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AERODYNAMIC HEATING OF THE
STRUCTURE OF THE BLUE STREAK
BALLISTIC MISSILE INCLUDING THE
THERMODYNAMICS OF THE
PROPELLENT TANKS

by

J. Porter, M.Sc. (Eng.)

NOVEMBER, 1961

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AERODYNAMIC HEATING OF THE STRUCTURE OF THE BLUE STREAK BALLISTIC MISSILE INCLUDING THE THERMODYNAMICS OF THE PROPELLENT TANKS

by
J. Porter, M.Sc.(Eng.)

SUMMARY

An analysis has been made of the effects of kinetic heating during the boost phase on the main structure of Blue Streak. Three representative trajectories were investigated.

The thermodynamic processes occurring within the propellant tanks were considered, both as a part of the general problem of structural heating and as specific problems in their own right. This last analysis produced information concerning pressurisation of the tanks and boil-off from the liquid oxygen surface.
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1 INTRODUCTION

The main object of the investigation was to estimate the variations in wall temperatures of the Blue Streak ballistic missile during the powered flight phase. For this purpose the missile was divided into three sections: the guidance and transition bays, the liquid oxygen (lox) tank and the kerosene tank. The propulsion bay was not considered.

The guidance and transition bays presented a relatively simple problem; it was assumed that all of the aerodynamic heating entering the walls was either absorbed or re-radiated; the bays being unpressurised, heat transfer to internal gases was neglected.

The lox tank however presented a more complex problem and, to arrive at a solution, it was necessary to consider heat transfer to the internal fluid together with such factors as boil-off of lox from the liquid surface and blow-off of oxygen gas from the tank vent valve.

The kerosene tank presented fundamentally the same problem as the lox tank but boil-off had no longer to be considered. The presence of external stringers introduced a complexity not encountered in the case of the lox tank.

All of the calculations were programmed for the Ferranti Pegasus Digital Computer, assuming a missile system as shown in Fig.1. Three different trajectories were considered and these it is convenient to express as 20/0.7/36, 20/0.7/25 and 20/0.8/36 (the first number gives the vertical rise time after launch in seconds, the second number gives the subsequent turn over rate in degrees per second and the third is the final attitude measured with reference to the horizontal at launch).

2 METHOD OF ANALYSIS

2.1 The guidance and transition bays

An approach similar to that described in Ref.1 was used to estimate the kinetic heating of the guidance and transition bays. There was assumed to be no conduction along the length of the walls and no heat transfer to the internal fluid. The flow properties of the external boundary layer were computed assuming Newtonian Flow around the nose matched to a Prandtl-Meyer expansion to the final cone angle. The Newtonian pressure distribution was matched to the Prandtl-Meyer flow at the 60 degree point of the nominal hemisphere implied by the assumption of Newtonian Flow.

The heat transfer equation is then:-

\[ \rho_w C_w x_w \left( \frac{dT_w}{dt} \right) = h_{rw} (T_r - T_w) - \varepsilon B T_w^4. \] (1)*

The external heat transfer coefficient \( h_{rw} \) and the recovery temperature \( T_r \) were evaluated by the reference temperature method of Eckert².

*See List of Symbols.
2.2 The lox tank

In this case, heat and mass transfers inside the tank were considered, in addition to external heat transfer. As well as providing more realistic data on wall temperatures this broadened the scope of the investigation to include consideration of such factors as boil-off from the liquid surface and venting of the tank.

Fig. 2 shows diagrammatically the various heat and mass transfers considered in the case of the lox tank. These may be summarised:-

(i) Aerodynamic heat enters the wall, some is re-radiated,

(ii) The aerodynamic heat which is not re-radiated enters the wall and is either absorbed causing a change in temperature or it passes through the wall to the internal fluid.

(iii) The heat passing through the wall enters either the lox or the gas depending on the liquid level.

(iv) Heat entering the lox raises the temperature of the lox and may also cause boil-off. The gauge pressure within the tank being maintained constant, the process of boil-off is assisted by the falling absolute pressure, and thence lox saturation temperature, within the tank.

(v) Heat entering the ullage space from the walls is but one factor contributing to the heat balance of the pressurising gases. As well, there are heat inputs due to boil-off from the liquid surface and due to the in flow of pressurising gases. Heat is lost by venting and by condensation to the liquid surface.

