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A RUBIDIUM MAGNETIC DIRECTION-FINDER
FOR ASW APPLICATION (U)

-TECHNICAL REPORT-
R68-1

L. M. Vallese

May 1968

Contract N00r 3358(00)

Office of Naval Research
Department of the Navy
Washington, D.C. 20360

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FOREWORD

The work described is based on investigations conducted at ITT AVIONICS/Nutley, New Jersey under Contract NONr 3358(00). Mr. Franklin Reick helped in taking the experimental data and in the design of the Rubidium optical system. The support of the Office of Naval Research (Dr. A. Shostak, Code 427) is gratefully acknowledged.
SUMMARY

An investigation of the Dehmelt effect in optically pumped Rubidium cells is presented. It is shown that, under impulsive magnetization reversal, an enhancement of the direct ground-state excitation transitions occurs; this results in a modification of the optical transient transmission response. The application of the impulsive Dehmelt effect for the design of a Rubidium magnetic compass is presented; such a compass may find application in ASW.
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A RUBIDIUM MAGNETIC DIRECTION-FINDER
FOR ASW APPLICATION (U)

L. M. Vallese

1. INTRODUCTION

Double-resonance phenomena in optically pumped spin systems, such
as Rubidium alkali vapor, metastable Helium, have been used success-
fully to design highly sensitive magnetometers which yield information
about the amplitude of the magnetic field, but not about its direction.
In these systems, there is generated a Larmor resonance signal whose
frequency is directly proportional to the intensity of the magnetic
field and whose amplitude is a function of the angle $\theta$ between said
field and the optical-pumping axis; in particular, the amplitude varies
as $\cos^2 \theta$ or $\sin \theta \cos \theta$, depending upon the configuration utilized.

The sensitivity of these magnetometers is of the order of $\pm 0.5$ gamma
(1 gamma = $10^{-5}$ Oe), and the minimum measurable field is of the order of
2 gammas. When these instruments are flown over the area under investi-
gation, they provide a recording of the intensity of the magnetic field
along the flight path; from these values, maps of constant $H$ contours
are derived. The shapes of these contours depend upon the geographical
coordinates, the geological features, the mineral deposits, etc., and are
subject to erratic changes in time. The $H$ lines (lines tangential to $H$)
follow roughly the distribution which would correspond to a linear mag-
net having a length about one-third that of the diameter of the earth,
and along a direction about 20 degrees from that of the main axis of the
earth; both the direction and the magnitude of the magnetic field are
affected by the presence of high-permeability materials.

For applications to search of submerged submarines, a magnetometer
capable of giving the direction as well as the magnitude of the magnetic

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field would be highly useful. Such device would provide an instantaneous indication of the presence of a magnetic anomaly, and also would add important information about its location, its general shape, its motion, etc.

The design of a rubidium magnetic direction-finder is described in the following; its operation is based on the application of the Dehmelt method\(^{(1)}\) of detection of the spin relaxation of optically polarized rubidium atoms, using impulsive magnetic-field reversal. The Dehmelt effect is dependent upon the angle \(\theta\) between magnetic field and optical pumping axis and exhibits a singular behavior when such angle is zero (180°). A clear evidence of such dependence may be obtained by recourse to impulsive magnetic-field excitation. Such an approach is illustrated in the following, along with its application to the design of a submarine-detection system.

2. OPTICAL PUMPING OF RUBIDIUM\(^{87}\) VAPOR

The energy-level diagram of Rb\(^{87}\) is shown in Fig. 1; Rb\(^{87}\) has a nuclear spin \(I = 3/2\), ground state \(5S_{1/2}\), states \(5P_{1/2}\) and \(5P_{3/2}\) excited by optical-resonance radiation respectively of wavelengths 7947 Å (D\(_1\) line) and 7800 Å (D\(_2\) line), etc.

The states are split by hyperfine interaction, with separate levels characterized by different \(F\) values (\(F\) is the sum of the nuclear angular momentum and of the resultant electron spin), and with transitions \(\Delta F = \pm 1, \Delta m_F = 0\) at microwave frequencies. Under the action of an external magnetic field, Zeeman-splitting with ground-state transitions \(\Delta F = 0, \Delta m_F = \pm 1\) at frequencies 700 H KHz (\(H\) is gauss), with transitions \(\Delta F = \pm 1, \Delta m_F = \pm 1, 0\) at frequencies (6834.6 ± kHz) MHz (where \(k = 0.7, 1.4, 2.1\), depending on the levels involved), etc., occur.

