Best Available Copy
NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
RECENT ADVANCES IN THE MEASUREMENT OF STRUCTURAL IMPEDANCE

by

Fred Schloss

HYDROMECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

January 1963
RECENT ADVANCES IN THE MEASUREMENT OF STRUCTURAL IMPEDANCE

by

Fred Schloss

January 1963

Report 1584
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>DEVELOPMENT OF MECHANICAL IMPEDANCE HEAD</td>
<td>1</td>
</tr>
<tr>
<td>FINAL DESIGN OF MECHANICAL IMPEDANCE HEAD</td>
<td>5</td>
</tr>
<tr>
<td>MAXIMUM IMPEDANCE</td>
<td>5</td>
</tr>
<tr>
<td>SUSPENSION OF SHAKER</td>
<td>7</td>
</tr>
<tr>
<td>RESONANT FREQUENCY OF ACCELEROMETER</td>
<td>7</td>
</tr>
<tr>
<td>TRANSVERSE SENSITIVITY OF ACCELEROMETER</td>
<td>8</td>
</tr>
<tr>
<td>ROCKING SENSITIVITY</td>
<td>8</td>
</tr>
<tr>
<td>BENDING STRAIN SENSITIVITY</td>
<td>10</td>
</tr>
<tr>
<td>ACOUSTIC SENSITIVITY</td>
<td>11</td>
</tr>
<tr>
<td>LINEARITY OF ACCELEROMETER</td>
<td>11</td>
</tr>
<tr>
<td>FORCE GAGE CHARACTERISTICS</td>
<td>12</td>
</tr>
<tr>
<td>DETERMINATION OF WEIGHT BELOW FORCE GAGE</td>
<td>12</td>
</tr>
<tr>
<td>LINEARITY OF STRUCTURES</td>
<td>13</td>
</tr>
<tr>
<td>EFFECT OF TEMPERATURE AND HUMIDITY</td>
<td>13</td>
</tr>
<tr>
<td>SMALL IMPEDANCE HEAD AND TRANSFER ACCELEROMETERS</td>
<td>13</td>
</tr>
<tr>
<td>SYSTEM CALIBRATION</td>
<td>13</td>
</tr>
<tr>
<td>DRIVING POINT IMPEDANCE MEASUREMENTS</td>
<td>13</td>
</tr>
<tr>
<td>TRANSFER IMPEDANCE MEASUREMENTS</td>
<td>18</td>
</tr>
<tr>
<td>AUTOMATIC SYSTEM CALIBRATION</td>
<td>18</td>
</tr>
<tr>
<td>APPLICATION PROCEDURES</td>
<td>19</td>
</tr>
<tr>
<td>CHOICE OF IMPEDANCE HEAD</td>
<td>19</td>
</tr>
<tr>
<td>ATTACHMENT OF HEAD TO STRUCTURE</td>
<td>19</td>
</tr>
<tr>
<td>CHANGE OF BOUNDARY CONDITIONS</td>
<td>19</td>
</tr>
<tr>
<td>CAUSE OF DISTORTION IN ACCELEROMETER SIGNAL</td>
<td>20</td>
</tr>
<tr>
<td>EXACT MEASUREMENTS AT MAXIMA OF IMPEDANCE OR ANTIRESONANCES</td>
<td>20</td>
</tr>
<tr>
<td>EXACT MEASUREMENTS AT MINIMA OF IMPEDANCE OR RESONANCES</td>
<td>21</td>
</tr>
<tr>
<td>EXCITATION BELOW 25 CPS</td>
<td>21</td>
</tr>
<tr>
<td>USE OF OSCILLOSCOPE</td>
<td>21</td>
</tr>
<tr>
<td>ELECTRICAL DESIGN CONSIDERATIONS</td>
<td>21</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Frequency Response of Three Velocity Pickups</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>First Design of TMB Model 1 Impedance Head</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Final Design of TMB Model 1 Impedance Head</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Final Design of TMB Model 2 Impedance Head</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>TMB Impedance Head Model 2 Calibration on Lumped Weights</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>Vibration Generator Suspensions</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>Transverse Response of Accelerometer of TMB Z Head 1</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>Rocking Sensitivity of Accelerometer in Impedance Head 18, Model 2</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>Rocking Sensitivity of Force Gage in Impedance Head 18, Model 2</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>Large and Small Impedance Heads with Accelerometers for the Measurement of Transfer Impedance</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>One-Decibel Confidence Limits of Large TMB Impedance Head and Electronic Mass Cancellation</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>One-Decibel Confidence Limits of Small TMB Impedance Head with Small Shaker and Electronic Mass Cancellation</td>
<td>16</td>
</tr>
<tr>
<td>13</td>
<td>Noise Level with 1000 Picofarads across Input of Various Commercial Quiet Transducer Preamplifiers</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>Block Diagram of Instrumentation Used in Reactance or Resistance Nulling Technique</td>
<td>26</td>
</tr>
<tr>
<td>15</td>
<td>Instrumentation Used in Reactance Nulling Technique</td>
<td>26</td>
</tr>
<tr>
<td>16</td>
<td>Phase Determination</td>
<td>28</td>
</tr>
<tr>
<td>17</td>
<td>Accelerometer Preamplifier, Tektronix Type 122 Low-Level Preamplifier, Modified</td>
<td>29</td>
</tr>
<tr>
<td>18</td>
<td>Force Preamplifier, Tektronix Type 122, Modified</td>
<td>29</td>
</tr>
<tr>
<td>19</td>
<td>Feedback Amplifier, Scott Type 140A, Modified</td>
<td>30</td>
</tr>
</tbody>
</table>
Figure 20 – Twin Power Supply for Two Type 122 Tektronix Preamplifiers ................................................................. 30

Figure 21 – Automatic Impedance Magnitude Plotter Using Commercially Available Equipment ................................................. 31

Figure 22 – Inexpensive Automatic System to Record Magnitude and Phase of the Mechanical Impedance Using Commercially Available Equipment ................................................................. 32

Figure 23 – Automatic Impedance, Mobility, or Equivalent Mass Plotter, under Construction ................................................................. 33

Figure 24 – Automatic Phase and F/A Recording System with Graphical and Digital Readout (5–25,000 Cycles per Second) ................................................................. 35

LIST OF TABLES

Table 1 – Magnetic Sensitivity of Transducers at 60 Cycles per Second ................................. 4

Table 2 – Bending Strain Sensitivities of Various Accelerometers ............................................. 11

Table 3 – Prime Characteristics of Two TMB Impedance Heads ............................................. 14
ABSTRACT

Two devices for measuring mechanical impedance have been developed, one for heavy structures capable of measuring a stiffness of $140 \times 10^6$ lb/in. over the frequency range of 5 to 4000 cps and the other for lighter structures capable of measuring a stiffness of $5 \times 10^6$ lb/in. over the frequency range of 20 to 8000 cps. Phase angles are measured in the presence of noncorrelated vibrational and electrical noise and distortion. Each device uses two piezoelectric ceramics to sense the applied force and the acceleration response of the structure. The mass associated with the coupling between the impedance transducer and the structure is cancelled electronically. Novel details of the design are presented, such as the elimination of magnetic pickup by the piezoelectric transducers and the reduction of mechanical crosstalk. Manual and automatic phase and impedance measuring systems having a high degree of accuracy are discussed. The report includes procedures for determining the acoustic sensitivity, transverse sensitivity, rocking sensitivity, and strain sensitivity of accelerometers and practical considerations for making mechanical impedance measurements.

INTRODUCTION

The David Taylor Model Basin first became interested in constructing transducers to measure mechanical impedance for use in the evaluation of resilient mountings. The concept of mechanical impedance is having more and more significance in the field of mechanical shock and vibration because it offers an efficient method for expressing the dynamic characteristics of a system. It may be used to evaluate the properties of materials, to make more accurate high-frequency acceleration measurements,\(^1\) to investigate damping treatments, or to characterize mechanical 4-terminal networks, such as resilient mountings, shaft couplings, pipe couplings, or hull coatings.

DEVELOPMENT OF MECHANICAL IMPEDANCE HEAD

Since mechanical impedance is defined as the complex ratio of the force phasor to the velocity phasor, the design of a mechanical impedance head seemed simple enough at first glance; namely, to incorporate in one integral unit a force sensor, a velocity sensor, and a driver or vibration generator.