Moving now from this general picture to a more detailed consideration of the processes involved it is possible to formulate equations:-

(i) $h_{rw}$ and $T_w$, the characteristics of the external flow, are calculated using the methods of Ref.1 and assuming free stream flow properties.

(ii) A heat balance equation for the walls may be written:

\[
\frac{dT}{dt} = h_{rw} (T - T_w) + h_{wg} (T - T_g) + \varepsilon B T_w^4 + \rho_w C_w \frac{dT_w}{dt} \quad (2)
\]

where the skin is in contact with gas.

When the skin is in contact with lox it is assumed that its temperature remains the same as the lox (approximately true if boiling is by a nucleate mechanism). Then the heat absorbed by the wall is negligible in comparison with the heat passing through the wall to the lox.

(ii) The equations governing heat flow from the wall to the lox and from the wall to the gas are:-
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\[ q_L = h_{rw} (T_r - T_w) - e B T_w^4 \]  \quad (3)

\[ q_g = h_{wg} (T_w - T_g) \]  \quad (4)

(iv) In calculating skin temperatures it was assumed that all of the heat entering the lox increased its bulk temperature until saturation conditions were reached; at which point boil-off commenced.

Two mechanisms of heat transfer are possible between the wall and the lox: nucleate boiling or film boiling. Under normal conditions nucleate boiling (i.e. boiling by the formation of many bubbles which do not adhere to the heating surface) is stable at heat inputs up to about 5CHU/sq ft/sec. If the heat input exceeds this value then the nucleate method of boiling becomes unstable and the many small bubbles degenerate to form a stable film of vapour next to the wall causing the heat input to the liquid to fall sharply and the wall temperature to rise.

On the basis of experimental evidence\(^1\), it seems likely that stratification effects produce an approximately linear axial temperature gradient in the lox bulk, where the temperature rise of the surface layers is twice the average temperature rise over a small time interval. An analysis of nucleate boiling was made using this model; further, it was assumed that when the surface layer of lox reached saturation conditions, boil-off would occur and that all of the heat subsequently entering the lox would be used to boil-off more lox and not divided, partly increasing the bulk temperature and partly causing boil-off\(^2\). This was a rather severe assumption.

A more severe assumption is that vibration of the tank walls will induce film boiling at heat inputs very much less than 5CHU/sq ft/sec. In calculations based on this model the change in wall temperature became significant and the heat entering the lox was less than in the case of nucleate boiling but all of this heat was used to produce boil-off there being no change in the lox bulk temperature.

(v) The heat balance equation for the pressurising gases, treated as an ideal, quasi-static gas is:

\[ \text{Heat supplied} = \text{aerodynamic} + \text{heat supplied by pressurising gas} + \text{boiling lox} \]

\[ - \text{heat lost by condensation} - \text{heat lost by blow-off} \]

\[ = \text{gain in internal energy} + \text{work done by the gas} \]

Thus (assuming \( C_p \) is constant):

\[ E_g + C_p \frac{dM_g}{dt} - C_p T_g \frac{dM_g}{dt} - C_p T_g \frac{dM_g}{dt} + C_p T_s \frac{dM_s}{dt} \]

\[ = M_g C_v \frac{dT_g}{dt} + C_v T_g \frac{dM_g}{dt} + \frac{P_g}{J} \frac{dV}{dt} \]

\[ - 6 - \]

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and for continuity:

\[ \frac{dh}{dt} = \frac{dv}{dt} - \frac{dM}{dt} - \frac{dv}{dt} + \frac{dM}{dt} \]

Whence:

\[ \frac{dV}{dt} - \frac{E}{C_v} + (\gamma T - T_g) \frac{dM}{dt} + (\gamma T_s - T_g) \frac{dh}{dt} - (\gamma - 1) T_g \left( \frac{dM}{dt} + \frac{dM}{dt} \right) \]

where \( E \) is the heat input rate from the walls integrated over the relevant surface area.

These equations were programmed for the Ferranti Pegasus Digital Computer. The constants and physical properties used are listed in Appendix 1 and illustrated in Figs. 1 and 3.

