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SOME ASPECTS OF THE DESIGN OF STAR-CENTRE SOLID PROPELLENT ROCKETS CHARGES

by

K. E. SILMAN

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AUGUST, 1960

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II Title: Star-centered solid propellants
III
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V Project No.
VI Contract No.

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SOME ASPECTS OF THE DESIGN OF STAR-CENTRE SOLID PROPELLANT ROCKET CHARGES

by

K. E. Silman

SUMMARY

A mathematical treatment of a family of suitable designs is presented in the form of charts showing the inter-relation of density of loading, sliver fraction and web thickness.
LIST OF CONTENTS

1 INTRODUCTION 3
2 EXPLANATION OF NOMENCLATURE 3
3 THE DESIGN OF STAR-CENTRE CHARGES 4
  3.1 Typical charge 4
  3.2 The effect of the number of lobes 4
4 DESIGN CHARTS 5
  4.1 General 5
  4.2 Six-pointed star-centre designs with values of $a$ other than $\pi$ 5
  4.3 Approximate equations for six-pointed star-centre designs 6
REFERENCES 7
ADVANCE DISTRIBUTION 8
TABLES I AND II 9
APPENDIX 10-14
ILLUSTRATIONS Fig. 1-11 -
DETACHABLE ABSTRACT CARDS -

LIST OF TABLES

Table
I Conditions for neutral burning 9
II Maximum density of loading for neutral burning and $P = 2aR$ 9

APPENDIX

Geometry of star-centre charge designs 10-14

LIST OF ILLUSTRATIONS

Typical star-centre charge design Figure 1
Segment of star-centre charge showing notation 2
Neutral burning star-centre design, $N = 5$, $P = 2aR$ 3
Neutral burning star-centre design, $N = 6$, $P = 2aR$ 4
Neutral burning star-centre design, $N = 7$, $P = 2aR$ 5
Neutral burning star-centre design, $N = 8$, $P = 2aR$ 6
Variation of sliver with web thickness 7
Neutral burning 6-point star-centre design, $P = 2aR$ 8
Neutral burning 6-point star-centre design, $P = 2aR$ 9
Possible end of burning conditions 10
Neutral burning star-centre design, web and star-centre burn out together, i.e. $P_A = P_w = P_o = 2a^2R$, variation of $a^* \text{ with } N \text{ and } \phi/\beta$ 11

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1 \textbf{INTRODUCTION}

Designing solid propellant charges is an art. If this is recognised then the methods now presented will be accepted in their right perspective as another attempt to formulate a rational basis for designing charges in which burning occurs at approximately constant pressure on the surface of a central conduit of star form.

The condition of constant burning pressure demands constant surface area of burning except for the initial period in which erosive burning may occur.

Already several reports have been written on the mathematical approach to star-centre charge design. Two of the earliest reports known to the writer are those of references 1 and 2, whilst later reports giving more arithmetical results are listed as references 3, 4 and 5. The treatment presented in this paper is new and the results are in a convenient form for assessment work.

2 \textbf{EXPLANATION OF NOMENCLATURE}

A typical form of star-centre charge is shown in Fig. 1 and some of the defining quantities are shown in Fig. 2. These, and others not capable of pictorial presentation, are given below.

The mathematical exercise is feasible only if the surface area of burning is confined to the conduit; that this surface is the product of the axial length of the conduit and the perimeter of the conduit in the plane normal to the axis of the conduit, and that the axial length is constant. Burning at constant pressure, now defined as "neutral burning", therefore implies that the perimeter of the conduit shall be constant.

