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METHODS OF CALCULATING NUCLEAR REACTORS
(SELECTED PARTS)

By
G. I. Marchuk
UNEDITED ROUGH DRAFT TRANSLATION

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English Pages: 42

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Date: 21 Dec 1962
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METODY RASCHETA YADERNYKH REAKTOROV

Gosudarstvennoye Izdatel'stvo Literatury
V Oblasti Atomnoy Nauki I Tekhniki
Moskava 1961
Pages: 2-17, 647-661

FTD-TT-62-1314/1
Methods of Calculating Nuclear Reactors

by

G.I.Marchuk

Annotation

In the monography are explained basic methods of physical calculation of nuclear reactors. Developed were algorithms for practical utilization of mathematical methods for solving various problems, originating during the planning of experimental and atomic energy installations. Given are calculated values of critical masses of reactors, which can be used in practical operation.

The book is intended for students-physicists, scientific workers and engineers, specializing in the field of atomic energy.

Foreword

This monography is devoted to methods of calculating nuclear reactors of different spectra. The first effort of giving a systematic explanation of reactor calculation methods was made by the author in his book entitled "Numerical Methods of Calculating Nuclear Reactors" published before the Second International Conference on Peaceful Utilization of Atomic Energy (Geneva, 1958). However, since this book appeared on the library shelves many changes have taken place in the methods of calculating reactors. This circumstance prodded the author into writing a monography, in which is described the present day status of problems connected with physical calculation of nuclear reactors. Of course, not all mathematical problems of physical calculation are sufficiently well explained in this book, but in all instances the author tried to explain all the data with consideration of a certain perspective for further development of methods. Main attention in the monography was devoted to problems of mathematical description of the processes occurring in reactors; problems
concerning the theory of reactors have been touched upon only in minimum extent.

In the book have been reflected problems developed by the author together with a group of co-workers: Sh.S. Nikolayshvili, Ye.I. Pogudalina, V.V. Smelov, G.A. Ilyasov, F.F. Mikhaylus, V.P. Kochergin, A.I. Vaskin, V.P. Il'in, B.L. Gavrilin, V.V. Varkomeyeva, Z.N. Milyutina and others.

Calculations of critical masses, mentioned in the book, were carried out under the supervision of author A.I. Nevinnitsa and O.P. Uznadze with active participation of V.P. Kochergin.

Chapter 28 of the book, upon appeal by the author, was written by V.F. Turchin.

Methods of calculating reactors, as explained in the book, have been developed and improved under the influence of functions, carried out under the scientific guidance of academician of the Academy of Sciences Ukr-SSR A.I. Leypunskiy.

During all the stages of work the author maintained scientific communication with physicist-theoreticians: L.N. Usachev, V.V. Orlov, T.N. Zubarev, S.B. Shikhov, V.F. Turchin, V.Ya. Pukko, Ya.V. Shevelev, B.F. Grinov, G.I. Toshinskiy, V.V. Chekunov and others, and also with physicists-experimenters: I.I. Bondarenko, V.A. Kuznetsov, V.I. Mostov, B.G. Dubovskiy, A.I. Mogil'ner, M.B. Yegiazarov, V.S. Dikarev, E.A. Stumbur and others. The scientific communication with physicists-theoreticians and experimenters has in many instances overcome the trend of work connected with the development of reactor calculation methods.

The author is deeply indebted to A.I. Leypunskiy for numerous evaluations of various problems concerning the theory of nuclear reactors, which aided in successful development of mathematical methods.

The author is deeply indebted to V.S. Vladimirov for effective evaluation of problems, concerning the transfer of neutrons, as well as to V.F. Turchin, who wrote chapter 28 of the book.

The author is grateful for the evaluation of various mathematical problems concerning the theory of neutron transfer, to Ye.S. Kuznetsov, N.N. Yaneiko, N.I. Bulyev.
Great assist in preparing the book for print was given by V.F.Ilin, A.I.Nevinnitsa, and O.F.Uznadze, to whom the author expresses his profound thanks.

G.I.Marchuk

Introduction

Broad application in the calculation of high speed computers created favorable conditions for the development of many fields of science and technology. Newest achievements of computer mathematics enabled to reexamine algorithms for the solution of many problems of mathematical physics and to develop new algorithms, based on the use of numerical methods, submitting to effective realization on computers.

