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INVESTIGATION OF A LASER SYSTEM FOR AIR-TO-SEA
SUBMARINE DETECTION AND COMMUNICATION (C)

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Contract NOur 3358(00)
-For Period July 1966 to June 1967-

Office of Naval Research
Department of the Navy
Washington, D.C.

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This report covers the work of analysis and experimentation conducted at ITT Federal Laboratories, Nutley, New Jersey under Contract N00r 3358(00) - Task "Investigation of a Laser System for Air-to-Sea Submarine Detection and Communication". The work was performed during the period July 1966 to June 1967. Dr. A. Shostak, Head of the Electronics Branch, Office of Naval Research, and Lt. Ronald Troutman, Project Officer, were the U.S. Navy Technical monitors for the Contract. Dr. L. M. Vallese, Mr. J. Bula, Mr. R. Lachenauer were the ITTFL personnel engaged in the project work.

The support and guidance offered by Dr. A. Shostak in the performance of this work are sincerely appreciated.
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The following report summarizes the investigations conducted during the period July, 1966 to June 1967 under Contract NOnr 3358(00). The report comprises three sections. Section A, entitled "Laser Detection of Submarine Signatures", discusses the fundamentals of mechanical wave propagation in the sea both for transverse-type waves and for longitudinal-type waves, and the corresponding surface-wave distributions; these characterize respectively the "optical signature" and the "acoustic signature" of submarines and torpedoes. Two separate methods which utilize laser techniques for readout of such signatures are described; experimental results of the investigation pertaining to readout of the acoustic signature are presented. The material of Section A has also appeared as a paper of the same title in the Proceedings of the Third DOD Laser Technology Conference, Pensacola, Florida, April 1967.\(^1\)

Section B discusses a method of detection of the optical signature of submarines and torpedoes using a pulsed infrared laser beam for illumination of the sea surface, gating techniques for extended range operation, TV-type image display of the sea surface, automatic readout by spatial filtering techniques.

Section C discusses a method of two-way communication between an airplane and a submerged submarine using laser and sonar collimated beams to "write" and to "readout" messages on the sea surface.

\(^{1}\) Laser Technology Conference, Pensacola, Florida; April 18-20, 1967.
SECTION A - LASER DETECTION OF SUBMARINE SIGNATURES

Introduction

Detection of submerged submarines by direct illumination with a high-power pulsed laser, operating in the water transmission window, is conceivable, although the associated technical difficulties are formidable. Another approach is based on the use of lasers to detect the "signatures" that submerged submarines leave at the sea surface; as an example, as shown by Hall, wakes may be detected with considerable enhancement of their signal-to-noise ratio by recourse to coherent optical processing.

The perturbation represented by wakes may be defined as an "optical signature". It consists of certain transverse-type waves and is associated with the motion of the body in the water. Another surface perturbation of great interest is that associated with the longitudinal-type waves generated by the vibrations within the moving body and may be defined as an "acoustic signature"; these two types of waves have quite different characteristics.

An investigation of the feasibility of application of lasers for real-time readout of optical and acoustic signatures is being conducted under the sponsorship of the Office of Naval Research; in this paper, a status report of the work accomplished so far is presented. The extension of the method of readout of acoustic signatures to the design of a communication link from underwater to air, and in particular of an IFF system, is also discussed.

Mechanical Wave Motions in the Sea

As is well known, two types of mechanical wave motions may be generated in an elastic medium such as the sea--i.e., the transverse type, in which the particle motion is perpendicular to the wave travel, and the longitudinal type, in which the particle motion is in the direction of the wave travel. Both types are characterized by the same wave equation:
where \( v \) is the wave velocity and \( A \) is the disturbance; however, in transverse-type waves, the velocity is a function of the wavelength, of the acceleration of gravity, of the surface tension, and of the water depth, while in longitudinal-type waves the velocity is a function of the compressibility and the density of the medium, and is independent of the wavelength \((A_3, A_4)\).

Transverse-type waves are produced at the water surface as a result of various disturbances within the medium. Since the flow is irrotational, the wave velocity may be expressed as the gradient of a scalar potential function which satisfies Laplace's equation. The boundary conditions express the fact that, at the sea bottom, the normal component of the velocity is zero, while at the sea surface ("free surface"), the vertical component of the acceleration is zero. As an example, taking the \( y \) axis vertical and the origin at the unperturbed sea surface, the potential function for a single wave traveling in the \( x \) direction is typically:

\[
\phi = A \cos (kx - \omega t) \cos h \kappa (y+h)
\]

In the latter equation, \( k = 2\pi / \lambda \), and \( \omega / k = v \). Indicating with \( h \) the water depth, with \( \gamma \) the water density, and with \( T \) its surface tension, the expression of the velocity is:

\[
v = \left[ \frac{\phi}{2w} + \frac{2\sqrt{T}}{\gamma \lambda} \right] \tan h \frac{2wh}{\lambda} \, \left| \frac{1}{2} \right.
\]

An example of \( v \) versus \( \lambda \) is plotted in Fig. 1; it is noted that a minimum velocity occurs in correspondence of \( \lambda \approx 2 \text{cm} \), and that, correspondingly, waves can be classified as capillary-type or ripples \((\lambda < 2 \text{ cm})\) and as gravity-type \((\lambda > 2 \text{ cm})\).
FIG. 1 - PLOT OF VELOCITY VERSUS WAVE-LENGTH FOR TRANSVERSE WATER WAVES
(60°F)

h = 30.5 cm
h = 12.2 cm
The rate at which energy is transported is the group velocity \( u \); for a single-wave train:

\[
 u = v - \lambda \frac{dv}{dA}
\]  

(4)

From this equation it is found that \( u \) is less than \( v \) for gravity waves and is larger than \( v \) for capillary waves. For a group of gravity waves, the velocity \( u \) depends on the water depth \( h \), approaching \( v/2 \) for very large depth; for a group of capillary waves, new waves appear at the group head and disappear at its end. Schooley \((A5, A6)\) has pointed out that, while small waves have approximately sinusoidal profiles, large gravity waves have a modified trochoidal shape, sharper near the crests and flatter in the troughs; and large capillary waves have sharp troughs and wide crests \((A7)\) and, in general, larger average slope than gravity waves \((\text{Fig. 2})\). The surface slope also depends upon the wind and statistically is peaked and skewed in the downwind direction \((\text{compared to a Gaussian distribution, Fig. 3})\).