2.3 The kerosene tank

The kerosene tank presented fundamentally the same problem as the lox tank. However, it was unnecessary to consider boil-off, and all of the heat entering the kerosene was assumed to heat its whole bulk. Condensation of the pressurising gas, nitrogen, onto the surface of the kerosene was not possible, but heat transfer from the nitrogen to the surface of the kerosene was allowed for by assuming (see Appendix 1) that the liquid was equivalent to a cold plate facing upwards gaining heat by a process of natural convection.

The kerosene tank has forty six external stringers, capped at their forward ends and secured to the tank by spot welds (see Fig. 1). In heat transfer calculations it was assumed that the effective periphery of free skin between stringers (i.e., that skin which is involved in heat transfer between the atmosphere and the kerosene) was the distance \( p \) of Fig. 1, so that the total effective periphery involved in heat transfer was taken to be 46p.

3 ANALYSIS AND INTERPRETATION OF RESULTS

3.1 The guidance and transition bays

In calculating the heat transfer to the guidance and transition bays, the flow properties were computed assuming a Newtonian pressure distribution around the nose matched to a Prandtl-Meyer expansion to a final cone semi-angle of 5 degrees. A typical point was considered which was seven feet aft of the stagnation point in the case of laminar flow and seven feet aft of the transition point when the flow was turbulent. The most likely starting point for a turbulent boundary layer was thought to be the abrupt change in slope at the front of the conical frustrum. However, because the heat transfer coefficient varies only as the fifth root of the reference length in the case of turbulent flow, a seven foot reference length to the typical point was retained.
The skin temperatures are plotted against time from launch, together with the rate of heat input, in Fig. 4. The surface Reynolds Number based on a seven foot reference length drops from about $2.4 \times 10^5$ at 60 seconds after launch to less than $10^5$ at 110 seconds after launch and less than unity 180 seconds after launch. It seems unlikely, therefore, that turbulent flow will persist throughout flight, and the turbulent flow curves probably represent an excessively severe case.

The curves obtained using a constant surface emissivity of 0.9 are included to show the probable effects of treating the skin (normal emissivity varies between 0.2 and 0.4) with a heat resistant, matt-black paint.

In order to assess the heating on the ten degree semi-angle section which joins the guidance bay to the tank section some calculations were performed with a fourteen foot run of turbulent boundary layer, the flow properties being computed as before, only this time using a ten degree semi-angle.

The results showed that the maximum rate of kinetic heat input occurs about 120 seconds after launch; the maximum skin temperatures, encountered towards the end of the powered flight phase, are summarised below:

<table>
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<th>Turbulent flow</th>
<th>Laminar flow</th>
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<td>Variable $\varepsilon$</td>
<td>$\varepsilon = 0.9$</td>
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<tr>
<td>20/0.7/36</td>
<td>483</td>
<td>500 *</td>
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<tr>
<td>20/0.7/25</td>
<td>588</td>
<td>600 *</td>
</tr>
<tr>
<td>20/0.8/36</td>
<td>555</td>
<td>570 *</td>
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All for seven feet run of boundary layer and 5° cone semi-angle except * for 14 ft run and 10° semi-angle.

3.2 The lox tank

3.2.1 Assumptions

As the calculations progressed a number of assumptions were made. If these assumptions are now collected together and summarised it will be easier to relate the results of the investigation to the assumptions.

(i) Flat plate flow properties were assumed for the outside of the tank.

(ii) Conduction along the walls was of a sufficiently small magnitude to be negligible.

(iii) The walls were sufficiently thin and had a high enough thermal conductivity to justify the assumption of no thermal gradient across the skin.

(iv) The top dome was assumed to be perfectly insulated, an assumption made possible, it is suggested, by the rarified atmosphere within the unpressurised guidance bay at altitude and by the very low radiation heat transfer.
(v) The bottom dome and the lox contained by it were assumed to insulate the bulk of the lox from the kerosene tank. Any heat passing into the lox by way of the inter tank dome will be absorbed by lox adjacent to the dome; most of this lox is then drawn off from the tank by the lox feed pipe (see Fig.1).

(vi) The lox tank was assumed to be pressurised by an inflow of hot oxygen gas.