\textbf{List of Symbols}

- $P'$ \hspace{1cm} Imaginary initial perimeter (see Fig.2)
- $P_1$ \hspace{1cm} Actual initial perimeter
- $P$ \hspace{1cm} Any intermediate perimeter
- $P_w$ \hspace{1cm} Perimeter at instant of web burn out
- $R$ \hspace{1cm} Outermost radius of charge (inner radius of motor case if the charge is case-bonded)
- $r_1$ \hspace{1cm} Initial radius of fillet at junction of lobe and web (see Fig.1)
- $r_2$ \hspace{1cm} Initial radius of lobe tips
- $N$ \hspace{1cm} Number of lobes
- $w'$ \hspace{1cm} Web thickness corresponding to imaginary initial perimeter
- $w$ \hspace{1cm} Web thickness corresponding to actual initial perimeter
- $a$ \hspace{1cm} Constant, where $P = 2aR$
- $a^*$ \hspace{1cm} Value of $a$ for simultaneous burn out of web and lobe
- $b'$ \hspace{1cm} The distance measured normal to the lobe face from the imaginary initial junction of the lobe face and the web to the centre line of the lobe (see Fig.2)
Specified as $b' - r_1$ (see Fig.2)

Segment angle (including semi-web and semi-lobe = $\pi/N$)

Design angle (when $2\phi$ includes one imaginary web portion)

Design angle (see Fig.2)

Angle subtended by circular arc of a semi-sliver at the imaginary
initial junction of lobe and web, at instant of web burn-out, for
the case of prior lobe burn through (see Fig.10a)

Sliver fraction, i.e. $\frac{\text{Total cross-sectional area of slivers at web burn-out}}{\text{Initial cross-sectional area of charge}}$

Density of loading, i.e. $\frac{\pi R^2}{\text{Initial cross-sectional area of charge}}$

Density of loading for the imaginary initial perimeter

THE DESIGN OF STAR-CENTRE CHARGES

3.1 Typical charge

The cross-section of a typical star-centre charge is shown in Fig.1, and it will be seen that there are no sharp corners. The apices of the lobes are rounded to reduce the initial burning perimeter and so reduce or eliminate the high initial pressure peak that would otherwise occur (an alternative treatment is suggested in reference 2). Fillets are provided at the junctions of the lobes and the webs to eliminate stress concentrations, which might lead to cracking of the charge during storage or firing.

A single charge segment is shown in Fig.2, together with generating lines and details of angles. The geometry of this type of charge design is investigated in the Appendix.

3.2 The effect of the number of lobes

The final perimeter of the conduit at web burn-out is likely to be only slightly greater than $2\pi R$, so the first designs investigated were those giving neutral burning for the greater part of the effective burning time, and having a conduit perimeter of $2\pi R$. The condition for neutral burning is derived in the Appendix and it will be seen that this is satisfied by a certain value of the angle $(\alpha + \phi)$; this has been found for values of $N$ from 2 to 16 and the results are given in Table I. For $N$ less than or equal to 5, Table I shows that the lobe side is re-entrant for neutral burning, that is $(\alpha + \phi)$ is greater than 90 degrees, which means that, for small values of $\phi$, and sharp corners, an impossible design could result with the apices overlapping. The maximum density of loading, the ratio $w/R$, and the sliver fraction, have been calculated for the above condition ($P = 2\pi R$), and are tabulated in Table II for $N$ from 2 to 16, for designs having sharp corners. These designs, for values of $N$ of 5 or less, have the lobe apices just meeting; the effect of rounding off the corners is dealt with in the Appendix.

It will be seen that the maximum density of loading occurs at $N$ equal to 6, with only a slightly lower value at $N$ equal to 5. This is not really surprising, since consideration of the angle $(\alpha + \phi)$ would show that a maximum density of loading equal to 1.0 would be achieved when $(\alpha + \phi)$ is equal to 90 degrees; for neutral burning, this would occur with a value of $N$ of 5.55 which is not possible.
4 DESIGN CHARTS

4.1 General

Detail charts have been produced for star-centre designs, to have neutral burning characteristics ($P = 2\pi R$), for values of $N$ of 5, 6, 7 and 8, and these are shown in Fig. 3, 4, 5 and 6. Basically, the density of loading is plotted against web thickness/charge radius ($w/R$) and the radius at the outer star point/charge radius ($r/R$). On this are superimposed lines of constant sliver area/motor area ($\lambda s$), which occur as straight lines, and lines of constant proportions of neutral burning ($b/w$). Also shown in a small picture is the ratio of the final perimeter at web burn-out to the normal perimeter ($P/w$) plotted against the design angle $\phi$; the value of $\phi$ can be found from the main chart, since the values are shown on the boundaries and a line of constant $\phi$ would appear as a straight line parallel to a line of constant $\lambda s$.