By optical pumping with one of the resonance radiations, the atoms of the ground state can be excited by absorption transition to one of the
upper levels, from which they can return to the ground state. If the light is in a definite state of polarization, the allowed absorption transitions must satisfy the condition of conservation of the angular momentum (i.e., \( \Delta m_F \) remains zero for plane-polarized light, and \( \Delta m_F = +1 \) or \( \Delta m_F = -1 \) respectively for right- or left-circularly polarized light). The graphs of Fig. 2 illustrate the optical-pumping transitions for the \( D_1 \) line and for the \( D_2 \) line respectively, in the case of negligible magnetic field; thus, considering \( D_1 \)-line pumping, with \( \sigma^+ \) radiation, the only resulting absorption transition is \( \Delta F = 1, \Delta m_F = +1 \), which excites the atoms of the \( F = 1, m_J = -1/2 \) level of the \( 5S_{1/2} \) state to the \( F = 2, m_J = +1/2 \) level of the \( 5P_{1/2} \) state. The excited atoms relax spontaneously to the ground state, reemitting a \( D_1 \) photon, following all allowed transitions; thus, a fraction of the originally excited atoms is transferred to the level \( F = 2, m_J = +1/2 \) of the ground state. Repetition of this process under the action of the optical pumping results in an equilibrium distribution which depends upon the existing relaxation phenomena; when the vapor has acquired its final polarization state, it exhibits minimum absorption (maximum transparency), and has a large number of its atoms in the \( m_J = +1/2 \) level, where the atoms' magnetic moment is parallel to the external magnetic field \( H \).

When the external field is not negligible, splitting of the above levels takes place; in this case, more than one optical absorption transition between levels with \( \Delta m_F = +1 \) is obtained with \( \sigma^+ \) optical-pumping radiation (Fig. 2). The excited atoms return to the ground state and approach asymptotically an equilibrium distribution corresponding to maximum vapor transparency. In general, when the magnetic field has a component perpendicular to the optical-pumping axis, a mixing of the atom populations of the various \( m_F \) levels of each \( F \) state occurs; as a result, the vapor transparency remains low. When the magnetic-field direction coincides with that of the optical axis, the equilibrium polarization and the vapor transparency are greatest.
Experimental evidence of the orientation and alignment of alkali-metal vapor atoms (sodium) by means of circularly polarized radiation was obtained by Hawkins\(^2\). Measuring the polarization ratio \(\frac{\sigma^- - \pi}{\sigma^+ + \pi}\) of the light scattered at 90° to the axis of quantization (pumping axis), and varying the amplitude of the axial component of the magnetic field, the results shown in Fig. 3 were found; in this case, the transversal component of \(H\) was reduced to a value below 10\(^{-3}\) gauss, but no compensation of a-c fields was provided. The vapor polarization is maximum for large values of the axial \(H\) components and goes through a minimum in correspondence of zero axial field.

3. TRANSIENT BEHAVIOR OF OPTICALLY POLARIZED ALKALI ATOMS

Dehmelt\(^1\) suggested a method for the investigation of the relaxation time constant of optically polarized alkali atoms, based on the study of their transient behavior under an abruptly reversed axial magnetic field. Allowing the vapor to reach its equilibrium polarization with an axial field of about 0.5 gauss produced with a ring coil energized from a short-time constant circuit (after cancelling the earth's field) and then reversing the latter by reversing the energizing voltage, the +m and the -m levels are interchanged simultaneously, thus resulting in an overpopulation of the more absorbing levels. The atom ensemble is no longer in equilibrium and, through the mechanism of excitation by means of the optical-pumping action as well as through its natural relaxation, approaches a new equilibrium with time constant depending upon the intensity of the light and upon the intrinsic relaxation properties of the system. If the pump-light intensity is reduced further and further, the decay time approaches asymptotically the value of the intrinsic relaxation-time constant of the atoms.

Typical experimentally determined transient responses taken from the cited Dehmelt reference and showing the exponential increase of the transmitted light intensity after an abrupt reversal of the axial \(H\) field are shown in Fig. 4, for various values of resonant light intensity \(I\); the
corresponding time constants are plotted versus \( I \) in Fig. 5 (these data were taken with a sodium cell).

Dehmelt developed a quantitative analysis of the phenomenon, assuming that the optical-pumping process coupled with collisions would result in a completely random redistribution of the excited hyperfine magnetic sublevels. On the other hand, an analysis based on pumping with a single circularly polarized \( D_1 \) line was developed by Franzen and Emslie(3). In general, indicating with \( p_k(t) \) the occupational probability of the \( k \)-th ground-state level (\( k = 1, 2 \ldots 8 \)), one can write the following system of differential equations:

\[
\frac{d}{dt} p_k = \sum_{i=1}^{8} (b_{ik} + w_{ik}) p_i - p_k \sum_{j=1}^{8} (b_{kj} + w_{kj}) \sum_{k=1}^{8} p_k = 1
\]

In these equations, the prime is used to indicate exclusion of the \( i=k \), \( j=k \) values in the summations; the coefficients \( b_{ik} \), \( b_{kj} \) represent the probabilities per unit time for transitions between levels of the ground state involving absorption or reemission of a photon; and the coefficients \( w_{ik} \), \( w_{kj} \) represent the similar probabilities per unit time for transitions involving only direct relaxation. The first summation in the set (1) gives the rate of excitation of the atoms to the \( k \)-th level, and the second summation gives the rate at which the atoms leave the \( k \)-th level.

