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SAND MOVEMENT ON COASTAL DUNES

by

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Introduction

The problems of the supply and loss of sediment at a shoreline are of considerable importance at many localities along the coastline of the United States. One basic mechanism involved in this overall problem is the transportation of sand by wind action. For example, at many localities along the Washington, Oregon, and California coasts considerable amounts of sand apparently are moved inland each year by wind action. A study at the major dune areas of the California coast shows that three basic conditions are satisfied for the existence of a dune system — namely, (1) a large supply of sand from a nearby major stream, (2) a shoreline orientation approximately parallel to the crests of the prevailing wave condition, thus creating a favorable condition for a low littoral current and consequently a location for sediment accumulation, and (3) low topography back from the beach where the prevailing onshore winds can easily move the sand inland from the region of accumulation at the beach. Figure 1 is a typical example of a major dune area on the California coast.

To obtain reliable data on the approximate magnitude of the annual supply or loss of sand from a coastline by wind action, a program of laboratory studies was undertaken in a special wind tunnel with sands of various sizes to define the relationship between the rate of sand transport and the characteristics of the wind and the sand. This laboratory program was supplemented by a limited program of field studies which was made to define the extent of the dune areas, the characteristics of their sands, and the frequency of occurrence of coastal winds capable of moving sand. For example, to provide information on the character of the sands encountered in actual dune systems and therefore serve as a basis for the sand sizes which should be studied in the laboratory, a sampling program was conducted at most of the important dune areas on the California coast. Surface samples were taken at the mid-tide level on the beach face and extended inland in the direction of the prevailing onshore winds. Typical of such information is that shown in Figure 2, taken along the range indicated in Figure 1. This plot shows the mean grain size and sorting coefficient as a function of distance from the shoreline.

Considerable work has been done on the mechanics of sand movement by wind by numerous investigators. Emphasis in these past studies was on the distribution of wind speed above the sand surface (with and without sand movement) and the rate of sand movement as a function of the principal variables.
The shear stress, $\tau$, produced at the sand surface by air flow is one of the most important factors in the basic mechanics of sand movement by wind. When the shear stress exceeds a critical value, the sand particles start to move. The wind speed profile and the shear velocity, $U_*$, are the primary factors involved in sustaining sand movement - the shear velocity being defined as $\sqrt{\rho g}$ where $\rho$ is the density of the air. Bagnold in his studies of the initiation of sand movement derived the following expression for the threshold value of the shear velocity,

$$U_{*t} = A \sqrt{\frac{\rho g d}{\sigma}}$$

where $d$ is the grain diameter, $g$ is the acceleration of gravity, and $\sigma$ and $\rho$ are the densities of sand and air, respectively. Bagnold found that the coefficient $A$ had a nearly constant value of 0.1 for a sand diameter of 0.25 mm and greater.

Several investigators have developed expressions for the rate of sand movement as a function of certain variables. A few of these expressions are as follows:

**Bagnold formula**

The rate of sand movement per unit width and unit time, $q$, is given by

$$q = C \sqrt{\frac{d}{D}} \frac{\rho}{g} U_*^3$$

where $D$ is the grain diameter of a standard 0.25 mm sand, $d$ is the grain diameter of the sand in question, $\rho$ is the density of the air (in c.g.s. units $\frac{\rho}{g}$ is equal to $1.25 \times 10^{-6}$), $U_*$ is the shear velocity and $C$ has the following values:

1. 5 for a nearly uniform sand
2. 8 for a naturally graded sand
3. 8 for a sand with a very wide range of grain sizes.

**Kawamura formula**

The rate of sand movement, $q$, is given by,

$$q = k \frac{\rho}{g} (U_* - U_{*t}) (U_* + U_{*t})^2$$

where $\rho$ is the density of air, $U_*$ is the shear velocity, $U_{*t}$ is the threshold shear velocity, and $k$ is a constant which must be determined by experiment. For a sand of average grain size 0.25 mm, Kawamura obtained $k = 2.78$ in wind tunnel tests with all terms being in c.g.s. units.
Experimental results obtained in wind tunnel tests by Bagnold \( \frac{1}{1} \) and Kawamura \( \frac{6}{6} \) using the same sand grain diameter of 0.25 mm present widely differing results as shown in Figure 3. Also plotted in this figure are the results of wind tunnel tests by Zingg \( \frac{9}{9} \) and Horikawa and Shen \( \frac{4}{4} \) for the same grain diameter of 0.25 mm. From his results, Zingg \( \frac{9}{9} \) modified the Bagnold formula to

\[ q = C \left( \frac{d}{D} \right)^{3/4} \frac{\rho}{g} U^3 \]

where \( C \) has the value 0.83.