In Fig. 3, for a 5-point star design, it will be seen that the upper part of the diagram has been cut off at a density of loading of about 0.95. This is due to the fundamental requirement that the lobe apices cannot overlap.

The effect of rounding the lobe apex to reduce the initial perimeter will only reduce the density of loading $\lambda$, and will not affect the values of $\lambda s$, or $P/w$. The proportion of neutral burning will be reduced by $r_2/w$ at the beginning, when the burning is progressive.

Since the lines of constant sliver/motor area ($\lambda s$) are not evenly spaced across the diagrams, curves of sliver/motor area ($\lambda s$) against the parameter ($w + r_2)/R$ are shown in Fig. 7 for values of $N$ of 5, 6, 7 and 8.

For given web thickness/charge radius ($w/R$) and density of loading $\lambda$, the sliver/motor area ($\lambda s$) is very nearly independent of the number of star points. The proportion of neutral burning ($b/w$) reduces, and the final perimeter increases, with increase in the number of star points. Also, for a given web thickness/charge radius ($w/R$), the range of density of loading that can be considered is reduced as $N$ is increased, the maximum being reduced and the minimum increased. From these points of view, it is suggested that 5 or 6 pointed star-centre designs are likely to be the most useful.

4.2 Six-pointed star-centre designs with values of $a$ other than $a^*$

It was considered that some investigation should be made into the effect of varying the constant $a$. In Fig. 8, the density of loading with sharp corners ($\lambda'$) is shown as a function of web thickness/charge radius and the constant $a$. Also shown are lines of constant sliver area/motor area and lines of constant final perimeter/normal perimeter ($P/w$). The value of the design angle $\phi$ is shown in Fig. 9 against web thickness/charge radius and the constant $a$. With sharp corners, lines of constant proportions of neutral burning ($b'/w'$) are also shown.

As is shown in the Appendix, for $a$ greater than $a^*$, the web will burn out before the star side and there will be neutral burning until web burn-out. In the present case $a^*$ is slightly less than $3.4$.

Let us now consider the effects of varying the value of $a$. Suppose we start at a large value of $a$. At constant density of loading, as $a$ is reduced we have the following effects:

- 5 -
(i) the sliver fraction is reduced, since $\lambda$ is constant,
(ii) web thickness/charge radius is increased,
(iii) there is no change in the final perimeter/normal perimeter ($P/w/2aR$) until $a = a^*$, but as $a$ is further reduced, this ratio increases,
(iv) as an alternative to (iii) the proportion of neutral burning ($b'/w'$) reduces as $a$ is reduced below $a^*$.

Due to the method of filling rocket motor tubes with plastic propellant some increase in perimeter can be permitted if the burning surface is to be kept constant, since the initial length of the charge is greater than the final length, as the propellant is pressed into the head and rear domes as well as the parallel portion of the tube. Since the reduction in length is unlikely to exceed 10%, unless the charge length/diameter is low, the maximum value of final perimeter/normal perimeter would be 1.10. Also, there do not appear to be any very valid reasons for having $a > a^*$, so that the reasonable range of $a$ is from 2.9 to 3.4.

4.3 Approximate equations for six-pointed star-centre designs

For project assessment work, it was considered that it would be useful to have some approximate equations relating density of loading, web thickness, sliver, etc. From the earlier discussion, several reasons have been given for using 5 or 6-pointed star centres, and for this case the 6-pointed star was used, with a perimeter of $2KR$.

Assuming that a density of loading less than 0.6 would not be of interest, the following equations were obtained, and the R.M.S. error is also given:

As linear equations

| $\lambda'$ | 2.661888 $(w'/R)$ - 0.167889 | 0.004722 |
| $\lambda_e$ | 0.658981 $(w'/R)$ - 0.168513 | 0.004791 |
| $\lambda' - \lambda_e$ | 2.002907 $(w'/R)$ + 0.000624 | 0.000069 |

$w'/R = 0.009055 \phi + 0.428325$ for $\phi$ in degrees.