(vii) The gas within the ullage space was assumed to be an ideal $(PV = nRT)$, perfectly mixed gas.

(viii) Boiling, condensation and heat transfer coefficients are discussed in Appendix 1.

3.2.2 Skin temperatures

By virtue of assumption (i) above, the skin temperature predictions are independent of the shape of the missile forward of the lox tank. The effect of different nose shapes on the flow transition from laminar to turbulent would have to be considered in practice. Remembering, however, that the turbulent heat transfer coefficient varies as the fifth root of the distance aft of the transition point, it is apparent that nose shape is unlikely to have much effect on skin temperatures of the lox tank.

Fig. 5 shows the complete wall temperature history of the lox tank for a 20/0.7/36 trajectory with turbulent flow from the nose. Fig. 6 shows the axial variation of wall temperature 160 seconds after launch.

In a second case, considered in order to obtain the worst possible case of structural heating, it was assumed that transition to turbulent flow occurred at the forward edge of the lox tank. If the structure successfully withstands these conditions, then it may be considered safe for all other flow conditions.

Fig. 7 shows skin temperatures one foot aft of the leading edge of the lox tank. A comparison is made of 20/0.7/36, 20/0.7/25 and 20/0.8/36 trajectories for turbulent boundary layers extending first from the nose and then from the leading edge of the lox tank.

The maximum permissible temperature which the Blue Streak lox tank structure will withstand (based on the maximum factored hoop stress due to internal pressure) is plotted on Fig. 7 for comparison with the actual temperatures attained.

The main body of results were obtained with a pressurising gas input of 2 lb/sec at 500°K. A single case was considered in which a pressurising gas input temperature of 600°K was assumed. The effect of this variation on the skin temperatures is shown in Fig. 8; the increased input temperature is seen to cause a small but not insignificant change in wall temperature.

The heat exchanger (see Figs. 1 and 2) is designed to produce gas at 2 lb/sec at a temperature of 600°K. Although no detailed calculations have been done, it is thought that the tank inlet temperature will be lower than the temperature out of the heat exchanger unit, so that the upper curve of Fig. 8 represents a worst possible case from the structural aspect.

Also plotted in Fig. 8 is a curve in which heat transfer from the skin to the internal gases is totally neglected; this curve agrees quite closely with the curve obtained with a pressurising gas input of 2 lb/sec at 500°K.
This approximation was used for a third case in which a transition Reynolds's Number of $3.10^6$ was assumed and heat transfer to the internal gases was again neglected. Fig. 9 compares the maximum skin temperatures reached assuming either all turbulent flow from the nose or a transition Reynolds's Number of $3.10^6$.

Transition Reynolds's Numbers as high as $10^7$ have been recorded on Black Knight which has a low drag head. However, for bluff bodies, recorded temperatures seem to agree more with predictions made assuming turbulent flow from the nose.

3.2.3 Pressurisation

Pressurisation of the tank maintains structural integrity. It is therefore essential to supply sufficient pressurising gas. On the other hand, to supply too much is a wasteful process. Pressurising oxygen is supplied throughout flight at a nominally constant rate; the gauge pressure of the ullage space is maintained constant by blowing surplus gas through a vent valve. If oxygen boils-off from the liquid surface then the pressurisation gas demands of the tank are unlikely to become critical because the tank is, to some extent, self pressurising; but if little or no lox is boiled off then the amount of gas vented will be reduced and the pressurisation supply may become critical.

From the calculations discussed in Section 3.2.2 it was ascertained that the rate of gas venting was least during the first thirty seconds of flight i.e. the maintenance of tank pressure is most critical during this period. A closer investigation of this region produced the results of Fig. 10, in which the blow-off mass rate is plotted for various inlet conditions. These results indicate that an inlet temperature of 400°K at a flow rate of 2 lb/sec is necessary to maintain the tank pressure.

Fig. 11 is an alternative plot showing the amount of gas which must be supplied at particular temperatures and the required input temperatures at particular constant flow rates.

3.2.4 Boil-off

Boil-off of lox is of considerable importance because it lowers the ullage temperature and this results in a high all burnt weight, thus adversely affecting performance. (At 160 seconds after launch the mass of gas in the tank is 200 lb if the ullage temperature is 240°K and 400 lb if the ullage temperature is 220°K.)