In this way, analytical methods of solving problems have in many instances given place to more effective numerical methods.

The use in calculations of high speed computers enabled to bring forth new problems of science and technology, a greater volume of calculation work of which has previously appeared practically unrealizable. As result such conditions have been created, at which it was found possible to solve more difficult mathematical and logical problems. In this connection, basic importance was acquired by problems, connected with mathematical arrangement of the problems and with the development of effective calculation algorithms.

Successes in the field of computational mathematics have produced a greater effect on the development of the theory and methods of calculating nuclear reactors. After the book by Glesston and M.Edlund[79] which discussed the elementary bases of the theory of thermal neutron reactors, appeared monographs by A.D.Gelanin[59], devoted to problems of further developing the theory of thermal neutron base-reactors.

General scientific and theoretical bases of atomic energy were explained in books by A.Veynberg and Ye.Vigny[48], as well as by B.Davisson[102]. Finally, in the monography[172] are discussed numerical methods of calculating nuclear reactors.

Up to this day there is no loss in theoretical value of reports of earlier periods

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of development of atomic energy. These are reports by Ye. Vigneron [44, 47], G. Plachak [227, 228, 231], Peyerls [224], R. Marchak [187, 190], N. N. Bogolyubov [23], I. I. Gurevich and I. Ya. Pomeranchuk [87], G. Vik [48, 49], A. Veynberg [44, 86], G. Bethe [18] and others.

Further development of the theory of nuclear reactors and calculation methods was prompted in the lectures presented at the First and Second International Conferences on Peaceful Utilization of Atomic Energy in Geneva.

Broader development of the theory and methods of calculating nuclear reactors was attained in the USSR in reports by I. V. Kurchatov [57], A. P. Aleksandrov [4], A. I. Alikhanov [57], D. I. Blokhintsev [19, 21], V. S. Fursova [296], I. I. Gurevich and I. Ya. Pomeranchuk [87], A. K. Krasin [145, 146], S. M. Feynberg [282, 286], A. D. Galanin [59, 66], V. I. Mostovoy [208], V. V. Orlov [218, 221], B. G. Dubovskiy [97, 179], V. F. Turchin [276, 277], V. V. Smelov [177, 252, 253] and others. On the basis of these reports have been scientifically founded projects of nuclear reactors for a greater number of power generating installations, such as the Voronezh, Byeloruskaya atomic electric power stations, the ice breaker LENIN and others, as well as a greater number of experimental reactors with various decelerators and heat carriers [67, 259, 284-286].


Methods of calculating fast neutron reactors have been developed by A. I. Leypunskiy [63, 164], D. I. Blokhintsev [22], L. N. Usachev [278, 280], G. D. Kazachkovskiy [124, 163, 164], I. I. Bondarenko [24, 163, 164], S. B. Shikhov [216, 310], Yu. A. Stavisskiy [163, 164], as well as V. S. Vladimirov [57, 58], Yu. A. Romanov [239] and others. The mentioned reports enabled to scientifically found plans for atomic power generating stations BR-1 and BR-2 built and put into operation in 1957 and 1959 respectively.

A review of reports of foreign scientists on the theory and methods of calculating

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Substantial development in recent years has been gained also by computational mathematics, the basic results of which were obtained by the group at the V.A. Steklov Math. Inst. at the Academy of Sciences USSR under scientific guidance of academician N.V. Keldysh. The maximum number of results from these investigations pertains to the solution of finite-difference equations of elliptical type, as well as to the formulation of finite-differential equations, approximating differential equations with disruptive coefficients.

To solve three-point finite-differential equations of elliptical type I.M. Gel'fand and O.V. Lokutsiyevskiy [166], independently, A.S. Kronrod [59] and Stark [214], formulated a factorizing method, which enabled to reduce a marginal problem to a subsequent solution of three finite-differential (difference) equations of the first order.

