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DESIGN CONSIDERATIONS IN SELECTING MATERIALS FOR LARGE SOLID-PROPELLANT ROCKET-MOTOR CASES

DEFENSE METALS INFORMATION CENTER
Battelle Memorial Institute
Columbus 1, Ohio
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1. To collect, store, and disseminate technical information on the current status of research and development of the above materials.

2. To supplement established Service activities in providing technical advisory services to producers, molteners, and fabricators of the above materials, and to designers and fabricators of military equipment containing these materials.

3. To assist the Government agencies and their contractors in developing technical data required for preparation of specifications for the above materials.

4. On assignment, to conduct surveys, or laboratory research investigations, mainly of a short-range nature, as required, to ascertain causes of troubles encountered by fabricators, or to fill minor gaps in established research programs.

Contract No. AF 33(616)-7747
Project No. 2(8-6975)
DESIGN CONSIDERATIONS IN SELECTING MATERIALS
FOR LARGE SOLID-PROPELLANT ROCKET-MOTOR CASES

by

C. W. Bert and W. S. Hyler

to

OFFICE OF THE DIRECTOR OF DEFENSE
RESEARCH AND ENGINEERING

DEFENSE METALS INFORMATION CENTER
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The authors wish to acknowledge the helpful discussions held with R. J. Runck and E. W. Camthorne of the Defense Metals Information Center Coordination Staff. Acknowledgment is also made to the following personnel of the Department of Mechanical Engineering for reviewing the manuscript: J. M. Allen, T. J. Atterbury, F. L. Bagby, R. J. Favor, J. L. Harp, and G. M. McClure.
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DESIGN CONSIDERATIONS IN SELECTING MATERIALS FOR LARGE SOLID-PROPELLANT ROCKET-MOTOR CASES

SUMMARY

Seven major design considerations in selecting materials for large solid-propellant rocket-motor cases are covered in this report: (1) missile types, mission profiles, loads, and environment, (2) stresses and design factors, (3) structural safety and reliability, (4) static behavior, (5) fatigue behavior, (6) environmental effects, and (7) structural weight.

Two different loading situations govern the design of motor cases:

1. Internal Pressure. This loading results in a high tension stress in the circumferential direction; this tends to produce failure in tension. The structural index which governs the motor-case weight when internal pressure is critical is $F_{tu}/d$ — the ratio of the ultimate tensile strength to density. The higher this ratio, the lower is the weight.

2. Buckling Due to External Loading. This type of failure may be produced by axial compression, by bending moments, or by a combination of the two. When buckling due to external loads is critical, the structural index which governs the motor-case weight is $\sqrt{E/d}$ — the ratio of the square root of the effective elastic modulus to the density. Again, the higher the ratio, the lower is the weight.

On the basis of the two structural indexes mentioned above and certain assumptions, the weight per unit of enclosed volume for motor cases can be predicted quantitatively as a function of mission staging and type of construction and materials. The results of such predictions are shown in Figures 15 and 16 (pages 51 and 52) of this report. They can be summarized as follows:

1. For any given combination of loads, there is a composite structure which is lighter than the lightest monolithic structure. However, there is no single type of composite construction which is best for all ranges of external loads.

2. For monolithic construction (excluding beryllium), the titanium alloys result in the lightest motor case, except in the case of very high external loadings, where aluminum alloys look promising. In all instances, steel motor cases are heavier than titanium-alloy cases on a weight-volume basis.

3. For relatively low external loads representative of ballistic-missile applications, a glass-filament-wound resin-bonded composite results in the lightest motor case per enclosed volume.

4. For the intermediate load range associated with boosters for earth-orbit vehicles and some stages of earth-escape boosters, the composite of glass filaments resin bonded to an aluminum alloy base gives the lightest motor case.
(5) For the very high loads associated with the intermediate stages of an earth-escape booster, the sandwich-type composite of glass-filament facings and an aluminum-alloy honeycomb core appears to be the most promising from a weight-volume standpoint.