Previously built impedance heads\(^2,3\) had been used to investigate such diverse problems as the mechanical properties of the mastoid bone of the ear and characteristics of the soil. However, these devices, as well as an instrument designed specifically for

\(^1\)References are listed on page 36.
testing heavy structures, were not able to meet our primary performance requirements, which are as follows:

- Frequency range: 5–5000 cps
- Minimum detectable acceleration: $10^{-5}$ g
- Relative phase shift between transducers: a constant $\pm 0.5$ deg
- Mechanical crosstalk: when driving a 50-ton weight with a 1-lb force, the resulting acceleration signal should be no more than $1.1 \times 10^{-5}$ g

Based on these considerations, electromagnetic velocity transducers could not be used because of their poor phase and level response, as shown in Figure 1. Such devices are used above the fundamental resonant frequency and below the first standing-wave frequency. The ratio of these two frequencies is inherently limited by the velocity of sound of the material of the suspension system. In addition, an electromagnetic transducer is highly susceptible to magnetic pickup, particularly if it is located near a vibration generator. Another reason for its impracticability will be discussed later.

![Figure 1 - Frequency Response of Three Velocity Pickups](image-url)
The first Taylor Model Basin impedance head is shown in Figure 2. The force gage [3]* consists of a radially polarized annular ring of piezoelectric ceramic material mounted on a conical base, in the center of which is a piezoelectric ceramic accelerometer assembly. A cylindrical inertial mass of tungsten [7] constructed in the form of an inverted cup is mounted on the ceramic disc in the accelerometer. The material and shape of the mass provided a low center of gravity, thus lowering the transverse response of the accelerometer. It should be noted that there is no mechanical connection between the inertial mass and the surrounding housing [10]. Such a connection, even if very compliant in the axial direction, would give erroneous results in the measurement of large impedances. To drive the impedance head, a small vibration generator [24] was selected; thus the device was portable and easily detachable. The 1.5-lb shaker has a permanent magnet to supply the necessary field; thus the need for a field power supply is eliminated. The vibration generator drives the impedance head along its axis through the rod [19], which is fastened solidly to the armature of the generator and to the center of the cover [14] of the impedance head. The stator of the vibration generator is resiliently mounted [22]. The system has a natural frequency of 25 cps with the base [2] of the impedance head mounted to a large impedance. With such an arrangement, the maximum force output is about 1.5 lb from 30-5000 cps.

The first impedance head measured small impedances satisfactorily, but not large impedances. It was found that the ceramic disc in the accelerometer has a large sensitivity to strain produced by the application of a force to the base through the force sensor. For the impedance head to work satisfactorily, this strain on the accelerometer ceramic must be reduced to less than 10^-14. It was therefore necessary to incorporate in the second design a mechanical strain isolator, as shown in Figure 3. The isolation is accomplished by placing the accelerometer on a pedestal, which has a circular slot to further reduce transverse strains. The three screws fastening the portion of the pedestal above the slot to the lower pedestal do not transmit any appreciable horizontal stresses, since they pass through a relatively long

---

*Bracketed numbers refer to the numbers in Figure 2.
clearance hole and serve only to prevent rocking of the accelerometer assembly about the lower pedestal in the frequency range of interest.

This model (second design) worked satisfactorily except that, for high impedances, the output of the accelerometer was influenced by magnetic pickup. Magnetic sensitivity of piezoelectric accelerometers had first been observed at the Taylor Model Basin about 1952. The phenomenon, not yet explained, may be observed by moving a permanent magnet near an accelerometer even though the accelerometer is constructed of nonmagnetic materials and fastened to a heavy nonmagnetic base. No signal will be observed if a capacitor is substituted for the accelerometer crystal. The magnetic sensitivities of various transducers have been measured and are listed in Table 1.

Figure 3 – Final Design of TMB Model 1 Impedance Head

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Equivalent Acceleration Signal with the Transducer at Rest for a Flux Density of 1 Gauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer A</td>
<td>$200 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>B</td>
<td>$20 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>C</td>
<td>$30 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>D (ADP Crystal)</td>
<td>$30,000 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>DTMB (unshielded)</td>
<td>$200 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>DTMB (shielded)</td>
<td>$7 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>Electromagnetic Velocity Pickup E</td>
<td>$200,000 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>Electromagnetic Velocity Pickup F</td>
<td>$20,000 \cdot 10^{-6}$ g</td>
</tr>
<tr>
<td>Electromagnetic Velocity Pickup G (shielded)</td>
<td>$3,000 \cdot 10^{-6}$ g</td>
</tr>
</tbody>
</table>

4
Figure 4 shows the impedance head in its final form. The accelerometer is magnetically shielded and the manner of strain isolation has been changed to increase the effective stiffness of the head. The strain isolation is achieved by an annular groove in the base of the impedance head between the annular force gage and the accelerometer. No magnetic shielding of the force gage is necessary, since its output is always relatively large compared to the a-c field generated by the vibration exciter.

**MAXIMUM IMPEDANCE**

In the measurement of impedances of large seismically suspended, lumped-steel masses, curves as shown in Figure 5 were obtained. Above the natural frequency of the mass on its suspension, it was found that the impedance increased initially in proportion to the frequency as expected. However, depending on the size of the mass, an antiresonance occurred at some higher frequency, beyond which the impedance appeared to be springlike. For masses of the same material, the impedance above the antiresonance asymptotically approached the same stiffness line; for equal masses of different materials, the impedance asymptotically approached different stiffness lines, whose ratio was approximately the ratio of their Young's moduli. It was found that further deepening of the groove in the impedance head—even severing the accelerometer from the force gage or changing the material of the base of the impedance head from aluminum to steel—made no change in the results. It was therefore concluded that the unexpected behavior of the impedance of the mass resulted from the local deformation of the mass beneath the impedance head.

The limiting stiffness of the mass can be calculated from the theory which treats the problem of the deformation of a semi-infinite elastic body by an annular die and by an annular pressure distribution. If the dimensions of the mass are several times the diameter of the impedance head, it can be shown that the limiting stiffness \( k_{\text{lim}} \) (a transfer stiffness between the annular ring of the impedance head base, which transmits the force, and the accelerometer in its center) is given approximately by
where $E$ is Young's modulus of the specimen and $d$ is the mean diameter of the force gage.

This is an order of magnitude lower than the transfer stiffness of the impedance head placed against a mass of infinite rigidity, theoretically calculated to be $6 \times 10^8$ lb/in. The curves in Figure 5 show a transfer stiffness of $6.5 \times 10^7$ for steel, which is close to the theoretical value of $7 \times 10^7$.

For the final version of the large impedance head, this limiting stiffness is $7 \times 10^7$ lb/in. for a steel mass and $2.3 \times 10^7$ lb/in. for an aluminum mass. To be confident within 1 db that the local deformation does not affect the measurement, an upper confidence limit of one-tenth this stiffness must be used. Larger impedances must be measured either by the transfer method with an external accelerometer or by a larger impedance head in which
the force gage has a larger mean diameter. Tests of impedance heads should always be con-
ducted on large masses because such tests show many common types of design or construction
deficiencies in the instrument.

**SUSPENSION OF SHAKER**

The suspension of the small vibration generator has been modified considerably as
shown in Figure 6. Figure 6a, the first modification, shows a pair of flexible mounting rings
designed to give the assembly a large axial and a small radial flexibility. However, spurious
phase shifts occurred in the measurements of large impedances because of rocking resonant
frequencies of the vibration generator in its suspension. This led to the design shown in
Figure 6b and to the final design shown in Figure 6c, which embodies a double mounting
system. A new small shaker is presently being developed.

It should be noted that the tests performed on the final model are similar to tests which
should be conducted on accelerometers. It would be helpful if manufacturers of commercial
accelerometers would test their transducers as extensively as has been done here. In ad-
dition to the determination of magnetic sensitivity and tests on large lumped masses, the
following tests, details of which will be published in a future report, were conducted.

**RESONANT FREQUENCY OF ACCELEROMETER**

The resonant frequency is the frequency at which the mechanical impedance looking
into the base of the accelerometer is a maximum. It is generally not recognized that this
frequency is dependent on the electrical termination of the transducer. For example, the
open-circuit resonant frequency $f_0$ is related to the short-circuit resonant frequency $f_s$ by
the equation where $k$ is the coupling coefficient of the crystal material

$$f_0 = f_s \left(1 + \frac{1}{k^2}ight)$$

In practice, the value falls between $f_0$ and $f_s$, depending on the capacity of the cable
and the input impedance of the preamplifier. For the accelerometer of Figure 4, $f_s$ was
12,000 cps and $f_0/f_s$ was 1.30.

