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BUCKLING OF A TRUNCATED HEMISPHERE UNDER AXIAL TENSION

John C. Yao

Prepared for
COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
INGLEWOOD, CALIFORNIA

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AEROSPACE CORPORATION
2400 East El Segundo Boulevard
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ABSTRACT

This report is concerned with the theoretical evaluation of the buckling strength of a truncated hemisphere under axial tensile load. The edges of the shell are assumed to be restrained from moving radially or from rotating. Theoretical results were obtained by Vlasov's small deflection theory and Galerkin's method. The quick convergence of the series solution is demonstrated. Comparison of theoretical values with a few available experimental results is given.
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SYMBOLS

$A_n, B_n$ = coefficients of series expansion for radial deflection and stress function, respectively

$a$ = mean radius (see Figure 2)

$E$ = Young's modulus

$l$ = $\sin \alpha$

$m$ = number of buckle waves in the circumferential, direction

$P$ = axial tensile load per unit circumferential length at the shell equator

$q$ = radial load per unit middle surface area

$T_\phi, T_\theta, T_{\phi\theta}, T_{\theta\phi}$ = additional force components in the buckled shell

$T_{\phi\phi}, T_{\theta\theta}$ = membrane force components prior to buckling

$t$ = shell thickness

$w$ = radial buckling displacement of a middle surface point

$x$ = $\sin \phi$

$\alpha$ = altitude angle of the truncated edge (see Figure 2)

$\theta, \phi$ = spherical coordinates (see Figure 2)

$\mu$ = Poisson's ratio

$\psi$ = stress function

$\nabla^2$ = Laplace's operator

$$\left[ \left( \frac{\partial^2}{\partial \alpha^2} \frac{\partial^2}{\partial \phi^2} \right) - \tan \phi \left( \frac{\partial}{\partial \alpha} \frac{\partial^2}{\partial \phi^2} \right) + \sec^2 \phi \left( \frac{\partial^2}{\partial \alpha^2} \frac{\partial^2}{\partial \phi^2} \right) \right]$$
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I. INTRODUCTION

A truncated hemisphere may buckle under an axial tensile load that exceeds a certain critical value (Figure 1) because of compressive hoop stresses. The buckled shape is similar to that of a cylinder buckled under radial pressure, with one half wave in the axial direction and many small waves in the circumferential direction. The present theoretical study treats the case where both the truncated edge and the edge at the equator are restrained from moving radially or from rotating.

In the analytic work, Vlasov's equations of equilibrium and compatibility condition (Reference 1) are used. These equations are satisfied by the use of Galerkin's method (Reference 2) in conjunction with expressions for the radial displacement and the stress functions which satisfy the approximate boundary conditions.

Numerical results of theoretical buckling loads for shells of various geometric configurations are given by curves as well as by simplified algebraic formulas, and compared with a few experimental data.

II. THEORY

Using spherical coordinate system $\phi$ and $\theta$, as shown in Figure 2, the equilibrium condition in the radial direction for a spherical shell element can be expressed by the following equation:

$$
\frac{1}{12 (1-\mu^2)} \left( \frac{r}{a} \right)^2 \frac{\partial^2 \psi}{\partial \theta^2} \left( \frac{w}{a} \right) - a^2 \frac{\partial^2 \psi}{\partial \phi^2} \left( \frac{\psi}{Ea_t^2} \right) \frac{\partial \psi}{\partial \theta} = 0 ,
$$

where

$$a^2 \frac{\partial \psi}{\partial \phi^2} = \frac{\partial^2 \psi}{\partial \theta^2} - \tan \phi \frac{\partial \psi}{\partial \phi} + \sec^2 \phi \frac{\partial^2 \psi}{\partial \theta^2},$$

$$\psi^4 = \psi^2 \psi^2$$

and $\psi$ is the stress function, in terms of which the additional force components in the buckled shell wall are given by
Fig. 1. Truncated Hemisphere Buckled by Axial Tension.
Fig. 2. Shell Geometry and Load.
Equations (2) approximately satisfy the equations of equilibrium for a shell element in both the meridional and the circumferential directions. The stress function $\phi$ and the radial deflection $w$ are also related by the compatibility equation

$$T_\phi = \frac{1}{a^2} \left( \sec^2 \phi \frac{\partial^2 \phi}{\partial \theta^2} - \tan \frac{\partial \phi}{\partial \phi} \right)$$

$$T_\theta = - \frac{1}{a^2} \frac{\partial^2 \phi}{\partial \phi^2}$$

$$T_{\phi \theta} = T_{\theta \phi} = \frac{1}{a^2} \sec \phi \left( \frac{\partial^2 \phi}{\partial \phi \partial \theta} + \tan \phi \frac{\partial \phi}{\partial \theta} \right)$$

Equations (2) are Vlasov's equations (Reference 1), expressed in spherical coordinates.