(6) Regardless of the combinations of loads considered, the music-wire resin-bonded composite is inferior, from a weight standpoint, to the other composites. However, a beryllium-wire resin composite may be promising.

The selection of rocket-motor-case materials involves other considerations in addition to structural-material-weight indexes based on uniaxial static material properties. It also involves other considerations in addition to structural-weight indexes based on simple pressure vessels subjected to the stresses discussed in this report. For example, nonpolar openings in the case, such as occur in four-nozzle rocket motors and in cases that contain thrust reverser ports, severely penalize the potential weight advantages of filament-wound structures because load-bearing members must be cut to accommodate the openings. During the time history of powered flight, aerodynamic and internal heating (from propellant burning), high-frequency random vibrations, and other effects imply that thermally dependent mechanical properties and fatigue should be considered. Also, internal pressure loading produces biaxial-stress effects, and difficulties in inspecting for small flaws make brittle fracture an important consideration. For a space booster traveling for some time in space, the effects of the space environment can become significant. Thus, for particular missions, the strength values used in a weight analysis are dependent upon these other material requirements which are also discussed in this report. Requirements for reliability, additional external insulation, radiation shielding, or bumpers for solid-particle shielding also should be considered in making trade-offs to arrive at an acceptable final design. Finally, it must be recognized that producibility, cost, and corrosion resistance, three key selection factors not considered in this report, are frequently as important in the selection of materials for large solid-propellant rocket-motor cases as are the design considerations covered by the report.
INTRODUCTION

The Defense Metals Information Center was given a special assignment by the Office of the Director of Defense Research and Engineering to collect, evaluate, and disseminate information on the design and fabrication of the inert parts of large solid-propellant rocket motors.

In this report, design considerations that influence the selection of materials for rocket-motor cases are discussed. The main intent of the report is to clarify considerations of design for the benefit of the materials specialists in the aerospace industry and for the benefit of the materials industries which supply the aerospace industry. The presentation starts with a brief discussion of types of missiles and mission profiles. This is followed by a discussion of the external and internal loads and environment to which the missile is subjected. The mechanical and thermal loads are reacted by the motor case acting as a structure; hence, a discussion of stresses and other design factors is included. Next, structural safety and reliability are discussed from a design point of view.

Since it is material behavior that must be made compatible (in a selection of structural materials) with the applied stresses to assure the highest possible performance consistent with an adequate reliability level for the vehicle, these aspects of basic material behavior are discussed in relation to motor-case design requirements: static mechanical behavior, fatigue behavior, thermal effects, and other environmental effects. Finally, in order to provide realistic criteria for materials selection, design for minimum weight is presented from the material-index point of view. Considerations not related directly to design, for example, producibility, cost, and corrosion resistance, are not included in this report.

MISSILES AND MISSIONS

In discussing design considerations, attention here is directed to large solid-propellant (SP) rocket-motor cases. No thought is given to such smaller rockets or boosters as may be associated with sounding rockets and short-range missiles. Large boosters may be used for the following missions:

1. Ballistic missiles, including both surface-launched and air-launched types (ALBM), of medium range (MRBM), intermediate range (IRBM), or intercontinental range (ICBM).
2. Earth-orbiting mission.
3. Lunar or satellite rendezvous.
4. Interplanetary missions.

[The report supersedes DMIC Memorandum 117, dated March 8, 1962. Addendum: Interceptor missiles (ALBM), with large size, have very high accelerations which introduce problems of a special nature that are considered to be outside the scope of this report, since it is limited to large SP rockets of low and intermediate acceleration.]
Booster for these missions usually are multistage vehicles, and in the past, generally have consisted of one rocket motor per stage (Figure 1). In an effort to obtain large total thrust to place very heavy payloads in orbit or on an earth-escape mission, a trend now is developing in the direction of clusters of simple or segmented solid-propellant rocket motors in each stage. In the future this trend could result in requirements for relatively few designs of very large rocket motors. Such motors will have to be carefully conceived, in order to have the versatility that would be required by the variety of mission requirements. This follows, because past experience has been that, in general, the vehicle configuration and its major subsystems have been closely related to the requirements of specific missions.