![Figure 6a](image1.png) ![Figure 6b](image2.png) ![Figure 6c](image3.png)

**Figure 6 — Vibration Generator Suspensions**

7
The resonant frequency of an accelerometer may also be determined by electrically driving with a constant current generator the transducer rigidly attached to a heavy mass. The generator voltage and the voltage across the accelerometer are observed on the X-Y axes of an oscilloscope. If the frequency is slowly increased, the first sudden phase shift occurs at resonance. Minor housing resonances are difficult to detect by this method.

Impact tests, which are often used by commercial manufacturers of accelerometers, tend to give higher resonant frequencies than those obtained by the above two methods. This may occur if the anvil impedance is too low or if under high impact the accelerometer becomes "free" due to elastic deformation of the mounting screw or stud in tension. It has been observed that the resonant frequency listed by manufacturers may be as much as twice the true resonant frequency. The sensitivity at frequencies less than 0.75 of the resonant frequencies is approximately \[
\frac{1}{1 - \left(\frac{f}{f_r}\right)^2} \times \text{low frequency sensitivity}
\]
where \(f_r\) is the resonant frequency.

\[
\left(\frac{f_0}{f_s}\right)^2 = \frac{1}{1 - k^2} \quad [2]
\]

TRANSVERSE SENSITIVITY OF ACCELEROMETER

The transducer was excited along three axes, 120 deg apart, in a plane perpendicular to the axis of the impedance head, at discrete frequencies from 10 to 5000 cps (Figure 7). The development of a continuous calibration method above 1000 cps is highly desirable. Commercial transducers, unfortunately, are usually tested at a low frequency only. The sensitivity at any frequency in any of the three transverse directions was less than 3.1 percent of the axial sensitivity.

ROCKING SENSITIVITY

Reference 6 has shown that some accelerometers are highly sensitive to rocking about an axis perpendicular to the main axis of the transducer (Figures 8 and 9). The impedance head was rocked about three such axes, 120 deg apart, lying in the plane of the bottom of the base over a frequency range from 35 to 800 cps in half-octave steps. The head was attached to the center of a 2- by 2- by 10-in. aluminum bar driven by two vibration generators. The two driving points were in the same plane as the base of the impedance head. The amplitudes and phase of the vibration generators were adjusted independently to null the sum of the
Figure 7 – Transverse Response of Accelerometer of TMB Z Head 1

Figure 8 – Rocking Sensitivity of Accelerometer in Impedance Head 18, Model 2
outputs from two equally amplitude- and phase-sensitive accelerometers mounted equidistant from the center of the bar, thus assuring pure rotation. The normalized rocking sensitivity was less than 0.04 in./radian for the accelerometer and less than 0.02 in./radian for the force gage.

BENDING STRAIN SENSITIVITY

If an accelerometer is firmly attached to a vibrating beam, the base of the accelerometer is alternately bent and stretched, causing the piezoelectric element of the accelerometer to deform and to generate a pseudoacceleration signal. Reference 7 and tests conducted at the Taylor Model Basin have shown that some commercial accelerometers have a strain sensitivity over 1000 times higher than the accelerometer in the impedance head. The strain sensitivity of all accelerometers, as well as accelerometers in impedance heads, should be determined.

To check bending strain sensitivity, a steel bar, \( \frac{3}{8} \) in. thick and 36 in. long, was supported and driven at the center. At the first antiresonant frequency of 49.4 cps, the motion at the center was very small; however, the outer fiber strain was rather high (7.032 \( ty/L^2 \)), where \( t \) and \( L \) are the thickness and length of the bar, and \( y \) is the deflection at the ends, causing a large strain which was transmitted frictionwise to the accelerometer base. The deflection \( y \) was observed optically. An accelerometer, whose base was not strained,
TABLE 2
Bending Strain Sensitivities of Various Accelerometers

<table>
<thead>
<tr>
<th>Accelerometer</th>
<th>Equivalent Acceleration for an Outer Fiber Strain of $10^{-6}$ g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer in Large DTMB Impedance Head</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Accelerometer in Small DTMB Impedance Head</td>
<td>$24 \times 10^{-4}$</td>
</tr>
<tr>
<td>Large DTMB Transfer Accelerometer</td>
<td>$3.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Small DTMB Transfer Accelerometer</td>
<td>$90 \times 10^{-4}$</td>
</tr>
<tr>
<td>B 1</td>
<td>$4.17 \times 10^{-4}$</td>
</tr>
<tr>
<td>C 7</td>
<td>$5.33 \times 10^{-4}$</td>
</tr>
<tr>
<td>E 3</td>
<td>$5.01 \times 10^{-4}$</td>
</tr>
<tr>
<td>E 8</td>
<td>$1.34 \times 10^{-4}$</td>
</tr>
<tr>
<td>M 7</td>
<td>$5.80 \times 10^{-4}$</td>
</tr>
<tr>
<td>M 8</td>
<td>$8.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>W 13</td>
<td>$5.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>W 34</td>
<td>$1.10 \times 10^{-4}$</td>
</tr>
<tr>
<td>W 71</td>
<td>$1.00 \times 10^{-4}$</td>
</tr>
<tr>
<td>W 72</td>
<td>$5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

measured the motion at the center. For the accelerometer at rest and for an outer fiber strain of $10^{-6}$, the equivalent acceleration signal was less than $2.5 \times 10^{-3}$ g. The bending strain sensitivities of TMB accelerometers and some commercial accelerometers are shown in Table 2.

ACOUSTIC SENSITIVITY

To obtain the true acoustic sensitivity, the transducer should be subjected to a sound pressure with its base at rest. This was accomplished by mounting the impedance head on a heavy steel block which was completely covered with pressure release material except over the area of attachment. The acoustic sensitivity thus obtained was less than 15 microvolt/dyne/cm² from 100–5000 cps. Accelerometers are often acoustically tested by suspending them seismically in the sound field. This is not a proper test, since the accelerometer output will then be proportional to the pressure gradient above the resonant frequency of the suspension system.

LINEARITY OF ACCELEROMETER

The output from the accelerometer is linear from $10^{-6}$ to $10^3$ g.
FORCE GAGE CHARACTERISTICS

Few tests, other than impedance measurements on large masses and rocking sensitivities, were conducted on the force gage because its output is relatively high and therefore not greatly influenced by side effects. The stiffness of the force gage is $3 \times 10^7$ lb/in., and the output from the gage is linear within 1 db up to static loads of 1500 lb.

The output from the force gage in the large impedance head is linear between 250-lb tension and 1500-lb compression. (Static plus dynamic load.) The maximum static load in compression is 10,000 lb and in tension, 250 lb. If the compressional load exceeds 1500 lb, the system (force gage, accelerometer, and preamplifiers) will have to be calibrated at these loads. This can be accomplished by driving a known mass in a universal testing machine, the machine applying the desired static compressional load. The mass should be isolated by a thick compliant layer from the platen of the machine and the driving frequency should be considerably above the natural frequency of the mass-layer system. The armature suspension of the driver must be sufficiently stiffened to support the large static load. The calibration procedure is similar to that described under System Calibration.

It is important in the design that the accelerometer is mounted on the "specimen end" of the force gage. Impedance measurements will then be independent of the stiffness of the force gage—even if the specimen is in resonance with the force gage—if the frequency is low enough for it to act as a simple spring. The dimensions of the force crystal are such that wave effects occur at much higher frequencies.

DETERMINATION OF WEIGHT BELOW FORCE GAGE

For system calibration it is necessary to determine the weight $W_X$ below the force gage. The impedance head is driven at any frequency between about 50 and 2000 cps by seismically suspending or supporting the assembly. The natural frequency of the suspension system should be well below 50 cps. If the electrical output from the force gage amplifier is $E_0$ without a specimen and $E_W$, after firmly attaching a weight of $W$ lb, at the same value of acceleration, then

$$\frac{W_X}{W_X + W} = \frac{E_0}{E_W}$$

[3]

and

$$W_X = \frac{E_0 W}{E_W - E_0}$$

[4]

The external metal weight should have a diameter of 2 in., a thickness of 1/2 in., and should have one end machined flat with a central 3/8-in. NC 16 tapped hole. For the impedance head shown in Figure 4, the weight below the force gage is about 1/3 lb.
LINEARITY OF STRUCTURES

It has been found that the impedance of welded, homogeneous structures is linear with amplitudes to within 1 db provided the maximum stress does not exceed 2 percent of the elastic limit of the material of the structure. The behavior of riveted, laminated, honeycombed, etc., structures must be investigated on an individual basis.