The mathematical description of the surface-wave distribution accompanying a body in motion at the surface of the sea has been developed by Lord Kelvin \((A8)\); waves due to impulses generated beneath the surface have been discussed by Lamb \((A9, A10)\). Modern extensive studies of the wake pattern due to a submerged body have been made at the David Taylor Model Basin \((\text{see, for example, Refs. A11, A12})\). In general, the surface-wave distribution consists of a local disturbance, which moves with the speed of the submerged body \( c \), and of a V-shaped system of waves, respectively divergent and transverse, which accompanies the body and travels downstream with group velocity \( c/2 \). The shape of the latter system is approximately independent of the speed \( c \) \((\text{provided this is a constant vector})\) and of the depth of the submerged body; examples are shown in \(\text{Fig. 4}\) for the case of a straight course at constant speed, and \(\text{Fig. 5}\) for the case of a circular course.
FIG. 2 - PROFILES OF CAPILLARY WAVES (Refs. 5, 6, 7)

FIG. 3 - STATISTICAL DISTRIBUTION OF WAVE SLOPES (Ref. 5)
FIG. 4 - WAKE WAVES FOR STRAIGHT COURSE AT CONSTANT SPEED (Ref. 3)

FIG. 5 - WAKE WAVES FOR CIRCULAR COURSE AT CONSTANT SPEED (Ref. 3)
at constant speed. The waves are loci of constant phase, for a number of phase values differing by $2\pi$; for a straight-course constant-speed case, their representation in parametric form, referred to coordinate axes with the origin on the surface moving with the body, is (Fig. 6(A3)):

$$x = \frac{a}{2} \left( 2 \cos \varphi - \cos^3 \varphi \right)$$
$$z = \frac{a}{2} \cos^2 \varphi \sin \varphi$$

where $a$ is a length parameter proportional to $c^2$ and to the phase (for a surfaced moving body, $a = (2 c^2 \varphi)^{-1}$, and $\varphi$ is an angular parameter representing the angle between the tangent to the curve and the $x$ axis. The amplitudes of the various waves of the wake decrease as the inverse square root of the horizontal distance to the moving body; the waves are contained within a sector of constant half-angle $\varphi = \sin^{-1} \frac{1}{3} = 19^\circ 28'$. 

Longitudinal-type waves consist of alternating pressures (expansion-compression), as well as of longitudinal particle displacement or velocity oscillations, traveling at constant velocity $v = \sqrt{k/\varphi}$. Assuming plane-wave propagation along the $x$-axis direction, the velocity potential $\phi$ is obtained from the wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} = v^2 \frac{\partial^2 \phi}{\partial x^2}$$

which yields for a single periodic perturbation of frequency $\omega$:

$$\phi = B \cos (k x - \omega t) = B \cos k (x - vt)$$

where $k = 2\pi/\lambda$. There follows also for the longitudinal velocity:
FIG. 6 - CONSTANT-PHASE CURVES
$$\frac{\partial}{\partial x} = -B \sin k (x - vt)$$  \hspace{1cm} (8)$$

and for the wave pressure:

$$p = \frac{\partial}{\partial t} \Phi = -B v \sin k (x - vt) = \Phi'$$  \hspace{1cm} (9)$$

Thus it is seen that the pressure and the velocity functions are in phase.

(2) We consider now the case of a point source at depth $d$; as first approximation, the sea surface may be considered as a fixed specular reflector of the pressure waves, and its effect on the field distribution may be computed by recourse to the image theory (Fig. 7(A13)). Using spherical coordinates, the velocity potential is expressed as follows:

$$\Phi = \frac{B}{r_1} \epsilon j (k r_1 - \omega t) - \frac{B}{r_2} \epsilon j (k r_2 - \omega t)$$  \hspace{1cm} (10)$$

Assuming that the receiver is at depth $h$ and at large distance from the source (i.e., $r_2 \gg d$, $r_1 \gg d$), the above expression may be simplified (A1b), letting $r_2 - r_1 \approx 2 h d / r$, and

$$\Phi = 2 j \frac{B}{r} \sin \frac{k h d}{r} \epsilon j (k r - \omega t)$$  \hspace{1cm} (11)$$

Finally, one finds that the vertical component of the displacement is:

$$\Phi_{v} = 2 j \frac{k d B}{\omega r^2} \cos \frac{k h d}{r} \epsilon j (k r - \omega)$$  \hspace{1cm} (12)$$

Thus, the vertical component of the acoustic displacement wave is largest at $h = 0$ (receiver at the sea surface) and is inversely
FIG. 7 - POINT SOURCE OF AC PRESSURE WAVES IN SEA MEDIUM
proportional to the frequency $\omega$; as the depth $d$ varies, the vertical displacement remains proportional to the factor $d \cos \frac{k h d}{r}$, and thus tends to increase as $d$ increases, if the receiver is at the sea surface. However, the displacement is also inversely proportional to the square of the distance $r$. Finally, it should be noted that, at the sea surface, the horizontal component of the wave displacement and the wave pressure vanish; the wave vertical displacement is in quadrature with the wave pressure, and is maximum at $h = 0$. Using Eq. 8, it is noted that $\xi_m = \frac{p}{\omega v}$ (i.e., letting $p = 1 \, \text{dy/cm}^2$, $v = 1500 \, \text{m/sec}$, and $\omega = 2 \times 10^6$, one has $\xi = 10^{-9} \, \text{cm}$). Thus, recapitulating, acoustic perturbations caused by longitudinal waves generated by vibrations at the submerged body are characterized by wave displacements which have a very small amplitude, proportional to the acoustic pressure and inversely proportional to $\omega r^2$.

(C) A-3 Detection of the Optical Signature by Coherent Optical Processing

(C) The problem of the enhancement of wake patterns from photographic displays by recourse to spatial filtering with matched filters has been discussed extensively by Hall (A1). Our own investigation has been directed to the realization of a real-time system and has not yet been completed with respect to the optical-processing components.

(C) The experimental laboratory model built is shown in Fig. 8 in block-diagram form; it consists of laser illuminator, beam scanner, receiver-collector and photodetector, display scope, scan-rate clock and synchronized staircase-type voltage sources. Additional blocks in Fig. 8 indicate the optical-processing components and the electronic-processing components for the enhancement of the wake pattern respectively at the optical level and at the electronic level.
A CW HeNe laser ($\lambda = 6328\text{Å}, P_{\text{out}} \approx 5 \text{ mw}$) was used for this application; the mechanical scanner consisted of a pair of reflecting prisms with axes perpendicular to each other (Fig. 9). The prisms are driven by step-motors which are actuated by suitable pulse trains; repetition rates of up to 45 step/min are used for the line scanner, and $1/15$ of the latter rate for the frame scanner. The angle of rotation per step depends upon the number of prism faces and must be made small in order to cover a limited range of sea surface per scan; however, for initial laboratory experimentation, an angle of $3^\circ$ was used. The pulse trains are generated by a clock consisting of a free-running multivibrator and of a monostable multivibrator triggered periodically by one of the pulses of the first train, and such that a fixed ratio of 15:1 is maintained between the two rep rates.

The clock generator also is utilized to produce a synchronized stair-case voltage used for the control of the two-dimensional scanning of the readout scope. Photographs of the matrix display ($15 \times 15$), of the rack-mounted equipment and of the laser, mechanical scanner and small bench water tank are shown in Figs. 10, 11, and 12 (Al5).

In experiments conducted in the laboratory, patterns obtained by reflection of the laser beam from the water surface have been obtained. Further work is expected to deal with: a) the design of a matched filter of the wake to enhance recognition of the wake pattern in the presence of extraneous scatter; b) the design of an electronic edge-enhancement network to improve the intensity of the discontinuities in the display of the scope; c) the design of a mechanically stabilized platform to permit repetitive scanning of the given area of the sea surface.