In dealing with the problem of stability of a spherical shell, we must take into account the radial component of stress existing prior to buckling. With the assumption that these prebuckling stresses are adequately represented by the membrane state of stress, we have

$$T_{\phi o} = - T_{\theta o} = P \sec^2 \phi$$

During buckling, because of curvature changes, these finite membrane forces contribute a radial component equal to

$$q = \frac{P}{a^2} \left( \frac{\partial^2 w}{\partial \phi^2} - \sec^2 \phi \frac{\partial^2 w}{\partial \theta^2} + \tan \phi \frac{\partial w}{\partial \theta} \right) \sec^2 \phi$$

which is used in Equation (1).
The boundary conditions chosen to be satisfied by the radial deflection \( w \) and the stress function \( \phi \) are those corresponding to shell edges that are restrained from moving radially or from rotating, motion in the meridional and circumferential directions being restrained insofar as rigid body motion is concerned. These conditions may be expressed as

\[
\frac{\partial w}{\partial \phi} = T_\phi = T_\phi \theta = 0 \quad \text{at} \quad \phi = 0, \alpha , \quad (6a)
\]

or, for Equation (2), as

\[
\frac{\partial w}{\partial \phi} = \phi = \frac{\partial \phi}{\partial \phi} = 0 \quad \text{at} \quad \phi = 0, \alpha . \quad (6b)
\]

A possible method of solving the problem would be to first assume an arbitrary radial deflection function that satisfies the radial deflection boundary conditions (6); then Equation (3) could be solved for the stress function \( \phi \) in terms of a particular solution involving \( w \) and the general solution of the homogeneous equation, the constants of integration being determined by the stress function boundary conditions. Finally, the radial deflection function and the derived stress function can be substituted into Equation (1), which then could be solved by means of the Galerkin method (Reference 2). Because the solution for the stress function in terms of the radial deflection function is difficult, however, this method of solution can be replaced by the equivalent process of choosing arbitrary functions which satisfy the appropriate boundary conditions for both the stress function \( \phi \) and the radial deflection function \( w \) and solving both Equations (1) and (3) by the Galerkin method. Thus,

\[
\int_0^a \int_0^{2\pi} \left\{ \frac{1}{12 (1-\mu^2)} \left( \frac{t}{a} \right)^2 a^4 v^2 \left( \frac{w}{a} \right) - a^2 v^2 \left( \frac{\phi}{E^2 t} \right) \right. \\
\left. - \frac{P}{Et} \left( \frac{\partial^2}{\partial \phi^2} + \tan \phi \frac{\partial}{\partial \phi} - \sec^2 \phi \frac{\partial^2}{\partial \theta^2} \right) \left( \frac{w}{a} \right) \sec^2 \phi \right\} \delta \left( \frac{w}{a} \right) a^2 \cos \phi \, d\theta \, d\phi = 0
\]  

\( (7a) \)
\[ \int_0^a \int_0^{2\pi} \left[ a^4 \varphi^4 \left( \frac{\dot{\varphi}}{Ea^2} \right) + a^2 \varphi^2 \left( \frac{\dot{\varphi}}{a} \right) \right] \delta \left( \frac{\dot{x}}{Ea^2} \right) a^2 \cos \phi \, \sin \phi \, \sin \phi \, \phi = 0 \]  

(7b)

where \( w \) and \( \phi \) are subjected to the boundary conditions given by Equation (6b).