EFFECT OF TEMPERATURE AND HUMIDITY

The temperature range of the two impedance heads is -65 to 180 deg Fahrenheit, the maximum temperature limited by the type of cement, solder, and cabling used and not by the ceramic piezoelectric material (Curie Point over 300 deg Centigrade). High temperature units could easily be made.

Although the impedance heads are hermetically sealed, the humidity may affect the external cabling and change the low frequency response, although no such difficulties have as yet been reported. The resistance across the signal leads should always measure more than 100 megohms.

SMALL IMPEDANCE HEAD AND TRANSFER ACCELEROMETERS

The previous discussion has been concerned with an impedance head able to measure point impedances encountered in full-scale submarines. However, a smaller head and transfer accelerometers are needed for point impedance measurements on models and smaller structures and for transfer impedance measurements. A smaller head and the accelerometers were therefore designed and built; photographs are shown in Figure 10. The proper choice of impedance head is discussed in the section on Application Procedures.

Table 3 is a summary of the important characteristics of the two heads. Figures 11 and 12 show the impedances which may be measured with a confidence of 1-db accuracy. The specimen and the dimension of the base of the impedance head limit the maximum stiffness which may be measured with the large head. The mass lines of Figures 11 and 12 give the smallest impedances which may be measured with electronic mass cancellation, as discussed in the next section.

SYSTEM CALIBRATION

DRIVING POINT IMPEDANCE MEASUREMENTS

The system, which includes the head and electronic amplifying equipment, may be calibrated without determining individually the sensitivities of the force gage and accelerometer or the gains of the amplifiers. The weight $W_X$ below the force gage (previously determined) serves as the calibrating weight. The calibration should be performed immediately before and immediately after impedance measurement. The impedance head is driven
Figure 10 – Large and Small Impedance Heads with Accelerometers for the Measurement of Transfer Impedance

TABLE 3

Prime Characteristics of Two TMB Impedance Heads

<table>
<thead>
<tr>
<th></th>
<th>Large Head</th>
<th>Small Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity of Accelerometer, mv/g</td>
<td>200</td>
<td>25</td>
</tr>
<tr>
<td>Sensitivity of Force Gage, mv/lb</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Electrical Capacitance of Accelerometer, pf</td>
<td>900</td>
<td>700</td>
</tr>
<tr>
<td>Electrical Capacitance of Force Gage, pf</td>
<td>900</td>
<td>700</td>
</tr>
<tr>
<td>Resonant Frequency of Accelerometer, cps</td>
<td>12,500</td>
<td>25,000</td>
</tr>
<tr>
<td>Frequency Range, cps</td>
<td>5–4000</td>
<td>20–8,000</td>
</tr>
<tr>
<td>Stiffness of Head, lb/in.</td>
<td>&gt;5 \times 10^8</td>
<td>5 \times 10^6</td>
</tr>
<tr>
<td>Weight below Force Gage, lb</td>
<td>0.35</td>
<td>0.06</td>
</tr>
</tbody>
</table>

at any frequency between about 50 and 2000 cps without a specimen by seismically suspending or supporting the assembly. The natural frequency of the suspension system should be well below 50 cps. The frequency-independent ratio of the voltage \( E_{A_{cal}} \) at the output of the acceleration channel to the voltage at the output of the force channel \( E_{F_{cal}} \) should be noted.

If the impedance head is subsequently attached to a specimen without changing gain settings on the amplifiers, it can be seen readily that the “equivalent weight” of the specimen, which includes the weight below force gage, is
Figure 11 — One-Decibel Confidence Limits of Large TMB Impedance Head and Electronic Mass Cancellation

Specimen and not the impedance head limits the maximum stiffness which can be measured.
Figure 12 – One-Decibel Confidence Limits of Small TMB Impedance Head with Small Shaker and Electronic Mass Cancellation
where \( E_F \) and \( E_A \) are voltages appearing at the output of the force and acceleration channel when driving the specimen.

The magnitude of the impedance is

\[
\frac{2\pi f}{g} \frac{E_F}{W_X} \frac{E_A\text{ cal}}{E_F\text{ cal}}
\]

[6]

where \( F \) is frequency and \( g \) is acceleration of gravity.

The magnitude of the true point impedance can be computed by subtracting the reactance of the weight below the force gage and is

\[
(R^2 + X^2)^{1/2}
\]

[7]

where

\[
R = \frac{2\pi f}{g} W_X \frac{E_A\text{ cal}}{E_F\text{ cal}} \frac{E_F}{E_A} \sin \phi
\]

[8]

and

\[
X = \frac{2\pi f}{g} W_X \left( \frac{E_A\text{ cal}}{E_F\text{ cal}} \frac{E_F}{E_A} \cos \phi - 1 \right)
\]

[9]

where \( \phi \) is the angle by which the signal in the acceleration channel leads the signal in the force channel. The phase angle of the impedance is \( \tan^{-1} R/X \).

Fortunately, the effect of the weight below the force gage (plus any other additional weights, such as attachment blocks) may be cancelled electronically. A simple version of this cancellation has been used previously. The force which must be subtracted from the force channel equals \( W_X a_c \), or a constant times the acceleration. It is therefore only necessary to subtract electronically from the force signal an acceleration signal of a properly adjusted constant amplitude. Note that this electronic subtraction would be difficult to accomplish with a transducer other than an accelerometer.

Immediately after calibration of the system, without driving a specimen, a linear potentiometer in the "mass cancelling" circuit is adjusted to give a minimum signal in the force channel. A different setting of the potentiometer will be necessary to cancel the weight below the force gage and an attachment block. The magnitude of the impedance with this electronic subtraction is

\[
\frac{2\pi f}{g} W_X \frac{E_F}{E_A} \frac{E_A\text{ cal}}{E_F\text{ cal}} = c_1 \frac{E_F}{E_A}
\]

[10]

where \( c_1 \) is the constant.
The phase angle between the two channels will be the phase angle between the applied force and the acceleration provided there is no relative phase shift in the electronics. For ease of operation, the signal processing should be accomplished in the preamplifier circuits at points prior to the gain controls.

**TRANSFER IMPEDANCE MEASUREMENTS**

To perform the mass cancellation in transfer impedance measurement, it will be necessary to have three transducers and preamplifiers, two accelerometers, and one force gage. However, the transfer impedance is usually of such magnitude that the error introduced by not cancelling is usually small. It is necessary to recalibrate with the external accelerometer attached, then to readjust for mass cancellation without the external accelerometer.

\[
Z_{\text{transfer}} \approx \frac{2\pi f}{g} \left( W_X + W_{\text{acc}} \right) \frac{E_{A \text{ cal}}}{E_{F \text{ cal}}} \frac{F_F}{F_A}
\]

where \( W_{\text{acc}} \) is the weight of the external accelerometer and \( E_{A \text{ cal}}/E_{F \text{ cal}} \) is the ratio of the signal from the external accelerometer to the force signal with the external accelerometer attached to the base of the head.

In transfer impedance measurements, it is necessary to correct for the weight of the external accelerometer if the impedance of the accelerometer \( Z_{\text{acc}} \) is of the same order of magnitude or higher than the point impedance of the specimen at the accelerometer location \( Z_p \). The correct transfer impedance is

\[
Z_{t \text{ correct}} = \frac{Z_p}{Z_{\text{acc}} + Z_p} Z_t
\]

where \( Z_t \) is the measured transfer impedance.

**AUTOMATIC SYSTEM CALIBRATION**

If an automatic impedance measuring system is to be calibrated, it will be more convenient to use an external weight of 1 lb after electronic mass cancellation than to use the internal weight below the force gage. By using decade attenuators in the acceleration channel, calibration lines of 1 lb, 10 lb, and so on, can be plotted. The care to be used in making measurements cannot be overstated. The operator of the equipment should be thoroughly familiar with the behavior of structures. 8, 9
APPLICATION PROCEDURES

CHOICE OF IMPEDANCE HEAD

The impedance range for which valid impedance measurements can be made is determined not only by the head itself but also by the specimen being investigated. The part of the head between the force gage and the specimen adds weight to the specimen (which can be corrected for electronically with a 1-db confidence as shown by the mass lines in Figures 11 and 12) and also may locally stiffen the specimens under test. The method of attachment of the head must be varied according to the nature of the specimen being tested. If the structure is very stiff and massive compared to the head, the head may be attached directly. However, if the head is much stiffer than the structure, either the small head should be used or the large head must be attached with a conical reducing adapter. This adapter reduces the stiffness of the head considerably.