The design of a wake matched filter may be obtained following the techniques developed by VanderLugt and others (A16, A17); a complex-valued function represented by the transmittance function:
FIG. 11 - RACK-MOUNTED EQUIPMENT (RATE CLOCK, STAIRCASE GENERATOR, AMPLIFIER, RATE COUNTER, POWER SUPPLY) AND DISPLAY SCOPE.

FIG. 12 - LASER, MECHANICAL SCANNER, BENCH WATER TANK WITH WAVE MAKER. Also, a Photomultiplier and a Fresnel Lens Collector are shown.
\[ N(p, q) = k \frac{S^*(p, q)}{S(p, q)} , \quad (13) \]

where \( S^*(p, q) \) is complex-conjugate of the Fourier transform of the signal \( s(x,y) \), and \( N(p, q) \) is the spectral density function of the autocorrelation function of the \( n(x,y) \), may be represented in photographic film converting the phase information into intensity information by recourse to a modulation process in a modified Mach-Zender or Rayleigh interferometer.

(c) Electronic enhancement is a process well known in the study of TV-bandwidth reduction systems and is based on the differentiation of the video signal obtained from scanning a picture, and on the enhancement of the high-frequency components corresponding to edge information.

(c) A\textsuperscript{4} Detection of the Acoustic Signature by Laser Readout

(c) If a laser beam is directed at the sea surface where acoustic displacement waves are generated as a result of alternating pressure sources within the medium, modulation of the reflected beam with the signals associated with the acoustic signature occurs. The basic phenomena involved are:

a) Frequency modulation due to doppler effect
b) Spatial angular modulation due to variation of orientation of the reflecting surface
c) Polarization modulation due to variation of the reflection coefficient for different components of polarization

(c) In addition, amplitude modulation may be associated with the variation of the reflection coefficient. For example, Shostak (Al8) has pointed out that the physical parameters of sea water (conductivity, dielectric constant) and therefore the reflection coefficient at the sea surface vary with the acoustic pressure set up in the medium, although the relative modulation depth is very small.

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The detection of the frequency modulation at optical frequencies presents considerable difficulties, especially in connection with field operations; a simpler approach may be based on the detection of the frequency modulation of a microwave signal modulating the laser carrier. In the latter case, optical heterodyning is not required; however, high-efficiency microwave modulators and very fast and sensitive detectors must be utilized.

Simpler methods of demodulation may be implemented in connection with the phenomena of spatial angular modulation and of polarization modulation; in either case, suitable converters to amplitude modulation may be used.

An example of realization of the first system is shown in Fig. 13, where the spatial angularly modulated beam is made to scan a matrix of small apertures behind which a photodetector surface receives optical pulses which are modulated in time with the desired acoustic spectral information. A polarization analyzer and a transmission grating may also be added. An example showing the signals found at the IF stage of a superheterodyne receiver, tuned at the acoustic frequency, is presented in Fig. 14; in this case, the acoustic source was pulsed at 60-c/s repetition rate, and its carrier frequency was 60 Kc.

Systematic tests have been conducted using a plastic acoustically damped cylindrical tank of 2-1/2-ft diameter and 2-1/2-ft height; a commercial-type transducer fed with a CW sinusoidal signal of frequency 25 Kc to 250 Kc and input power varying from tenths of milliwatts to watts was used. The power data were computed from impedance and voltage measurements at the transducer input; assuming an average transducer efficiency of 0.4, power levels into the water were evaluated. In Figs. 15 and 16, the S/N ratios, measured at the input to a superheterodyne receiver tuned to the acoustic frequency are plotted versus the power level for the various frequencies, with the transducer placed at depths of 1" and 8' from the water sur-
FIG. 13 - LASER DETECTION SYSTEM OF ACOUSTIC SIGNATURE EXPERIMENTAL SET-UP
NOISE -
LASER OFF; TRANSDUCER OFF
.2v/cm; 5ms/cm

NOISE -
LASER ON; TRANSDUCER OFF
1v/cm; 5ms/cm

SIGNAL -
20v/cm; 5ms/cm

SIGNAL -
20v/cm; 5ms/cm
10-sec time exposure

FIG. 14 - IF Signal Scope Display, Transducers Operated at 60 kc and Pulsed at 60 cps
face. The numerical data depend upon the conversion efficiency from angular modulation to amplitude modulation. Although the design of the latter has not yet been optimized, it is of interest to note that, in agreement with Eq. 12, for a given power level, the S/N ratio appears to vary roughly in inverse proportion with the frequency.

(C) In practice, the optimum parameters of the simple aperture matrix-converter shown in Fig. 13 are a function of the acoustic frequency; for the data of Figs. 15 and 16, aperture diameters of .046" and aperture separation of 1/16" were used.

(C) A more appropriate converter design is one based on an adaptive-type diaphragm, such as an acoustic diffraction grating produced in a transparent ultrasonic delay line by employing the Debye-Sears effect\(^\text{(A19)}\). In this case, the intensity of the diffracted laser beam is modulated at the frequency of the signal injected into the ultrasonic delay line; it is therefore possible, by recourse to a signal proportional to a reference acoustic signature of submarines or torpedoes, to obtain a modulation of the laser beam depending on the product of the actual signature and of the reference signature and then to utilize methods of cross-correlation to enhance the sensitivity of the detection (see Fig. 17).

(C) Extensions of the above-described technique to the establishment of a communication link between airplanes and submerged submarines are also possible; in particular, this provides a convenient method of design of a Friend Identification system. With respect to communication from air into the water, an approach which is being studied is based on the use of a high-power pulsed beam impinging on the sea surface. It is known\(^\text{(A20)}\) that acoustic pressure waves may be generated in liquids as a result of transient heating effects associated with high-power laser pulses operating outside the optical water transmission band.
FIG. 17 - BLOCK DIAGRAM OF LASER SYSTEM FOR DETECTION OF SUBMARINE ACOUSTIC SIGNATURE
BIBLIOGRAPHY


(A2) Contract N0003358(00) - Office of Naval Research, Electronics Branch.


(A10) H. Lamb - Hydrodynamics - Dover Publications (1945), Ch. IX.


Bibliography (2)


SECTION B - LASER DETECTION OF THE OPTICAL SIGNATURE OF TORPEDOES AND SUBMARINES (U)

Introduction

The detection of torpedoes and submarines is an important problem of naval warfare, for which a fully reliable solution is needed. Although sonar techniques appear as the most direct approach, several difficulties are encountered with this method in general, due, for example, to reflections and propagation deviations of acoustic waves at sea-air interface; in addition, in the case of torpedoes, the sonar constitutes a powerful acoustic source on which the torpedo may "home". Undoubtedly these difficulties may be reduced or overcome with ingenuity of design; it is, however, desirable to investigate additional approaches which may be utilized concurrently (or separately) with sonar techniques and which may provide an added dimension of reliability of detection. In the following, a brief introductory discussion of the methods of detection based on the investigation of the optical signature is presented.