0.003530

As parabolic equations

| $\lambda'$ | 3.363424 $(w'/R)^2$ + 0.254098 $(w'/R)$ + 0.257747 | 0.000752 |
| $\lambda_e$ | 3.412416 $(w'/R)^2$ - 1.783882 $(w'/R)$ + 0.263323 | 0.000766 |
| $\lambda' - \lambda_e$ | - 0.048993 $(w'/R)^2$ + 2.037979 $(w'/R)$ - 0.005576 | 0.000044 |

$w'/R = -0.000211\phi^2 - 0.005599\phi + 0.421195$ for $\phi$ in degrees.

0.000191

Now, as shown in the Appendix, the actual density of loading is given by:
\[ \lambda = \lambda' - (2r/R) - \left(\frac{r_2}{R}\right)^2 \]

and

\[ \frac{w'}{R} = \frac{w}{R} + \frac{r}{R} \]

and

\[ \left(\frac{r_2}{R}\right)^2 = (1 - \frac{P_1}{P})^2 \]

e.g., it may be found that for a given propellant under certain conditions, a value of \( P_1/P \) of 0.9 will be sufficient to remove an initial pressure peak and a value of 0.005 for \( r_1/R \) will be adequate to safeguard against charge cracking. Then

\[ \lambda = \lambda' - 0.02 \]

\[ \frac{w}{R} = \frac{w'}{R} - 0.005 \]

and

\[ \lambda - \lambda_e = (\lambda' - \lambda_e) - 0.02 \]

It will be realised that \((\lambda - \lambda_e)\) represents the charge area burnt away at burn-out, and the equations given represent extremely good approximations to this.

**LIST OF REFERENCES**

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| 2   | Piasecki, L.         | Notes on rocket charge design 1 The geometrical characteristics of charges with star centre and burning inhibited on the outer surface  
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Tables I and II
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- 8 -

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### TABLE I
Conditions for neutral burning

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Maximum density of loading for neutral burning and \( P = 2\pi R \)

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APPENDIX

GEOMETRY OF STAR-CENTRE CHARGE DESIGNS

Some consideration has already been given to this type of design but with various limitations, such as $\phi = 0$, $r_p = 0$, or $r/R = 0.06$. To simplify the equations we will assume initially that $r_2 = 0$ and use $w'$ instead of $(w + r_1)$.

The initial perimeter $P'$ is given by

$$
P' = (R - w') \phi + \frac{(R - w') \sin (\beta - \phi)}{\cos (\alpha + \phi - \beta)}
$$

and after a distance $x$ has burnt the perimeter $P_x$ is given by

$$
\frac{P_x}{2N} = (R - w' + x) \phi + xx + \frac{(R - w') \sin (\beta - \phi)}{\cos (\alpha + \phi - \beta)} - x \tan (\alpha + \phi - \beta)
$$

provided that the star side has not burnt out.

Until the star side or the web burns out, whichever occurs first, we have, by differentiating (2)

$$
\frac{1}{2N} \phi \left( \frac{d}{dx} \frac{P_x}{dx} \right) = \alpha + \phi - \tan (\alpha + \phi - \beta).
$$

Density of loading

The cross-sectional area of the charge ($\pi R^2 \lambda'$) can be shown to be

$$
N \left[ R^2 \beta - (R - w')^2 \phi - \frac{(R - w')^2 \sin (\beta - \phi) \cos \alpha}{\cos (\alpha + \phi - \beta)} \right]
$$

or

$$
\lambda' = 1 - \left( 1 - \frac{w'}{R} \right)^2 \phi - \left( 1 - \frac{w'}{R} \right) \frac{\sin (\beta - \phi) \cos \alpha}{\beta \cos (\alpha + \phi - \beta)}.
$$