Speaking rather loosely, for valid impedance measurements the regions of contact between the head (or its adapter) and the test specimen should be considerably smaller than the linear dimensions of the specimen that are most important in determining its vibratory behavior.

ATTACHMENT OF HEAD TO STRUCTURE

The best procedure is to drill and tap the specimen and to attach the impedance head directly by a stud. A layer of heavy grease should be applied between the mating surfaces. As an alternative, a tapped block, at least 1 5/8 in. in diameter for the large impedance head and 1/2 in. in diameter for the small head, may be cemented to the specimen. The thickness of the hard cement coat should not be more than 0.005 in. The compliance of the block and cement will lower the maximum measurable stiffness. To check the validity of the results, it is advisable to measure the transfer impedance at very low frequencies between the force gage and an external accelerometer placed near the head or on the opposite side of the structure. At sufficiently low frequencies, the driving point and transfer impedance should be equal.

CHANGE OF BOUNDARY CONDITIONS

If the rotational impedance is low compared to the axial impedance, some inconsistencies in results are obtained. It is thought that, whenever rotation exists, the head and shaker increase the rotational inertia of the system and, in effect, change the boundary conditions. For example, the low-frequency stiffness at the end of a simple cantilever is \( \frac{6EI}{l^3} \). If the head is constrained to move in the axial direction, it will measure a low-frequency stiffness of
In actual measurements, the stiffness was always found to be greater than \( \frac{3E}{l^3} \), the magnitude depending on the shaker, the head, and the geometric arrangement. This is a serious problem, since measurements are often made at points or frequencies where the rotational impedance is low; for example, on flanges of stiffeners of submarine hulls or on the hull near stiffeners. A possible solution to this problem is presently under study.

**CAUSE OF DISTORTION IN ACCELEROMETER SIGNAL**

A large amount of harmonic distortion, which is not due to malfunctioning of the accelerometer, may exist in the accelerometer signal at discrete frequencies. If the impedance of the specimen is low at a frequency \( f \) and relatively high at a frequency \( f/n \) (where \( n \) is an integer), a relatively small amount of generation of harmonics in either the oscillator, power amplifier, or vibration generator is sufficient to cause considerable distortion of the accelerometer signal when the system is driven at frequency \( f/n \). This distortion is unavoidable, and it is important that the acceleration signal be filtered.

**EXACT MEASUREMENTS AT MAXIMA OF IMPEDANCE OR ANTIRESONANCES**

The frequency at which a maximum in impedance occurs can be determined by observing a minimum in the filtered acceleration signal, since the vibration generator tends to act as a constant force generator in the vicinity of antiresonances. The exact frequency should be read with an electronic counter.

At antiresonances, especially at low frequencies, the slight strain sensitivity of the accelerometer (see section on final design) causes an error in frequency, magnitude, and phase, depending on the magnitude of the impedance. For example, an actual impedance of 1500 lb-sec/in. at an antiresonance of 51 cps may measure as an impedance of only 400 lb-sec/in. at an apparent antiresonance of 50 cps.

To obtain precise values, a transfer measurement may be made with an accelerometer that has low strain sensitivity attached through a block or, preferably, held by a thin layer of grease to the other side of the structure coaxial with the impedance head. (At an antiresonance, the impedance head is at a node and the transfer accelerometer must also be placed at the node.) No additional adjustment of the mass cancellation circuit is necessary at antiresonances, since the impedance of the specimen is high compared to the impedance of the attachments.

The properties of light samples of materials may be more accurately determined at antiresonances than at resonances, as will be explained subsequently.
EXACT MEASUREMENTS AT MINIMA OF IMPEDANCE OR RESONANCES

The frequency at which a minimum in impedance occurs can be determined by observing a minimum in the filtered force signal, since the vibration generator tends to act as a constant acceleration generator in the vicinity of resonances. The exact frequency should be read with an electronic counter.

At resonances, the impedance of the structure may be lower than the lower limit shown in Figures 11 and 12. The limit in these figures is based on the electronic mass cancellation, which may be achieved over the total frequency range without readjustment. At resonance, greater cancellation can be achieved by further amplitude and phase adjustment once the approximate frequency is known.

EXCITATION BELOW 25 CPS

At frequencies below 25 cps, the impedance head should be driven by a larger shaker, such as Goodmans 390A. The shaker should be connected to the head by a thin rod, and the shaker itself should be suspended from a nearby structure by very soft strings. The natural frequency of the shaker on its suspension should be below 3 cps.

For impedance measurements at low frequency, as, for example, surface ships, submarines, or missiles, a mechanical reaction-type vibration generator should be used. Low-frequency measurements on ships have been reported and presented graphically as functions of excitation force and displacement. These data might be plotted as mechanical impedance.

USE OF OSCILLOSCOPE

No measurements should be made without using an X-Y or dual-channel oscilloscope. The outputs from the force and acceleration preamplifiers should be displayed so that the oscilloscope will readily indicate overloading of the preamplifiers or any other malfunctioning of the equipment.

ELECTRICAL DESIGN CONSIDERATIONS

The impedance heads were designed to have the capacitance of the force gage nearly equal to the capacitance of the accelerometer. Equal electrical capacitances assure small relative phase shifts at low frequencies between the transducers, if identical preamplifiers are used, and they obviate the necessity for use of cathode followers, which are usually noisy.

The output of the large impedance head is balanced (neither terminal of sensor is grounded). A balanced transducer, if connected through a two-conductor shielded cable to a proper differential preamplifier, will eliminate ground loop problems and will greatly reduce power frequency pickup. A balanced or ungrounded transducer also lends itself easily to
electrical calibration, in which an electrical signal is inserted through the transducer and preamplifier. The electrical system used here, particularly the use of differential preamplifiers rather than cathode followers, is a departure from previously accepted systems for piezoelectric transducers. Admittedly, these preamplifiers have a lower input impedance than the usual cathode followers. However, if the transducers are designed to have capacitances equal within 1 percent and, if the time constant of the system is greater than 30 msec, the relative phase shift at 5 cps will be less than 0.5 deg. For the small head, the time constant of the system is 12 msec, and the relative phase shift is less than 0.3 deg at its lower frequency limit of 20 cps.

Figure 13 shows the advantages of using low noise differential or single-ended amplifiers over conventional cathode followers. It can be seen that at low frequencies the noise level for a cathode follower is much higher than for a conventional high-impedance, low-noise amplifier. For example, at 10 cps the minimum detectable acceleration signal is at least ten

![Figure 13 - Noise Level with 1000 Picofarads across Input of Various Commercial Quiet Transducer Preamplifiers](image-url)
times lower with a low-noise amplifier than with a cathode follower. The commercial preamplifiers shown in curves C and D (Figure 13) have a differential input with an impedance of 20 megohms paralleled by 25 pf. They may be used with the large impedance head, and accelerations as low as $10^{-5}$ g with a 10-db signal to noise ratio may be detected. The equivalent input noise of the single-ended amplifier having an input impedance of 10 megohms paralleled by 50 pf is shown in curve E.

A new quiet transistor preamplifier with an input impedance of 100 megohms has recently been marketed. The noise level, with a 1000-pf source capacitance, is similar to curve D in Figure 13.

### IMPEDANCE MEASURING SYSTEMS

#### GENERAL

The automatic and nonautomatic or manual systems described here are designed to measure the magnitude and phase or the reactance and resistance of the impedance. Even though our experience tends to favor these systems, it is by no means implied that other solutions are not possible or preferable. Only recently, a promising system for measuring the reactive and resistive components of the impedance, requiring a great amount of development of its components, has come to our attention. Other proposed systems have been omitted because they have been proved by experience to be unworkable or because too much time and effort would be required to explore their feasibility. Some of the systems may also be used to make a variety of measurements such as transmissibility, correlation, and so on.