Optical Signature of Torpedoes and of Submarines

The motion of a body below the surface of the sea is accompanied by transverse-type waves which appear at the surface and which consist of a local disturbance, moving with the velocity of the submerged body c, and of a V-shaped system of waves ("wakes"), respectively divergent and transverse, which accompanies the body and travels downstream with group velocity c/2. For example, in Fig. B-1, a representation of the surface perturbation after one second from start is shown. The shape of the wake is approximately independent of the speed c (provided this is a constant vector), and of the depth of the submerged body. The waves are loci of constant phase, for a number of phase values differing by 2π; for a surfaced body in a straight-course, constant-speed case, the representation of the loci in parametric form referred to coordinates axes...
FIG. B1 - SURFACE-WAVE PATTERN OF A SUBMERGED BODY STARTED IMPULSIVELY FROM REST
with the origin on the surface moving with the body is (Fig. B2):

\[
\begin{align*}
x &= \frac{a}{2} (2 \cos \theta - \cos^3 \theta) \\
z &= \frac{a}{2} \cos^2 \theta \sin \theta
\end{align*}
\]

where \(a\) is a length parameter proportional to \(c^2\) and to the phase, and

\(\theta\) is an angular parameter representing the angle between the tangent to the curve and the \(x\)-axis.

The amplitudes of the various waves of the wake decrease as the inverse square root of the horizontal distance to the moving body; the waves are contained within a sector of constant half-angle \(\gamma = \sin^{-1} \frac{1}{3} = 19° \ 28'\).

(U) The analytical representation of the surface elevation \(\eta\) of the waves is very complex, and in general is given as an integral expression which must be computed by approximations or by numerical procedures; the amplitude of the waves is found to decrease with the distance from the moving body and with its depth.

(C) The wake has proportions which are approximately standard and independent of the velocity, depth or dimensions of the moving body; for example, the angle in which the wake is contained is constant (58° 56'), the shape of the loci of divergent and transverse waves is a constant. These features prove very helpful in the application of optical processing techniques for the detection of the presence of the wake. These techniques have been discussed in some detail in a previous report, "Detection of Optical Signature of Submarines by Laser Techniques (U)" (Ref. 1). The techniques are based on the Fourier transformation properties of lenses and on the utilization of special masks acting as "matched filters" for a particular pattern; these masks modify the amplitude and the phase of the light distribution in such a manner that a peak luminous
FIG. B2 - WAKE WAVES FOR STRAIGHT COURSE AT CONSTANT SPEED
signal appears at the output if and only if a signal matching the particular pattern is present. Matched filters for wake pattern can be designed experimentally or from theoretical considerations; for example, it can be shown that the matched filter corresponding to the loci of Eqs. (B1) is represented by the following locus in polar coordinates:

\[ r = \frac{2}{a} \sec^2 \theta \]  

A representation of this locus is given in Fig. B3. Recapitulating, the optical signature of a torpedo or of a submarine moving under water has standard characteristics, which may be utilized either for a direct visual detection or for an automatic detection by recourse to optical processing techniques. A coherent light source must be used in connection with optical processing techniques.

(C) B-3 Laser Detection of the Optical Signature

(C) The realization of an operational laser system for the detection of the optical signature must be made taking into account the various practical environmental conditions expected in naval warfare. These include mechanical instabilities of the system platform (naval ship or aircraft), atmospheric turbulence and scatter of the optical beam, etc. As pointed out in Ref. Bl, the use of optical heterodyning techniques for the direct readout of the optical signature is impractical; similarly, the application of spatial filtering techniques to the reflected laser beam is not advisable, because of difficulties encountered in maintaining the optical alignment and of the poor signal-to-noise conditions. On the other hand, a high-power pulsed collimated infrared laser beam may be used with considerable advantage to illuminate the sea surface at large range; by recourse to range-gating techniques and to polariza-
FIG. B3 - FOURIER TRANSFORM OF WAKE
tion techniques, the reflected signal may be resolved clearly, even in the presence of atmospheric turbulence and fog scatter. The detected signal may be converted and displayed by means of closed-circuit television using an infrared pick-up camera of the image orthicon or vidicon type in which an automatic stabilization system, such that the effect of the motion of the platform on the image is compensated, may be introduced.

(C) The detection of the optical signature in the display may be made either by direct visual observation or by automatic matched spatial filtering techniques.

(C) In Fig. B4, a block diagram of the direct visual system is presented; this consists of a television-type camera tube (image-orthicon or Vidicon suitable for infrared viewing) whose input light is obtained by illuminating the sea surface with a pulsed, high-peak-power infrared laser. Gating techniques are used at the camera tube in order to allow detection only at the time of arrival of photons from the sea surface, thus excluding the light scattered from intermediate fog, rain, or turbulence. For this purpose, the pulses that are controlling the operation of the laser illuminator are delayed suitably and are used to control a gate within the camera tube.

(U) A diagram of an Image Orthicon is shown in Fig. B5; this shows that the light from the scene is imaged on a semitransparent conducting photocathode deposited on the inside face of the tube, and the resulting photoelectrons are focused with a magnetic field and are imaged on a thin semiconducting target. A gate may be inserted in the interspace to control the acceptance of electrons as previously indicated. The electron image formed on the semiconducting target is amplified by secondary emission using a fine target mesh maintained at a slightly higher potential; finally, a low-velocity electron beam is used to scan the reverse side of the target. This beam
Fig. B4 - Optical-Signature Display with Laser Illuminator
ALIGNMENT COIL
ELECTRON IMAGE SECTION
ELECTRON MULTIPLIER
FOCUSING COIL
DEFLECTION YOKE
RETURN BEAM
SCANNING BEAM
WALL COATING (50 V)
PHOTOCATHODE (~300 V)
GUN CATHODE (ZERO VOLTS)
TARGET MESH (2V)
THIN SEMICONDUCTING TARGET

FIG 5 Diagram of an Image Orthicon.
discharges the target, readying it to accept new photoelectrons and at the same time is reflected back towards the gun carrying a video modulation signal. This latter signal is amplified and displayed on the screen of a cathode-ray tube.

(U) A diagram of a Vidicon is shown in Fig. B6; in this case, the light from the scene is focused on a thin layer of photoconductor deposited on the inside face of the tube, which has been previously covered with a transparent conducting coating such as tin oxide. This transparent conductor acts as the signal plate and is maintained at a positive potential (10 - 40 V) with respect to the gun cathode. An acceptance gate for the incoming light may be introduced by pulsing the target voltage or by interposing a controlled transparency or reflectance. As shown in the diagram of Fig. B6, an electron beam is again used to scan the target; at each point of the latter where the arriving photoelectrons cause the photoconductors to rise a few volts positive proportionally to the light intensity, the electron beam returns the potential to ground, thus generating a video signal in the signal-plate lead. The latter signal is amplified and displayed on a suitable cathode-ray-tube screen.

(C) Thus, recapitulating, by illuminating the sea surface with a pulsed, high-peak-power infrared laser beam and by gating the light returned to the TV camera tube, a visual representation of the illuminated sea-surface area is obtained. This may be observed visually for detection of wake signatures.