If the optical-pumping effect is negligible, equations (1) can be written as follows:

\[
\frac{d}{dt} p_k = \sum_{i=1}^{8} w_{ik} p_i - p_k \sum_{j=1}^{8} w_{kj}
\]

Assuming in first approximation that the values of \( w_{ik} \) are the same for all \( k \) levels, one has:
\[
\sum_{i=1}^{G} w_{ik} = 8w = 1/T
\]  
\[ (3) \]

i.e., \( w_k = 1/8T \) where \( T \) is the spin-relaxation time for the atoms in the ground state.

From Eq. (2), there follows:

\[
\frac{d}{dt} P_k = \frac{1}{8T} - \frac{P_k}{T} \]  
\[ (4) \]

This set of equations shows that, within the assumptions made, in equilibrium the populations of all levels are equal and no orientation or alignment is obtained. In practice, the populations would differ in accordance with the Boltzman distribution.

Returning to the consideration of the general case of Eq. (1), and maintaining the above assumptions, one finds the following results:

\[
\frac{d}{dt} P_k = \sum_{i=1}^{G} b_{ik} P_i - P_k \sum_{j=1}^{G} b_{kj} + \frac{1}{8T} - \frac{P_k}{T} \]  
\[ (5) \]

From these equations, the time dependence of the occupational probabilities can be computed, provided the values of the coefficients \( b_{ik} \) are known.

When the magnetic field is negligible, the analysis is simplified considerably, since only two levels need be considered; specifically, one finds the following results:

\[
\frac{d}{dt} P_1 = b_{21} P_2 - b_{12} P_1 + \frac{1}{2T} - \frac{P_1}{T} \]  
\[ (6a) \]

\[
\frac{d}{dt} P_2 = b_{12} P_1 - b_{21} P_2 + \frac{1}{2T} - \frac{P_2}{T} \]  
\[ (6b) \]
Using the relationship $p_1 + p_2 = 1$, these are written:

$$\frac{dp_1}{dt} + (b_{12} + b_{21} + \frac{1}{T}) p_1 = b_{21} + \frac{1}{2T}$$

$$\frac{dp_2}{dt} + (b_{12} + b_{21} + \frac{1}{T}) p_2 = b_{12} + \frac{1}{2T}$$

wherefrom:

$$p_1(t) = A e^{-(b_{12} + b_{21} + \frac{1}{T})t} + \frac{1 + 2b_{21} T}{2} \left[ 1 + (b_{12} + b_{21})T \right]$$

$$p_2(t) = 1 - p_1(t)$$

The coefficient $A$ is determined from initial conditions. As an example, assuming that Level $I$ saturation is reached at $t = 0$, one has:

$$p_1(0-) = p_1(0+) = 1$$

$$p_2(0-) = p_2(0+) = 0$$

Substituting in Equation 8, one finds:

$$p_1(t) = \frac{1 + 2b_{21} T}{2} \left[ 1 - e^{-(b_{12} + b_{21} + \frac{1}{T})t} \right] + e^{-(b_{12} + b_{21} + \frac{1}{T})t}$$

$$p_2(t) = 1 - p_1(t)$$

When optical pumping is performed with $\sigma^+ D_1$ radiation, only the transition $\Delta m = +1$ is allowed, and $b_{21} = 0$ (this assumes that no mixing occurs in the excited states). A graphical representation of Eqs. (10) is given in Fig. 6.
Experimentally, one detects the intensity of the optical-pumping radiation $I_t$ transmitted through the cell. This is a function of the average probability that the atoms will absorb light--i.e., $I_t = I_p - I_a$, where $I_p$ and $I_a$ are respectively pumping and absorbed light intensities, and

$$I_a(t) = \beta(t) = \frac{1}{2} (b_{12} P_1 + b_{21} P_2) \quad (11)$$

In the above-considered case, the latter relationship reduces to the following:

$$I_a(t) = \beta(t) = \frac{b_{12}}{2} P_1 = \frac{b_{12}}{2} (1 - P_2) \quad (12)$$

Thus, the time dependence of the absorbed light intensity is the same as that of the occupational probability $P_1(t)$, and exhibits a time constant

$$\gamma = \frac{T}{1 + \frac{b_{12}}{T}} \quad (13)$$

These considerations are applicable to the graphs of Fig. 4.