All systems should incorporate a circuit to cancel the mass below the force gage by subtracting a proper fraction of the acceleration signal from the force channel, as described. All systems should have a signal to drive the vibration generator through a power amplifier at a frequency at which the transducer signals will be analyzed. To do this conveniently requires a special analyzer which has an oscillator output at a frequency to which the filter is tuned automatically. The output should be independent of the magnitude of the incoming signal. Two such analyzers are available commercially, Radiometer FRA2T and Hewlett Packard 302A. Without such an instrument or its equivalent, it would be extremely difficult to find resonances and antiresonances and it would be laborious to make measurements.

#### REQUIREMENTS OF IMPEDANCE MEASURING SYSTEMS

1. Frequency range: 10–5000 cps
2. Accuracy of phase measurement: $1/4$ deg desired
3. Dynamic range: 60 db without switching
4. Noise and distortion: the apparatus must be able to measure signal voltage over a 60-db amplitude range in the presence of discrete frequency tones one octave or farther away in frequency—ten times the signal
amplitude; it must also measure signals over this amplitude range in the presence of wide-band noise with a spectrum level one-tenth the signal level.

Requirement 2 imposes severe demands on the system and Requirement 3 precludes the use of multipliers presently available for automatic systems. Requirement 4 necessitates the use of narrow-band filters or equivalents and excludes the use of most commercially available phasemeters, unless the signal is previously filtered.

The frequency requirements on all electronic equipment, other than equipment operating at fixed frequencies, are severe. For example, the frequency range (the two extreme frequencies at which the response has changed by 3 db) of a preamplifier should be about 0.05–1,000,000 cps if an absolute phase accuracy of 1/4 deg is desired from 10 to 5000 cps. Since the interest is in the relative phase angle between the force and acceleration signals, the requirements would be less stringent were it not for the additional circuitry. The mass cancellation circuit, for example, may connect the amplifiers at different stages, yet this cancellation must be exact, especially at high frequencies. Another example is a nonautomatic system in which the two channels are connected at different stages to make phase measurements; hence the phase shift of the preamplifiers at the connection points must be identical and the circuitry between the two channels must have absolute phase requirements.

All manual systems except the second may be automated, with further and extensive development and design of new equipment.

The dynamic range of all automatic systems may be extended by plotting the impedance, mobility, or equivalent mass on two recorders with different settings of input attenuators.

Greater accuracy and a greater impedance range can be obtained with sinusoidal excitation of the specimen, and other types of excitation should be used only if the time allowed for tests is limited. The use and limitations of magnetic tape recorders are discussed in Reference 11.

In automatic systems, the frequency sweep rate must be commensurate with the mechanical $Q$ of the specimen. A logarithmic sweep is preferable, and various sweep times ranging from about 5 min to 1 hr for the frequency range of 10–5000 cps should be available.

In automatic measurements of mechanical structures, it has proved more feasible to keep the force signal at a constant level. However, this may present a problem at resonant frequencies of an extremely undamped structure.

**MANUAL SYSTEMS**

**Sum and Difference Technique**

This method has been used successfully at the Taylor Model Basin for measuring the transmissibility of mountings. The operation of the phase measurements depends on the fact that the vector sum of two alternating voltages is a function of their individual amplitudes.
and the phase angle between them. Therefore, by arranging that the filtered voltages are adjusted to the same predetermined level, the vector sum or difference becomes a function of the phase angle only. Since the signals are filtered after the vector addition or subtraction, no error is introduced by phase shifts in the filter. The accuracy of this system is described in Reference 13. This system is commercially available from Muirhead and Company, Limited, Beckenham, Kent, England, but it is recommended that one of the analyzers described previously be used rather than the Muirhead analyzer.

**Oscilloscope Technique**

This method is very inexpensive and is based on the fact that the force signal is a pure sinusoid.\(^1\)\(^4\) At first, the force signal is split and fed through both preamplifiers. One preamplifier is attached through a filter to the X-axis of the scope and the other directly to the Y-axis of the scope. After the filter is adjusted to give a 0-deg phase shift on the scope, the acceleration signal is switched into one preamplifier-filter channel with the force signal passing through the other channel. A calibrated phase-shifting network is adjusted again to indicate 0-deg phase shift on the scope. The phase angle may then be obtained from the adjustments on the phase-shifting network. An accuracy of ± 1 deg is claimed.

**Reactance or Resistance Nulling Technique**

This system, designed by the Taylor Model Basin and shown in Figures 14 and 15, has been used extensively to make mechanical impedance measurements. The operation depends on the fact that the reactive or resistive components of one of the vectors may be cancelled. The amplitude of the remaining vector and the amplitude of the original vector are functions of the phase angle between the vectors. The acceleration and force signals pass through attenuators to two preamplifiers having identical input stages. The attenuators for both channels consist of capacitances and resistances, which may be paralleled with the transducers and cable to attenuate the signal without introducing phase shifts. In other words, the time constant for any transducer for any attenuator is held constant. A signal from Output 1 of the acceleration preamplifier is fed through a feedback amplifier and added to the force gage signal in the force gage preamplifier. The sum of these signals is measured by the Radiometer analyzer, which has a 4-cps constant bandwidth, reading the voltage at the frequency of interest. The feedback amplifier contains an integrator or 90-deg phase shifter which may be switched in or out with Switch 3. The signal from the feedback amplifier can be added to or subtracted from the force signal by the position of Switch 4. In other words, a portion of the acceleration signal—this portion being adjusted by Potentiometer 2—may be added to the force signal in phase or 90, 180, or 270 deg out of phase. By turning Potentiometer 2, there are two combinations of the possible four switch conditions of Switches 3 and 4 whereby a minimum signal is measured by the analyzer at Output 4. The phase angle
Figure 14 — Block Diagram of Instrumentation Used in Reactance or Resistance Nulling Technique

Figure 15 — Instrumentation Used in Reactance Nulling Technique
may then be computed from the amplitude of this minimum $F_{\text{min}}$ which is the reactive or resistive component of the force signal, and the original force signal $F$ can be obtained by turning off the added signal with Switch 1. With Switch 1 turned off, the impedance magnitude can be determined by measuring the signals from the force and acceleration preamplifiers. A signal is fed from Output 2 of the acceleration preamplifier to Potentiometer 1 in the force preamplifier to cancel the mass below the force gage. The potentiometer is adjusted as described in the calibration procedure.

Figure 16 shows the formulas for obtaining the phase angles from the ratio $F_{\text{min}}/F$ as a function of the switch position. It also indicates which of the two switch conditions possible in each quadrant should be chosen to obtain the greatest accuracy. For example, if a minimum can be obtained by switch conditions B and C, the acceleration must lead the force by 90 to 180 deg. (The term "masslike" or "springlike" of Switch 4 indicates masslike or springlike behavior only in point impedance measurements without using the 90-deg phase shifter.) If, in switch condition C, $F_{\text{min}}/F = 0.5$, the acceleration must lead the force by $180 - \sin^{-1} 0.5$, or 150 deg. Switch condition C is more accurate than switch condition B in this octant. For point impedances rather than transfer impedances, where the acceleration may lead the force by 0 to 180 deg, switch condition D is not necessary. If less accuracy is permissible near 90 deg, no 90-deg phase shifter is necessary for point impedance measurements. The 90-deg phase shifter is always necessary for transfer impedance measurements.

The phase accuracy is somewhat better than in the sum or difference technique, since always only one critical measurement, $F_{\text{min}}$, is necessary. For some angles in the sum or difference technique, two vectors are adjusted consecutively to equal amplitude. In the latter technique, a small error is introduced if the gain of the amplifiers or any other components varies with time or if signals are not exactly adjusted to equal amplitudes. Generally speaking, the accuracy from 30 to 2000 cps at angles $\pm 10$ deg from the coordinates is within $\pm 4$ deg. The magnitude and phase of the impedance can be measured in about 45 sec.

The necessary modifications made on commercially available components for this system are shown in Figures 17–20. A twin power supply for the preamplifiers has been marketed by Tektronix recently.

Another nonautomatic system using the same technique has been specifically designed for the small impedance head. The frequency range has been extended to 10,000 cps and the size and weight of the equipment have been considerably reduced.
Phase Determination

<table>
<thead>
<tr>
<th>Switch Condition</th>
<th>Position of Switch 3</th>
<th>Position of Switch 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

In each quadrant a maximum can be reached in two specific switch conditions as shown below. The quadrant can therefore be determined. For greater accuracy the switch positions shown inside the circle are recommended. (Whenever $F_{\text{min}}/F$ is greater than 0.707, greater accuracy may be obtained by changing to other switch condition of the same quadrant.)