Next M.V. Keldysh, I.M. Gel'fand and O.V. Lokutsiyevskiy together with K.I. Babenko, V.V. Rusanov and N.N. Chentsov [72] have substantially developed methods for solving finite-difference equations, having developed an apparatus of matrix factorization applicable to systems of difference equations. In particular, this allowed to formulate a new method of solving multidimensional differential equations of elliptical type. The method of matrix factorization has produced a decisive influence on the development of numerical methods for solving equations a method of spherical harmonics applicable to problems of atomic energy.

To the study of finite-difference equations, approximating differential equations of elliptical type, have been devoted the investigations by A.N. Tikhonov and A.A. Samarskiy [270-273]. A.N. Tikhonov and A.A. Samarskiy made a thorough investigation of problems concerning the formulation of three-point finite-difference schemes, given are practical algorithms of most rational approximations of differential equations with finite-difference ones, and established were also the general aspects of solutions.
of difference equations. They introduced the concept of integral accuracy of a solution of a finite-difference equation relative to the solution of a differential equation and have proven theorems on the convergence of solutions of finite-difference equations. They have proven, in particular, that the formulated by them difference equations in the class of disruptive coefficients have the second integral order of accuracy.

Achievements of computational mathematics have been widely discussed in scientific literature, as well as at conferences and conventions.

All the above mentioned reports on calculations methods prepared a base for further development of methods for calculating nuclear reactors. These methods constitute the basic content of this monograph.

The book consists of eight parts. The first part devoted to problems of neutron diffusion. Investigation of the problems is conducted under the assumption, that all neutrons have identical velocity and differ only in direction of the velocity vector.

In chapter 1 is given a brief summary of single speed neutron transfer equations. Derived was an integro-differential equation in various geometries, discussed is the integral equation by Rayersls, corresponding to the equation of particle transfer. Special attention was devoted to transport approximation. In conclusion were formulated problems on critical dimensions of a nuclear reactor.

In chapter 2 are analyzed results of solving transfer equations in case of a point source of neutrons in an infinite medium. The discussion is carried on at first for the case of isotropic diffusion of neutrons.

Next is investigated the problem of asymptotic density of neutrons at greater distances from the source. Using a single speed problem as an example was investigated the method of moments, as well as the method of approximation, developed by Sh.S. Nikolayshvili. In many instances these methods allow to obtain solutions for problems regarding the transfer of neutrons and gamma-quanta.

In chapter 3 are explained numerical methods of solving kinetic equations.
Much attention has been devoted to the best known in physical calculations of nuclear reactors numerical methods by V.S. Vladimirov [57] and B.Karlson [129]. Thoroughly investigated is the problem of improving the convergence of iteration processes, which appear to be the component element of algorithms for solving kinetic equations by the Vladimir and Karlson method. Together with the well-known method of L.A.Lyusternik, is explained a new method of improving the convergence of the subsequent approximation method, formulated by V.P. Morozov [206]. In conclusion of the chapter is described the variation method for solving neutron transfer equations introduced by V.S. Vladimirov [55].

In chapter 4 are discussed methods of solving integral equations by Payerl. At first is discussed the Krylov-Bogolyubov method, which acquired broad application in solving integral equations. This method is illustrated on a problem concerning the distribution of neutrons within a flat layer. Then special attention is devoted to the method of degenerated nuclei. This method is illustrated by an example of calculating the critical dimension of a homogeneous sphere in case of isotropic, as well as in a simple case of a nonisotropic indicatrix of neutron diffusion.

In chapter 5 is given a general description of the spherical harmonics method, developed by G.Ya. Rumer, R.Marshak [177, 189, 199], S.Mark [171], B.Davison [100, 102, 103], L.N.Usachev [277], M.Van and Ye.Gut [40], S.Chandrasekar [202] and others, given are also the basic features of the method, supplemented by concrete applications to the case of plane-parallel and spherically-symmetrical systems. Discussed are monodimensional cylindrical systems. Special attention is devoted to $P_1$-approximation and to diffusion approximation as well.

G.Ya. Rumer is first called attention to the possibility of practically utilizing in calculations of even approximations the method of spherical harmonics, whereby he placed special emphasis on the employment of the $P_2$-approximation in calculations. It should be pointed out, that G.Ya. Rumer also developed the method of spherical harmonics for arbitrary geometry in problems of setting up
concrete boundary conditions $[241, 242]$. 