In addition to the above formulae, O'Brien and Rindlaub \( \frac{7}{7} \) proposed the following formula from data derived by field tests:

\[ G = 0.036 U^3_5 \text{ (for } U_5 > 20 \text{ ft/sec)} \]

where \( G \) is the rate of transport in pounds per day per foot width, and \( U_5 \) is the wind velocity 5 ft. above the sand surface in ft/sec.

Confirmation of these formulae by field results is not particularly good, but since there is considerable scatter in the experimental data, these formulae are still useful in the description of a particular condition when a suitable constant is chosen.

**Experimental Program**

In order to reconcile some of the apparent differences in the various existing relationships for the rate of sand movement and to consider those conditions usually found in the field, a program of laboratory studies were made at the University of California under sponsorship of the Beach Erosion Board. The first tests in this program were made to develop a suitable sand trap for measuring the rate of sand transport in the field. These studies have been reported elsewhere \( \frac{4}{4} \) and are not summarized in this paper. Subsequent to these earlier tests which were conducted in a wind tunnel 12 in. x 15 in. in cross section, a larger and more suitable tunnel 4 ft. wide by 2.5 ft. high and 100 ft. long (Fig. 4) was constructed and used in the laboratory tests discussed below \( \frac{2}{2} \). Wind in the tunnel was generated by a variable speed fan at the exit end. Wind speeds up to a maximum of about 40 ft/sec could be attained. Vertical and horizontal velocity distributions during tests were made with a standard Prandtl-type pitot tube connected to a previously calibrated manehelic gage having a range of one-half inch of water and graduated into 0.02 inch divisions. During all tests, sand was spread over a length of about 62 ft. of the flume with a depth of 2 inches. A hopper to feed sand into the flume automatically was placed near the entrance to the tunnel (Fig. 4). The rate of sand feed was adjusted to be equal to the rate of sand transport as measured by the sand traps which consisted of the vertical trap developed by Horikawa and Shen \( \frac{4}{4} \) and a horizontal trap. This latter trap consisted of 18 compartments permanently fixed at the end of the sand bed (Fig. 4). In order to eliminate side-wall effects with the horizontal trap the rate of sand transport was measured only over a width of 2 ft. in the central part of the flume. Sand was removed from the compartments...
in the horizontal trap at the conclusion of each run with a vacuum cleaner. Comparison of the rate of transport as measured by the vertical trap with that measured with the horizontal trap showed that the vertical trap is close to 100 percent efficient at wind speeds higher than about 27 ft/sec, but becomes much less efficient at lower speeds. For high wind speeds, the grain size distribution of the sand caught in the vertical trap was very close to the grain size distribution of the bed except for the very largest grains. The relative absence of these larger grains probably is caused by the platform of the trap which sometimes can be an obstacle to surface creep. For wind speeds near the threshold value, the grain size distribution of sand in the vertical trap shows a distinct lack of the larger grains. This condition cannot be attributed entirely to the inefficiency of this trap but probably indicates the manner in which sand is moving near the threshold; i.e., the larger grains are not in general movement.

The mechanical composition of the three sands used in the tests is shown in Figure 5. The characteristics of these sands are as follows:

<table>
<thead>
<tr>
<th>Sand</th>
<th>Mean Grain Diameter (mm)</th>
<th>Sorting Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.44</td>
<td>1.23</td>
</tr>
<tr>
<td>B</td>
<td>0.30</td>
<td>1.15</td>
</tr>
<tr>
<td>C</td>
<td>0.145</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Experimental Results