**Figure 16 – Phase Determination**
Figure 17 – Accelerometer Preamplifier, Tektronix Type 122 Low-Level Preamplifier, Modified

Figure 18 – Force Preamplifier, Tektronix Type 122, Modified
Figure 19 – Feedback Amplifier, Scott Type 140A; Modified

\[ R_1 = R_4 = 1,000 \, \Omega \, \frac{1}{2} \text{ Watt} \]
\[ R_2 = 54,000 \, \Omega \, \frac{1}{2} \text{ Watt} \]
\[ R_3 = 20,000 \, \Omega \, \frac{1}{2} \text{ Watt} \]
\[ R_5 = 160 \, \Omega \, 7 \text{ Watt} \]
\[ C_1 = C_2 = 1,000 \, \text{mfd} \, 150 \, \text{v} \, \text{(Mallory 15010)} \]
\[ P_1 = \text{Transpac Reg. P.S. (CV 250 Case D)} \]
\[ P_2 = \text{Transpac P.S. (F 12)} \]

Note: Will not operate with one amplifier.

Figure 20 – Twin Power Supply for Two Type 122 Tektronix Preamplifiers
AUTOMATIC SYSTEMS

Simple Impedance Magnitude Plotter

This plotter is shown in Figure 21. No phase information is obtained. It functions properly only if the force signal is a relatively pure sinusoid. The acceleration signal passes through the preamplifier and motor-driven wave analyzer to the level recorder. The other recorder is connected to automatically control the voltage to the power amplifier, thus keeping the output from the force preamplifier constant. A recorder used as an automatic volume control, although more costly, offers greater dynamic range and accuracy than other types. The signal to the potentiometer of the recorder is obtained from the audio output of the analyzer. The level recorder plots the ratio of acceleration to force, but various modifications may be made to plot equivalent mass (force/acceleration), impedance, or mobility. A log amplifier and d-c recorder may be substituted for the plotter. Impedance accuracy depends on the choice of the potentiometers of the recorder.

If a Hewlett Packard Model 302A analyzer is used in this system, it may be modified to have an output signal at the frequency of interest and proportional to the filtered acceleration. (This analyzer uses crystal filters and a crystal oscillator.) If a commercial phasemeter with recorder is added to the system, phase measurements may be made. A phase shifter must be included in the IF circuit of the analyzer.

![Diagram]

1 Circuit to Cancel Mass below Force Gage Electronically
2 Wave Analyzer, Radiometer FRA 2 T or HP 302A
3 Level Recorder, B & K or General Radio, to Keep Force Constant
4 Level Recorder, B & K or General Radio, to Record Impedance

Figure 21 – Automatic Impedance Magnitude Plotter Using Commercially Available Equipment

31
Inexpensive Automatic System to Record Magnitude and Phase of the Mechanical Impedance

In this and the following systems, two band-pass filters, one for each channel, are used. Since the phase shift over the pass band is at least 180 deg and, since the noise requirements make a narrow pass band of 10 cps or less necessary, the filters must be chosen carefully. For a phase accuracy of 1/4 deg, the drift of the filter must be less than $1/4 \times 10/180 = 1/72$ cps. If a simple heterodyne circuit is used, the intermediate frequency should be at least twice the maximum audio frequency, or 10,000 cps. Both the filter and the fixed-frequency oscillator stability must therefore be better than one part in 720,000.

The stability required for the local variable frequency oscillator depends on the $Q$ of the mechanical systems. Therefore, temperature-controlled crystal filters and crystal oscillators should be used.

This system is shown in Figure 22. After both signals are heterodyned and filtered in the wave analyzers using a common local oscillator, they are fed through logarithmic amplifiers and the difference is recorded. The logarithmic amplifiers, which have a d-c output proportioned to the logarithm of the a-c input, should have a dynamic range of at least 60 db.

---

1 Circuit to Cancel Mass below Force Gage Electronically
2 Wave Analyzers HP 302A or Equivalent Modified to Have
   a. Common Local Oscillator
   b. IF Output from Crystal Filters
   c. Preferably Carefully Matched Crystal Filters
3 Sanborn or Moseley
4 Phase Shifter (to Operate at One Frequency Only) to Make
   Up for Constant Phase Shift between Crystals
5 Phasemeter to Have Large Dynamic Range at IF

Figure 22 - Inexpensive Automatic System to Record Magnitude and Phase of the Mechanical Impedance Using Commercially Available Equipment
The outputs from the wave analyzer are at a fixed frequency and have the same phase relationship as the original signal except for a constant frequency-independent phase shift caused by the slight unavoidable mismatch of the filters. The phase shifter corrects for this phase shift. The phasemeter d-c output, which is proportional to the phase angle between the two signals, is recorded by a d-c recorder. The accuracy of the phase is ±3 deg and the accuracy of the magnitude is ±1 db.

**Automatic Impedance, Mobility, or Equivalent Mass Plotter**

This system, shown in Figure 23, was conceived by TMB and is presently being constructed. In the preceding automatic systems, the analyzers and phasemeters had to operate over the complete dynamic range of 60 db, and it had been observed that small phase shifts occurred as a function of signal strength. In this instrument, the force signal at the frequency of interest is held constant to within 0.5 db by a servomechanism, and the acceleration signal at the frequency of interest is held constant to within 0.8 db at a point just before entering the voltage amplifier and thereafter. Therefore, mixers, filters, and phasemeter operate at a constant amplitude. The servomechanisms used in this system are General Radio recorders wired to automatically control the volume with an analyzer inserted in the circuit. Using an 80-db potentiometer in the acceleration channel, a 75-db dynamic range can be achieved. (The dynamic range of the preamplifiers is 75 db.) Stray capacitances in the acceleration

![Figure 23 - Automatic Impedance, Mobility, or Equivalent Mass Plotter, under Construction](image-url)
potentiometer cause a maximum phase shift of $1/4$ deg at 5000 cps. The phase accuracy
could be improved by using a specially made low-impedance potentiometer. The logarithmic
potentiometer in the accelerometer channel is specially wired to record the reciprocal of the
acceleration, thereby directly recording equivalent mass without the integrator in the circuit,
or impedance with the integrator. Mobility may be plotted by using the conventional potenti-
ometer. Integrator may be switched in or out of circuit.

**Automatic Recording System with Graphical and Digital Readout**

This system, shown in Figure 24, conceived and presently under development by the
Taylor Model Basin, is a refined version of the system just described. The signal generation
for driving the shaker is identical. The signal in each channel at the input of the first mixer
at the frequency of interest is held constant within $\pm 1$ db by the servomechanisms of two
General Radio type 1521-A recorders, thereby increasing the precision of subsequent cir-
cuity. After passing through the first set of mixers and crystal filters, the signals are het-
erodyned a second time by a crystal oscillator which differs in frequency from the set of
crystal filters or the crystal oscillator in the driving circuit by 277.78 cps. The crystal
oscillators and filters are stable to at least one part in $10^7$. At the output of the low-pass
filters, the signals from the two channels have constant amplitude, constant frequency of
277.78 cps, and the same phase relationship as the inputs to the preamplifiers except for a
fixed-phase shift caused by the mismatch of the two crystal filters. A phase shifter that
needs only initial adjustment corrects for this shift, which is independent of the audio drive
frequency. The phase shift may now be read directly by a time interval meter, since one
period at 277.78 cps equals 3600 $\mu$sec. Using a meter capable of measuring an interval of
1 $\mu$sec, the phase angle may be determined to an accuracy of $\pm 1$ count or $\pm 0.1$ deg. With
present commercially available recorders, this accuracy cannot be realized beyond about
10,000 cps because of the stray capacity in the 10,000-ohm potentiometer of the servomechanism
in the acceleration channel. A potentiometer having an impedance of 1000 ohms or less should
be used for greater accuracy at higher frequencies.

The recorder in the acceleration channel plots the magnitude of the equivalent mass or,
with an additional integrator, plots mobility or impedance. The accuracy depends on the number
of contacts and the type of potentiometers used in the recorders. It can be improved by substi-
tuting the filtered rectified force signal for the internal d-c standard, operating the recorder as
a ratiometer. By superimposing on the a-c signal from a constant voltage source on the poten-
tiometer in the acceleration channel, a d-c output proportional to the equivalent mass, impedance,
or mobility is obtained after filtering. This quantity and the frequency and phase angle may be
printed once each second by digitizing the corresponding signals.
Servo 1 has special low-impedance pot.
Servo 1 + 2 provide constant signal amplitude to mixers.