A study of the problem concerning the solving of transfer equations in case of a point source in an infinite medium with the aid of the spherical harmonics method was carried out by Sh. S. Nikolayshvili $[175, 176]$. 

At the conclusion of the chapter is formulated the variational principle introduced by V. S. Vladimirov $[50, 51, 55]$. On the basis of the variation principle is made a conclusion of equations of spherical harmonics for various geometries in case of isotropic diffusion indicatrices and simultaneously are obtained boundary equations, coinciding with R. Harshak conditions.

It should be pointed out, that the methods discussed in the first part of the book, are of general nature, and can be applied without any difficulty for solving more difficult problems. Particularly, they can serve as basis for solving problems concerning critical dimensions of a reactor.

Part two of the book is devoted to problems of formulating finite-difference equations of elliptical type, having greater importance in calculating nuclear reactors.

In chapter 6 are discussed finite-difference equations of diffusion for mono-dimensional zones. The explanation is done on the basis of above mentioned results obtained by A. N. Tikhonov and A. A. Samarskiy $[270-273]$. 

Discussed are simple finite-difference equations of diffusion in the class of discontinuous coefficients. Pointed out are ways of improving finite-difference equations. Formulated were difference equations for systems of two diffusion equations of first order. Introduced into the investigation were finite-difference balance equations, enabling to control the correctness of solving problems in finite-difference formulation. Finally, notes are given on the accuracy of the numerical calculations by finite-difference systems, and a relationship is established between finite-difference equations of continuous calculation and difference equations, formulated with the aid of a method of introducing fictitious points.

In chapter 7 are analyzed the results of formulating finite-difference equations.
for two-dimensional zones. Into investigation is introduced a method of formulating difference equations for multidimensional zones in the class of discontinuous coefficients. Described is a matrix form of recording finite-difference equations for two-dimensional zones, and corresponding balance ratios have been formulated.

In chapter 8 is described the formulation of finite-difference equations by the method of spherical harmonics. This formulation is advisable when employing matrix recording of the initial system of ordinary differential equations, constituting together with the boundary conditions a boundary problem. Finite-difference equations formulated for plane-parallel zones with spherical-symmetrical and cylindrical geometries. It should be underlined, that the use of new methods of solving finite-difference equations enabled to considerably raise the effectiveness of applying the spherical harmonics method for solving a wide class of reactor problems.

In chapter 9 are discussed methods of solving finite-difference equations. The most effective method of solving difference equations of elliptical type is the factorization method. The method of linear finite-difference factorization allows to effectively solve many problems connected with physical calculation of reactors. Next are described methods for solving finite-difference diffusion equations for two-dimensional geometry. A corresponding problem could be formulated in vector-matrix form, formally coinciding with a three point linear matrix system. A solution to this problem is found with the aid of the matrix factorization method.

It should, however, be pointed out, that when solving a finite-difference elliptical equation by the matrix factorization method at a greater number of unit points it is necessary to deal with matrices of high order. This considerably reduces the possibility of effectively applying matrix factorization for solving the mentioned problems.

N.I. Buleyev made a successful attempt to develop and iteration method, which appears to be a combination of the relaxation process with the method of linear factorization.
This method in its simplicity is in no way inferior to the simplest iteration processes of Liebman and Seidel, but converges many times faster.

Among other methods, used successfully for solving two-dimensional finite-difference equations, should be mentioned the method of longitudinal-lateral linear factorization, discussed by J. Douglas [98, 99], N. M. Yamenko [216, 317] and V. K. Seulyev [286, 247].

We must mention, that the effective solution of finite-difference equations of spherical harmonics is realized with the matrix factorization method.

The third part of the book is devoted to problems of slowing down (decelerating) neutrons.

Chapter 10 has an introductory nature. In are described the elementary processes, taking place in the nuclear reactor, the structure of the sections where neutrons react with nuclei, estimated and formulated are physical bases for the calculation of a nuclear reactor.

In chapter 11 is given the product of kinetic equation of a reaction. Formulated is an equation for slow down and boundary conditions are fixed. Discussed is a partial case of kinetic equation of a reactor, and namely, problem with constant sections and problem with energy dependence.