(C) Stabilization of the image for the purpose of compensating automatically for the platform movements may be accomplished by recourse to derotation prisms; these are inserted in the optical path and are driven by servo motors controlled by voltages proportional to the deviation of the platform from the horizontal reference plane.
Diagram of the Vi-Com photoconductive camera tube.
A system for the application of optical spatial filtering for the enhancement of the detection of the optical signature is shown in Fig. B7. This consists of a coherent optical correlator in which a laser beam is modulated with the video signal from the TV camera tube and is made to scan synchronously with the electron beam of the latter; the resulting modulated coherent light is collimated and transmitted through an optical processing system in which a two-dimensional matched filtering occurs such that an output luminous spot is obtained if, and only if, a signal pattern (wake) of the type corresponding to the filter used is present. Therefore, by visual observation or by recourse to a photodetector, an indication of the presence of the signal pattern is obtained.

The principles of the optical processing system are illustrated in Fig. B8; a coherent light beam is collimated with lens $L_c$ and illuminates the plane $P_1$, where it presents a light distribution corresponding to the display of the sea-surface area under investigation. As previously indicated, this light distribution is obtained by modulating the laser output with the video signal from the TV camera tube, and by making the beam scan the plane $P_1$ synchronously with the scan function of the camera tube.

Let us indicate with $f(x,y)$ the said resulting light distribution at $P_1$; the lens $L_1$ forms a new light distribution at plane $P_2$ (planes $P_1$ and $P_2$ are the principal focal planes of $L_1$), which is the Fourier transformation of $f(x,y)$; i.e.,

$$F(p,q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j(px + qy)} \, dx \, dy \quad (B3)$$

A transparency having a distribution $H(p,q)$ corresponding to the matched filter of the signal pattern (wake) is placed at $P_2$; the resulting light distribution at $P_2$ is $F(p,q) \cdot H(p,q)$. Lens $L_2$ now forms a Fourier transform of $F(p,q) \cdot H(p,q)$ at plane $P_3$:  

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FIG. B7 - WAKE-SIGNAL ENHANCEMENT AND AUTOMATIC WAKE RECOGNITION

Fig8 A coherent optical system.
\[ r(x, y) = \frac{1}{4\pi^2} \int \int F(p, q) H(p, q) e^{j(px + qy)} dx dy \] (B4)

(C) The distribution \( r(x, y) \) is the result of the filtering; in particular, if the transmittance \( H(p, q) \) is a filter matched to the signal \( s(x, y) \) or to the signal-to-noise ratio \( s(x, y)/n(x, y) \), the distribution \( r(x, y) \) maximizes respectively \( s(x, y) \) or \( s/n \).

(C) Similar filters have been built and applied for such operations as optical character recognition (Ref. B2). In general, the output light distribution \( r(x, y) \) of the optical correlator does not present the shape of the signal pattern; instead, it shows a signal consisting of an intense luminous spot if, and only if, the signal pattern corresponding to the matched filter is present at plane \( P_1 \). Thus, detection of the signal may be made by measuring the integral of the output light distribution with a suitable photodetector. This provides a means for the automatic recognition of the presence of the signal pattern (wake); the output voltage from the photodetector may be utilized to operate an alarm, or to provide a continuous recording, etc.

(C) B-4 Conclusions

(C) In the previous pages, the general problem of the detection of the optical signature of torpedoes and submarines has been discussed. It has been shown that the signature may be detected by recourse to laser techniques. As a first step of design of a Signature Laser Detector, one may develop a system which permits visual observation of a selected area of the sea surface; such a system consists of a television-camera tube (Image Orthicon or Vidicon), receiving illumination from the light scattered by a pulsed, high-power infrared laser, and gated suitably to exclude most of the extraneous light scattered from fog, rain, or turbulence. This equipment would provide a continuous display of the area under investigation for direct observation.

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As a second step of design of an advanced Automatic Signature Laser Detector, one may develop a system which permits the enhancement of the wake pattern, and the automatic recognition of its presence. This system consists of a scanning laser beam, modulated with the signal taken from the TV camera tube, and transmitted through an optical processor for optimum filtering.

REFERENCES


SECTION C - LASER-SONAR COMMUNICATION SYSTEM

C-1 Introduction

A communication link between a submerged submarine and an aircraft may be established by application of a technique similar to that used for readout of the acoustic signature: the submerged submarine directs a collimated sonar beam to the sea-air interface about an axis approximately vertical, and the aircraft reads the corresponding acoustic message by means of a collimated laser beam, whose reflected rays are angularly modulated.

On the other hand, communication from the aircraft to the submerged vessel may be established by recourse to a pulsed high-peak-power laser beam impinging on the sea surface and generating pressure waves which propagate within the sea medium. These may be received directly at the submarine hydrophones; it may also be possible to "read out" these acoustic perturbations generated at the sea-air interface by recourse to a collimated sonar beam, directed from the submarine to the said surface and reflected back towards the submarine. The reflected beam is expected to be cross-modulated by the laser-produced surface waves in a manner analogous to the modulation of the reading laser beam in air. Both techniques above described have been tested successfully in laboratory experiments; however, the technique of cross-modulation of sonar beams scattered from the sea surface, where localized pressure waves are generated with an impinging pulsed laser beam, has not been investigated so far.

C-2 Propagation of Sound Waves in the Sea

The propagation of longitudinal waves (sound waves) in the sea has been discussed in Section A, Par. 2 of this report. A more detailed review is presented in the following.

In general, the sound field is characterized by a scalar potential function $\phi$; assuming a plane harmonic wave, with time
dependence \( \exp(-j\omega t) \), the velocity and the pressure are defined with the following relations:

\[
\vec{v} = -\text{grad} \phi, \quad p = -j\omega \phi
\]  

(C1)

where \( \phi \) is the medium density. In the following, we shall assume that the sea is infinitely deep and shall investigate only the effect of the plane surface interface with air. At the plane surface of separation between sea and air, the reflection and the refraction depend upon the angle of incidence as well as upon the densities and the wave velocities of the two media. With reference to Fig. Cl, indicating with subscript 1 the quantities pertaining to air, one has:

\[
\phi = A \exp jk (x \sin \Theta - z \cos \Theta)
\]

\[
\phi_{\text{refl}} = r A \exp jk (x \sin \Theta + z \cos \Theta)
\]  

(C2)

\[
\phi_1 = \frac{Z}{\rho} A \exp jk_1 (x \sin \Theta_1 - z \cos \Theta_1)
\]

Using the boundary conditions which express the continuity of the pressure and of the normal component of the velocity at the \( z = 0 \) plane, one has:

\[
\phi(x, 0, t) = \phi_1(x, 0, t)
\]

\[
\left( \frac{\phi}{z} \right)_{z=0} = \left( \frac{\phi_1}{z} \right)_{z=0}
\]  

(C3)

Substituting in the previous equations, and letting \( n = \frac{k_1}{k} \), \( m = \frac{\zeta_1}{\zeta} \), one obtains the following relationships:

\[
\frac{\sin \Theta}{\sin \Theta_1} = \frac{k_1}{k} = n
\]  

(C4)
FIG. C1 - REFLECTION AND REFRACTION OF PLANE ACOUSTIC WAVE.