For sufficiently moderate value of the pumping intensity $I_p$, the coefficient $b_{12}$ is proportional to $I_p$; therefore, measuring $\gamma$ for decreasing values of $I_p$, the spin-relaxation time $T$ is obtained (see the example of Fig. 5).

More generally, if the magnetic field is not negligible, the ground-state levels involved are eight, and the set of Equation (11) applies. Considering the case of $\sigma + D_1$ pumping, Franzen and Emslie(3) have computed the time-dependent occupational probabilities. The relative absorption probability $(1/\beta_o) \sum b_{kj}$ for each level $k$, where

$$\beta_o = \frac{1}{8} \sum \sum b_{ij} \quad (14)$$

is the average light-absorption probability per unit time, are:
TABLE I
RELATIVE ABSORPTION PROBABILITIES

<table>
<thead>
<tr>
<th>F, m_F</th>
<th>k</th>
<th>( \frac{1}{\beta_o} \sum b_{kj} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, -1</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>1, 0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1, +1</td>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>2, -2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2, -1</td>
<td>5</td>
<td>3/2</td>
</tr>
<tr>
<td>2, 0</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2, +1</td>
<td>7</td>
<td>1/2</td>
</tr>
<tr>
<td>2, +2</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \beta(t) = \sum_k \sum_j b_{kj} \rho_k \]  \hspace{1cm} (15)

Assuming no mixing within the levels of the excited state, and writing the set of Equation (1) as follows:

\[ \frac{d \rho_k}{dt} = \frac{\beta_o}{\beta} \sum_{i=1}^{8} B_{ik} \rho_i + \frac{1}{\beta T} \]  \hspace{1cm} (16)

where \( k = 1, 2, ... 8 \), and

\[ B_{ik} = \frac{\beta}{\beta_o} b_{ik} \text{ for } i \neq k \]

\[ B_{kk} = \frac{-\beta}{\beta_o} \left[ \frac{1}{T} + \sum_j b_{ij} \right] \]  \hspace{1cm} (17)

one can compute the transition probabilities \( B_{ik} \) (see Table II):
### Table II

<table>
<thead>
<tr>
<th>$(F, m_F)$</th>
<th>$(1, -1)$</th>
<th>$(1, 0)$</th>
<th>$(1, 1)$</th>
<th>$(2, -2)$</th>
<th>$(2, -1)$</th>
<th>$(2, 0)$</th>
<th>$(2, 1)$</th>
<th>$(2, 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(F, m_F)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1, -1)$</td>
<td>-9p-11/3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(1, 0)$</td>
<td>2/3</td>
<td>-9p-19/3</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>5/3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(1, 1)$</td>
<td>1/3</td>
<td>5/3</td>
<td>-9p-6</td>
<td>0</td>
<td>1</td>
<td>5/3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$(2, -2)$</td>
<td>0</td>
<td>0</td>
<td>-9p-28/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(2, -1)$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>10/3</td>
<td>-9p-9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(2, 0)$</td>
<td>2/3</td>
<td>5/3</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>-9p-19/3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(2, 1)$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>-9p-10/3</td>
<td>0</td>
</tr>
<tr>
<td>$(2, 2)$</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4/3</td>
<td>-9p</td>
</tr>
</tbody>
</table>

For Sodium Optically Pumped with Circularly Polarized $D_1$ Radiation. No reorientation in the excited state.
A similar computation may be made in the case in which complete mixing of the populations of the excited levels takes place (see Table III). Such mixing would occur when the magnetic field has a component normal to the axis of quantization. In Ref. 3, graphs depicting the time dependence of the functions $P_k(t)$ for the two cases above considered are given; these graphs are shown in Figs. 7 and 8.

Finally, Franzen and Emslie have also computed the special case in which the spin-relaxation time is exceptionally long; corresponding equilibrium values of the various $P_k$ are given in Table IV, where $Q = 1/\beta_0 T$. They have also pointed out that the light-absorption probability approaches the values $7Q\beta_0 = 7/T$ and $3.63/T$ respectively for the cases of no excited level mixing or complete mixing. Thus, the percentage of light absorbed is independent of $I_p$ and is inversely proportional to $T$.

4. IMPULSIVE DEHMELT EFFECT

In the previous Section, the transient behavior of optically pumped alkali atoms, which occurs when the sign of the axial magnetic field is reversed, has been reviewed.