Another way in which large SP rockets can be used is in conjunction with liquid-propellant rockets, as exemplified by the Titan III (Figure 2), which is to be used for launching the X-20 (Dyna-Soar) glide vehicle.

For the four mission types cited above, there are certain similarities during the first portion of the flight (powered flight) that are important in considering rocket-case design. For a ground launch, the vehicle starts with a short vertical rise during which launching transients damp out. During this period, some missiles are rolled to obtain the proper flight azimuth. After a given rise, the missile begins a programmed pitch maneuver (zero lift or gravity turn) to minimize aerodynamic forces in the sensible atmosphere (vehicle axis is kept oriented along the flight path in this maneuver). Above about 200,000 feet, the missile trajectories are shaped further in accordance with mission requirements so that, at last-stage burnout, the vehicle will have reached desired velocity, altitude, and flight-path angle.

From lift-off to about 200,000 feet, aerodynamic forces and maneuvering loads are important to the structural design. Maximum dynamic pressure usually occurs in the vicinity of 35,000 to 40,000 feet. Above the sensible atmosphere, both aerodynamic and maneuver loads are low. Hence, for ground launching, it is the early part of powered flight that is of design importance.

Certain vehicle missions have other requirements later in flight that can influence the problem of rocket-case design. For example, some vehicles may be inserted initially into a near or far earth orbit (that may or not involve eclipses from the sun) for some indefinite time period. Subsequently, a payload could be fired (1) toward the ground, (2) toward some other orbiting target, or (3) into lunar or interplanetary space. In addition, design requirements exist for the return of manned vehicles that have landed on the lunar or planetary surfaces.

In these instances, the effects of the space environment on material behavior have to be considered as part of the design of rocket-motor cases. The effects that may be important include the following:

1. Electromagnetic radiation

2. Particle radiation (galactic cosmic radiation, solar-particle radiation, trapped-particle radiation)

3. Solid-particle impingement (meteoroids)
FIGURE 1. TWO CURRENT MISSILES POWERED BY SOLID PROPELLANTS
FIGURE 2. AIR FORCE TITAN III BOOSTER WITH X-20 MANNED BOOST-GLIDE SPACECRAFT
(4) Vacuum environment

(5) Unusual atmospheres.

These environments are discussed in the next section.

Since it is not currently envisioned that recoverable boosters will be of the solid-propellant type, re-entry problems are not included in this discussion.

**Loads and Environment**

The loads acting on a missile determine the type and magnitude of stresses; the environment also has a strong bearing on the selection of motor-case materials. These topics are discussed below under separate headings.

**Loads**

Load sources acting on a missile structure may be categorized into two types: static loads and dynamic loads. Static loads are of particular use in evolving the preliminary structural design considering the missile to react as a rigid body. However, once a preliminary design has been achieved, it is possible to compute the dynamic characteristics of the missile. With these characteristics, it is then necessary to consider effects of the dynamic loads on the structural behavior. This infers study of the interactions between the control system, propulsion system, aerodynamic effects, etc., and the dynamic behavior of the structure.

Although many load sources are known to contribute to the total load environment that a missile experiences, all of these loads do not act simultaneously. It is necessary to establish which are the significant loads, based upon their higher probability of occurrence throughout the time history of the mission. To a large extent, this decision will be related to the basic mission of the system. In the case of loads pertinent to the rocket-motor cases, the decision will be affected by the location of the motor case in the sequence of stages.