FIG. C2 - REFLECTION AND REFRACTION COEFFICIENTS FOR ACOUSTIC POTENTIAL WAVE INCIDENT FROM SEA.

FIG. C3 - REFLECTION AND REFRACTION COEFFICIENTS FOR ACOUSTIC POTENTIAL WAVE INCIDENT FROM AIR.
\[ r = \frac{m \cos \theta - n \cos \theta}{m \cos \theta + n \cos \theta} = \frac{m \cos \theta - \sqrt{n^2 - \sin^2 \theta}}{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \]

\[ \gamma = \frac{2 \cos \theta}{m \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \]  
\[(c5)\]

These relations are general and apply for waves incident either from sea onto air or from air onto the sea provided the proper numerical values of \( m \) and of \( n \) are used in each case; since the wave velocities and the densities of these two media are respectively

\[
\begin{align*}
\text{air} & : c = 333 \text{ m/sec}, \quad \gamma = 1.3 \times 10^{-3} \text{ gm/cc} \\
\text{sea} & : c = 1500 \text{ m/sec}, \quad \gamma = 1 \text{ gm/cc}
\end{align*}
\]

one obtains the following values of the quantities \( m \) and \( n \) : for waves incident from the sea:

\[
\begin{align*}
m & = 1.3 \times 10^{-3} , \quad n = 4.5 \\
\end{align*}
\]

for waves incident from the air:

\[
\begin{align*}
m & = 768 , \quad n = 0.222 \\
\end{align*}
\]

It should be noted that two critical values of the angle of incidence may be defined, respectively the angle of Brewster (for which \( r \) is 0, \( \gamma \) is maximum), and the angle of total internal reflection (above which \( r \) and \( \gamma \) become complex).

(U) The Brewster angle is defined by the relation:

\[
\theta_B = \sin^{-1} \sqrt{\frac{m^2 - n^2}{m^2 - 1}} \]

\[(c9)\]

This value is real provided one of the following conditions is satisfied:

-45°
Neither of these conditions is satisfied for the sea-air case, and thus no Brewster phenomena occur.

\[(U)\] The angle of total internal reflection is defined by the relation:

\[\theta^* = \sin^{-1} n\] (C11)

This value is real provided \( n < 1 \); thus, the phenomenon of total internal reflection occurs in the case of waves incident from air onto the sea.

\[(U)\] In general, it may be noted that, in the case of waves incident from the sea, the relations that express \( r \) and \( \tau \) may be simplified approximately as follows:

\[r \simeq -1, \quad \tau \simeq \frac{2 \cos \theta}{\sqrt{20.25 - \sin^2 \theta}} = 0.445 \cos \theta\] (C12)

Similarly, in the case of waves incident from the air, one has:

\[|r'| \simeq 1, \quad |\tau'| \simeq 2.6 \times 10^{-3}\]

\[\theta^* = \sin^{-1} 0.222 \simeq 13^\circ 42'\] (C13)

The coefficients \( r' \) and \( \tau' \) are complex for \( \theta > \theta^* \); there follows that, for \( \theta > \theta^* \), the acoustic potential function of the refracted wave is attenuated exponentially.
\[ |\varphi_1| = \frac{2}{m} A \exp (-i \beta) \]  

where \[ \beta = k \sqrt{\sin^2 \theta - n^2} = \frac{\omega}{c} \sqrt{\sin^2 \theta - n^2} \]. Substituting the numerical values of \( m \) and \( n \) for this case, one has:

\[ \varphi_1 = 0.445A \exp (-\beta d) \]  

\[ \beta = \frac{\omega}{333} \sqrt{\sin^2 \theta - 0.222} \text{ meters}^{-1} \]

The above relations are illustrated in Figs. C2, C3, and C4.

Returning to the consideration of the case of waves incident from the sea, we obtain for the total field of incident and reflected waves in the sea the following expression:

\[ \varphi = A \left[ \exp (-j k z \cos \theta) + \frac{\omega}{c} \exp (j k z \cos \theta) \right] \exp (j k x \sin \theta) \]  

The corresponding values of the pressure, velocity components and displacement components are respectively:

\[ p = \rho \frac{\partial \varphi}{\partial t} = -j \omega \gamma \varphi \]

\[ v_x = -\frac{\partial \varphi}{\partial x} = -j k \sin \theta \varphi \]

\[ v_z = -\frac{\partial \varphi}{\partial z} = -A j k \cos \theta \left[ \exp (-j k z \cos \theta) - \frac{r}{\omega} \exp (j k z \cos \theta) \right] \exp (j k x \sin \theta) \]  

\[ \xi_x = \int v_x \, dt = -\frac{1}{j \omega} v_x \]

\[ \xi_z = -\frac{1}{j \omega} v_z \]
FIG.C4 - DEPTH OF PENETRATION OF SOUND WAVE INTO SEA FOR INCIDENCE AT TOTAL INTERNAL REFLECTION.
At the boundary between sea and air, the total acoustic pressure is continuous; since the reflection coefficient is \( r = -1 \) and the transmission coefficient is \( T = 0.445 \cos \theta \), the acoustic pressure in air is much smaller than that in the incident wave. Conversely, in the case of waves incident from air, since \( r' = +1 \), the pressure in the sea is approximately twice as large as that in the incident wave.

The power density of a plane wave is expressed as:

\[
P = \frac{|p|^2}{\frac{2}{\gamma} c} = \frac{(\omega A)^2}{2 \gamma c} \]

Hence, the power densities of incident, reflected, and refracted waves are respectively:

\[
P_{\text{inc}} = \frac{(\omega A)^2}{2 \gamma c}
\]
\[
P_{\text{refl}} = |r|^2 \frac{(\omega A)^2}{2 \gamma c}
\]
\[
P_{\text{refr}} = |T|^2 \frac{(\omega A)^2}{2 \gamma_1 c_1} = |1 + r|^2 \frac{(\omega A)^2}{2 \gamma_1 c_1}
\]

The ratio between the normal component of transmitted and incident power densities is:

\[
\frac{P_{\text{refr}} \cos \theta_1}{P_{\text{inc}} \cos \theta} = \frac{1 + r}{\gamma_1 c_1} \frac{c \cos \theta_1}{\cos \theta} \]

At normal incidence (\( \theta = \theta_1 = 0 \)), this ratio may be written as follows:

\[
\frac{P_{\text{refr}}}{P_{\text{inc}}} = \frac{4 m^2}{(m+n)^2}
\]
The above expression remains unchanged if \( m \) and \( n \) are replaced respectively with \( 1/m \) and \( 1/n \); therefore, the power transfer has the same value for waves incident normally from the sea or from the air.

At oblique incidence, Equ. C20 may be written as follows:

\[
\frac{P'_{\text{refr}} \cos \theta_1}{P_{\text{inc}} \cos \theta} = \frac{4m n \cos \theta \cos \theta_1}{(m \cos \theta + n \cos \theta_1)^2} \tag{C22}
\]

In this case, if \( m, n_1, \theta \) and \( \theta_1 \) are interchanged with \( 1/m, 1/n, 1, \text{ and } 0 \) respectively, the relation acquires the following value:

\[
\frac{P'_{\text{refr}} \cos \theta_1}{P'_{\text{inc}} \cos \theta} = \frac{4m n \cos \theta_1 \cos \theta}{(m \cos \theta + n \cos \theta_1)^2} \tag{C23}
\]

Therefore, in general, the ratio of the normal components of the power density flux is invariant with respect to the direction of incidence of the wave onto the sea-air interface.