The results obtained by Dehmelt and by Franzen and Emslie show that the phenomena involved may be monitored through the corresponding variations of the intensity of the transmitted light; these are exponential and exhibit time constant and asymptotic amplitude both dependent upon the spin relaxation time of the alkali atoms and upon the optical pumping transition probabilities. In particular, the asymptotic amplitude is maximum (i.e., the corresponding light absorption is minimum) when the magnetic-field direction coincides with that of the quantization axis. Thus, a magnetic compass could be designed using a servo system capable of controlling the direction of the quantization axis so that a maximum light transmission is obtained; by recourse to a periodic reversal of the axial component of the magnetic field, an AC signal of amplitude proportional to the intensity of the transmitted light may be derived. More
| $|F, m_F|$ | $(1, -1)$ | $(1, 0)$ | $(1, 1)$ | $(2, -2)$ | $(2, -1)$ | $(2, 0)$ | $(2, 1)$ | $(2, 2)$ |
|---|---|---|---|---|---|---|---|---|
| $(1, -1)$ | 1 | -8p-3.5 | 1 | 1.5 | 2 | 1.5 | 1 | 0.5 | 0 |
| $(1, 0)$ | 2 | 0.5 | 0 | 1.5 | 2 | 1.5 | 1 | 0.5 | 0 |
| $(1, 1)$ | 3 | 0.5 | 0 | 1 | -8p-10.5 | 2 | 1.5 | 1 | 0.5 | 0 |
| $(2, -2)$ | 4 | 0.5 | 0 | 1 | 1.5 | -8p-14 | 1.5 | 1 | 0.5 | 0 |
| $(2, -1)$ | 5 | 0.5 | 0 | 1 | 1.5 | 2 | -8p-10.5 | 1 | 0.5 | 0 |
| $(2, 0)$ | 6 | 0.5 | 0 | 1 | 1.5 | 2 | 1.5 | -8p-7 | 0.5 | 0 |
| $(2, 1)$ | 7 | 0.5 | 0 | 1 | 1.5 | 2 | 1.5 | 1 | -8p-3.5 | 0 |
| $(2, 2)$ | 8 | 0.5 | 0 | 1 | 1.5 | 2 | 1.5 | 1 | 0.5 | -8p |

For Sodium Optically Pumped with Circularly Polarized D_1 Radiation. Complete reorientation in the excited state.
TABLE IV

Steady-state occupation probabilities of the eight magnetic sublevels of the ground state of the sodium atom optically pumped with circularly polarized D$_2$ radiation. The occupation probabilities are expressed in terms of $P = 1/\beta_o T$ which is assumed to be small compared to unity. The last column gives the steady-state value of the ratio $\beta/\beta_o$ of the average absorption probability per atom per unit time to its value at thermal equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$\beta/\beta_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Orientation</td>
<td>0.384p</td>
<td>0.398p</td>
<td>0.941p</td>
<td>0.107p</td>
<td>0.194p</td>
<td>0.398p</td>
<td>1.45p</td>
<td>1-3.87p</td>
<td>3.63p</td>
</tr>
<tr>
<td>Complete Reorientation</td>
<td>2p</td>
<td>p</td>
<td>2/3p</td>
<td>1/2p</td>
<td>2/3p</td>
<td>p</td>
<td>2p</td>
<td>1-7.89p</td>
<td>7p</td>
</tr>
</tbody>
</table>
specifically, assume that the magnetic field has no component transversal
to the quantization axis, and that it has a superimposed sinusoidal time
variation; i.e.,

\[ H = H_0 + \Delta H_m \cos \omega t \]  

(18)

If \( \Delta H_m \) is less than \( H_0 \), the magnetic-field direction remains constant,
the atom system remains polarized in the same direction, and the trans-
mittted light exhibits constant intensity; if \( \Delta H_m \) is larger than \( H_0 \),
the magnetic-field direction is reversed twice during each period, the
atom system correspondingly reverses its polarization, and the transmitted
light exhibits the transient variations due to sudden absorption, excita-
tion, and repumping effects as illustrated by Dehmelt. On the other hand,
if the magnetic-field axis does not coincide with the quantization axis,
a mixing of the populations of the excited state takes place, and the
transmitted-light intensity exhibits correspondingly a modulation; when a
reversal of the axial magnetic field occurs, the latter intensity exhib-
its, in addition, transient variations similar to those previously dis-
cussed. These phenomena are illustrated schematically in the graphs of
Fig. 9.

Experimental data, taken with a \(^{87}\)Rb cell (with 5mm Ar as buffer gas),
pumped with a \( \sigma^+ D_1 \) beam, are shown in Fig. 10. The quantization axis
was made to coincide with the direction of the local earth field; a low-
frequency \( (35 \text{ Hz}) \) sinusoidal component was superimposed to the latter.
The photographs show the intensity of the transmitted light for the cases
in which no reversal or reversal of the polarization occurs; the light
exhibits lamp noise with superimposed transient phenomena of exponentially
decaying absorption in correspondence of the polarization reversals. When
the axes of quantization and of magnetization diverge, the light exhibits
in addition the familiar sinusoidal modulation.