Tables 1 and 2 list possible static and dynamic loads to which a rocket case may be subjected and the types of stresses that would result from the load sources. Those entries with an asterisk are considered the load sources that should be examined in every instance.

From these tables, and recognizing that the solid-propellant rocket-motor case is a part of the missile structure until stage separation, it is possible to list the important load sources that affect the basic design of the case.

(i) Axial load. The major load on the missile is the thrust load which is reacted by the inertial load of the missile mass and the aerodynamic drag.

---

*For an extensive discussion of loads may be found in Reference (1); Chapter 1 covers flight loads, Chapter 3 discusses axial, lateral, and crosswind loads, and Chapter 10 treats dynamic loads.*
<table>
<thead>
<tr>
<th>Item</th>
<th>Load Source</th>
<th>Type of Resulting Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>Control system — vanes or nozzle swivel</td>
<td>Bending</td>
</tr>
<tr>
<td>2*</td>
<td>Atmospheric lift and drag</td>
<td>Bending, axial</td>
</tr>
<tr>
<td>3*</td>
<td>Unilateral loading — gust, wind shear, etc.</td>
<td>Bending due to unilateral pressure</td>
</tr>
<tr>
<td>4</td>
<td>Inertial loading — ground handling, erection, launch, staging, space maneuvering, etc.</td>
<td>Various types of stresses (including longitudinal and torsional shear at the case-propellant interface)</td>
</tr>
<tr>
<td>5*</td>
<td>Thrust loading</td>
<td>Axial, bending</td>
</tr>
<tr>
<td>6</td>
<td>Weapon effects</td>
<td>Bending due to unilateral pressure</td>
</tr>
<tr>
<td>7*</td>
<td>Dynamic pressure</td>
<td>(Forward portion only)</td>
</tr>
<tr>
<td>8*</td>
<td>Wind-induced loads — post erection, prelaunch</td>
<td>Bending</td>
</tr>
<tr>
<td>9*</td>
<td>Internal pressure</td>
<td>Hoop and axial stress (biaxial)</td>
</tr>
<tr>
<td>10*</td>
<td>Thermal — propellant burning, aerodynamic heating</td>
<td>Biaxial stress</td>
</tr>
<tr>
<td>11*</td>
<td>Shunts in center of gravity and center of pressure</td>
<td>Bending</td>
</tr>
<tr>
<td>12</td>
<td>Spin-stabilization loads</td>
<td>Torsion</td>
</tr>
<tr>
<td>Item</td>
<td>Load Source</td>
<td>Type of Resulting Stress</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>1*</td>
<td>Control system — coupling with structural modes</td>
<td>Bending</td>
</tr>
<tr>
<td>2*</td>
<td>Unilateral loading — gust, wind shear, etc.</td>
<td>Bending</td>
</tr>
<tr>
<td>3*</td>
<td>Inertial loadings — ground handling, staging, space maneuvering, etc.</td>
<td>Various types of stresses</td>
</tr>
<tr>
<td>4*</td>
<td>Weapon effects — Blast Neutron heating Fragment impact</td>
<td>Bending Thermal Localized point loads</td>
</tr>
<tr>
<td>5*</td>
<td>Thrust</td>
<td>Axial, bending</td>
</tr>
<tr>
<td>6*</td>
<td>Wind-induced loads — post erection, prelaunch</td>
<td>Bending</td>
</tr>
<tr>
<td>7*</td>
<td>Acoustical</td>
<td>Panel bending, vibration</td>
</tr>
<tr>
<td>8</td>
<td>Shifts in center of gravity and center of pressure</td>
<td>Bending</td>
</tr>
<tr>
<td>9</td>
<td>Short-period rigid-body rotation</td>
<td>Bending</td>
</tr>
<tr>
<td>10</td>
<td>Spin-stabilization</td>
<td>Torsion</td>
</tr>
</tbody>
</table>
Bending Loads. These loads arise from a variety of sources. However, generally they result from the lateral aerodynamic force (or lift), the lateral component of the thrust vector, and inertia forces reacting to the former. Except at hypersonic velocities, aerodynamic lift on bodies of revolution occurs only when there is an angle of attack between the longitudinal axis and the relative air velocity. This is why wind effects, particularly at about 35,000 feet (where wind velocity is greatest), are important. Since dynamic pressure also is near maximum at this altitude, maximum bending moments occur in the vicinity of this level.

