig. 6 Tank target: lethal radii for near misses

(1) O.E.G. Study: Complete or partial lack of fire power due to direct hit on vulnerable area (R = 1.3c \( \frac{1}{3} \))

(2) O.B. & A.O.R.G. (R = 2.6c \( \frac{1}{3} \))

(3) O.E.G. Study: Immobilization from near miss (R = 3.2c \( \frac{1}{3} \))

(4) R.A.R.D.E: Extrapolated from shell damage: Irreparable by crew in the field (R = 4.9c \( \frac{1}{3} \))

--- Notes blast criterion (R = 24c \( \frac{2}{3} \))
Fig. 7 Aircraft target vulnerability curve

Required weight (lb) of H.E. for blast curves of total warhead for continuous rod curves.

Major structural damage to targets on ground.

Blast warhead
- Hard region of target
- Soft region of target

Continuous rod warhead
- 1" rod
- 3/8" rod

Distance from tail of fuselage (ft)

Edge of bomber
Edge of fighter
100 200 ft

Bomber
Direct hit region
Near miss region

Fighter
Fig. 8 Bridge target: direct hits: conditional kill probability estimates for H.E. warheads

Curve based on U.S. estimates (Navweps rep. 7915) for single lane all truss bridge

Simple girder based on N.W.T.P - 20-1

Continuous truss based on N.W.T.P - 20-1
Fig. 9 Ship target: direct hits: conditional kill probabilities for various types of ships (British estimates)
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**Bisby, F.**

**Warhead Size Determination**

Royal Aircraft Establishment Technical Report 68254, October 1968

A hypothesis is suggested for guidance in weapon design, that the best warhead size for the attack of a specified target is the threshold size required for a direct hit to produce a kill. It is based upon the fact that this threshold size is one or more orders smaller than that required to cater for near-misses and that this economy in warhead size can be exploited to increase the chance of getting a direct hit, either by increasing delivery accuracy or by deploying more weapons, thereby leading to greater weapon effectiveness. Multi-weapon deployment modes are discussed, leading in particular to the further principle (over)

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that for weapons which have to be delivered in one attack opportunity, e.g., unguided air-to-surface weapons, the most efficient attack is with a spaced salvo of threshold weapons.

It is not claimed that the hypothesis is universally true. Whether it leads to a more effective weapon in any specific case will always require evaluation. Furthermore, it may be more applicable to air-to-surface weapons, mainly referred to as examples in this paper, than to other weapon types, such as anti-aircraft weapons, for which the warhead is a less dominant component.

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