In chapter 12 are formulated conjugated equations of a reactor. Given is a general theory of conjugated equations, developed by the author together with V. V. Orlov [173] which generalizes somewhat the known results by Ye. Vigner [43, 44], L. M. Usachev [278, 380], N. A. Dmitriyev [273], N. Fuks [295], B. B. Kadomtsev [120] and B. Davison [120].

Introduced into investigations are important linear functionals and formulated are conjugated equations with respect to the given functionals. Examination of the theory of conjugated equations is completed by a conclusion of formulas of the theory of perturbation.

Chapter 13 is devoted to diffusion approximation. Following in the steps of R. Marshak [199] the approximated solution of a kinetic equation is sought with the aid of two members of the series in accordance with specific functions. As result we arrive
at two integro-differential equations, forming a closed system of reactor equations. For moderators with $M \gg 1$ the solution of moderation equations is sought in form of a series by lethargy. This allows to free our hands in equations from the integral operator and as consequence the problem is reduced to solving a system of differential equations for functions, depending upon spatial coordinates and lethargy. At the end of the chapter are discussed problems of defining more closely the diffusion-age theory, investigated by Ye. Greyling and D. Gertsel (79).

In chapter 14 are explained methods of solving reactor equations in $P_n$-approximation allowing to define more closely the solutions obtained in diffusion approximation. Formulated are basic and conjugated equations in $P_n$-approximations and corresponding boundary conditions. Special attention is devoted to $P_3$-approximation, presenting greater interest in problems concerning critical mass of reactor.

In fourth part of the book are discussed methods of group representation of reactor equations.

Chapter 15 is devoted to the conclusion of multigroup system of basic and conjugated kinetic reactor equations. The method of group averaging is applied in such a way, that during transition from basic problem to multigroup the most essential functional remained unchanged. If in the role of problem functional is considered the critical dimension of the reactor, then during the formulation of a multigroup system is necessary that the critical mass of the reactor should remain unchanged. If in the role of functional is selected, for example, a dosage of radiation coming out from the reactor, as is the case when calculating the protection against radiation, then during the transition to multigroup problem it is necessary, that the dosage of outcoming radiation should remain unchanged. Naturally, that during the averaging, intended for the preservation of these or any other functionals, there is a change in form of conjugated functions, which are also taken with respect to the fixed functional. If the number of groups increases then the method of averaging appears to be indifferent and, in this way, there is the possibility of compiling universal multigroup constants.

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which do not depend upon the neutron spectrum in the reactor. This, in any event, is valid for problems, connected with the calculation of critical mass or reactor reactivity. As to problems pertaining to the passing of neutrons through the protection, then for them, apparently, it is practically impossible to compile a universal multigroup system of constants, valid for calculations of small scale and asymptotic larger protections as well.

In chapter 16 are explained the general algorithms for the obtainment of multigroup reactor equations in $P_1$-approximation. Explained are methods of averaging physical constants in the ultra-warm zone, given are transforms of formulas for averaged physical constants and simple averaging methods are given.

It should be pointed out, that the problem of most rational averaging of physical constants has received sufficiently great attention in literature. These problems, in particular, have found reflection (mention) in reports of R. Erlikh and G. Gurvits [314], S. Glesston and M. Edlund [77], L. N. Usachev [279, 283], G. I. Marchuk [172, 178, 180] and others. In many of the mentioned reports physical constants are averaged for reactors with moderators consisting of a mixture of elements, having no hydrogen.

The method of averaging constants by groups for mixtures containing hydrogen nuclei, was introduced by G. I. Marchuk and N. Ya. Lyashchenko [286]. Further on this method has been substantially developed by N. Ya. Lyashchenko and applied to a broad class of problems.

In the monography by the author [272] was formulated an approximation calculation method for uranium-water reactors, which can also be applied for physical calculations. In this monography, when discussing problems of multigroup representation of reactor equations the author selected a new way, based on the use of the apparatus of the theory of perturbations, allowing to average the constants with consideration of statistical weights, which appear to be solutions of a multigroup system of conjugated reactor equations with respect to the fixed functional.
In conclusion of chapter 16 are basic and conjugated reactor equations, which are changed into form convenient for carrying on practical calculations. Great attention is also devoted to the consideration of resonance effects during multigroup calculation.