As previously pointed out, the transient phenomena observed by Dehmelt
represent the response of a first-order differential system characterized
by differential equation of type:

-14-
\[ H = H_0 (1 - e^{-\beta t}) U(t) \]  

(25)

where \( \beta \) is the time constant of the Helmholtz coil pair; hence, letting \( \alpha = 1/T + 1/T_p \), the solution of Eq. (24) is written:

\[ N_z = \left( \frac{X_o H_0}{T \alpha} + \frac{M_p}{T_p \alpha} \right) (1 - e^{-\alpha t}) - \frac{X_o H_0}{T (\alpha - \beta)} \left( e^{-\beta t} - e^{-\alpha t} \right) \]  

(26)

Thus, the transient response as shown by the variation of the intensity of the transmitted light exhibits characteristics depending upon the time constants of the spin system and of the Helmholtz coil pair. In particular, if \( \beta \gg \alpha \), Eq. (26) may be approximated as follows:

\[ N_z = \left( \frac{X_o H_0}{T \alpha} + \frac{M_p}{T_p \alpha} \right) (1 - e^{-\alpha t}) - \frac{X_o H_0}{T \beta} e^{-\alpha t} \]  

(27)

where the last term of the second member provides a correction only in the vicinity of the origin.

Similarly, the solution for the case

\[ H = H_0 + \Delta H \sin \omega t \]  

(28)

may be derived. An interesting case of transient response of Rubidium optically pumped cells was noted some time ago by F. Reick at ITT AVIONICS. Reversing abruptly the direction of the axial magnetic field, he found that, when the axes of magnetization and of quantization were coincident, an anomalous transient response occurred. The clarification of this unexplained result was undertaken in the present report.

Consider the transient response of the system represented by Eq. (24) under applied magnetic fields as follows:

\[ H = H_1 U(t) + H_0 (1 - e^{-\beta t}) \]  

(29)

and

\[ H = H_1 \delta(t) + H_0 (1 - e^{-\beta t}) \]  

(30)
\[ H = H_0 (1 - e^{-\beta t}) U(t) \]  

(25)

where \( \beta \) is the time constant of the Helmholtz coil pair; hence, letting \( \alpha = 1/T + 1/T_p \), the solution of Eq. (24) is written:

\[
N_z = \left( \frac{X_0 H_0}{Ta} + \frac{M}{T_p \alpha} \right) \left( 1 - e^{-\alpha t} \right) - \frac{X_0 H_0}{T(\alpha - \beta)} \left( e^{-\beta t} - e^{-\alpha t} \right) 
\]

(26)

Thus, the transient response as shown by the variation of the intensity of the transmitted light exhibits characteristics depending upon the time constants of the spin system and of the Helmholtz coil pair. In particular, if \( \beta \gg \alpha \), Eq. (26) may be approximated as follows:

\[
N_z = \left( \frac{X_0 H_0}{T_x} + \frac{M}{T_p \alpha} \right) \left( 1 - e^{-\alpha t} \right) - \frac{X_0 H_0}{T \beta} e^{-\alpha t} 
\]

(27)

where the last term of the second member provides a correction only in the vicinity of the origin.

Similarly, the solution for the case

\[ H = H_0 + \Delta H \sin \omega t \]

(28)

may be derived. An interesting case of transient response of Rubidium optically pumped cells was noted some time ago by F. Reick at ITT AVIONICS. Reversing abruptly the direction of the axial magnetic field, he found that, when the axes of magnetization and of quantization were coincident, an anomalous transient response occurred. The clarification of this unexplained result was undertaken in the present report.

Consider the transient response of the system represented by Eq. (24) under applied magnetic fields as follows:

\[ H = H_1 U(t) + H_0 (1 - e^{-\beta t}) \]

(29)

and

\[ H = H_1 S(t) + H_0 (1 - e^{-\beta t}) \]

(30)
In the first case, the magnetic field exhibits a step-amplitude variation, and in the second case it exhibits an impulsive-amplitude variation. The corresponding solutions of Eq. (24) are obtained adding to Eq. (26) respectively the following terms:

\[
X_0 \frac{H_1}{T \alpha} (1 - e^{-\alpha t}) U(t) \quad (31)
\]

and

\[
X_0 \frac{H_1}{T \alpha} e^{-\alpha t} U(t) \quad (32)
\]

Thus, a step-amplitude variation of \( H \) does not result in a basic modification of the waveform of the transient response; however, an impulsive-amplitude variation of \( H \) adds a term which subtracts from the exponential variation observed by Dehmelt.

The experimental verification of this phenomenon was obtained by applying an impulsive magnetization to the Helmholtz coil pair previously described.