In the skin temperature calculations of Section 3.2.2, it was assumed that all of the heat entering the lox increased its bulk temperature. Under these conditions the lox never reached saturation conditions and thus there was no boil-off.

It has been suggested (Section 2.2) that boiling may be governed by a nucleate or a film boiling mechanism; and in the case of nucleate boiling it is further assumed that stratification effects cause a linear axial lox temperature gradient (see Section 2.2). The results of calculations based on each of these models are plotted as Figs. 12 and 13, respectively.

If boiling is by a nucleate mechanism, then tests have shown that after boiling starts the heat entering the lox will be divided: part causing an increase in the bulk temperature of the sub-surface strata of the lox and part supplying latent heat to boil off lox. Thus the predictions of Fig. 12 are probably pessimistic. However, if film boiling is initiated by vibration of the tank walls then it is possible that a case approaching the severity
of Fig. 13 may be encountered. Whether this is likely to happen in practice must depend to a large extent on the results of experimental investigation of the possibility of inducing film boiling, at low heat input rates, by vibration.

3.3 The kerosene tank

In the case of the kerosene tank boil-off was a factor which did not have to be taken into consideration. The two factors remaining to be investigated were skin temperature variations and tank venting. Using a similar approach to that employed in analysing the LOX tank the results of Figs. 14, 15, 16 and 17 were obtained.

Fig. 16 compares different trajectories and, as well, compares the free skin temperature with the stringer temperature at the same station. The stringer temperatures were calculated assuming that no heat was transferred from the stringer crown to the air beneath the top hat section.

Fig. 17 shows the venting rate for different pressurising gas input conditions. Allowance was made, in this computation, for heat losses to the LOX tank dome, it was assumed that the dome was, effectively, a cold plate facing downwards being heated by turbulent convection. The bulk of the LOX contained in the dome maintained a constant dome temperature. It seems, from these results, that the kerosene tank is pressurised adequately by 0.75 lb/sec of nitrogen supplied at 400°K. Design for excessive pressurisation gas flow carries an inherent weight penalty because the pressurising gas is supplied from a container which must hold sufficient nitrogen for the whole of the boost phase flight up to cut-off.

CONCLUSIONS

The problem of structural heating is complicated by many factors and any results obtained must be assessed with due regard to each of the assumptions made in the course of calculation. As far as possible, the aim of this investigation has been to present a clear picture of the assumptions made in the course of analysis so that the reader may assess the results with reference to these assumptions.

The problem was divided into two sections. The main aim was to investigate structural temperatures during the boost phase, but this end proved unattainable without considering, as well, the thermodynamic processes occurring within the propellant tanks. Thus, it became necessary to investigate the effects of LOX boil-off and pressurising gas input to the tanks. These two problems proved to have an importance in their own right as well as being useful auxiliaries to the structural heating analysis.

Boil-off of LOX, from the liquid surface carries a performance penalty. However, calculations based on a nucleate boiling mechanism suggest that boil-off is unlikely to be more than 200 lb, which is equivalent to an all burnt ullage weight penalty of about 140 lb.

If insufficient pressurising gas is supplied to the tanks then structural integrity is impaired. On the other hand, to supply too much pressurising gas carries an inherent performance penalty. The calculations show that the supply of pressurising gases is sufficient but not excessive.

The results of the calculations show how structural heating effects may be minimised by using slow turn-over rate trajectories; which keeps the velocity low while the missile is still within the denser layers of the atmosphere.
Suffixes have been employed in, as far as possible, a logical manner:-

- $g$ denotes the gas in the tank
- $l$ denotes lox
- $k$ denotes kerosene
- $w$ denotes the wall conditions
- $r$ denotes the external recovery conditions

