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Final Design and Construction Details for a 6000 BTU/hr. Thermoelectric Air Conditioner

Approved by: J. T. Burwell, Jr.

Date: December 31, 1961

This report is submitted as fulfillment of the requirements of Contract No. DA-44-009-Eng-4643. The report constitutes the details of final design and fabrication as per Phase II of the contract.

American-Standard Corporation
Research Division
Monroe and Progress Streets
Union, New Jersey

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The completed thermoelectric air conditioner design and construction details presented in this report were made possible through the coordinated efforts of the following Scientists:

Heat exchanger design and fabrication: N. Kosowski and P. Renzi
Air system design and fabrication: E. Tillman
Thermoelectric design: E. R. Boyko
Thermoelement bonding and thermoelectric fabrication: S. Swietluk
Coordinating Scientist: R. C. Roxberry

This report represents the joint authorship of the above personnel.
1. SUMMARY

The design details of a 6000 Btu/hr thermoelectric air conditioner designed and built by the Research Division of American-Standard Corp. and delivered to the Engineering Research and Development Laboratory of the Army Corps of Engineers on September 21, 1961, are presented in this report. The performance specifications are summarized and the design calculations are also outlined.

At design operating conditions, this air conditioner should produce 6000 Btu/hr of cooling with an overall coefficient of performance of 0.44. The unit is equipped with a number of small blowers which circulate air over the fins attached to the cold thermoelectric junctions and a second set of blowers which circulate air over the fins attached to the hot thermoelectric junctions. The air conditioner weighs approximately 85 pounds and is designed to fit conveniently in the wall of the enclosure to be cooled.

This report also deals with the practical aspects of fabricating sound mechanical and electrical junctions between the thermoelectric elements and the bases of the fin blocks which serve to connect one thermoelectric element to the next. Satisfactory junctions were produced by electroplating the thermoelectric elements with nickel, capping the nickel with a lead-tin alloy and then oven soldering these pellets to the tinned base of the aluminum fin blocks with a solder wafer placed between the capped element and the tinned base. A previously used technique of capping with indium was found to be unsatisfactory since interdiffusion of indium and bismuth-telluride from the thermoelectric elements resulted in low bond strength and high contact resistance. The electrical contact resistance of a single junction is of the order of one micro-ohm and the joint can withstand a tensile load of at least 50 lb/cm². In order to achieve this mechanical strength, it was found
that the edges of the thermoelectric elements had to be rounded before electroplating, thus reducing stress concentration and increasing contact area at the edge of the plated interface.

As assembled pair of thermoelectric elements was tested to determine the figure of merit. The figure of merit was estimated from these tests to be $2.28 \times 10^{-3} \cdot \text{C}^{-1}$, whereas the figure of merit calculated from the manufacturer's data is $2.72 \times 10^{-3} \cdot \text{C}^{-1}$.

While it was understood that the air conditioner could not be performance tested in our laboratory because of lack of a specification test facility before delivery to the Army, the unit was operated and its cooling capacity and power input were determined approximately. Within the accuracy of the tests performed, this unit appears capable of delivering the required 6000 Btu/hr at the design ambient conditions with a coefficient of performance very close to the predicted value.
2. INTRODUCTION

The thermoelectric air conditioner described in this report was built by the Research Division of American-Standard Corp. for the Army Corps of Engineers' Research and Development Laboratory at Fort Belvoir, Virginia. This report describes in detail the research, development and design efforts conducted by American-Standard in connection with the fabrication of this first prototype thermoelectric air conditioner.

In a previous report, the general design concept for the thermoelectric air conditioner was presented. The concept described in that report has been retained and, with minor modifications, is the basis for the final design presented here.

The air conditioner is composed of four identical sections each of which supplies one-quarter of the total cooling requirement of 6000 Btu/hr. A total of four hundred thermoelement pairs are connected electrically in series. When a direct current of the proper polarity is applied, heat will be withdrawn from the air blown over the cold junctions and this heat, together with the heat equivalent of the power input, will be dissipated to the air blown over the hot junctions.

Since the original design concept was established, several changes were made which affected both fan selection and heat exchanger geometry. These changes were instituted to increase the coefficient of performance of the original design as requested by the cognizant

---

engineer, Mr. R. Mehalik. The revised design of the air system and heat exchange surfaces required to achieve an improved coefficient of performance is described in this report. These changes resulted in a slightly increased cost and weight of the unit.

The thermoelectric air conditioner was built in accordance with the design presented in this report with only minor dimensional differences between the design geometry and the final product. These differences were required by factors arising during fabrication. Final detail dimensions should be obtained from the detail drawings since these may differ slightly from those shown in the calculations in this report. The differences are not sufficient to warrant a revision of the calculations since they are well within the error associated with such analyses.
3. AIR CONDITIONER SPECIFICATIONS

The air conditioner specifications as noted in Ref. 1 have been revised. The design changes were made in order to improve the overall coefficient of performance. These new specifications are listed in Table I. The design changes have also increased the weight of the unit to a measured value of approximately 85 pounds.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specifications</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Cooling Capacity:</th>
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<tbody>
<tr>
<td>Net</td>
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<tr>
<td>Gross</td>
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</tbody>
</table>

<table>
<thead>
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<th>Power Input:</th>
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<tbody>
<tr>
<td>Thermoelectric Circuit</td>
</tr>
<tr>
<td>Fans</td>
</tr>
<tr>
<td></td>
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<td>Total</td>
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</table>

<table>
<thead>
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<th>Coefficient of Performance:</th>
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<td>Thermoelectric</td>
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<tr>
<td>Overall</td>
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</table>

<table>
<thead>
<tr>
<th>Air Circulated:</th>
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</thead>
<tbody>
<tr>
<td>Hot Side</td>
</tr>
<tr>
<td>Cold Side</td>
</tr>
</tbody>
</table>
Air Temperatures:

<table>
<thead>
<tr>
<th>Side</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Side Inlet</td>
<td>90°F</td>
</tr>
<tr>
<td>Hot Side Inlet</td>
<td>110°F</td>
</tr>
</tbody>
</table>

Thermoelectric Circuit Current: 95 amperes

4. HEAT TRANSFER ASPECTS

The hot and cold side heat exchangers consist of finned aluminum modules attached to each pair of thermoelectric elements. The modules are machined from solid aluminum blocks by cutting the spaces between fins with an assembly of milling cutters carefully spaced on the arbor of a horizontal milling machine. Although this procedure was found to be successful, quantity production would permit the use of more efficient techniques.

The details of these heat exchanger modules are shown in Figs. 2, 3, and 4. The complete heat transfer calculations for the prototype air conditioner are shown in Appendix I.

The air conditioner consists of four sections each containing one hundred pairs of thermoelectric elements. A detail of one quarter section showing the arrangement of hot and cold side exchanger modules is shown in Fig. 5. The exchangers serve to connect the thermoelectric elements electrically as well as to dissipate or absorb heat. Specially designed heat exchanger modules are required at the crossover points from one row of thermoelectric elements to the next. In all, three different styles are required: one for all the cold side modules, and two for the hot side modules.
In Appendix I, it is shown that the performance of the hot and cold side heat exchangers can be expressed as functions of the fin base temperature as follows:

\[ Q_c = A_1 - B_1 t_{BC} \]

\[ Q_H = -A_1 + B_1 t_{BH} \]

The factors A and B are constants for a particular exchanger configuration, inlet air temperature, and flow rate. They are respectively

\[ A = w_G C_p t_{in} \left[ 1 - \exp \left( \frac{-h_o A_{HT}}{w_G C_p} \right) \right] \]

\[ B = w_G C_p \left[ 1 - \exp \left( \frac{-h_o A_{HT}}{w_G C_p} \right) \right] \]

The pertinent parameters required to evaluate these constants are listed in Table II.