In Fig. 11, photographs of waveforms of the Helmholtz coils' current (exclusive of DC component) and of the intensity of the transmitted light are shown for two separate cases; the axes of the Helmholtz coils and of optical pumping coincide, but the direction of the earth's magnetic field makes an angle of about 10° with the latter in the case of Fig. 11a, and coincides with it in the case of Fig. 11b. The current waveform exhibits the impulsive components impressed, but not the DC components.

Transients of the light intensity which appear in correspondence of the applied impulsive reversal in Fig. 11a are partly missing in the case of Fig. 11b. By careful adjustments of the alignment, it is possible to obtain the cancellation of both transients, although this condition is found to be critical.

Recapitulating, under impulsive transient conditions, the transient response of the Dehmelt effect may be modified such that, when the axes of quantization and of magnetization coincide, the spin polarization occurs
mainly by direct ground-state transitions, and the optical excitation process is decreased. This effect may be utilized to design a Rubidium magnetic compass, consisting of a servo system whose control signal is obtained from detection of the intensity of the transmitted light.

5. APPLICATION TO THE DESIGN OF A RUBIDIUM MAGNETIC COMPASS

As previously pointed out, the transient response phenomena of Rubidium optically pumped cells may be utilized for the design of a magnetic compass. Such a device would be very useful in ASW, giving instantaneous information about the direction of the magnetic field and about its rate of change in time.

A magnetic compass may be designed as a dynamic or as a static device. In the first case, the system would be rotated in space until its optical axis is oriented along the direction of the resultant magnetic field; and, in the second case, external magnetic-field components would be added by means of three pairs of Helmholtz coils, so that the net field is parallel to the optical axis. In either case, the electrical control is performed by means of a signal obtained from the detector output.

The static magnetic direction finder appears more convenient than the dynamic type for practical applications. This system, combined with a conventional magnetometer, would provide information about the magnitude and the direction of the earth's magnetic field, and thus would simplify the investigation of magnetic anomalies.

It is also possible to incorporate the principle of direction detection with that of magnitude detection. For example, a Rubidium magnetometer which oscillates with a frequency proportional to the magnetic field may be modified with the addition of the periodic switching of the axial magnetic field. The oscillator becomes then frequency-modulated, exhibiting the above-described transient phenomena as modulation. The unit may be operated intermittently as a conventional single-frequency oscillator and as a frequency-modulated oscillator, thus providing in a single package information about the magnitude and the direction of the magnetic field.
An example of realization of the above-described magnetic direction-finder is shown in Fig. 12.

The basic electro-optic system consisting of Rubidium cell, Rubidium lamp, collimator, filter and polarizer, detector and pair of coils coaxial with the optical axis is shown in Box A; the output of the detector is amplified, passed through a narrow bandpass filter $F$, detected and differentiated. Three pairs of Helmholtz coils arranged along perpendicular axes provide magnetic field components such that the net resultant field (including the earth's field) is directed along the optical-pumping axis. The vertical-axis coil is fed with a saw-tooth waveform as described later; and the horizontal-plane coils are fed with an AC voltage and provide a resultant field which rotates counterclockwise or clockwise. The procedure of automatic control of the three currents to obtain the axial alignment above indicated will be discussed in the following.

A pair of coils inside Box A generate a magnetic field along the optical axis; these coils are supplied with a square-wave current of frequency $f$ and generate at the detector signals of the type shown in Fig. 11. When the alignment of net magnetic field and optical axis is realized, the signal obtained at the detector is zero; in all other conditions, the signal consists of pulses at repetition rate $f$. Thus, if the bandpass filter has center frequency $f$, the signal output $V_1$ is null when the magnetic field is aligned with the optical axis.

The wave shapes of the signals $V_1$ and $\frac{dV_1}{d\theta}$ are plotted in Fig. 13 as functions of the azimuth $\theta$. (Note that $\theta$ varies with $t$.) It is seen that the sign of signal $\frac{dV_1}{d\theta}$ changes from negative to positive when $\theta$ crosses from values below to values above the critical angle $\theta_c$ at which alignment with the optical axis occurs.

Assume that the net vertical component of the magnetic field is zero; the resultant field $H$ rotates in the horizontal plane and, on crossing the $\theta_c$ direction, produces a signal at $\frac{dV_1}{d\theta}$. As shown in Fig. 12, this signal is utilized to operate a switch $S$; thus, if $H$ is rotated clockwise initially, its direction of rotation is reversed as soon as it
crosses $\theta_0$. Also, $H$ crosses $\theta_0$ a second time and again generates a signal $\frac{dV_1}{d\theta}$ of such polarity that the field direction is reversed, and so on. In conclusion, the signal $\frac{dV_1}{d\theta}$ appears as an alternating voltage, and the field $H$ acquires a hunting angular motion around the direction $\theta_0$.