In chapter 17 are formulated multigroup systems of reactor equations in diffusion-approximation. Formulas are derived for a multigroup system of constants and ways are discussed for practical realization of the algorithm. Much attention was devoted to the approximated method of calculating spatial-energy distribution of neutrons in the reactor. The obtained neutron spectrum in the reactor can be utilized for multigroup averaging of constants and for more accurate solution of the problem. In conclusion is given a method of calculating the effective addition to the active zone on account of the reflector.

Chapter 18 is devoted to the utilization of the multigroup method to problems in P₂-approximation. Given are effective methods for calculating a multigroup of constants, based on the utilization of the neutron spectrum, obtained in diffusional or P₁-approximations. A special place in the chapter is devoted to P₂-approximation, which in its realization is no more complex, than P₁-approximation.

In the fifth part of the book is developed an effective single-group theory and methods of small group approximation.

In chapter 19 are discussed monogroup kinetic equations, obtained on the basis of rational methods of averaging a multigroup system of kinetic equations. For monogroup averaging used methods of the theory of perturbations so that single group kinetic equations lead to the very same functional values, as does a multigroup system. Discussed are two averaging methods, based on the use of subsequent approximation methods. On the basis of single group theories is considered the problem of calculating reactor dimensions without reflectors.

In chapter 20 are investigated problems of monogroup theory applicable to diffusional and P₁-approximations. Obtained was a system of basic and conjugated reactor...
equations and compiled were formulas for the obtainment of a monogroup system of constants. The corresponding formulas are of simple form and are suitable for practical utilization. In paragraph 20.3 are explained effective methods for calculating critical masses. This part of the chapter appears to be the central one, because in it is formulated the method of defining calculations of critical masses of a reactor, obtained in multigroup $P_1$-approximation. The task of closely defining consists in the fact, that the multigroup calculation in $P_1$-approximation allows to find single-group constants, which are later used in solving a single group problem in $P_n$-approximation. Furthermore, the use in calculations of the single-group theory considerably cuts the time of calculating the critical mass of the reactor. The fact is, to calculate the critical mass of a reactor, it is necessary to employ the method of probes, with the aid of which it is possible to find the dimension of the reactor, corresponding to the value $K_{eff} = 1$. To do this it is necessary to make a series of multigroup calculations. If the single group theory is used, then it is sufficient to solve the system of multigroup reactor equations for one value $K_{eff}$ and average the physical constants, so as to obtain coefficients for single group reactor equations. Further calculation is conducted with the aid of the single group theory. As results is found the critical mass of the reactor. Critical mass calculations by the single group theory may have a certain error, because single group constants were obtained by the use of the neutron spectrum in a noncritical reactor $K_{eff} \neq 1$. To define the calculations it is necessary at values, obtained with the aid of single group theory, of critical dimensions and concentration of the spallating isotope, to make a multigroup calculation, and then again average the constants, and with the aid of the single group theory, to define the value of critical parameters of the reactor. Methods based on use of single-group theories for calculating critical masses of reactors, have been developed by the author together with Ye.I. Pogudalina.

In conclusion of the chapter is described a simple, but sufficiently effective approximation method of calculating critical masses of a reactor in single group.
approximation, developed by S.B. Shikhov and A.I. Novozhilov [216].

In chapter 21 are developed methods of small-group approximation. It is known that when calculating nuclear reactors two- or three-group methods are used quite often. In paragraph 21.2 and 21.4 is given the theoretical basis for employing two-group and three-group methods of calculating reactors. Of special importance here is the three-group method, which allows to combine all neutrons of the fission spectrum in one group, neutrons of resonance energies in another group and thermal energy neutrons in a third one. This offers the possibility of obtaining not only proper values of critical mass, but also the function of the number of secondary neutrons $Q(r)$. If the three-group system of constants is formulated in diffusional or $P_{1}$-approximation, then it can be used further on to define calculations, carried out, for example, within the frame works of $P_{n}$-approximations.

In paragraph 21.4 is described an approximate method of two-group calculation of reactors, developed by S.B. Shikhov and V.B. Troyanskiy. This method is highly effective in application to the calculation of reactors of different spectra.