**TABLE II**

**HEAT EXCHANGER DESIGN PARAMETERS**

<table>
<thead>
<tr>
<th></th>
<th>Cold Side</th>
<th>Hot Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{HT}, \text{ ft}^2 )</td>
<td>58.3</td>
<td>82.3</td>
</tr>
<tr>
<td>( h_o, \text{ Btu/hr ft}^2 \text{ °F} )</td>
<td>10.82</td>
<td>13.4</td>
</tr>
<tr>
<td>( t_{in}, \text{ °F} )</td>
<td>92.5*</td>
<td>110.0</td>
</tr>
<tr>
<td>( W_G, \text{ lb/hr} )</td>
<td>2250.0</td>
<td>4240.0</td>
</tr>
</tbody>
</table>

*Based on 396 watts fan input and 90°F room air with fans placed between room air and heat exchangers.
The resulting heat flow equations using these parameters are:

\[ Q_c = 34,415.5 - 372.06 \ t_{BC} \ \text{(Btu/hr)} \]
\[ Q_H = -74,101.6 + 673.65 \ t_{BH} \ \text{(Btu/hr)} \]

Expressed in terms of heat flow in watts and base temperature in °K, these equations become:

\[ Q'_c = 60,181.67 - 196.28 \ T_{BC} \ \text{(watts)} \]
\[ Q'_H = -112,419.9 + 355.38 \ T_{BH} \ \text{(watts)} \]

In the derivation of the constants A and B it was assumed that all the heat exchanger modules of the air conditioner operate with the same \( T_{BC} \) and \( T_{BH} \). Although this is not actually the case, the correction expected for non-uniform base temperature can be shown to be small. In Appendix II of Ref. 1, an analysis of this effect was made. This analysis indicated that a heat exchanger having a segmented base should provide performance differing only slightly from the performance of a heat exchanger having an integral base construction.

5. AIR FLOW CONSIDERATIONS

The fans used to circulate air over the hot and cold side heat exchangers were selected considering weight and size as factors of major concern. The power requirement of these fans was also of importance although it could not take precedence over the size and weight considerations. In addition, it was decided that the fans should be commercially available.
at the present time. These considerations narrowed the choice to small, high speed fans of the vane-axial type.

The performance characteristics and specifications of the fans selected for this air conditioner are shown in Figs. 6 and 7. The assembly drawing, Fig. 1, shows eight fans for the hot side and twelve for the cold side. It should be emphasized that the fans especially designed for the particular application would probably be lighter and more efficient than the ones selected.

Because the air paths to the hot and cold side fins involve a number of duct transitions and turns, a careful analysis of the air system pressure losses was necessary. These calculations are shown in Appendix II.

At design conditions, to provide 6000 Btu/hr of cooling, the cold side of the unit will require a total of 520 CFM provided by twelve Rotron Aximax 2 Fans*. The total pressure loss through the cold side flow passages is calculated to be 1.59 inches of water not including a filter. At the design flow rate of 43.3 CFM per fan, the fans develop 1.70 in. of water pressure (see Fig. 6) which permits a filter to be added at the cold side inlet taking as much as .11 in. of water pressure drop.

A total of 1024 CFM of air flow is required on the hot side of the air conditioner. Eight Aximax 3 fans are provided - each delivering 128 CFM at a pressure of 2.56 in. of water (see Fig. 7). The system air resistance is calculated to be 2.55 in. of water which means that only .01 in. of water can be used through screen on the hot side air inlet.

The total power consumption of hot and cold side fans is calculated to be 1460 watts. This is made up of 396 watts for the cold side fans and 1064 watts for the hot side fans.

*Manufactured by Rotron Manufacturing Co., Inc., Woodstock, New York
In the following sections, the performance of the heat exchange and fan systems will be combined with the performance characteristics of the thermoelectric circuit to establish the operating current in the thermoelectric circuit which will produce the required 6000 Btu/hr of cooling. The required gross cooling capacity of the unit must be greater than 6000 Btu/hr (1760 watts) by the amount of heat put into the air in passing over the cold side fans and by the amount of heat conducted through the plastic insulation surrounding the thermoelectric elements. These two quantities are 396 watts and 24.3 watts respectively. Hence, the gross cooling provided should be 2180.3 watts to deliver the required one-half ton of net cooling.

6. ELECTRODE SPECIFICATIONS

The techniques used in the multi-step process of connecting the radiating fins to the thermoelements proper deserve close attention. The specifications for these connections (herein referred to as the electrode) are listed to clarify all the aspects and considerations given.

The requirements for an acceptable electrode are:

1. High mechanical strength (above 50 lbs/cm² in tension).

2. Low electrical resistance. (Below 3% of the total unit resistance, i.e. below 6 X 10⁻⁶ ohms.)

3. Good heat transfer in and out of the thermoelement junctions.

4. Electrode material must be chemically stable under operating temperatures and conditions. It must not form undesirable intermetallic compounds.
5. Formation of the junction must not lead to layers of depleted carrier density or intermediate layers of high resistance.

6. The expansion coefficients of electrode materials and the thermoelement must be comparable in order to avoid shearing stress during thermal cycling.

7. The interdiffusion of thermoelement and electrode materials should not cause a physically weak phase within the plane of the junction.

8. Ease of fabrication.

7. METHODS OF ELECTRODE FABRICATION

Several methods and techniques in fabricating electrodes were tried and evaluated:

I. Soldering of different metals and alloys to thermoelements using:

   a. Direct heat
   b. Reducing atmosphere
   c. Ultrasonic techniques

II. Electroplating of the thermoelements.
The following metals and alloys were used:

1. Tin - melting point 232°C  
2. Bismuth-tin - melting point 139°C (eutectic)  
3. Bismuth - melting point 271°C  
4. Lead-tin (60/40) - melting point 183°C (eutectic)  
5. Lead-Antimony - melting point 252°C (eutectic)  
6. Lead-Tin-Zinc - melting point below 183°C  
7. Tin-A-Lum* - melting point 210°C  
8. Indium - melting point 156°C  

Indium Capping

The bismuth-telluride thermoelectric units were purchased from the Cominco Products, Inc. The contract called for the units to be indium capped. It was found that the the capping melted at 97°C. The indium melting point is considered to be 156°C. This condition proved to be due to the diffusion of indium into the bismuth-telluride semi-conductor. The indium combines with Bi₂Te₃ to form an indium telluride alloy and free bismuth. The free Bi then combines with excess In to form an In-Bi alloy, the eutectic of which has a melting point below 100°C.

The problem of diffusion was more pronounced in the case of the N type material. This can be explained by the difference in the amount of impurities added to this type. Both the P and N type materials contain a certain amount of selenium and antimony. The amount of antimony in the P type (relative to N type) is greater, thus preventing the formation of Bi-In alloy. In view of these difficulties, it was agreed to abandon the indium capping of both the P and N type materials and find another method of producing an acceptable electrode.

*Tin-A-Lum - A patented aluminum solder manufactured by Production Metals, Inc., 299 Pavonia Avenue, Jersey City 2, New Jersey
Bismuth Capping

Out of the three methods noted above, Ia, Ib, and Ic, the reducing atmosphere method, Ib, (hydrogen flame) seemed to give the best results - especially in the case of pure bismuth. In using other metals, there is considerable difficulty experienced as far as the wetting of the thermoelectric material surface is concerned.