For convenience, the following substitutions are introduced:

\[ x = \sin \phi, \quad t = \sin \alpha, \]
\[ \frac{\partial}{\partial \phi} = \sqrt{1 - x^2} \frac{\partial}{\partial x}, \text{ etc.} \]
\[ a^2 \varphi^2 = (1 - x^2) \frac{\partial^2}{\partial x} - 2x \frac{\partial}{\partial x} + \frac{1}{1 - x^2} \frac{\partial^2}{\partial \phi^2} \]

The boundary conditions (6b) then become

\[ w = \frac{\partial w}{\partial x} = 0 \quad \text{at} \quad x = 0, \ell \]
\[ \phi = \frac{\partial \phi}{\partial x} = 0 \quad \text{at} \quad x = 0, \ell \]

Suitable expressions for \( w \) and \( \phi \) that satisfy conditions (9) may be taken as

\[ \frac{w}{a} = \sum_{n=2, 3, \ldots}^N A_n x^n (x-\ell)^n \cos \theta \]

\[ \frac{\dot{x}}{Ea^2} = \sum_{n=2, 3, \ldots}^N B_n x^n (x-\ell)^n \cos \theta \]

The substitution of Equations (10) into Equation (7) then leads to

\[ \sum_{n=2}^{\infty} \left( A_n e_{nj} + B_n d_{nj} \right) = 0 \]

\[ \sum_{n=2}^{\infty} \left[ \frac{1}{12 (1-\mu^2)} \left( \frac{t}{a} \right)^2 d_{nj} - \frac{P}{E a f_{nj}} \right] A_n - B_n e_{nj} \right) = 0 \]

\( j = 2, 3, 4, \ldots N \)
where

\[
\begin{align*}
(-1)^{n+j} d_{nj} &= \frac{2n^2}{(2n+2j-3)!} \xi^{2n+2j-3} \left[ \frac{n-3}{n} (n+j)! (n+j-4)! - 4(n+j-1)! (n+j-3)! \right] \\
&\quad + 3 \frac{n-1}{n-2} (n+j-1)! (n+j-2)! \\
&\quad - \frac{2n (n-1)}{(2n+2j-1)!} \xi^{2n+2j-1} \left\{ (7n^2 - 5n - 2)(n+j)! (n+j-2)! \right\} \\
&\quad - \left[ 4n (n+1) + \frac{n}{n-1} (6 + 2m_n^2) \right] \left[ (n+j-1)! \right]^2 - 4(n^2 - n - 2)(n+j+1)! (n+j-3)! \\
&\quad + (n-1)(n-2)(n-3)(n+j+2)! (n+j-4)! \\
&\quad + \frac{n}{(2n+2j+1)!} \xi^{2n+2j+1} \left\{ \left( \frac{m^4 - 2m_n^2}{n} + n^3 + 2n^2 + n \right) \left[ (n+j)! \right]^2 \right\} \\
&\quad - (4n^3 + 12n^2 - 14n + 4)(n+j+1)! (n+j-1)! + (6n^2 + 18n + 14)(n-1)(n+j+2)! (n+j-2)! \\
&\quad - 4(n-1)(n^2 - 4)(n+j+3)! (n+j-3)! + (n-1)(n-2)(n-3)(n+j+4)! (n+j-4)! \\
&\quad + (2m^4 - 6m_n^2)(n+j)! (n+j+2)! \frac{\xi^{2n+2j+3} + (3m^4 - 10m^2)(n+j+4)! (n+j)!}{(2n+2j+3)!}. \tag{12}
\end{align*}
\]

\[
\begin{align*}
(-1)^{n+j} e_{nj} &= \frac{2n}{(2n+2j-1)!} \xi^{2n+2j-1} \left\{ (n-1)(n+j)! (n+j-2)! - n \left[ (n+j-1)! \right]^2 \right\} \\
&\quad - \frac{n}{(2n+2j+1)!} \xi^{2n+2j+1} \left\{ \left( n+1 + \frac{m_n^2}{n} \right) \left[ (n+j)! \right]^2 - 2(n+1)(n+j+1)! (n+j-1)! \\
&\quad + (n-1)(n+j+2)! (n+j-2)! \right\} \\
&\quad - \frac{m^2 (n+j+2)! (n+j)! \xi^{2n+2j+3} - m \frac{2(n+j)! (n+j+4)!}{(2n+2j+3)!} \xi^{2n+2j+5}}{(2n+2j+3)!}. \tag{13}
\end{align*}
\]
\begin{equation}
(-1)^{n+j} f_{nj} = \frac{2n}{(2n+2j-1)!} \xi^{2n+2j-1} \left\{ (n-1)(n+j)! \cdot (n+j-2)! - n \left[ (n+j-1)! \right]^2 \right\} \\
+ m^2 \frac{[(n+j)!]^2}{(2n+2j+1)!} \xi^{2n+2j+1} + m^2 \frac{(n+j+2)! \cdot (n+j)!}{(2n+2j+3)!} \xi^{2n+2j+3} \\
+ m^2 \frac{(n+j+4)! \cdot (n+j)!}{(2n+2j+5)!} \xi^{2n+2j+5}
\end{equation}

For a nontrivial solution for \( A_n \) and \( B_n \), the determinant of their coefficients in Equations (11) must vanish, yielding the stability criterion for the problem. The lowest root of the determinant equation is the critical value of the load parameter \( P/E_t \).

