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Typical Values of Plasma Parameters Around a Conical Re-Entry Vehicle

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Prepared by D. M. Dix
Physical Research Laboratory

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California
TYPICAL VALUES OF PLASMA PARAMETERS AROUND A
CONICAL RE-ENTRY VEHICLE

Prepared by
D. M. Dix
Physical Research Laboratory

AEROSPACE CORPORATION
El Segundo, California

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ABSTRACT

The characteristics of the plasma around a slender, slightly blunted cone during re-entry are discussed in relation to the plasma sheath transmission problem. The flow field around the aforementioned vehicle is qualitatively described, and the plasma and collision frequency distribution around a perfectly sharp cone during re-entry is presented graphically. The methods employed in computing the presented values of the plasma parameters are outlined and referenced in the Appendix.
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NOMENCLATURE

f_p plasma frequency, cps
h enthalpy
H total enthalpy = h + u^2 / 2
Le Lewis number
M Mach number
p static pressure
Pr Prandtl number
R gas constant
r recovery factor
Re Reynolds number
T temperature
u velocity parallel to surface
x distance along cone surface from vertex
y distance normal to cone surface
δ boundary layer thickness
θ_c cone semivertex angle
θ_sh shock angle
μ viscosity
ν electron collision frequency, collision/sec

Subscripts

l laminar boundary layer
t turbulent boundary layer
w wall conditions
δ inviscid surface conditions
∞ free stream conditions
I. INTRODUCTION

The characteristics (i.e., plasma and collision frequency distributions) of the plasma around a slender, slightly blunted cone during re-entry are discussed in relation to the plasma sheath transmission problem. The flow field around the aforementioned vehicle is qualitatively described, and the plasma and collision frequency distribution around a perfectly sharp cone during re-entry is presented graphically.

II. QUALITATIVE DESCRIPTION OF FLOW FIELD

The flow field about a slender, slightly blunted re-entry vehicle can be roughly divided into five regions, as shown in Fig. 1. With the exception of the shock wave, the boundaries between the regions are, of course, not sharp, well defined ones, but this fact should not detract from the following discussion.

Region 1 is the undisturbed atmosphere. Region 2 contains air which has passed through the strong, nearly normal portion of the shock wave and is, therefore, at a higher pressure, temperature, and density than the free stream and possesses a significant electron density. The gradients of the properties in this region are such that diffusive, viscous, and thermal conduction effects may become appreciable, particularly about bodies with small nose radii, although these effects are usually neglected. Region 3 contains air which has passed through the conical portion of the shock wave. For the slender bodies and flight conditions of interest here, this air does not contain an appreciable number of electrons, although it is of course at a higher temperature, pressure, and density than the free stream. The gradients of the properties in this region are such that the dissipative effects are negligible.
Region 4 is the laminar boundary layer, and Region 5 is the turbulent boundary layer. Both of these regions are characterized by large gradients in the plasma properties (except the pressure, which is constant) in the direction normal to the body surface and small gradients in the streamwise direction. The diffusive and dissipative effects are of prime importance in this region. In addition to air, these boundary layers may also contain vaporized ablative material and/or other contamination from the body surface. The point on the body at which transition from laminar to turbulent flow occurs is a function, primarily, of the Reynolds number of the flow outside the boundary layer and is such that at high altitudes the boundary layer tends to be completely laminar, whereas at low altitudes it tends to be completely turbulent.
A significant feature of the flow field is that at some distance downstream of the nose, Region 2 disappears; the air in this region has been consumed by the boundary layer, and the flow field resembles that around a perfectly sharp body. The character of the boundary layer changes markedly depending upon whether the air is supplied by Region 2 or Region 3. When air is supplied by Region 2, the maximum temperature and electron density occur at the outer edge of the boundary layer, as heat transfer to the body more than offsets viscous heating effects. When the air to the boundary layer is supplied by Region 3 (the case for a perfectly sharp body), the peak temperature and electron density occur within the boundary layer and are governed by a balance between viscous heating and heat transfer to the body.

The chemical processes occurring in the flow vary considerably over the regions of interest. At high altitudes, collision frequencies are very low, and essentially no chemical reactions occur (frozen flow). As the altitude decreases, collisions become more frequent, and chemical reactions occur to an appreciable extent (nonequilibrium flow). At sufficiently low altitudes, the collision frequencies are adequate for chemical equilibrium to be attained (equilibrium flow). The effects on the plasma parameters of flow in the first two regimes are far beyond the scope of this report; for present purposes, it is sufficient to state that the transition to equilibrium flow occurs between 100,000 and 150,000 ft.