Several thermoelements were produced with bismuth caps using hydrogen flame as a heat source as well as a reducing agent. Different soldering fluxes were tried. The best results were obtained with the Kester No. 44 rosin flux. The bismuth capped units were tested for mechanical strength. In tension, the ultimate stress varied between 40 and 100 lbs/cm².

The electrical resistance of the joint was acceptable (less than 3% of the total unit resistance). The bismuth capping technique was nevertheless considered impractical from the standpoint of expedient production. Efforts were then made in the investigation of nickel electroplating technique.

Nickel Electroplating, Lead-Tin Capping

The nickel plating method lends itself easily to high quantity production. Several thermoelements were produced this way using a low plating current density of 5 ma/cm² with the following bath composition:

- 115 cc of water
- 2.3 grams Ammonium Chloride
- 11.7 grams Nickel Sulphate
- 17.0 grams Sodium Sulphate
- 2.3 grams Boric Acid
The semi-conductor button was prepared for plating by roughening the flatted surfaces. The surface should, in general, be sand blasted. But in view of our lack of this equipment, the surface was roughened with sandpaper. The corners of each thermoelement were also rounded in order to increase the mechanical bond strength of the nickel plating to the thermoelement. Each thermoelement was then capped with lead-tin solder (183°C eutectic) using Kester No. 44 rosin.

Several units of this type were tested electrically and mechanically. The results were very satisfactory. A few samples exhibited a tensile strength below 50 lbs/cm². This was attributed to improper surface preparation before electroplating.

At this stage, the results of investigations in this laboratory were transferred to Cominco Products, Inc. Subsequently, the electroplated and lead-tin capped samples were received directly from them. The samples were sand blasted with an S. S. White abrasive unit by Cominco before plating and then 10% of the batch were tested for electrical and mechanical parameters before being shipped to the American-Standard Company.

8. CONSTRUCTION OF A THERMOELECTRIC PAIR

Several experimental thermoelectric single units and pairs were constructed and tested. The following types of tests were performed:

1. Single units - contact resistance and mechanical strength measurement.

2. Pairs - junction pair figure of merit (Z_c) measurement.

*The Consolidated Mining and Smelting Company of Canada, Ltd., Montreal, Canada. (Electronic Materials Dept. - 933 West Third Ave., Spokane 4, Wash.)
Figures 9 and 10 show the curves taken during the figure of merit measurement. For maximum cooling to occur, the cold side temperature should be minimum, or

\[ \frac{dI}{dT} = 0 \]

where \( I \) = current applied to the pair (Amps)  
\( T \) = temperature of cold side (°K)

From Figure 9 we see that the minimum point has not been reached at \( I = 120 \) Amps.

By extrapolation of the curve, we may assume that the minimum point would occur at about 180 to 200 Amps with the minimum cold side temperature of about -40°C. At that time, the temperature difference (\( \Delta T_m \)) between cold and hot sides would be 62°C.

From this value, the figure of merit of the thermoelectric pair can be calculated:

\[ \Delta T_m = \frac{1}{2} Z_c T_c^4 \]

\[ Z_c = \frac{2 \Delta T_m}{T_c^2} \]

where \( T_c \) = temperature of cold side in °K  
\( Z_c = 2.28 \times 10^{-3} \text{°C}^{-1} \)
According to the data given by Cominco Products, Inc. on the thermo-electric material, the Z value of the material is calculated as follows:

\[ Z = \frac{S_n^2}{\rho K_T} \quad \text{(According to unpublished report 9-61-1, by Dr. D. H. Howling)} \]

where

- \( S_n \) = N type material Seebeck coefficient (V°C⁻¹)
- \( \rho \) = Resistivity (ohm-cm)
- \( K_T \) = Thermal conductivity (Watt cm⁻¹°C⁻¹)

Thus

\[ Z = \frac{(190 \times 10^{-6})^2}{8 \times 10^{-4} \times 1.68 \times 10^{-2}} \]

\[ Z = 2.72 \times 10^{-3} \text{C}^{-1} \]

The experimental value of \( Z \) is reasonable considering the measurement accuracies and the accuracies of the quantities given by Cominco.

The tolerances as given by Cominco are:

- \( S_n \) ± 5%
- \( K_T \) ± 5%
- \( \rho \) ± 3%
Figure 11 shows the curve taken during the contact resistance measurement. The measurement was performed by axially scanning a needle probe across the periphery of the contact made between an aluminum electrode and the thermoelectric material. The zero to ten mills distance on the curve represents the resistance of the contact. Note that the contact resistance is of the order of one (1) micro ohm.

Thus \[ R_c = 1 \times 10^{-6} \text{ ohms} \]

According to the design requirement,

\[ R_c \leq 6 \times 10^{-6} \text{ ohms} \]

The curve extending between 10 and 220 mills represents the resistance of the thermoelectric material.

9. COOLING FINS PREPARATION

The base surface of the aluminum cooling fins is prepared as follows:

1. The fins are heated up to about 220°C.
2. The surface is then covered with Tin-A-Lum.
3. A thin layer of lead-tin eutectic is deposited on top of the Tin-A-Lum.
4. After cooling, the surface is leveled on the sander machine.
10. TECHNIQUE OF THE FINAL ASSEMBLY

There are 400 thermoelectric pairs in the air conditioner. The unit is divided into four thermoelectric quadrants, each quadrant consisting of 100 pairs of thermoelements. A quadrant consists of 10 strips, each strip containing ten thermoelectric pairs. Figure 12 shows one assembled quadrant and Fig. 14 shows all four quadrants assembled in the final unit.

The cooling fins are soldered to the thermoelements in single strip fashion (10 pairs at one time). A special jig was made for this purpose. The strip itself is made of laminated thermosetting plastic material which is a woven glass fabric base laminate bonded with melamine resin. There are 20 holes in the strip to accommodate 20 thermoelements (10 pairs).

The assembly process in fabricating a single strip is as follows:

1. Place the plastic strip into the jig.
2. Alternately place the capped N and P type thermoelements into the strip holes (refer Fig. 5).
3. Cover the capping of the thermoelements with Kester No. 44 rosin flux.
4. Place two wafers of indium-tin eutectic (each of 0.625 inches diameter and 0.004 inches thick) on each side of the thermoelements.
5. Cover the cooling fins' solderable area (prepared as previously described) with Kester No. 44 rosin flux.
6. Place the fins into the jig.
7. Assemble the jig and place into a pre-heated 130°C oven for a period of one hour. (Maintain oven temperature at 130°C.)
The melting point of the indium-tin wafers is approximately 117°C. An oven temperature of 130°C is sufficient to melt the wafer. The lead-tin capping (183°C eutectic) of the thermoelements will obviously not remelt at this oven temperature. Figure 15 shows one strip assembled in the jig with cover removed. Ten strips prepared this way are then assembled into one quadrant as shown in Figs. 12 and 13. Figures 16 and 17 show the hot side of the complete unit with fans. Figure 18 shows the cold side with covers on, exposing the blowers only.

11. FAN SPECIFICATIONS

The fans were purchased from Rotron Manufacturing Company, Inc., Woodstock, New York. The design performance curves for the fans are shown in Figs. 6 and 7.

The air conditioner utilizes 8 hot side fans and 12 cold side fans. The specifications are listed in Table III.