Consider now the control of the vertical-field component. The vertical-field coil is supplied with a saw-tooth voltage whose repetition frequency is much slower than that of the rotating horizontal field. The saw-tooth signal is generated with a conventional negative resistance and R-C oscillator, in which the resistance $R$ has a value which can be varied within a wide range; such resistance is obtained by means of a field-effect transistor whose bias is provided by rectifying the signal $\frac{dV_1}{d\theta}$ previously discussed. Thus, assume that the vertical field is varied slowly until the resultant is approximately compensated. The alternating signal $\frac{dV_1}{d\theta}$ then appears, and upon rectification changes the bias of the field-effect transistor. The resistance $R$ becomes large and causes the discharge of Capacitor $C$. The process continues indefinitely, and the current supplied to the vertical-field coil remains at the value for which compensation occurs.

Recapitulating, the system illustrated in Fig. 12 provides a means for generating a net field aligned along the optical axis. The direction and the magnitude of the earth's magnetic field can be compensated for readily from the values of the currents $I_1$, $I_2$, $I_3$ which flow through the coils $L_1$, $L_2$, $L_3$ of the system at equilibrium.

6. CONCLUSION

In this report, an investigation of the Dehmelt effect in Rubidium optically pumped cells has been discussed. It has been shown that, under impulsive magnetic-field switching, the transient response of the spin system exhibits enhanced direct polarizing transitions in the ground state, resulting in decreased optical absorption. In general, the Dehmelt effect
may be utilized for the design of a Rubidium magnetic compass because its response presents a singularity when magnetization and optical pumping coincide. An example of realization of one such type of magnetic compass has been discussed.

(c) This device could be utilized for the detection of submarines since it would permit detection of the magnetic-field direction and of its variations as a function of the motions of the magnetic source.
REFERENCES


FIG. 1 - ENERGY-LEVEL DIAGRAM OF $^{87}$Rb
FIG. 2 - ENERGY-LEVEL DIAGRAM WITH PUMPING TRANSITIONS VIA THE $5P_{1/2}$, $F = 2$ STATES

FIG. 3 - POLARIZATION RATIO OF THE SCATTERED RESONANCE RADIATION AS A FUNCTION OF THE APPLIED AXIAL FIELD COMPONENT, INCIDENT LIGHT CIRCULARLY POLARIZED
FIG. 4 - WAVEFORMS OF LIGHT-INTENSITY TRANSIENTS FOR VARIOUS PUMPING LEVELS (Dehmelt)

FIG. 5 - PLOT OF TIME CONSTANT $\tau^*$ VERSUS LIGHT INTENSITY (Dehmelt)
\[ \frac{1}{\tau^*} = \frac{1}{T} + b_{12} + b_{21} \]

**FIG. 6 - EXAMPLE OF REPRESENTATION OF** \( p_1(t), p_2(t) \)

(Equation 10)
FIG. 7 - PREDICTED RATES OF CHANGE OF OCCUPATIONAL PROBABILITIES FOR THE EIGHT SUBLEVELS OF THE GROUND STATE OF Na23. NO REORIENTATION IN THE EXCITED STATE.

FIG. 8 - PREDICTED RATES OF CHANGE OF OCCUPATIONAL PROBABILITIES FOR THE EIGHT SUBLEVELS OF THE GROUND STATE OF Na23. COMPLETE REORIENTATION IN THE EXCITED STATE.
FIG. 9 - RELATIONSHIPS BETWEEN AXIAL MAGNETIC FIELD AND INTENSITY OF TRANSMITTED LIGHT WHEN MAGNETIC FIELD IS PARALLEL TO OPTICAL AXIS. (Note: A sinusoidal modulation is superimposed on I when a component of H normal to the optical axis is present.)
FIG. 10 - MAGNETIZATION REVERSAL EQUIPMENT

a) Experimental Set-Up

b) Photographs of AC Magnetic Field Waveform and
   of Transmitted Light Intensity with $\Delta H_m < H_0$

c) Photographs of AC Magnetic Field Waveform and
   of Transmitted Light Intensity with $\Delta H_m > H_0$

(Note that a lamp-noise component is present on the transmitted light intensity.)
FIG. 11 - IMPULSIVE MAGNETIZATION REVERSAL
(Photographs of Helmholtz-coil impulses and of transmitted light intensity)

a) Earth's magnetic field at 10° to optical-pumping axis

b) Earth's magnetic field parallel to optical-pumping axis
FIG. 12 - PROPOSED DESIGN OF MAGNETIC DIRECTION FINDER
FIG. 13 - WAVEFORMS OF SIGNALS PERTINENT TO FIGURE 12

$V_1$ and $\frac{dV_1}{d\theta}$ shown for Equilibrium Condition