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<tr>
<td>$T_g$</td>
<td>gas temperature</td>
</tr>
<tr>
<td>$T_l(T_k)$</td>
<td>lox (kerosene) temperature</td>
</tr>
<tr>
<td>$T_w$</td>
<td>wall temperature</td>
</tr>
<tr>
<td>$T_r$</td>
<td>boundary layer recovery temperature</td>
</tr>
<tr>
<td>$T_s$</td>
<td>saturation temperature of lox</td>
</tr>
<tr>
<td>$T$</td>
<td>pressurising gas input temperature</td>
</tr>
<tr>
<td>$M_g$</td>
<td>mass of gas in tank</td>
</tr>
<tr>
<td>$M_l(M_k)$</td>
<td>mass of lox (kerosene)</td>
</tr>
<tr>
<td>$M_v$</td>
<td>mass of gas vented from tank</td>
</tr>
<tr>
<td>$M_b$</td>
<td>mass of lox boiled off from lox-gas interface</td>
</tr>
<tr>
<td>$M_c$</td>
<td>mass condensing from gas to lox surface</td>
</tr>
<tr>
<td>$M$</td>
<td>mass input by pressurising system</td>
</tr>
<tr>
<td>$P_g$</td>
<td>pressure of gas in tank</td>
</tr>
<tr>
<td>$V_g$</td>
<td>volume of ullage space</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heats</td>
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<td>$R$</td>
<td>characteristic gas constant</td>
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LIST OF SYMBOLS (CONT'D.)

L<sub>e</sub>  latent heat of lox  CHU/lb

C<sub>e</sub>(C<sub>k</sub>)  specific heat of lox (kerosene)  CHU/lb/°K

B  Stefan Boltzmann constant  CHU/sq ft/sec/°K<sup>2</sup>

e  emissivity of wall material

C<sub>w</sub>  specific heat of wall material  CHU/lb/°K

ρ<sub>w</sub>  density of wall material  lb/cu ft

x<sub>w</sub>  thickness of wall  ft

h<sub>rw</sub>  wall atmosphere heat transfer coefficient  CHU/sq ft/sec/°K

h<sub>wg</sub>  wall gas heat transfer coefficient  CHU/sq ft/sec/°K

h<sub>wk</sub>  wall lox (kerosene) heat transfer coefficient  CHU/sq ft/sec/°K

q<sub>e</sub>(q<sub>k</sub>)  heat input to lox (kerosene) from walls  CHU/sq ft/sec

q<sub>g</sub>  heat input to pressurising gases from walls  CHU/sq ft/sec/°K

β<sub>g</sub>  coefficient of volumetric expansion of gas

k<sub>g</sub>  thermal conductivity of gas  CHU/ft sec/°K

μ<sub>g</sub>  viscosity of gas  lb/ft/sec

ρ<sub>g</sub>  density of gas  lb/cu ft

r  radius of tank  ft

ν<sub>g</sub>  apparent axial acceleration of vehicle  ft/sec<sup>2</sup>
### LIST OF REFERENCES

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Appendix 1  
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Detachable Abstract Cards

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RESTRICTED
APPENDIX 1

DATA USED IN CALCULATIONS

1 INITIAL CONDITIONS

1.1 Guidance and transition bays
At 50 seconds after launch $T_w = 288^\circ\text{K}$.

1.2 Lox tank
Volume of gas space $V_g = 96$ cu ft
Temperature of gas $T_g = 240^\circ\text{K}$
Wall temperature next to lox $T_w = 93.5^\circ\text{K}$
Lox temperature $T_l = 93.5^\circ\text{K}$
Lox saturation temperature $T_s = 101.7^\circ\text{K}$
Pressure in the gas space $P_g = 42.7$ lb/sq in.
Mass of gas in the tank $M_g = 28.3$ lb
Input of pressurising gas (oxygen) $\frac{dM}{dt} = 2$ lb/sec at $500^\circ\text{K}$

1.3 Kerosene tank
Volume of gas space $V_g = 107$ cu ft
Temperature of gas $T_g = 288^\circ\text{K}$
Wall temperature next to kerosene $T_w = 288^\circ\text{K}$
Kerosene temperature $T_k = 288^\circ\text{K}$
Pressure in the gas space $P_g = 25.7$ lb/sq in.
Mass of gas in the tank $M_g = 13.9$ lb
Input of pressurising gas (nitrogen) $\frac{dM}{dt} = 0.75$ lb/sec at $600^\circ\text{K}$

2 CONSTANTS

2.1 General
$\gamma = 1.4$
$J = 14.00$ ft lb/CHU
$B = 1.58 \times 10^{-11}$ CHU/sq ft sec $(^\circ\text{C})^4$
$\rho_w = 490$ lb/cu ft
2.2 Lox tank