**TABLE III**

**HOT SIDE FANS' SPECIFICATIONS**

<table>
<thead>
<tr>
<th>Motor Type</th>
<th>Volts</th>
<th>Phase</th>
<th>CPS</th>
<th>RPM</th>
<th>Full Load Watts</th>
<th>Line Amps</th>
<th>Max. CFM</th>
</tr>
</thead>
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<td>341 QS</td>
<td>200</td>
<td>3</td>
<td>400</td>
<td>22,000</td>
<td>133</td>
<td>0.41</td>
<td>162</td>
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**COLD SIDE FANS' SPECIFICATIONS**

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<th>Volts</th>
<th>Phase</th>
<th>CPS</th>
<th>RPM</th>
<th>Full Load Watts</th>
<th>Line Amps</th>
<th>Max. CFM</th>
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</thead>
<tbody>
<tr>
<td>35 QA</td>
<td>200</td>
<td>3</td>
<td>400</td>
<td>20,000</td>
<td>33</td>
<td>0.14</td>
<td>60</td>
</tr>
</tbody>
</table>
12. PERFORMANCE CALCULATIONS

The thermoelectric and heat exchanger parameters specified for this design permit solution of the design equations for the equilibrium base temperature $T_{BC}$ and $T_{BH}$ as functions of $I$. The parameters $Q_c$, $E$, and $V$ are subsequently determined. The tabulations in Table IV list these quantities for values of $I$ ranging from 40 to 140 amperes. The tolerance of these calculations is considered to be $\pm 10\%$.

**TABLE IV**

<table>
<thead>
<tr>
<th>I</th>
<th>$Q'_c$</th>
<th>$E$</th>
<th>$V$</th>
<th>$T_{BC}$</th>
<th>$T_{BH}$</th>
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<tr>
<td>40</td>
<td>1012</td>
<td>2.12</td>
<td>11.9</td>
<td>301.5</td>
<td>320.5</td>
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<td>50</td>
<td>1279</td>
<td>1.75</td>
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<td>38.3</td>
<td>293.0</td>
<td>338.9</td>
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</table>

An inspection of the Table shows that the $Q'_c$ design specification of 2180 watts will be satisfied at approximately $I = 95$ amperes. The overall coefficient of performance is obtained from the ratio of net cooling to total input power. At 95 amperes we have:
Power input to thermoelectric section = 95 Amps. X 26.5 Volts = 2518 watts
Power input to fans = 12 X 33 = 396 watts
= 8 X 133 = 1064 watts
Total Power Input

Overall C. O. P. = \frac{\text{Net Cooling}}{\text{Total Input Power}} = \frac{1760 \text{ watts}}{3978 \text{ watts}} = 0.44

At the specified inside ambient conditions of 90°F (D. B.) and 75°F (W. B.), the dew point is found from an ASHRAE psychrometric chart to be 69°F. With operating current of 95 amperes, the $T_{BH}$ is 295.4°F or 72.4°F. Condensation cannot occur under these conditions. The sensible heat factor is therefore 100%.

13. FINAL REMARKS

The completed air conditioner was operationally tested at the laboratory. The D. C. power supply used was of our own design. The ripple was limited under load to less than 5%. The power factor for the total fan load is approximately 0.83 lagging. A 10 microfarad capacitor per phase correct the power factor to unity.

The rudimentary test results indicated that the air conditioner's performance was very satisfactory. An agreement was made with Mr. Mehalik to test and evaluate the performance of the unit by Ft. Belvoir personnel under properly controlled conditions. Mr. Mehalik also agreed to furnish American-Standard with the experimental data obtained from these tests.
14. NOMENCLATURE

A' Cross sectional area of thermoelements, sq in. or sq cm
A_{FR} Free flow frontal area, sq in.
A_{L} Heat transfer surface per unit length, sq ft/ft
A_{HT} Total heat transfer surface, sq ft
C_{p} Specific heat, Btu/lb°F
\(d\) Equivalent diameter of flow passage, in., ft
E Thermoelectric coefficient of performance, dimensionless
f Fluid flow friction factor, dimensionless
G Mass flow rate, lb/sec sq ft
A_{H} Gravitational acceleration
h Heat transfer coefficient, Btu/hr sq ft *°F
\(h_{o}\) for overall coefficient including fin efficiency, \(h_{o} = h_{B} + h_{B}^{*}\)
\(h_{B}\) for pure or uncorrected coefficient
I Current flow, amperes
j Heat transfer factor, \(j = N_{ST} N_{PR}^{L}\)
k Thermal conductivity, Btu/hr ft °F or watts/cm °K
K_{c} Entrance pressure loss coefficient
K_{e} Exit pressure loss coefficient
K Thermal conductance, Btu/hr °F or watts °K
L Length of thermoelements, in. or cm
LMTD Log mean temperature difference, °F
m Fin efficiency parameter, \(m = (H_{B}/ky_{o})\) dimensionless
N_{PR} Prandtl number, \((C_{p} \mu/k)\) dimensionless
N_{ST} Stanton number, \((h/GC_{p})\), dimensionless
N_{RE} Reynolds' number, \((Gd/\mu)\), dimensionless
NTU Number of heat transfer units, \((A_{HT} h_{o}/w_{G} C_{p})\), dimensionless
N Number of thermoelectric pairs
\( \text{P}_w \) - Wetted perimeter, in.
\( Q \) - Heat flow, Btu/hr; \( Q' \) heat flow, watts
\( Q_c \) or \( Q'_c \) - cold side heat flow
\( Q_H \) or \( Q'_H \) - hot side heat flow
\( Q_{AH'}, Q_{BH'}, Q_{CH} \) - three cases for hot side (A, B, and C)
\( R \) - Sum of electrical resistance of thermoelectric elements, ohms
\( S \) - Thermoelectric power, volts/°K
\( t \) - Temperature, °F
\( t_{in} \) for entering air temperature, °F
\( t_o \) for leaving air temperature, °F
\( t_B \) for fin base temperature, °F
\( T \) - Temperature, °K
\( T_{BH} \) for hot side fin base temperature, °K
\( T_{BC} \) for cold side fin base temperature, °K
\( T_{in} \) for inlet air temperature, °F
\( T_{in} \) for inlet air temperature, °K
\( T_{e} \) for cold side fin base temperature at inlet end, °K
\( V \) - Voltage drop across thermoelectric section, volts.
\( w_G \) - Weight flow, lb/hr
\( X \) - Fin length along flow direction, in. or cm
\( y_o \) - Half of fin thickness, in.
\( Z \) - Thermoelectric figure of merit, °K⁻¹; \( Z = S^2/RK \)
\( \delta \) - Ratio of free flow frontal area to total frontal area
\( \rho \) - Air density, lb/cu ft or resistivity, ohm - centimeters
\( \mu \) - Air viscosity, lb/ft sec
\( \eta \) - Fin efficiency, \( \eta = \frac{\tan h}{m} \)
\( \Delta \) - Pressure drop psi or in. H₂O
\( \Delta p_F \) - For pressure drop through length of fin
\( \Delta p_E \) - For pressure drop due to exit and entrance effects
\( \Delta p_T \) - For total pressure drop
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FIG. 6

MANUFACTURER  ROTRON MANUFACTURING COMPANY INC.
WOODSTOCK, NEW YORK

MODEL     AXIMAX 2
MOTOR SERIES  367 QS
VOLTS       200
PHASE       3
CYCLES      400
R.P.M.      20100
FIG. 7

MANUFACTURER: ROTRON MANUFACTURING COMPANY INC.
WOODSTOCK, NEW YORK

MODEL: AXI-MAX 3
MOTOR SERIES: 341QS
VOLTS: 200
PHASE: 3
CYCLES: 400
R.P.M.: 22000
SINGLE PAIR TEMPERATURE CHANGE AS A FUNCTION OF THE APPLIED CURRENT