Sliver area

The portion of the charge referred to as sliver is shown in Fig. 10a and Fig. 10b, where it will be seen that two equations are required, according to whether the star side burns out before the web or the converse. If the side has burnt out first as in Fig. 10a
Sliver area/N = $R^2(\beta - \phi) - (w')^2 a' - w'(R - w') \sin a'$

where

$$a' = (\beta - \phi) + \sin^{-1} \left\{ \frac{R - w'}{R - w_1} \sin (\beta - \phi) w' \right\}$$

or sliver area/motor area,

$$\lambda e = 1 - \frac{\phi}{\beta} - \left( \frac{w'}{R} \right)^2 \frac{a'}{\beta} - \left( \frac{w'}{R} \right) \left( \frac{1 - w'}{R} \right) \sin \frac{a'}{\beta}.$$  \hfill (5)

For the case shown in Fig. 10b, where the web burns out before the star-side, it can be shown that

Sliver area/N = $R^2(\beta - \phi) + (w')^2 \left\{ \tan (\alpha + \phi - \beta) - \alpha \right\}$

$$- \frac{(R - w') \sin (\beta - \phi)}{\cos (\alpha + \phi - \beta)} \left\{ (R - w') \cos \alpha + 2w' \right\},$$

or sliver area/motor area,

$$\lambda e = 1 - \frac{\phi}{\beta} + \left( \frac{w'}{R} \right)^2 \frac{\tan (\alpha + \phi - \beta) - \alpha}{\beta}$$

$$- \left\{ \left( \frac{1 - w'}{R} \right) \frac{\sin (\beta - \phi)}{\cos (\alpha + \phi - \beta)} \right\} \left\{ \left( \frac{1 - w'}{R} \right) \cos \alpha + 2\frac{w'}{R} \right\}. \quad \hfill (6)$$

Effect of rounding outer star point i.e. $r_1 \neq 0$

Decrease in density of loading = $- \Delta \lambda_1$

$$r_1 = \frac{1}{\pi R^2} \int_0^R P_x \, dx$$

$$= \frac{2}{\beta} \left( \frac{r_1}{R} \right) \left( 1 - \frac{w'}{R} \right) \left\{ \frac{\sin (\beta - \phi)}{\cos (\alpha + \phi - \beta)} \right\} + \left( \frac{r_1}{R} \right)^2 \frac{(\alpha + \phi) - \tan (\alpha + \phi - \beta)}{\beta}. \quad \hfill (7)$$
Effect of rounding lobe apex, i.e., $r_2 \neq 0$

Reduction in initial perimeter

$$= 2N r_2 \{\tan (\alpha + \phi - \beta) - (\alpha + \phi - \beta)\} \cdot (8)$$

Reduction in density of loading $= - \Delta \lambda_2$

$$= \left(\frac{N}{\pi}\right) \left(\frac{r_2^2}{R}\right) \{\tan (\alpha + \phi - \beta) - (\alpha + \phi - \beta)\} \cdot (9)$$

Final perimeter

If the star side burns out first, the final perimeter at web burn-out, $P_w$, is given by

$$P_w/2N = R(\beta - \phi) + w'\alpha' \cdot (10)$$

If the star side has not burnt out

$$\frac{P_w}{2N} = R\phi + w'\alpha + (R - w'\alpha) \sin (\beta - \phi) - w'\tan (\alpha + \phi - \beta) \cdot (11)$$

Neutral burning

In this case, it is necessary to make $dP_x/dx = 0$, or

$$(\alpha + \phi) = \tan (\alpha + \phi - \beta) \cdot (12)$$

where

$$\beta = \pi/N \cdot$$

Equation (12) has been solved by iteration for $N = 2, 3, 4 \ldots 9, 10, 12, 14, 16, 18$, and the values of $(\alpha + \phi)$ are given in Table I in both radian and circular measure.

For a given value of $N$, increasing the value of $(\alpha + \phi)$ from that for neutral burning will make $dP_x/dx$ negative, and reducing the value of $(\alpha + \phi)$ will give progressive burning.