- \( R = 87 \text{ ft lb/lb/°K} \) (oxygen)
- \( L = 50.9 \text{ CHU/lb} \)
- \( C_\ell = 0.4 \text{ CHU/lb/°K} \)
- \( p_\ell = 70.4 \text{ lb/cu ft} \)

2.3 Kerosene tank

- \( R = 99 \text{ ft lb/lb/°K} \) (nitrogen)
- \( C_k = 0.46 \text{ CHU/lb/°K} \)
- \( p_k = 50.3 \text{ lb/cu ft} \)

3 VARIABLES

3.1 It was assumed in calculation that the cylindrical portions of both the Lox and kerosene tanks were emptied during the first 160 seconds of flight, the liquid level was assumed in each case to fall linearly with time. Thus:

\[
V_g = 96 + \frac{22}{160} \pi r^2 t \quad \text{for the Lox tank}
\]

\[
V_g = 107 + \frac{14}{160} \pi r^2 t \quad \text{for the kerosene tank,}
\]

where \( t \) is the time from launch (secs).

Other variables are plotted in Fig.3.

Fig.3(i) shows the variation in ullage volume with time, the relationships given above were used to produce these curves.

3.2 Fig.3(ii) shows the variation of pressure within the tanks with altitude. The tank pressure at any altitude is simply the sum of the tank gauge pressure and ambient pressure. The pressure in the Lox tank was assumed to be a constant 28 p.s.i.g, and that in the kerosene tank constant at 11 p.s.i.g.

3.3 The variations of specific heat at constant volume of oxygen and nitrogen are plotted in Fig.3(iii) from data obtained from Ref.*4.

3.4 The heat transfer coefficient between the internal gases and the tank walls based on MoAdams^5 formula is:

\[
h_{wg} = 0.13 \frac{P}{k} \frac{C_p \rho_p^2 (ng) \Delta T}{k_g \mu_g}
\]

for turbulent natural convection to a vertical wall. This reduces to:
h_{wg} = H \left[ (ng) P^2 \Delta T \right]

where \( H \) is a function of a reference temperature only and \( \Delta T \) is the temperature difference between the wall and the gas. The value of \( H \) is plotted against temperature in Fig.3(iv).

3.5 Condensation is taking place from the hot gas to the surface of the lox. It was assumed that this condensation was induced by a process of turbulent convection above the lox surface which was taken as equivalent to a cold plate facing upwards. From unpublished work an expression was obtained for the heat transfer coefficient, \( h \):

\[
h = 0.000063 \Delta T^{5/4}
\]

where \( \Delta T \) is the difference in temperature between lox and gas.

Thence the heat passing is:

\[
0.000063 \Delta T^{5/4} \text{ Btu/sq ft/sec.}
\]

If it is assumed that this heat reduces the temperature of a mass of gas \( M \) to the temperature of the lox, which then condenses onto the lox surface, then:

\[
M C_p \Delta T + M_o C_\ell \Delta T = 0.00494 \Delta T^{5/4}
\]

where the constant is corrected to allow for an area of 25\( \pi \) square feet.

Thence

\[
M_o = \frac{0.00494 \Delta T^{5/4}}{(L_\ell + C_p \Delta T)}
\]

this relationship is plotted in Fig.3(v).

3.6 Fig.3(vi) is a plot of the variation of specific heat of the wall material against temperature.

3.7 Fig.3(vii) is an adaptation of experimental results obtained for the variation of emissivity of the wall material with temperature.

3.8 Heat transfers between the tank walls and the contained liquids are plotted in Fig.3(viii). For the kerosene the formula of Section 3.3 of this Appendix was used to predict heat transfer rates. For the lox experimental figures obtained by Sinha were used to predict the heat transfer rates with nucleate boiling.
3.9 Film boiling heat transfer rates between the wall and the lox are plotted in Fig. 3(ix). These results were obtained from the work of Bonchero, Barker and Bollé.

3.10 Fig. 3(x) shows the variation of lox saturation temperature against altitude. This curve is based on N.B.S. experimental data.
FIG. I.