Rate of sand transport. The amount of sand caught by the horizontal trap was measured for velocities varying from the threshold value to about 37 ft/sec. The feeding of sand into the wind tunnel at the upstream end of the sand bed was an important factor in sand movement for the lower wind velocities. Feeding sand into the tunnel during tests greatly lowers the threshold velocity, and at the same time changes the amount of sand transported for lower velocities. This effect is evident in Figure 6 which shows the rate of transport as a function of wind velocity (and shear velocity) with and without sand feed for Sand A (d = 0.44 mm). The apparent reversal of the curve obtained without sand feeding is perhaps due to the fact that the sand used in this study has a relatively wide range of grain sizes (Fig. 5). At, or near the threshold condition it is possible that the action of the smaller grains was impeded by the larger, thus modifying the over-all values for the threshold and the rate of transport. More precisely, near the threshold the sand grains move mainly by saltation. Since the surface layer remains practically immobile (no surface creep), the smaller grains are hidden by the larger ones and as a result the sand behaves as though it had a much larger mean diameter.

The experimental values for the maximum rate of transport (i.e., with a sand feed), $q$, are compared to the values predicted from the Bagnold and Kawamura formulas as follows:
Sand A (d = 0.44 mm). The experimental value of the threshold shear velocity (with sand feed) was 30 cm/sec which compares favorably with a value of 34 cm/sec as calculated by the Bagnold formula (Equation 1). For the rate of transport Bagnold (Equation 2) proposes a value of $C = 1.8$ for normally graded sand and $C = 2.8$ for a sand with a very wide range of grain size. Using a value of $C = 2.5$ for Sand A, the experimental and calculated values are as shown in Figure 7. It is evident from this figure that except for wind velocities near the threshold where the Bagnold formula is not applicable, the agreement is good. In the Kawamura formula (Equation 3) with a value of $U_{*t} = 30$ cm/sec combined with a value of $k = 3.1$ excellent agreement between experimental and calculated values was obtained over the entire range of shear velocity as shown in Figure 7.

Sand B (d = 0.30 mm). The experimental value of the threshold shear velocity (with sand feed) was 16 cm/sec which agrees fairly well with a value of 18 cm/sec as calculated by the Bagnold formula. With respect to rate of transport, Figure 8 was prepared to show a comparison of experimental values with the Bagnold and Kawamura data on a sand of 0.25 mm diameter. It is evident that the curves differ considerably, although the sand in each case had almost the same mean grain diameter. The sorting coefficient of the various sands possibly was quite different. In the study with Sand A (d = 0.44 mm) the Bagnold and Kawamura formulas could be used to describe the experimental data; however, with Sand B no values for the constants $C$ and $k$ could be found to describe the rate of transport over the entire range of shear velocities. Those values that most closely described the experimental data are shown in Figure 9.

Sand C (d = 0.145 mm). The experimental value of the threshold shear velocity (with sand feed) was 22.0 cm/sec and that calculated by the Bagnold formula was 17.8 cm/sec. This relatively large difference between the observed and calculated values of the threshold velocity is expected from the work of Bagnold $^{1/}$ wherein it was shown that for such small grain sizes as Sand C the coefficient $A$ is no longer constant and the shear velocity reaches a minimum. For the rate of transport, Figure 10 shows the experimental values compared with the Bagnold and Kawamura formulas. In this comparison, a value of $C = 1.5$ was used in the Bagnold formula. This is the lower limit of the value of $C$ recommended by Bagnold and applies to a uniform grain size. For the Kawamura formula, a value of $k = 1.0$ had to be selected in order that the calculated curve would pass near the experimental points.

Response time of sand bed to change in wind speed. In order to investigate the response time of the bed to a change of wind speed, some tests were made in which the wind was first allowed to blow over the sand surface (using Sand A) at a velocity of 31.5 ft/sec for a relatively long time (sufficiently long to observe a duplication of the results on the amount of sand transported, both in the vertical and horizontal traps). The wind speed then was suddenly changed to a value of 35 ft/sec and the sand collected in the vertical trap was weighed every two minutes, until a new constant rate
of transport was observed. The wind speed then was again adjusted to the first value, while the same measurements were made with the vertical sand trap. After a sufficiently long time, the wind speed was again adjusted to the higher value, and the same process repeated. There was some scatter especially for the higher velocity. This dispersion probably was due to some extent to the inaccuracy of the wind speed readings, these speeds being slightly different in corresponding runs. But as the important fact is the rate of transport with respect to time, the part of the dispersion due to differences in the mean wind speed can be eliminated by considering the discrepancy between the measurements made within the first few minutes of each run, and the average of the last measurements, when equilibrium is reached. Figure 11 shows the rate of sand transport as a function of time with this correction. The dispersion was greater at the beginning than at the end of a run, but except for a slight increase during the first 4 minutes in most of the runs no trend could be observed. In general it appears that the rate of transport adjusts itself rather quickly to a new wind speed. Consequently, under field conditions a knowledge of the actual duration of wind speeds of various magnitudes apparently should permit the calculation of the probable volume of sand moved on a given dune system in a given period of time.