Crystal filters are matched.
Crystal oscillators stable to one part in ten million per hour.
Phase accuracy ±0.1 deg 5 to 10,000 cps.
F/A accuracy ±3 percent.
At 277.8 cps, one period = 3600 μsec.

Figure 24 – Automatic Phase and F/A Recording System with Graphical and Digital Readout (5–25,000 Cycles per Second)
ACKNOWLEDGMENTS

This paper is based on investigations performed at the Taylor Model Basin, which were sponsored by the Ship Silencing Branch of the Applied Research Division of the Bureau of Ships under Project S-F013 11 01.

The author would like to express his appreciation for the helpful and considerate suggestions and criticism of Mr. A.O. Sykes and Dr. M. Strasberg.

REFERENCES


INITIAL DISTRIBUTION

Copies

10 CHBUSHP5S
  3 Tech Info Br (Code 335)
  1 Applied Sci Br (Code 342)
  2 Ship Silencing Br (Code 345)
  1 Mach Des Br (Code 430)
  1 Mach Sci & Res Sec (Code 436)
  1 Sci & Res Sec (Code 442)
  1 Sub Br (Code 525)

1 CHBUWEPS

2 CHONR
  1 Fluid Dyn Br (Code 438)
  1 Struct Mech Br (Code 439)

5 CO & DIR, USNEES
  1 Dr. K. Moeller
  1 Mr. J. Vallillo

3 CO & DIR, USNUSL
  1 Mr. T. Bell

3 CO & DIR, USNEL
  1 Mr. Coleman (Code 563)

2 CDR, USNOL

6 DIR, USNRL
  1 Libr (Code 2020)
  1 Sound Div (Code 5500)
  2 Mech Div (Code 6200)
  1 Dr. Robert O. Belsheim, Mech (Code 6260)
  1 Mr. Walter Young, Nucleonics (Code 7271)

3 NAVSHIPYD PUG
  2 Mr. Carl Newstrom
  1 Mr. Peterson (Carr Inlet Acoustic Range)

2 NAVSHIPYD MARE, Rubber Lab
  1 Mr. Ross Morris

1 NAVSHIPYD CHASN

1 NAVSHIPYD PTSMH

1 NAVSHIPYD NYK

1 NAVSHIPYD BSN

1 NAVSHIPYD SFRAN

1 CO, USNUOS

1 DIR, USNUSRL

5 SUPSHIP, EB Div, Gen Dyn Corp
  1 Mr. William Ezell
  1 Mr. R. Collier
  1 Mr. T. Baraclough

1 Dir, U.S. Waterways Exper Sta

1 CMDT, USCG

1 CHOFORD, Dept of Army, ORDTX

1 CG, Aberdeen PG, BuOrd Tech Liaison Off

1 CG, ENGRESDEVLAB

3 CDR, W-PADEVCECN, Air Res & Dev Com

9 DIR, Natl BuStand
  2 Off of Basic Instrumentation
  1 Sound Sec
  1 Thermal Metallurgy, Mr. Samuel J. Rosenberg
  2 Mr. Walter Koidan, FW Bldg
  1 Dr. Walter Ramberg, Chief, Mech Div
  1 Mr. Seymour Edelman, Sound Chamber
  1 Dr. R.S. Marvin, Rubber Sec

1 USAEC
  1 Dir of Operations Analysis & Planning

1 MARAD
  1 Mr. H.A. Sullivan, Chief, Sci Br

1 US Dept of Agr
  1 Mr. C.O. Badgett, Agr Res Admin
    Bur of Agr & Indus Chem, Philadelphia

10 CDR, ASTIA

2 DIR, ORL

1 DIR, Acoustics Res Lab, Harvard Univ

1 Illinois Inst of Tech, Dept of Mech Engin
  Tech Ctr, Chicago

2 Univ of Michigan, Engin Res Inst, Ann Arbor

1 Librarian, Carnegie Inst of Tech, Pittsburgh

1 Librarian, CIT, Pasadena

1 Librarian, Franklin Inst, Philadelphia

1 Librarian, Cornell Aero Lab, Inc, Buffalo

1 Forrestal Res Ctr, Princeton

39
1 Librarian, Bell Tel Lab, Inc, Murray Hill
1 Librarian, Accessory Turbine Engin Sec
   Aircraft Gas Turbine Div, Gen Elec Co
   West Lynn
1 Librarian, Reed Res, Inc
1 Librarian, NNSB & DD Co, Newport News
4 The Barry Controls, Inc, Watertown
   Attn: Staff Engr
2 Chrysler Corp, Detroit Universal Div, Dearborn
1 Bur of Reclamation, Denver Fed Ctr, Denver
2 New York Shipbldg Corp, Camden
2 Goodyear Tire & Rubber Co, Akron
2 Gen Motors Corp, Cleveland Diesel Engin Div
   Washington
   1 Mr. Kornichuk
   1 Mr. Melton
   Attn: Miss Mason
2 U.S. Rubber Co, Tire Div, Detroit
1 Gen Elec Co, Genl Engin Lab, Schenectady
1 Gen Elec Co, West Lynn
2 Gen Elec Co, Fitchburg
3 Westinghouse Elec Corp, Res Labs, E. Pittsburgh
   1 Mr. A.C. Hagg
   1 Mr. Wright
1 Lockheed Aircraft Corp, Missile Systems Div
   Dept 75-45, Van Nuys
2 Lockheed Aircraft Corp, Calif Div, Burbank
   1 Mr. F. Mintz, Group Engr, Acoustics
1 Lockheed Aircraft Corp, Sunnyvale, California
   Attn: Dr. T.A. Perls
1 Space Technology Lab, Rodondo Beach, California
   Attn: Dr. C.T. Molloy
1 Geiger and Hamme, Ann Arbor
1 Eclipse-Pioneer Div, Bendix Aviation Corp
   Teterboro
Two devices for measuring mechanical impedance have been developed, one for heavy structures capable of measuring a stiffness of $10^6$ lb in. over the frequency range of 5 to 4000 cps and the other for lighter structures capable of measuring a stiffness of $5 \times 10^5$ lb in. over the frequency range of 20 to 8000 cps. Phase angles are measured in the presence of non-correlated vibrational and electrical noise and distortion. Each device uses two piezoelectric ceramics to sense the applied force and the acceleration response of the structure. The mass associated with the coupling between the impedance transducer and the structure is cancelled electronically. Novel details of
the design are presented, such as the elimination of magnetic pickup by the piezoelectric transducers and the reduction of mechanical crosstalk. Manual and automatic phase and impedance measuring systems having a high degree of accuracy are discussed. The report includes procedures for determining the acoustic sensitivity, transverse sensitivity, rocking sensitivity, and strain sensitivity of accelerometers and practical considerations for making mechanical impedance measurements.
Two devices for measuring mechanical impedance have been developed, one for heavy structures capable of measuring a stiffness of 140 \times 10^6 lb in. over the frequency range of 5 to 4000 cps and the other for lighter structures capable of measuring a stiffness of 5 \times 10^6 lb in. over the frequency range of 20 to 8000 cps. Phase angles are measured in the presence of non-correlated vibrational and electrical noise and distortion. Each device uses two piezoelectric ceramics to sense the applied force and the acceleration response of the structure. The mass associated with the coupling between the impedance transducer and the structure is cancelled electronically. Novel details of...
the design are presented, such as the elimination of magnetic
pickup by the piezoelectric transducers and the reduction of
mechanical crosstalk. Manual and automatic phase and imped-
ance measuring systems having a high degree of accuracy are
discussed. The report includes procedures for determining the
acoustic sensitivity, transverse sensitivity, rocking sensitivity,
and strain sensitivity of accelerometers and practical consider-
ations for making mechanical impedance measurements.

the design are presented, such as the elimination of magnetic
pickup by the piezoelectric transducers and the reduction of
mechanical crosstalk. Manual and automatic phase and imped-
ance measuring systems having a high degree of accuracy are
discussed. The report includes procedures for determining the
acoustic sensitivity, transverse sensitivity, rocking sensitivity,
and strain sensitivity of accelerometers and practical consider-
ations for making mechanical impedance measurements.