Consider now the case of spherical waves sinusoidally varying with time. The acoustic potential function corresponding to a point source is written as follows:

\[
\phi = \frac{A}{r} \exp j (k r - \omega t) \tag{C24}
\]

Such a wave may be expanded into plane waves by recourse to the integral relations:

\[
\frac{e^{jkr}}{r} = \frac{2}{2\pi} \int_0^{2\pi} \int_0^{\pi} \exp j (k_x x + k_y y + k_z z) \sin \theta \, d\theta \, d\phi \tag{C25}
\]
where the + sign applies for the range \( z \geq 0 \) and the - sign applies for the range \( z \leq 0 \). In the above expression, one has:

\[
k_x = k \sin \theta \cos \varphi, \quad k_y = k \sin \theta \sin \varphi, \quad k_z = k \cos \theta \quad (C26)
\]

Therefore, the analysis developed for the case of plane waves is readily applicable to spherical waves.

(U) When a plane boundary of separation between sea and air is present, the computation of the resultant field is in general rather cumbersome; however, in the case in which the boundary is a perfect reflector, a considerable simplification may be introduced by recourse to the theory of images. Consider, for example, the case of Fig. C5, where a point source is located at \( 0, -h \) in the sea; the corresponding image source having the same intensity is located at \( 0, +h \). Thus, the resulting acoustic potential at any point within the sea medium is:

\[
\phi = \frac{A}{r_1} \exp j (k r_1 - \omega t) - \frac{A}{r_2} \exp j (k r_2 - \omega t) \quad (C27)
\]

Assuming that \( r_1, r_2 \) are much larger than \( h \), and letting

\[
r_2 - r_1 \approx \frac{2 \text{hz}}{r} \quad (C28)
\]

one may write Equ. C27 as follows:

\[
\phi = \frac{A}{r_1} \left[ \exp - \left( \frac{j k \text{hz}}{r} \right) - \exp \frac{j k \text{hz}}{r} \right] \exp j (k r - \omega t) = -\frac{A}{r} \left[ 2j \sin \frac{k \text{hz}}{r} \right] \exp j (k r - \omega t) \quad (C29)
\]

The corresponding expressions of the pressure, of the velocity, and of the displacement are computed from Equ. C29 as follows:
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FIG. C5 - PROPAGATION OF SOUND WAVE WITHIN THE SEA

FIG. C6 - COMMUNICATION FROM SUBMARINE TO AIRCRAFT
The vertical component of the displacement is evaluated approximately with the expression:

\[ \xi_z = \frac{-1}{j\omega} \psi_z = \frac{1}{j\omega} \frac{\partial \phi}{\partial z} = \frac{-2\pi k h z}{\omega \rho^2} \cos \frac{k h z}{\rho} \exp j(k r - \omega t) \]  

(C31)

The latter equation shows that the vertical component \( \xi_z \) is largest at the sea-air interface \( (z = 0) \). On the other hand, it is noted that the horizontal component of the displacement at the sea surface, given approximately by \( \psi_r \), becomes zero; this result is in agreement with that derived earlier for the case of plane waves. In fact, it has been shown (Eq. C12) that, at grazing incidence, the coefficient of reflection of a plane wave incident from the sea is \(-1\) and the coefficient of transmission is zero; thus, in this case, the incident and the reflected waves completely cancel one another. Therefore, it is concluded that the horizontal component of the displacement is zero at the plane boundary of separation between sea and air.

(U) The total power corresponding to a point source of potential function \( \phi = \frac{A}{r} \exp j(k r - \omega t) \) is expressed as follows:

\[ P = \frac{\omega^2 q}{2c} | \phi |^2 \frac{1}{4 \pi r^2} = \frac{2\pi k}{c} A^2 \omega^2 \]  

(C32)

Thus, if the power level is maintained constant at various frequencies, the product \( A \omega \) is constant.
The expression of the vertical displacement (Equ. C31) may be modified by substituting $k = \omega/c$ as follows:

$$\xi_z = -\frac{2\omega}{\omega c r^2} \cos \frac{\omega h z}{r_c}$$ (C33)

If the power level of the source is maintained constant while the frequency is being varied, the vertical displacement is found to vary inversely with the frequency.

In general, the displacement is very small; for a plane of given pressure $p$ and frequency $\omega$, one has:

$$\xi = \frac{p}{\omega q c}$$ (C34)

At frequency $f = 1000$ Hz, pressure $p = 1$ dy/cm$^2$, one finds $\xi \approx 10^{-9}$ cm.

A communication link from a submarine to an aircraft is readily established by application of the laser readout technique of acoustic displacements at the sea surface already discussed. With reference to Fig. C6, if a plane acoustic wave is launched from the submarine in a vertical direction towards the sea-air interface, a corresponding displacement is produced at the latter boundary; from Eqs. C16 and C17, letting $x = 0$, $\theta = 0$, $r = 1$, one finds:

$$\varphi = -A 2j \sin (kz)$$
$$p = -2\omega q A \sin kz$$ (C35)
$$v_z = -2j A k \cos kz$$
$$\xi_z = \frac{2 Ak}{\omega} \cos kz = \frac{2 A}{c} \cos kz$$
The power density of \( t \) incident wave is:

\[
P_{\text{inc}} = \frac{(\omega A)^2 q}{2c} \tag{C36}
\]

i.e.,

\[
A \omega = \sqrt{\frac{2c P_{\text{inc}}}{q}} \tag{C37}
\]

There follows that, for given power level, the displacement at the sea-air interface is expressed as follows:

\[
\xi_z = \frac{2}{\omega} \sqrt{\frac{2 P_{\text{inc}}}{q c}} \tag{C38}
\]

For example, assuming a pressure \( p = 10^2 \text{ dy/cm}^2 \), a power density \( P_{\text{inc}} = 0.033 \text{ erg/cm}^2 \text{ sec} \), a frequency \( \omega = 2 \times 10^5 \text{ c/s} \), one has:

\[
\xi = 2.12 \times 10^{-9} \text{ cm} \tag{C39}
\]

Although this displacement is very small, it may be detected readily with the laser readout technique already described in Section A.

(C) A collimated laser beam incident approximately normally on the sea surface is reflected as a beam whose direction is angularly modulated in space at the rate of the vibrations occurring at the sea surface. In general, these vary in a complex manner with time, resulting from the linear superposition of a large number of perturbations; it is, therefore, necessary to make recourse to a narrow passband filter after demodulation of the received laser beam.