III. NUMERICAL RESULTS AND DISCUSSION

Computations for the buckling tension load were carried out for shells with \( \alpha = 20, 30 \) and 40 degrees and \( \alpha/t \) ranging from 200 to 1600. Four terms each were used in the summation of Equations (10). The output is given by the non-dimensional quantity \( P/E_t \cos^2 \alpha \) against \( \alpha/t \) as shown in Figure 3. It may be seen that within the range of \( \alpha \) (that is, \( \alpha \) is less than 40 degrees), the buckling strength of the shell is far less influenced by \( \alpha \) than by \( \alpha/t \). The curve shown in Figure 3 may also be expressed by the following approximate formula:

\begin{equation}
P/E_t \cos^2 \alpha = 1.57 \left( \frac{\alpha}{\alpha_a} \right)^{1.12}
\end{equation}

In the limiting case where the radius of the sphere approaches infinity, the problem is reduced to a narrow, infinitely long strip which is subjected to compression along the length and tension across the width of the strip. In this case, the buckling stress is of the form (Reference 3, p. 337)

\begin{equation}
P/E_t = k \left( \frac{t}{\alpha_a} \right)^2
\end{equation}

where \( k \) is a constant coefficient.
Fig. 3. Buckling Stress of Truncated Hemisphere ($\mu = 0.3$).
We recall (Reference 3, pp. 450, 496) that the classical buckling stress for cylindrical shells under radial pressure is

\[
\frac{\sigma_{cr}}{E} = \frac{1}{4(1-\mu^2)} \left(\frac{t}{a}\right)^2
\]  

and that the classical buckling stress for spherical shells under external pressure is

\[
\frac{\sigma_{cr}}{E} = \frac{1}{\sqrt{3}(1-\mu^2)} \left(\frac{t}{a}\right)
\]  

We may see from Equations (4) and (15) that the buckling characteristic of the present problem is similar to the sphere problem rather than the cylinder problem.

Test data for five samples of various geometry were furnished by Douglas Aircraft Company and are reproduced here in Table 1. The theoretical buckling load parameter \(P/Et\) obtained by using one term in the summation of Equation (10) as the first approximation and succeeding by using more terms in the summation as higher approximations is also shown. We note that the convergence of the series solution is good and that using four terms in the summation is adequate for practical purposes. The theoretical values are found to be about twice greater than the experimental values, a difference rather common in buckling problems. In Table 1, also given are the theoretical circumferential buckling wave numbers associated with the found theoretical minimum buckling loads. These numbers seem quite high in comparison with the wave numbers found by test. However, a study of the curves for the theoretical buckling load versus wave numbers in Figure 4 leads to the belief that the actual buckling wave number probably is determined by the initial imperfection of the shell since the curve in the neighborhood of the theoretical minimum point is rather flat.

The subject problem is an interesting one as it is a particular example that an elastic system is buckled by tension, although the real cause of the instability is the compressive hoop stress. Certainly, there remain many
Table 1. Comparison of Experimental and Theoretical Results.

<table>
<thead>
<tr>
<th>$\frac{P}{2} \times 10^3$</th>
<th>$\frac{\alpha}{\cos \alpha}$</th>
<th>$a/t$</th>
<th>$P/tE$ (test)</th>
<th>1st approx.</th>
<th>2nd approx.</th>
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<th>4th approx.</th>
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<td>0.73</td>
<td>23°30'</td>
<td>480</td>
<td>6.15 x 10^{-4}</td>
<td>13.5945 x 10^{-4}</td>
<td>13.5528 x 10^{-4}</td>
<td>13.4911 x 10^{-4}</td>
<td>13.4903</td>
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<td>7.53251</td>
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<td>4.06182</td>
<td>4.04427</td>
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</tbody>
</table>

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Fig. 4. Buckling Forces Versus Wave Numbers.
areas for future studies, among them determination of the buckling load by large deflection theory and consideration of initial imperfections. The testing results presented here are rather scarce and scattered, hardly enough to give any conclusive indication. Extensive and systematic experiments in this respect are recommended.
REFERENCES


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