**FIG. 9**

![Diagram showing temperature change as a function of current applied to a single pair.](image-url)

- Temperature in °C
- Current in Amperes
TEMPERATURE CHANGE AS A FUNCTION OF CURRENT FOR A SINGLE PAIR

**Fig. 10**

Graph showing the temperature difference between hot and cold side as a function of current. The x-axis represents current in amperes (0 to 120) and the y-axis represents temperature difference ($\Delta T$) in °C (0 to 70).
CONTACT RESISTANCE MEASUREMENT

CONTACT BETWEEN 0 AND 10 MILS
THERMOCOUPLE BETWEEN 10 AND 220 MILLS

FIG. 11

DISTANCE ALONG THE THERMOCOUPLE UNIT

RESISTANCE

1,000 
100 
10 
1
**Figure 12**
Assembled Thermoelectric Quarter Section (Top View)

**Figure 13**
Assembled Thermoelectric Quarter Section (Isometric View)
Figure 14
Assembly of All Thermoelectric Quarter Sections.

Figure 15
Assembly of a Single Thermoelectric Strip in the Fabricating Jig (Cover Removed)
**Figure 16**

*Assembly - Looking at Hot Side - Fans Installed (No Panel Covers)*

**Figure 17**

*Assembly - Looking at Hot Side - Two Panel Covers Installed*
Figure 18
Assembly - Looking at Cold Side - Panel Covers Installed
Fig. 22

COLD AIR OUT

HEAT TRANSFER FINS
INSULATION COVER
FIN COVER
FANS

TURNING VANES
PLENUM
HOT SIDE GRILL
MOUNTING WALL PARTITION

WARM AIR IN
FIN COVER

THermoelectric air conditioner
Cutaway view (cold side)
16. APPENDIX I

HEAT EXCHANGER CALCULATIONS

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**PROJECT 24**

Heat Exchanger Design and Calculations

1. **Basic Heat Exchanger Equations**

For air passing through heat exchanger:

\[\omega g c_p \frac{dt}{dx} = h_0 \frac{A_{HR}}{L} (t_B - t) dx; \quad (1)\]

or

\[\frac{dt}{dx} = \frac{h_0 A_{HR}}{\omega g c_p L} (t_B - t); \quad (2)\]

Since by definition

\[NTU = \frac{h_0 A_{HR}}{\omega g c_p} \quad (3)\]

\[\frac{dt}{dx} = \frac{NTU}{L} (t_B - t) \quad (4)\]

Integrating, with the boundary condition:

\[x = 0; \quad (t_B - t) = (t_B - t_{IN}); \quad (6)\]

the following equation is obtained:

\[-\ln (t_B - t) = NTU x + \ln C \quad (7)\]

which becomes:

\[\text{See Nomenclature at the end of the report}\]
\[ t_B - t = (t_B - t_{IN}) e^{-NTUV} \]  \hspace{1cm} (8)

which for \[ x=L \] gives
\[ t_B - t_0 = (t_B - t_{IN}) e^{-NTUV} \]  \hspace{1cm} (9)

and
\[ t_0 = t_B - (t_B - t_{IN}) e^{-NTUV} \]  \hspace{1cm} (10)

Substituting (10) into the equation for total heat:
\[ Q = \omega_G c_p (t_0 - t_{IN}) \]  \hspace{1cm} (11)

The following is obtained:
\[ Q = \omega_G c_p \left[ t_B - (t_B - t_{IN}) e^{-NTUV} - t_{IN} \right] \]  \hspace{1cm} (12)

or
\[ Q = \omega_G c_p (t_B - t_{IN}) (1 - e^{-NTUV}) \]  \hspace{1cm} (13)

And since by convention for this particular calculation heat \( Q \) should be positive when heat is removed or added:
\[ Q = \omega_G c_p \left[ t_B - t_{IN} \right] (1 - e^{-\frac{A_{hr} h_0}{\omega_G c_p}}) \]  \hspace{1cm} (14)
Another way of deriving the same equation follows for the cold side:

\[ Q_c = \omega g c_p \left( t_{in} - t_o \right) = A_{HT} h_o \ln \frac{t_{in} - t_o}{t_{in} - t_B} \]

But \[ \Delta t = \frac{Q_c}{\omega g c_p} \]

\[ \ln \frac{t_{in} - t_B}{t_{in} - t_B} = \frac{A_{HT} h_o}{\omega g c_p} = NTU \]

and

\[ t_{in} - t_B = \left( t_{in} - \frac{Q_c}{\omega g c_p} - t_B \right) e^{NTU} \]

\[ \left( t_{in} - t_B \right) e^{-NTU} = t_{in} - \frac{Q_c}{\omega g c_p} - t_B \]

\[ \frac{Q_c}{\omega g c_p} = \left( t_{in} - t_B \right) \left(1 - e^{-NTU}\right) \]

and

\[ Q_c = \omega g c_p \left( t_{in} - t_B \right) \left(1 - e^{-NTU}\right) \]

analogously for the hot side.

\[ Q_h = A_{HT} h_o \frac{t_o - t_{in}}{\ln \frac{t_o - t_{in}}{t_B - t_{in} + \Delta t}} \]
\[(t_B - t_{IN}) = (t_B - t_{IN} + \frac{Q_{in}}{\dot{m}\cdot Cp}) e^{-NTU} \quad (24)\]

since \[t_B - t_{IN} = \frac{Q_{in}}{\dot{m}\cdot Cp} \quad (25)\]

simplifying (24) and putting in an explicit form, the following is obtained:

\[Q_{sh} = \dot{m}\cdot Cp (t_B - t_{IN}) (1 - e^{-NTU}) \quad (26)\]

In accordance with the convention that both \(Q_{sh}\) and \(Q_c\) be positive, Equation (22) and (26) can be combined into a single one:

\[Q = \dot{m}\cdot Cp (t_B - t_{IN}) (1 - e^{-\frac{A_{HT}\cdot h_0}{\dot{m}\cdot Cp}}) \quad (24)\]

In the following calculations:
- \(Q\) is the dependent variable
- \(\dot{m}\) is fixed
- \(Cp\) is a constant
- \(t_B\) is the dependent variable
- \(t_{IN}\) is fixed
- \(A_{HT}\) is fixed and determined by the selected geometry
- \(h_0\) is determined as follows:
\[ h_0 = \eta h_b \quad (27) \]

\[ h_b = N_{st} G c_p \quad (28) \]

\[ G = \frac{\omega_a}{A_{FR}} \quad (29) \]

\( A_{FR} \) is fixed by the geometry of the exchanger.