Effect of moisture on sand transport. Generally moist sand found along coasts can be classified into two categories depending upon the origin of the moisture—namely:

(a) Moist sand which has collected moisture from the atmosphere. Unlike very fine particles such as dust or loess, the sand does not appear to readily absorb moisture. As indicated below, however, there exists a correlation between the water content of the sand and air humidity. Allowing for this fact, and since in the field it is easier to measure the air humidity than the sand water-content, the air humidity instead of the water-content of the sand was taken as the variable in the studies discussed below.

(b) Moist sand whose water comes from sources other than air humidity, such as rain, rising of underground water, and sea water remaining in the sand by wave or tide action. When a wind not saturated with water vapor blows over such a sand, it gradually dries out the surface layer of the sand bed, until an equilibrium is reached between humidity of the wind and the water content of the surface layer of the ground. The study of sand movement is relatively complex in this case where water contents of air and sand, as well as wind duration, are factors.

To obtain information on some of the above problems, Belly performed a series of experiments on Sand A (d = 0.44 mm) in a closed-circuit wind tunnel where humidity could be controlled. The first series of tests were concerned with the determination of the effect on the threshold velocity where the moisture content of the sand was relatively low. In these tests, \( U_* \) was determined from a series of vertical velocity distribution curves for different mean velocities and different air humidities, \( h \). Figure 12 shows the relationship between the shear velocity, \( U_* \), and the mean velocity, \( U \).
This relationship is quite consistent and can be expressed by a straight line. Thus, it can be stated that the wind drag is practically uninfluenced by air humidity. In the later tests, only the value of the mean velocity was recorded and the value of $U_*$ was calculated from this graph.

The water-content of sand obtained during the various runs is plotted against the corresponding air humidity in Figure 13. The scatter of data is important, but there is a general tendency for the points to follow along a straight line (which obviously should pass through the origin).

The threshold velocity was found by investigating the lower part of the curves for the rate of sand transport. The three sample curves in Figure 14 clearly indicate the change in the rate of sand transport with humidity. From such curves, the data on the variation of the threshold shear velocity distinctly show an increase with air humidity, and the relationship is nearly linear (Fig. 15).

The second series of tests by Belly on threshold velocity were concerned with sand of high water-content; that is, greater than 0.25%. The relationship between $U$ and $U_*$ under such conditions is shown in Figure 16 which appears to be linear and merely an extension of the data shown in Figure 12. During the tests in which these data were obtained, the air was maintained saturated with the water content, $w$, of the sand varied. The tests show that with a relatively high water-content of the sand (in excess of 1%) the wind speed necessary to initiate sand movement becomes increasingly important. One explanation for this increase is that the sand surface becomes very smooth under wind action. The water contained in the sand fills the interstices between the grains and makes the extraction of the grains by the wind much more difficult than when dry. The experiment could not be made for a water-content higher that 4% because the wind strength necessary to initiate the movement could not be obtained with the available equipment; however, it might be expected that the wind strength would increase very rapidly with an increase of water-content. It is even probable that for a very high water-content (flooded sand) the problem changes aspect and becomes closer to the problem of an interface between two fluids.

Using the relationship between water-content and air humidity in Figure 13, it is possible to complete the results of the study of low water-content and thus find a relationship (Fig. 17) between $U_*$ and $w$ for the total range of water content (0 to 4%). As to be expected, since the air humidity in itself does not play an important role in the sand movement, there is no break at the point which joins the two sets of data, that is at a threshold shear velocity of 40 cm/sec. The data follow a straight line, thus indicating that the relationship between $U_*$ and $w$ can be put into the form,

$$U_* = a \log_{10} w + b$$

where $a$ and $b$ are two constants obtained from the graph. It was found
that
\[ U_{*t} = 17(\log_{10} w + 5.1) \text{ cm/sec} \]
or,
\[ U_{*t} = 28 (0.6 \log_{10} w + 1.8) \text{ cm/sec} \]
where 28 is the value of \( U_{*t} \) in cm/sec given by the Bagnold formula.