In chapter 22 are discussed problems of the theory of perturbations in case of a single group approximation, as well as in the case of small-group approximations. Obtained were formulae of the theory of perturbations for practically important instances.

In chapter 23 are discussed methods of calculating critical dimensions of reactors in mono-group approximation. Discussed are problems of calculating critical masses of reactors without reflectors. Derived is an approximate formula for calculating critical reactor mass in case of anisotrophy in the diffusion of neutrons. Formulated is a variation method for calculating critical masses, given are basic formulae, necessary for calculating critical —-parameters of reactors with homogeneous infinite reflector.

Part six of the book is devoted to the study of heterogeneous —-effects in nuclear reactors.

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In chapter 24 is explained the theory of resonant entrapment. At first is discussed moderation and entrapment of neutrons in a homogeneous medium. Formulated is a conjugate problem relative to the entrapment of neutrons and under consideration are methods of solving the integral equation of moderating, developed by the author together with F.F. Mikheylus [181]. Next is discussed the problem of resonant absorption of neutrons in the block. The explanation is made on the basis of the theory of resonant entrapment (resonance capture), developed by V.V. Orlov [218] and, independently, by A.P. Rudik [59]. This theory is valid for the calculation of the effective resonance integral of blocks of any thicknesses and coincides with the theory of N.I. Gurevich and I.Ya. Pomeranchuk [87] for thin blocks and with the Ye. Vigner [79] theory for blocks of greater thicknesses. The dependence of resonance capture upon the temperature of the block has been theoretically investigated by S. Brimberg and Dalstrom [27]. Listed are temperature coefficients for the change in volumetric and surface parts of resonance absorption. Discussed is the problem of mutual screening of lamellar blocks. A corresponding formula to calculate the effective resonance integral was obtained by the author together with V.V. Orlov [172]. At the conclusion of the chapter is considered an important problem of numerically calculating the effective resonance integral in the absorbing block and in the lattice of blocks. This question was brought up by B. Spinrad, Zh. Chernik and N. Korngold [264]. The authors obtained an integral equation of moderation, which is used for finding a central stream of neutrons along the block, necessary to calculate the effective resonance integral. But the theory of B. Spinrad, Zh. Chernik and N. Korngold is applicable only to blocks of greater thicknesses. V.V. Orlov and A.A. Lukyanov [220] developed a theory of resonance capture, based on the use of the integral equation of neutron moderation for blocks of any given thicknesses. Corresponding results are discussed in par. 24.8.

In par. 24.9 are discussed results obtained by the author and concerning resonance absorption of lamellar systems.

In chapter 25 are weighed problems of effective boundary conditions. Discussed is
the problem of effective conditions on the surface of "black" blocks. A review of the method of effective boundary conditions was made by G.A. Bat and D.F. Zaretskiy [13]. When examining the problem of effective conditions on the boundaries with black bodies, the balance method developed by A.L. Brudno was used.

When discussing the boundary conditions for gray bodies, the method introduced by D.F. Zaretskiy and D.D. Odintsov [113] was used, further developed by the author together with T.K. Sedelnikov [172].

When calculating effective conditions on boundaries with "gray" bodies, it is very important to know the probability of neutron capture in the block. To this problem were devoted the investigations by I.I. Stuart [265, 267], the results of which are given in the monography. In the chapter under discussion are given applications of effective boundary conditions to the calculation of nuclear reactors. In this respect very essential results have been obtained by A. Amuval, F. B Pena, and D. Gorovits [8]. In conclusion is discussed the problem of mutual screening of absorbing blocks in the field of ultra-thermal neutron energies. This problem is of special importance for the obtanment of a proper coefficient of multiplication on fast neutrons of a heterogeneous reactor.