\( N_{st} \) is determined from

\[ \tau^* = N_{st} (\frac{h}{c_p G})^{2/3} N_{pr}^{2/3} \quad (30) \]

where for any \( N_{re} \)

\[ \tau^* = \frac{0.60}{\sqrt{x \cdot N_{re}}} \quad (31) \]

for \( \frac{x}{d} \leq 3.5 \)

\[ \tau^* = \frac{0.60}{\sqrt{3.5 \cdot N_{re}}} \quad (32) \]

for \( \frac{x}{d} > 3.5 \)

---

*Taken from NACA TECHNICAL NOTE 2237 by S.V. Mihson, "Correlations of Heat Transfer Data for Interrupted Plate Fins Staggered in Successive Rows."
\[ d = \frac{4A_{PE}}{P_W} \]  
\[ N_{Re} = \frac{Gd}{\mu} \]  
\[ \eta = \frac{\tan h m}{m} \]  

where
\[ m^2 = \frac{H^2 2h_b}{k 2y_0} \]

The total pressure drop through the heat exchanger consists of the exit and entrance losses, and of the straight section loss. For the straight section loss,

\[ \Delta P_s = \frac{4f G^2 L}{32gd^4} \]  

where*

\[ f = \frac{11.8}{\frac{x}{d} N_{RE}^{0.67}} \quad \text{for} \quad \frac{x}{d} \leq 3.5 \]  
\[ f = \frac{11.8}{3.5 N_{RE}^{0.67}} \quad \text{for} \quad \frac{x}{d} > 3.5 \]  
\[ f = \frac{0.38}{\frac{x}{d} N_{RE}^{0.24}} \quad \text{for} \quad \frac{x}{d} \leq 3.5 \]  
\[ f = \frac{0.38}{3.5 N_{RE}^{0.24}} \quad \text{for} \quad \frac{x}{d} > 3.5 \]

*(See footnote on page 5*
Entrance and exit effects are given by:

\[ \Delta P_E = \frac{G^2}{2g_s \rho_{en}} \left[ (K_c + 1 - \delta^2) - (1 - \delta^2 - K_e) \frac{\rho_{en}}{\rho_{ex}} \right] \]  

(39)*

Where \( K_c \) and \( K_e \) are determined from charts in W.M. Kays & A.L. London's "Compact Heat Exchangers" p. 46 Fig. 20.

\( K_c \) & \( K_e \) vs. \( \delta \) & \( N_{Re} \) where \( \delta = \frac{A_{fr}}{A_{total}} \) (40)

Since \( \frac{\rho_{en}}{\rho_{ex}} \approx 1 \), equation (39) may be reduced to

\[ \Delta P_E = \frac{G^2}{2g_s \rho_{en}} [K_c + K_e] \]  

(41)

And finally

\[ \Delta P_T = \Delta P_F + \Delta P_E \]  

(42)

*Taken from W.M. Kays & A.L. London's "Compact Heat Exchangers" p. 43 and 21
II. Cold Side Heat Exchanger

Air 520 CFM at 90°F & 14.7 psi

\[ \dot{W}_G = 520\ \text{CFM} \times 0.072\ \text{lb}_m\ \text{cu ft} \times 60\ \text{min}\ \text{hr} = 2250\ \text{lb}_m\ \text{hr} \]

\[ C_p = 0.24\ \text{Btu/lb} \cdot \text{°F} \]

\[ t_{in} = 90°F \]

The fans are positioned in such a manner that the entering air passes over them before flowing into the heat exchanger.

The air absorbs the 396 Watts fan heat output and its temperature is raised:

\[ t_{in} = 90°F + \frac{396\times8.41}{0.24 \times 2250} = 92.5°F \]

\[ A_{HT} = 4 \times \frac{100}{144} \times [0.100'' \times 5 \times 1.3 + 0.050 \times 1.3 + 1.3'' \times 1.3'' \times 2 \times 6] = 5.83\ \text{sq. ft} \]

\[ A_{FR} = 4 \times 7.9 \times 1.3 - 4 \times 10 \times 6 \times 0.025 \times 1.3 = 41.1 - 7.8 = 33.3\ \text{sq. in.} \]

\[ G = \frac{2250 \times 144}{33.3 \times 3600} = 2.70\ \text{lbs/sec ft}^2 \]

\[ d = \frac{4 \times 33.3}{4 \times [7.9 - 10 \times 6 \times 0.025 + 2 \times 10 \times 6 \times 1.3]} = 0.205\ \text{in.} \]

\[ N_{Re} = \frac{0.205 \times 2.70 \times 3600}{12 \times 0.046} = 3610 \]

\[ \frac{\alpha}{\alpha_i} = \frac{1.3}{0.205} = 6.35 \]

\[ \frac{A}{(3.5 \times 3610)^{\frac{1}{2}}} = 5.32 \times 10^{-3} \]

*See Fan Selection Calculations*
\[ H_{ST} = \frac{5.32 \times 10^{-3}}{(0.70)^{2/3}} = 6.75 \times 10^{-3} \]

\[ H_B = 6.75 \times 10^{-3} \times 2.70 \times 3600 \times 0.24 = 15.75 \text{ Btu/hr sq ft of} \]

\[ m^2 = \frac{(1.3)^2 \times 2 \times 15.75}{118 \times 12 \times 0.025} = 1.50 \]

\[ m = 1.224 \]

\[ \tan \theta m = 0.841 \]

\[ \eta = 0.687 \]

\[ H_0 = 15.75 \times 0.698 = 10.82 \text{ Btu/hr sq ft of} \]

\[ Q_c = 2.250 \times 0.24 \left( 92.5 - t_B \right) \left( 1 - e^{-\frac{58.3 \times 10^{-3}}{2250 \times 0.24}} \right) \]

\[ Q_c = 34,415.55 - 372,060 \text{ t} \]

\[ f = \frac{0.38}{3.5 \times (3610)^{0.24}} = \frac{0.38}{3.5 \times 7.15} = 0.0152 \]

\[ \Delta P_f = \frac{4 \times 0.0152}{144} \times \frac{13.9}{0.205} \times \frac{(2.7)^2}{0.072 \times 64.1 \times 62.4} = 1.25'' \text{ H}_2\text{O} \]

\[ \delta = \frac{33.3}{4 \times 7.9 \times 1.3} = 0.81 \quad K_e = -0.08 \quad K_c = 0.3 \]

\[ \Delta P_e = \frac{(2.70)^2}{0.072 \times 64.1 \times 144} \times [0.3 - 0.08] \times \frac{1728}{G24} = 0.067'' \text{ H}_2\text{O} \]

\[ \Delta P_T = 1.32'' \text{ H}_2\text{O} \]
III. Hot Side Heat Exchanger

Air: 128 CFM/fan \times 8\text{ fans};

\omega_a = (128 \times 8) \text{ CFM} \times 0.069 \text{ lb} / \text{cu ft} \times 60 \text{ min/hr} = 4240 \text{ lb/hr}

\text{t}_{IN} = 110^\circ \text{F}

A_{HT} = \left(9 \times 10 \times 4 \times 0.70'' \times \left[0.057'' \times 15 + 
+ 0.018'' + 0.059'' + 16 \times 2 \times 1.3''\right] + 
+ 9 \times 4 \left[0.067'' \times 7 + 0.047'' \times 8 \times 1.3''\right] \times \frac{1}{50}'\right)^2

A_{HT} = 82.3 \text{ sq ft}

A_{FR} = \left[4 \times 13.9'' - 4 \times 9 \times 16 \times 0.023'' - 
- 4 \times 2 \times 8 \times 0.023'' \right] \times 1.3'' = 53.14 \text{ sq in}

G = \frac{4240 \times 144}{53.14 \times 3600} = 3.18 \text{ lb/s sec sq ft}

d = \frac{4 \times 53.14}{40.88 + 2 \times 10 \times 16 \times 4 \times 1.3''} = 0.124 \text{ in}