(C) Demodulation of the laser is obtained by conversion of the spatial angular modulation into an amplitude modulation. One of the simplest types of converters consists of a row of apertures.
uniformly spaced and scanned by the light beam; a photodetector placed behind such apertures is illuminated by a succession of light pulses, which have a repetition rate proportional to the rate of spatial angular modulation. This type of converter has been built and has been used successfully. A matrix of holes of 0.046" diameter, spaced 1/16" apart, was placed in front of a photomultiplier (Type RCA 7102); a HeNe laser of 5-mw power with a collimated beam of 5mm diameter was directed to the surface of a water-filled tank. A commercial-type transducer (Type Heatkit) was placed within the water at a depth of 1" to 3". Feeding such transducer at constant power and at frequencies selected in the range 25 Kc to 200 Kc, detection of the laser angular modulation was readily obtained with high signal-to-noise ratio. A schematic of the experimental equipment is shown in Fig. C7; in particular, a superheterodyne receiver, Type Stoddart Mod NM10-A, was utilized. The results are shown in Figs. C8 and C9 (see also Section A, Figs. 15 and 16).

(C) A mathematical analysis of the performance of the modulation converter may be derived using a simplified model representation. As an example, in Fig. C10, the converter is represented with a single row of rectangular apertures, of width a, separation b, and the laser beam is assumed to have width c, where \( a < c < b-a \), and to scan the row of apertures at velocity \( v \). A photodetector, placed at the non-illuminated side of the converter, receives light pulses having duration \( T = \frac{c}{v} \) and period \( T = \frac{(b-a)}{v} \); more accurately, the pulses have trapezoidal wave-form, with rise and decay time given by \( a/v \).

(C) The velocity \( v \) is proportional to the distance \( R \) from converter to sea surface, and varies periodically between positive and negative values depending upon the net oscillation of the reflecting water facet. Therefore, the output of the photo-
FIG. 07 - LASER DETECTION SYSTEM -- ACOUSTIC SIGNATURE EXPERIMENTAL SET-UP
FIG. 10 - SCHEMATIC OF MODULATION CONVERTER
detector consists of a non-uniform succession of pulses whose position modulation exhibits a period twice that of the oscillating reflected laser beam. In particular, if the total scanning deviation of the latter beam is larger than the length of the converter, the succession of current pulses is discontinuous.

(C) An harmonic analysis of the photodetector output shows that a component at the frequency of vibration of the water surface is present. In general, this analysis is cumbersome and is of little value because it is based on a highly idealized model. It is useful to recall that, in the case of a uniform series of pulses of peak value $I_p$, of duration $t_0$, and of period $T$, one has the following Fourier expansion:

$$i = I_0 + I_1 \cos \omega t + I_2 \cos 2\omega t: \ldots$$

where

$$I_0 = I_p \frac{t_0}{T}$$
$$I_1 = 2 I_0 \frac{\sin \pi t_o/T}{\pi t_o/T}$$
$$I_2 = 2 I_0 \frac{\sin 2\pi t_o/T}{2\pi t_o/T}$$

etc.

(C) On the other hand, if the above series of pulses is position-modulated with a sinusoidal modulating wave:

$$V(t) = A_v \sin \omega_v t$$

such that the maximum excursion of a pulse from its unmodulated position is $\pm M T/2$, the resulting spectrum analysis gives the following representation:
\[ i = \sum \sum a_{m,n} \cos (m \omega + n \omega_0) t \]  
\hspace{1cm} (C43)

where:

\[ a_{m,n} = I_0(-1)^n \sin \left( \frac{m+n \omega_0}{\omega} \right) k \pi \frac{T_0}{T} \left[ j \pi \left( m+n \frac{\omega_0}{\omega} \right) \right] \]  
\hspace{1cm} (C44)

(C) A modulation converter of a different type may be designed by recourse to an ultrasonic delay line, into which a signal \( v_s(t) \) is injected as shown in Fig. C11. Assuming that the laser beam has a small diameter and scans the delay line back and forth, the output light beam will be modulated in phase depending upon the local variation of density of the delay line induced by the ultrasonic wave. Specifically, the phase of the input light beam is modified according to the transfer function

\[ T_s = \exp j \psi \]  
\hspace{1cm} (C45)

if it is assumed that the ultrasonic wave varies sinusoidally with time frequency \( \omega_s \), i.e.,

\[ v(t) = v_m \cos \omega_s t \]  
\hspace{1cm} (C46)

and propagates along the delay line with velocity \( c \), the index of refraction is modulated with a traveling wave, and the output light beam is correspondingly phase-modulated; the net modulation depends upon the relative velocities of light scanning and of ultrasonic wave propagation. Demodulation of the light may be obtained by a homodyne technique.

(C) Recapitulating, various designs of the modulation converter may be developed; by suitable selection of the dimensional or
To Homodyne Detector

Acoustic Delay Line

Laser Beam

Scan

FIG. C11 - MODULATION CONVERTER USING TRANSPARENT ACOUSTIC DELAY LINE
electrical characteristics, an optimization of the conversion efficiency with respect to the frequency of the sonar wave may be obtained. In the case of ultrasonic delay lines, a cross-correlation technique may be extended for the present application.

(C) The problem of establishing a communication link from air to submarine may be solved by recourse to a pulsed high-power laser beam directed from the aircraft vertically to the sea surface (Fig. C12). As shown by Carome et al.\(^{(1)}\), if the laser radiation is absorbed by water, transient heating effects are produced which generate acoustic waves. Neodymium lasers, which operate at 1.06 μ, are well suited for this application; in this case, a saturable-liquid Q-switch such as Kodak 9740 bleachable liquid may be utilized for the generation of high-peak-power pressure pulses. Another useful type of laser is the CO\(_2\)-N\(_2\)-He laser, which operates at 10.6 μ; this laser may be operated at higher efficiency and at higher average power level than the Neodymium laser. The absorption of the light pulse in the sea results in a rapid localized increase of the temperature; the gradients of the resulting thermal stress act as sources of acoustic waves. Experiments conducted with water tanks at limited height have shown that very high peak pressures, on the order of several thousand atmospheres, may be produced with this technique; extension to the case of an aircraft flying over the sea appears practical. However, the generated shock waves propagate as spherical waves rather than as collimated beams, so that their intensity decreases as the reciprocal of the square of the distance. If a sufficiently high signal-to-noise ratio cannot be realized by the above-described technique of direct propagation of shock waves in the sea, it may be possible to make recourse to a sonar method of readout of the perturbations produced by the laser beam at the sea surface. In fact, if a collimated sonar beam is directed from the submarine to the sea-air interface, its reflected beam is expected to exhibit a spatial angular modulation or a doppler frequency modulation corresponding to the
FIG. C12 - COMMUNICATION FROM AIRCRAFT BY PULSED LASER-INDUCED SHOCK WAVES

FIG. C13 - READOUT OF SURFACE INDUCED WATER WAVES BY SONAR BEAM
time variation of the surface perturbation (Fig. C13). Such modulation may be converted to amplitude-modulation type by means of a suitable converter.

(C) Recapitulating, a two-way communication link may be established between an aircraft and a submerged submarine, by recourse to techniques utilizing laser readout of sonar-generated surface signatures and acoustic shock waves produced in the sea by means of high-power pulsed laser beams. The feasibility of these techniques has been demonstrated in laboratory experiments; however, an extensive engineering-type effort is required to render these techniques field-operable with the desirable levels of reliability and safety.

REFERENCES