GEOMETRY OF VEHICLE

TRANSITION AND GUIDANCE BAYS
60 EQUI-SPACED LONGITUDINAL STRINGERS

LOX TANK.

46 STRINGERS EACH 1.4" HIGH BY 2.9" ACROSS THE FLANGES CAPPED AT LEADING EDGE

KEROSENE TANK WITH 46 LONGITUDINAL STRINGERS EQUALLY SPACED AROUND THE TANK PERIPHERY.

FIG. I. GEOMETRY OF VEHICLE
FIG. 2. SCHEMATIC DIAGRAM OF LOX TANK
FIG. 3. PHYSICAL PROPERTIES ASSUMED IN CALCULATION.
FIG. 4.

WALL TEMPERATURES ATTAINED BY THE SKIN OF THE TRANSITION BAY (0.010 INCHES THICK)

All temperatures are for a 7 ft. run of boundary layer.

A 5" cone semi-angle.
FIG. 5

WALL TEMPERATURE HISTORIES FOR LOX TANK.
FIG. 6. AXIAL WALL TEMPERATURE DISTRIBUTION FOR THE LOX TANK 160 SECONDS AFTER LAUNCH.
FOR A POINT 15 FEET AFT OF THE NOSE TURBULENT FLOW WITH:
* TRANSITION AT THE NOSE
+ TRANSITION AT THE LEADING EDGE OF THE LOX TANK.
PRESSURISING GAS 21b/SEC. AT 500°K.

MAXIMUM PERMISSIBLE WALL TEMPERATURE

FIG. 7 WALL TEMPERATURES ONE FOOT AFT OF THE LEADING EDGE OF THE LOX TANK—TURBULENT FLOW. (SKIN 0.019 IN THICK)
20/0·7/36 trajectory for a point 15 feet aft of the nose. Turbulent flow from the nose.

**FIG. 8**

Variation in wall temperatures with different internal heat transfer assumptions.
20/0-7/36 TRAJECTORY
NO HEAT TRANSFER TO INTERNAL GASES.
FOR A POINT 15 FT. AFT OF THE NOSE.

FIG. 9. EFFECT OF TRANSITION REYNOLD'S NUMBER
ON WALL TEMPERATURES 15 FT. AFT OF NOSE.
TURBULENT FLOW FROM THE NOSE.
20/0.7/36 TRAJECTORY.

FIG. 10

FIG. 10. VARIATION OF LOX TANK BLOW-OFF MASS WITH DIFFERENT PRESSURISING GAS INPUT CONDITIONS.
REQUIRED MASS FLOW RATE TO PRESSURISE THE TANK WITHOUT VENTING - FOR DIFFERENT CONSTANT INPUT TEMPERATURES.

REQUIRED INPUT TEMPERATURE TO PRESSURISE THE TANK WITHOUT VENTING FOR DIFFERENT MASS FLOW RATES.

**FIG. II. LOX TANK PRESSURISING GAS REQUIREMENTS**
FIG. 12. NUCLEATE BOILING WITH LINEAR STRATIFICATION OF LOX.
FIG. 13. FILM BOILING OF LOX
FIG. 14. WALL TEMPERATURE HISTORIES FOR KEROSENE TANK.
FIG. 15. AXIAL WALL TEMPERATURE DISTRIBUTION FOR THE KEROSENE TANK-160 SECONDS AFTER LAUNCH.
FIG. 16. TURBULENT FLOW FROM THE NOSE.
PRESSURISING GAS 0.75lb/sec. AT 600 °K.
WALL 0.019 INCHES THICK
STRINGERS 0.028 INCHES THICK.

FIG. 16. WALL AND STRINGER TEMPERATURES
NEAR THE FRONT OF THE KEROSENE TANK.
FIG. 17. VARIATION OF KEROSENE TANK BLOW-OFF MASS WITH DIFFERENT PRESSURISING GAS INPUT CONDITIONS
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AD#: AD328279
Date of Search: 11 December 2008

Record Summary: AVIA 6/21514
Title: Aerodynamic heating of the structure of the BLUE STREAK ballistic missile including the thermodynamics of propellant tanks
Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
Former reference (Department) TECHNICAL NOTES GW 598
Held by The National Archives, Kew

This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

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