Assuming that moisture affects the movement of sand of different grain sizes in the same manner, the Bagnold formula for the threshold shear velocity may be modified as follows,
\[ U_{*t} = A \sqrt{\frac{g \rho \phi}{p}} g d (1.8 + 0.6 \log_{10} w) \]
where \( A \) approximates 0.1 and \( w \) is expressed in percent.

**Conclusions**

1. The Bagnold formula for the threshold shear velocity appears adequate in giving a value for this term for use in calculating the rate of transport of sands normally existing on the coastal dunes of California.

2. The Bagnold formula for calculating the rate of transport appears superior to the Kawamura formula in that the value of the coefficient \( C \) in the Bagnold formula is better defined and more limited in range than the coefficient \( k \) in the Kawamura formula. Also, the latter formula includes the threshold shear velocity which introduces a further uncertainty in the calculations of transport rates especially since this factor is influenced by the moisture content of the sand.

3. With a relatively high water-content of the sand the wind speed necessary to initiate sand movement becomes increasingly important. This is especially true for the coastal dunes of California where the higher wind speeds, which usually occur during storms, are also accompanied by considerable rain.

**Acknowledgments**

These investigations were sponsored by the Beach Erosion Board, U. S. Army Corps of Engineers. Laboratory studies were conducted by Pierre-Yves Belly and Abdel L. A. Kadib, graduate students in hydraulic engineering, University of California.
References


Fig. 1. Example of usual shoreline configuration for major dunes on the California coast.

Fig. 2. Variation of mechanical composition of dune sand with distance from beach (near Santa Maria River mouth).
Fig. 3. Experimental data on sand transport determined by previous investigators

Fig. 4. Wind tunnel used in sand transport studies
Fig. 5. Mechanical composition of sands tested

Fig. 6. Experimental data on the rate of transport of Sand A (d = 0.44 mm) with and without a sand feed
Rate of sand transport $q$ in g/cm-sec.

Fig. 7. Comparison of experimental data with the Bagnold and Kawamura formulas for Sand A ($d = 0.44$ mm).

Fig. 8. Comparison of experimental data on Sand B ($d = 0.30$ mm) with the experiments of Bagnold and Kawamura on a sand 0.25 mm in diameter.
Fig. 9. Comparison of experimental data with Bagnold and Kawamura formulas for Sand B ($d = 0.30 \text{ mm}$)

Fig. 10. Comparison of experimental data with Bagnold and Kawamura formulas for Sand C ($d = 0.145 \text{ mm}$)
Amount of sand collected every 2 minutes, in grams

\[ \text{Sand A} \]

Average for \( t > 4 \)

N.B. The average of all the values obtained on the amount of sand collected for a particular velocity, has been taken as a reference in plotting the discrepancies. This has been done in order to compare the magnitude of the discrepancies to the absolute values of the amount of sand collected, but has no quantitative value.

Fig. 11. Change in rate of transport with time following a change in wind velocity

**AIR HUMIDITIES**

- \( h = 45\% \)
- \( h = 54\% \)
- \( h = 57\% \)
- \( h = 68\% \)
- \( h = 73\% \)
- \( h = 100\% \)

Note: The shear velocity \( U_\theta \) was calculated from the Prandtl equation with \( k = 0.40 \): 

\[ U_\theta = 5.75 \cdot U_\phi \log \frac{h}{k} \]

Fig. 12. Relationship between mean velocity and shear velocity for various air humidities
Fig. 13. Relationship between water content of sand and air humidity

Fig. 14. Variation of rate of sand transport with air humidity
Fig. 15. Relationship between threshold shear velocity and air humidity (Sand A)

Fig. 16. Relationship between shear velocity and mean velocity for high water contents
Fig. 17. Variation of the threshold shear velocity with moisture content (from Figs. 12 and 16)