The foregoing discussion indicates that for sufficiently low altitudes at distances several nose radii back from the blunted portion of a slender body, the calculation of the properties of laminar and/or turbulent boundary layers for the flow of clean air in thermodynamic equilibrium over the related sharp body yields a reasonable estimate of the significant features of the plasma found around the body during re-entry.
III. RESULTS FOR CLEAN AIR IN THERMODYNAMIC EQUILIBRIUM

As an aid in gaining some appreciation for the magnitudes of the plasma parameters in the vicinity of a blunt nose, the plasma and collision frequencies occurring behind a normal shock wave are shown in Fig. 2 as a function of vehicle altitude and velocity. A typical trajectory for a re-entry vehicle of the type considered is also shown in this figure. The significant features are first that $v < f_p$ at the higher altitudes, with $v = f_p$ at approximately 75,000 ft; and second, as will be seen, the magnitudes of $f_p$ and $v$ are of the order of $10^2$ times those found in the boundary layer far back from the blunted nose. It should be pointed out that the units employed herein for plasma and collision frequency are those most amenable to physical interpretation and are not consistent ones for the determination of the dielectric properties. The consistent units are either $\omega$(rad/sec) and $v$(collisions/sec) or $f$(cps) and $v$(collisions/sec)/2$\pi$.

![Fig. 2. Plasma Characteristics in Stagnation Region](image)

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The remainder of the results presented here refers to a sharp cone with a 10 deg semivertex angle and a wall temperature of 2000°K. These conditions are representative of a re-entry vehicle of the type considered. (See the Appendix for the methods employed in computing the presented values of the plasma parameters.)

In Fig. 3, the maximum values of plasma and collision frequencies occurring in either laminar or turbulent boundary layers are shown as a function of the vehicle altitude and velocity. It is to be noted that these peak values are not a function of the distance from the cone vertex. Again, at higher altitudes, \( v < f_p \), with \( v = f_p \) at approximately 125,000 ft. The sensitivity of these results to the cone angle and wall temperature is not great; the plasma frequency decreases by approximately 5-10 per cent per degree of decrease in cone angle (the greater decrease at higher velocities), while it decreases by approximately 25 per cent per 1000°K decrease in wall temperature. The collision frequency is somewhat less sensitive than the plasma frequency.

![Graph](image)

**Fig. 3. Plasma Characteristics of Sharp Cone (Peak Boundary Layer Condition)**
The laminar boundary layer thickness (in the form $\delta / \sqrt{x}$) is plotted as a function of vehicle altitude and velocity in Fig. 4. This figure also shows the distances from the vertex at which transition to turbulence may be expected to occur. The significant features here are the $\sqrt{x}$ growth rate and the relative insensitivity of the thickness to vehicle velocity. It should be pointed out that the point at which transition occurs may be only crudely estimated.

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**Fig. 4.** Laminar Boundary Layer Thickness and Transition Distance for Flow Around a Sharp Cone
The turbulent boundary layer thickness (in the form $\delta_t/x^{0.8}$) is shown in Fig. 5 as a function of vehicle velocity and altitude. The significant features are the faster growth rate than the laminar case ($x^{0.8}$ versus $\sqrt{x}$) and the relative insensitivity of the thickness to vehicle velocity.

Fig. 5. Turbulent Boundary Layer Thickness on a Sharp Cone

Figures 6 and 7 show typical profiles of the plasma and collision frequencies found in the laminar and turbulent boundary layer as a function of normalized distance from the cone surface ($y/\delta$). In Fig. 6, the effect of vehicle altitude is indicated, while Fig. 7 shows the effect of vehicle velocity. The significant features are that the maximum condition in the boundary layer occurs much closer to the surface in the turbulent case, and that the collision frequency is relatively constant across the boundary layer.
Fig. 6. Collision and Plasma Frequency Profiles of Laminar and Turbulent Boundary Layers on a Sharp Cone Indicating the Effect of Vehicle Altitude
Fig. 7. Collision and Plasma Frequency Profiles of Laminar and Turbulent Boundary Layers on a Sharp Cone
APPENDIX

METHODS AND SOURCES FOR CALCULATIONS

The methods employed in computing the presented values of the plasma parameters are outlined herein.