H_{RE} = \frac{0.124 \times 3.18 \times 3600}{12 \times 0.048} = 2.470

x = \frac{0.70}{0.125} = 5.6

\delta = \frac{0.60}{(3.5 \times 2470)^{1/2}} = 6.45 \times 10^{-3}

H_{ST} = \frac{6.45 \times 10^{-3}}{(0.70)^{2/3}} = 8.19 \times 10^{-3}

H_B = 8.19 \times 10^{-3} \times 3.18 \times 3600 \times 0.24 = 22.5 \text{ B/ hr sq ft °F}
\[ m^2 = \frac{1.69 \times 2 \times 22.5}{118 \times 12 \times 0.023} = 2.34 \]

\[ m = 1.53 \]

\[ \tan h m = 0.9104 \]

\[ h = 0.596 \]

\[ h_0 = 0.596 \times 22.5 = 13.40 \text{ Btu/hr \cdot ft} \]

\[ Q_{\text{h}} = 4240 \times 0.240 \times (t_B - 110^\circ) \left( 1 - e^{-\frac{82.3 \times 13.4}{4240 \times 0.24}} \right) \]

\[ Q_{\text{ch}} = 673.651 \text{ t}_B - 74,101.610 \]

\[ f = \frac{11.58}{3.5} \left( \frac{2470}{0.61} \right)^{0.6} = 0.0178 \]

\[ \Delta P_c = \frac{0.0178 \times 4}{144} \times \frac{7.9}{0.124} \times \frac{(3.18)^2}{0.069 \times 64.9} \times \frac{1728}{62.9} = 1.985 \text{ in H}_2\text{O} \]

\[ \delta = \frac{53.1}{72.3} = 0.73 \]

\[ K_c = -0.05 \quad K_e = 0.35 \]

\[ \Delta P_e = \frac{(3.18)^2}{0.069 \times 64.4 \times 144} \left( 0.35 - 0.05 \right) \times \frac{1728}{62.9} = 0.131 \text{ in H}_2\text{O} \]

\[ \Delta P = 2.116'' \text{ H}_2\text{O} \]
IV. Weight of Heat Exchangers

Cold Side Heat Exchanger

Fins: \(400 \times 6 \times 1.3'' \times 1.3'' \times 0.025'' \times 0.0975 = 9.90 \text{ lbs}\)

Plate: \(1.3'' \times 0.7'' \times 400 \times \frac{1}{16} \times 0.0975 = 2.22 \text{ lbs}\)

Total: \(12.12 \text{ lbs}\)

Hot Side Heat Exchanger

Fins: \([360 \times 0.70'' \times 1.3'' \times 16 \times 0.023'' + 36 \times 8 \times 1.50 \times 1.3'' \times 0.023''] \times 0.0975 = 13.0 \text{ lbs}\)

Plate: \([360 \times 0.70 \times 1.3 + 36 \times 0.70 \times 1.50] \times \frac{1}{16} \times 0.0975 = 2.23 \text{ lbs}\)

Total: \(15.23 \text{ lbs}\)

Total Weight of Heat Exchangers

\(12.12 + 15.23 = 27.35 \text{ lbs}\)
Conversion of Equations into Watts & K

In Btu/hr & °F

\[ Q_c = 34,415.55 - 372.060 T_{80°F} \]

\[ Q_h = -74,101.610 + 673.651 T_{80°F} \]

1 Watt = 3.412 B/hr; \( t^o = \frac{(T_{OK} - 273) \times 1.8}{3.412} + 32 \)

The general equation form is

\[ Q = A' + B't \]

\[ \frac{Q}{3.412 \text{ Watts}} = \frac{A'}{3.412} + \frac{B'}{3.412} \left[ \frac{(T_{OK} - 273) \times 1.8}{3.412} + 32 \right] \]

The equation form in Watts & °K is

\[ Q = A + B T_{80K} \]

Where

\[ A = \frac{A'}{3.412} - \frac{B' \times 273 \times 1.8}{3.412} + \frac{B' \times 32}{3.412} = \]

\[ = \frac{A'}{3.412} - 134.6424 B' \]

\[ B = B' \frac{1.8}{3.41} = 0.52754 B' \]

\[ Q_c + 10,086.62 + 50,045.05 \]

\[ 60,131.67 \]

\[ Q_h - 21,717.94 - 90,701.99 \]

\[ -112,419.93 \]

\[ Q_c = 60,131.67 - 196,2765 T_{80K} \]

\[ Q_h = -112,419.93 + 355,3778 T_{80K} \]
Heat Transfer through Plate Separating the Hot and Cold Side Heat Exchangers

Overall Plate Dimensions: 13.9" x 7.9"

\[ t_{BH} = (330.7^\circ K - 273^\circ) \times 1.8 + 32^\circ = 57.7 \times 1.8 + 32 = 135.9^\circ F \]
\[ t_{BC} = (295.7^\circ K - 273^\circ) \times 1.8 + 32^\circ = 22.7 \times 1.8 + 32 = 72.9^\circ F \]
\[ \Delta t = t_{BH} - t_{CH} = 135.9 - 72.9 = 63.0^\circ F \]
\[ A_{HT} = \frac{1}{144} \times [13.9 \times 7.9 - \frac{\pi}{4} \left( \frac{1.5 \text{ cm} \times 1 \text{ in}}{2.54 \text{ cm}} \right)^2] \times 200 = \frac{1}{144} \times [110 - 54.6] = \frac{4 \times 55.4}{144} = 1.53 \text{ sq ft} \]
\[ K = 7.0 \times 10^{-4} \text{ Cal} - \text{cm} \text{ sq cm} \times \text{sec} ^\circ C = 7.0 \times 10^{-4} \times 241.9 = 0.169 \text{ Btu-ft/hr-sq ft x}^\circ F \]
\[ Q \text{ Btu/hr} = \frac{K}{x} \times A_{HR} \times \Delta t = \frac{0.169 \times 2.54 \text{ cm}}{2 \times 0.5 \text{ cm} \times 1 \text{ in}} \times 1.53 \times 63 = 82.9 \text{ Btu/hr} \]
\[ 1 \text{ Watt} = 3.412 \text{ Btu/hr} \]
\[ Q \text{ Watts} = 24.3 \]

Heat loss through the plate section not covered by heat exchangers was not computed. Appropriate insulation can be used here to minimize heat flow. For design purposes a total heat flow of 30 watts will be used.
VII. Drawings

FIG. 1 Cold Side Heat Exchanger Element  p. I-16

FIG. 2 Hot Side Heat Exchanger Main Element  p. I-17

FIG. 3 Hot Side Heat Exchanger Crossover Element  p. I-18

FIG. 4 Cold Side Heat Exchanger Assembly  p. I-19

FIG. 5 Hot Side Heat Exchanger Assembly  p. I-19

FIG. 6 Top View of Hot Side Heat Exchanger: Diagram of Electrical Connections p. I-20
FIG.1 COLD SIDE HEAT EXCHANGER ELEMENT

Not to scale

Material: Aluminum
Quantity: 400 elements required
5 x 0.125" + 0.025" + 0.050" = 0.7" 6 fins/element  NK 11-15-60
FIG. 2 HOT SIDE HEAT EXCHANGER MAIN ELEMENT

Material: Aluminum
Quantity: 360 elements required

16 x 0.08" + 0.023" + 0.059" + 0.018" = 1.3"
16 fins/element

NK 3-15-60
FIG. 3 HOT SIDE HEAT EXCHANGER CROSSOVER ELEMENT

Not to scale

Material: Aluminium
Quantity: 36 elements required
7 x 0.09 + 0.023 + 0.047 = 0.7"
8 fins/element
FIG 4  COLD SIDE HEAT EXCHANGER ASSEMBLY

Not to scale: Four such plates required

DIMENSIONS: All main elements: 1.3" x 0.7"
All crossover elements: 1.5" x 0.7"
All intervals between vertical rows: 0.08"
All intervals between horizontal rows: 0.10"

FIG 5  HOT SIDE HEAT EXCHANGER ASSEMBLY

Not to scale: Four plates in module  NK 11-15-60
DOTTED LINE REPR. ELECTRICAL CONNECTION

TO ELECTRICAL OUTLET

FIG. 6 TOP VIEW OF HOT SIDE HEAT EXCHANGER

DIAGRAM OF ELECTRICAL CONNECTIONS OF FOUR PLATE ASSEMBLY
## APPENDIX II

**FAN CALCULATIONS**

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Calculation of Air Pressure Drop

(1) Cold Side

Side view of duct
Total air flow 520 cfm from 12 fans
Power required, 33 watts/ton
The total pressure drop may be broken down as follows for the duct system.