In chapter 26 are formulated effective conditions of homogenizing a heterogeneous reactor. Discussed are two methods of approaching the solution of this problem. The first one of the methods is based on studying the problem on the passing of neutrons in direction, parallel and perpendicular to the generatrix of the block. In diffusion approximation for a plane-parallel medium this problem was investigated by Ya.V. Shevelev [305]. In a more accurate set up it was solved by N.I. Leletin [159], who also made an effort to develop effective methods of homogenization for cylindrical cells. The second method, developed by V.V. Smelov [252], is based on the preservation of the reactor's multiplication coefficient during the transition from the heterogeneous system to an equivalently homogeneous. An analogous approach to the solution of the problem of homogenization was developed by A.D. Calanin [66].
In the seventh part of the book are discussed problems of thermalizing neutrons. The first results in thermalization within the frameworks of a model of a monoatomic gas moderator for an infinitely homogeneous medium were obtained by B.N. Davydov in 1937 [33]. These problems were then thoroughly investigated by G. Gurvits [39], Wilkinson [137], Ye.Kogen [137, 138] and others.

In chapter 27 is described a model of a monoatomic gas moderator. Kinetic equations of moderation are discussed. The function of neutron diffusion, constructed by V.V. Smelov and L.V. Mayorov, selected with consideration of the mechanism of elastic diffusion of neutrons on moving nuclei, distributed by velocities in accordance with the Maxwell law. Executed was the transition from kinetic reactor equation to diffusional equations. Formulated was a problem concerning the spectrum of slow neutrons in infinitely homogeneous medium. In conclusion is presented a problem concerning the spectrum of neutrons in a cell of a heterogeneous reactor and listed are results of calculations, carried out by A.D. Galanin [65].

Chapter 28 was written by V.F. Turchin and represents a review of theoretical results on the scattering of slow neutrons in solid bodies and liquids. Most outstanding results in this direction were obtained by V.F. Turchin [276, 277], M.V. Kazarnovskiy and F.L. Shapiro [121, 123], M. Nelkin [210-212], Ye. Kogen [138, 139] and others. Discussed was the diffusion of neutrons by the isolated nucleus and combination of atoms. Great attention has been devoted to the case of incoherent scattering of neutrons. The central points of this chapter are paragraphs 28.5 and 28.6, in which is discussed the theory of diffusion of slow neutrons in crystals and in liquid. The results, formulated in these paragraphs, are of greater practical interest and are used in calculations.

In chapter 29 are discussed methods of multigroup solving equations for the spectrum of slow neutrons. An essential feature here is, that the mathematical apparatus, developed for this purpose by the author together with V.V. Smelov [177, 178, 254], is
not connected with the concrete selection of neutron diffusion functions and appears to be universal. On the basis of the theory of perturbations is formulated a single-group theory. Developed was a method for calculating the spectrum of slow neutrons in $P_3$-approximation of the method of spherical harmonics.

Finally, the eighth part of the book is devoted to critical masses of homogeneous reactors. This part consists of two chapters.

In chapter 30 are considered methods of physically calculating nuclear reactors; formulated are typical problems for the calculation of critical masses and spectra of neutrons; given are practical algorithms for the averaging of group constants; numerous problems were set up which take into consideration the use of small group approximations. In conclusion are discussed problems of calculating heterogeneous reactors and compensation systems. Practically speaking, this chapter has a majority of problems concerning physical calculation and considered are practical algorithms for their solution on the basis of results from previous chapters of the book.

In chapter 31 are presented results of calculating critical masses of reactors in a wide range of neutron spectra: from fast to thermal. The first effort to make a systematic generalization of results obtained from calculation of critical masses of homogeneous reactors was made by V.Ya.Pukhko [235, 236] and by T.N.Zubarev, G.I.Marchuk and A.K.Sokolov [115].

In report [115] are listed critical masses of uranium-beryllium, uranium-graphite, and uranium-water reactors, as well as of reactors, consisting of mixtures of $U^{235}$ and $Pt^{239}$, calculation of which was carried out by the author together with V.P.Kochergin, Ye.I.Pogudalina, V.V.Kolesov, G.A.Ilyasov and L.I.Kuznetsova [183-186].

In recent years these investigations have been repeated by the author together with V.P.Kochergin, A.I.Nevinitsa and O.P.Uznadze on the basis of using a more accurate 21-group system of constants. In addition, a greater number of new variants are considered, essential for the planning of reactors of variegated spectra.

In the appendix to the book are included certain data, useful for practical uti-
lization of methods, analyzed in the monography.
LITERATURE


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