A. CONE SURFACE

For the purposes of this computation, values of the fluid properties at the cone surface are required (e.g., \( u_\delta \), \( h_\delta \), \( p_\delta \)). The inviscid, supersonic, axisymmetric flow about a sharp cone has been solved for both perfect gases and air in thermodynamic equilibrium. Solutions for perfect gases appear in Refs. 1, 2, and 3, while solutions for air are given in Ref. 4. For present purposes, the conical shock wave is usually of insufficient strength for dissociation to be appreciable (\( M_\infty \sin \theta_c < 5 \)). In these cases, the shock angle and surface pressure coefficient are obtained from Ref. 3 (perfect gas, \( \gamma = 1.4 \)), and the enthalpy at the cone surface was computed approximately by the relation

\[
\frac{h_\delta}{h_\infty} = 1 + 0.202 M_\infty^2 \sin^2 \theta_{sh}
\]

These approximations do not contribute significant error to the results at the small cone angle of interest. For shock waves for which \( M_\infty \sin \theta_c > 5 \), the properties at the cone surface were obtained from Ref. 4.

B. LAMINAR BOUNDARY LAYER

Unfortunately, exact solutions for the laminar boundary layer over a sharp cone for air in thermodynamic equilibrium are not available. Work relevant to this problem is summarized in Ref. 5. Results for a gas satisfying the ideal gas law, \( p = \rho RT \), and variable properties are given in Refs. 6 and 7; results for air in thermodynamic equilibrium with \( Le = 1 \) are available in
Ref. 8, although the value for the dissociation energy of nitrogen employed in the computation is incorrect. Fortunately, in the region of interest of the present computations, the dissociation effects do not affect the boundary layer thickness appreciably. The thickness for cones is calculated from the empirical relation presented in Ref. 9,

\[ \frac{\delta f}{x} \sqrt{Re_\delta} = \frac{5.20}{\sqrt{3}} \left( \frac{T_{\text{max}}}{T} \right)^{0.8} \]

which correlates the data of Ref. 7 in the region of interest and also is in agreement with the results of Ref. 8. The thickness, \( \delta f \), is defined here as the point where the velocity in the boundary layer reaches 99.5 per cent of the inviscid surface value.

The velocity profile is computed approximately from the cubic equation:

\[ \frac{u}{u_6} = \frac{3}{2} \left( \frac{\gamma}{\delta} \right) - \frac{1}{2} \left( \frac{\gamma}{\delta} \right)^3 \]

which in the region of interest yields results within 20 per cent of those of Refs. 6 and 7.

The total enthalpy profile is computed from the usual Crocco relation

\[ \frac{H - H_w}{H_6 - H_w} = \frac{u}{u_6} \]

which is exact for \( Pr = Le = 1 \) and is a fair approximation in the present case, although the enthalpy tends to be overestimated.
C. TURBULENT BOUNDARY LAYER

The turbulent boundary layer thickness is estimated from the empirical correlations, based on the reference enthalpy method, developed in Ref. 10:

\[
\delta_t = 0.0194 \frac{\theta_t}{(Re_x)^{0.8}} \left( \frac{\rho^*}{\rho_0} \right)^{0.8} \left( \frac{\mu^*}{\mu_0} \right)^{0.2}
\]

\[
\frac{\delta_t}{\theta_t} = 8 + \left( 1.29 \frac{h_w}{h_r} + 1 \right) \left( 1 + 1.51\beta \right) \quad \beta < 4
\]

\[
\frac{\delta_t}{\theta_t} = 8 + \left( 1.29 \frac{h_w}{h_r} + 1 \right) \left( 1 + 3.02\sqrt{\beta} \right) \quad \beta > 4
\]

\[
\beta = \frac{u_0^2}{2h_0}
\]

\[
h_r = h_0 + r \frac{u_0^2}{2}
\]

where \( \rho^* \) and \( \mu^* \) are to be evaluated at the surface pressure and reference enthalpy, which are defined by

\[
\frac{h^*}{h_0} = 0.5 + 0.5 \frac{h_w}{h_0} + 0.22r\beta
\]

The accuracy of this correlation is difficult to evaluate, although it does correlate available experimental data (for \( M < 10 \)) within 40 per cent.

The velocity profile is assumed to be given by

\[
\frac{u}{u_0} = \left( \frac{\gamma}{\delta} \right)^{1/7}
\]
which is a good approximation for low speed flow; also, there is evidence that, at the high Mach numbers of interest here, this approximation is poor.

The total enthalpy profile is again assumed to be given by the Crocco relation; again, there is evidence that this approximation becomes poor at high Mach numbers.

D. AIR

The thermodynamic properties of air at high temperatures are obtained from Ref. 11; the transport properties (only viscosity is required for the present computations) are obtained from the compilation of Ref. 12.

The plasma frequency (or electron density) for air in thermodynamic equilibrium is obtained from unpublished data by Bleviss.* The collision frequency is obtained from Ref. 13.

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*Z. O. Bleviss, Private Communication, Physical Research Laboratory, Aerospace Corporation.
REFERENCES


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