I Expansion - fan to elbow  \( A_1 \) to \( A_2 \)
II Contraction in elbow  \( A_2 \) to \( A_3 \)
III Turning loss in elbow  \( A_2 \) to \( A_3 \)

All calculations are based on standard air and the methods of Chapter 21, ASHRAE Guide, 1960

Standard air:
Dry air
\( p = 0.075 \text{ lb/ft}^3 \)
\( T = 70.0 \text{ °F} \)

I Expansion Loss
\[
He = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{V_1}{4005}\right)^2
\]

\( A_1 = \text{Fan open area, 3 fans per elbow} \)
\[
A_1 = 3 \left[ \frac{\pi}{4} (1.890)^2 \right] = 0.0584 \text{ ft}^2
\]

\( A_2 = \frac{(2.5)(8.0)}{144} = 0.139 \text{ ft}^2 \)

\( V_1 = \frac{Q}{A_1} = \frac{520}{(4)(0.0584)} = 2220 \text{ ft}^3/\text{min} \)

\[
He = \left(1 - \frac{0.0584}{0.139}\right)^2 \left(\frac{2220}{4005}\right)^2
\]

\[
= (1 - .420)^2 (.555)^2 = (.580)^2 (.555)^2
\]

\( He = 0.104 \text{ inches of water} \)

II Contraction Loss
\[
He = C_3 \left(\frac{V_3}{4005}\right)^2
\]
A_3 = \frac{(8)(1.3)}{144} = 0.0722 \text{ ft}^2

Use coefficient for abrupt contraction.

\[ \frac{A_1}{A_2} = \frac{0.0722}{0.139} = 0.52 \]

From Table 4

\[ C_3 = 0.20 \]

\[ V_3 = \frac{Q}{A_3} = \frac{520}{0.0722} = 1800 \text{ ft/min}. \]

\[ H_c = 0.20 \left( \frac{1800}{4005} \right)^2 \]

\[ H_c = 0.041 \text{ inches of water} \]

III Turning Loss

\[ H_T = C \left( \frac{V_3}{4005} \right)^2 \]

From Table 3 for a miter elbow with vanes

\[ C = 0.35 \]

\[ H_T = 0.35 \left( \frac{1800}{4005} \right)^2 \]

\[ H_T = 0.071 \text{ inches of water} \]

Fin Pressure Drop

From previous calculations the pressure drop through the fins is

\[ H_{fin} = 1.320 \text{ inches of water} \]

This is for 520 cfm at 92.5°F and standard barometer. This \( H_{fin} \) must be corrected to standard conditions

\[ H_{fin, std} = 1.320 \left( \frac{460 + 92.5}{460 + 70} \right) = 1.320 \left( \frac{552.5}{530} \right) \]

\[ H_{fin, std} = 1.375 \]
Total Pressure Drop - Cold Side

\[
\begin{align*}
I & = 0.104 \\
II & = 0.041 \\
III & = 0.071 \\
Fins & = 1.375 \\
1.591 & \text{ inches of water}
\end{align*}
\]

From Rotron performance curves

\[
p_{\text{static}} = 1.70 \text{ at } 43.3 \text{ cfm/Fan}
\]

\[
1.70 - 1.59 = 0.11 \text{ inches of water available for a filter.}
\]
Top View of Duct

Total air flow, 1024 cfm from 8 fans
Power required, 133 watts/fin.
The total pressure drop may be broken down as follows for the duct system:

I Turning loss in elbow  \( A_1 \) to \( A_2 \)

II Expansion loss  \( A_1 \) to \( A_2 \)

III Contraction loss  \( A_2 \) to \( A_3 \)

I Turning Loss

\[
H_T = C \left( \frac{V_1}{4005} \right)^2
\]

\[
A_1 = \frac{(1.30)(1.40)(2)}{144} = 0.253 \text{ ft}^2
\]

\[
V_1 = \frac{Q}{A_1} = \frac{(4)(125)}{0.253} = 2030 \text{ ft/min}
\]

From Table 3 for a miter elbow with varics,

\[
C = 0.35
\]

\[
H_T = 0.35 \left( \frac{2030}{4005} \right)^2 = 0.090 \text{ inches of water}
\]

II Expansion Loss

\[
H_e = \left( 1 - \frac{A_1}{A_2} \right)^2 \left( \frac{V_1}{4005} \right)^2
\]

\[
A_2 = \frac{(1.40)(3)}{144} = 0.292 \text{ ft}^2
\]

\[
H_e = \left( 1 - \frac{0.253}{0.292} \right)^2 \left( \frac{2030}{4005} \right)^2 = 0.133^2(0.507)^2
\]

\[
H_e = 0.005 \text{ inches of water}
\]

III Contraction Loss

\[
H_c = C \left( \frac{V_3}{4005} \right)^2
\]

\[
A_3 = (4)(0.0368) = 0.147 \text{ ft}^2
\]

\[
\frac{A_3}{A_2} = \frac{0.147}{0.292} = 0.503
\]

From Table 4 for an abrupt contraction,

\[
C = 0.10
\]
\[ V_3 = \frac{Q_3}{A_3} = \frac{(4)(128)}{.147} = 3480 \text{ ft/min} \]

\[ H_e = .20 \left( \frac{3480}{4005} \right)^2 \]

\[ H_e = 0.151 \text{ inches of water.} \]

**Fin Pressure Drop**

From previous calculations the pressure drop through the fins is

\[ \Delta P = 2.116 \text{ inches of water} \]

This is for 1024 cfm at \( p = 0.069 \text{ lb/ft}^3 \)

\[ \Delta P_{sw} = \Delta P \left( \frac{\rho_{sf}}{p} \right) \]

\[ \Delta P_{sw} = 2.116 \left( \frac{0.075}{0.069} \right) \]

\[ = 2.300 \text{ inches} \]

**Total Pressure Drop - Hot side**

\[ I = 0.090 \]

\[ II = 0.005 \]

\[ III = 0.151 \]

\[ \text{Fins} = 2.300 \]

\[ \frac{2.300}{2.546} \text{ inches of water} \]

From Rotron Performance Curves

\[ P_{static} = 2.56 \text{ at 128 cfm/fan} \]

\[ 2.56 - 2.55 = 0.01 \text{ inches of water available for a screen.} \]
Nomenclature

A. Frontal area for air flow, $\text{Ft}^2$

C. Pressure loss coefficients

H. Static pressure loss, inches of water

P. Static pressure output of fans, inches of water

V. Air velocity, $\text{Ft/min}$

P. Density of air, Pounds/$\text{Ft}^3$
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