Initial perimeter $= 2aR$

Substituting this value in equation (1) we obtain
Now \( \alpha + \phi \) and \( \beta \) are constants for given \( N \), and equation (13) enables the value of \( 1 - \frac{w'}{R} \) to be found for various values of \( \phi \).

**Effect of rounding outer star points and lobe apices**

In this case, when \( r_1 \) is small, \( P_x \) is constant at \( 2aR \). Hence equations (7) and (9) become

\[
- \Delta \lambda_1 = 2a \frac{r_1}{\pi R} \quad (7a)
\]

\[
- \Delta \lambda_2 = \left( \frac{a}{\pi} \right)^2 \left( 1 - \frac{P_x}{P} \right)^2 \quad (9a)
\]

**Density of loading, \( \lambda \)**

This quantity is found from equation (4) using the value of \( \frac{w'}{R} \) from equation (13), and equations (7a) and (9a).

**Sliver fraction**

The value of sliver fraction is obtained by using equation (5) or (6) together with the density of loading found above. The value of \( a \) will determine whether equation (5) or equation (6) should be used. The star side will burn out at the same time as the web, if the length \( b' \) in Fig. 2 is equal to \( w' \). That is, if

\[
\left( R - w' \right) \sin (\beta - \phi) = w' \sin (\alpha + \phi - \beta),
\]

and this will be satisfied when

\[
a = a^* = \frac{\sin (\beta - \phi) + \phi \cos (\alpha + \phi - \beta)}{\sin (\beta - \phi) + \sin (\alpha + \phi - \beta)}. \quad (4a)
\]

The value of \( a^* \) has been evaluated for \( N \) between 2 and 16 and \( \phi \) between 0 and \( \beta \) and the results are shown in Fig. 11. Also shown in this figure is a line for \( \alpha = 90 \) deg. which occurs when the star points just meet for \( r_1 \) and \( r_2 = 0 \). If \( a \) is less than \( a^* \), the side will burn out before the web, and if \( a \) is greater than \( a^* \), the web will burn out first.

**Proportion of neutral burning**

This is the ratio \( \frac{b'/w}{w'/r_1} \) or \( \frac{b' - r_1}{w' - r_1} \).
Now

\[ b' = \frac{(R - w')}{{\sin(\alpha + \phi - \beta)}} \]

and, therefore,

\[ \frac{p}{v} = \frac{(R - w') \sin(\beta - \phi) - x_1 \sin(\alpha + \phi - \beta)}{(w' - x_1) \sin(\alpha + \phi - \beta)} \]  (15)
FIG. 1: TYPICAL STAR-CENTRE CHARGE DESIGN

FIG. 2: SEGMENT OF STAR-CENTRE CHARGE DESIGN SHOWING NOTATION
This chart is for $r_2$ zero.
For $r_2$ not zero
\[ \Delta \lambda = -\left(\frac{r_2}{R}\right)^2 \]
Initial perimeter $P_1 = 2\pi (R - r_2)$
--- Lines of constant $\lambda\varepsilon$
--- " " " $b/\omega$

Figure 4 Neutral Burning Star-Centre Charge Design $N = 6$, $P = 2\pi R$
Fig. 5
Neutral Burning Star-Centre Charge Design
N = 7, P = 2πR

This chart is for \( \gamma_2 \) zero.
\[ \Delta \alpha = \frac{\gamma_2}{R} (0.75 - \gamma_2) \]

Initial perimeter \( p = 2 \pi (R - \gamma_2) \)

Lines of constant \( \frac{\lambda \varepsilon}{b/\omega} \)

Density of Loading
This chart is for $r_2$ zero.
For $r_2$ not zero
$\Delta \lambda = - (r_2 / R)^2$
Initial perimeter = $2\pi (R - r_2)$

Fig. 6 Neutral Burning Star-Centre Charge Design $N=8$, $P = 2\pi R$
FIG. 7 VARIATION OF SLIVER WITH WEB THICKNESS
FIG. 10 POSSIBLE END OF BURNING CONDITIONS
Fig. 11. Neutron burning star—centre charge design. Web and lobe burn-out together.

\[ \frac{\phi}{\beta} \]
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