Internal Pressure. This loading results in a biaxial-stress state in the case shell. As discussed later in the section on Stresses and Design Factors, the effect of pressure is considered as a load and as a load-relieving effect.

Thermal Loading. Thermal loading arises from internal heating due to propellant burning and external heating by aerodynamic friction. In space flight, thermal loading arises from solar radiation. Two effects result: biaxial thermal stresses are set up in the case, and mechanical strength of the material may be decreased.

Ground Loads. Examinations of ground loads usually are made less from the standpoint of missile design than from the interaction with ground-support equipment. Then the ground-support equipment is designed to accommodate to these loads in order not to penalize the missile with additional weight.

Acoustical Loads. Acoustical loads may be transmitted to the structure as pressure fluctuations of random, high-frequency content or the loads may be transmitted through the structural members themselves.

Space Environment

For those missions that involve launch from an orbiting "platform" or launch from the lunar or planetary surface, some of the effects of the environmental factors peculiar to space have to be considered in the design of rocket-motor cases.

Within the last 2 years, there have been a number of excellent summaries of the factors involved in "space environment". To this body of information is being added, continually, vital knowledge of the environment that will be helpful in material selection. Accordingly, a detailed discussion of the environment in this report is not warranted. Instead, a brief statement of the nature of the environment is presented. Its possible effect on materials and design is treated in the section on Environmental Effects.

Electromagnetic Radiation. The chief source of electromagnetic radiation in the solar system is the sun. Its intensity varies inversely as the square of the distance from the sun (hence at a Venus orbit the intensity will be about 1.9 times that at earth orbit, at Mars, about 0.4 times). The spectrum of electromagnetic radiation ranges from long wavelength radio waves to very short wavelength gamma radiation. About half of the
sun's energy lies in the infrared and radiofrequency regions (>7000 A). Forty per cent of the sun's energy is in the visible range (4000 to 7000 A) and the remaining 10 per cent is in the ultraviolet and shorter wavelength regions of the spectrum (<4000 A). Wavelengths shorter than 3000 A comprise 1 per cent of the total energy of the sun. This latter radiation is absorbed in the atmosphere; however, it will be encountered in space.

In addition to solar radiation, an orbiting vehicle close to the earth or to another solar body will be subjected to direct reflected sunlight and electromagnetic radiation from the body. These latter two effects are small in comparison with that of direct solar radiation and their effect decreases with distance from the body.

Electromagnetic radiation may affect materials in several ways. Radiation impinging on a motor case may be absorbed, and the energy converted to thermal energy or heat. Very short wavelength radiation, which is encountered only in space, may cause atomic displacement or ionization of case materials. Depending upon the material, radiation damage may occur.

**Particle Radiation.** Particle radiation has three main origins: (1) galactic cosmic radiation, (2) solar particle radiation, and (3) geomagnetically trapped particle radiation radiation.

Galactic cosmic radiation (primary cosmic rays) consists primarily of high-energy positively charged particles with a continuous energy spectrum up to 10^{18} ev. Ionized nuclei of elements from hydrogen up to iron have been observed; however, the majority of the particles are hydrogen nuclei.

Solar particle radiation consists of a base-level radiation — solar wind — and radiation from solar flares. The proton density at earth orbit is estimated to be in the range 10 to 100 protons per cubic centimeter. These particles have much lower energy than primary cosmic radiation, in the range 1 to 5 kev. During a period of solar activity, the density of the particles at earth-orbit altitudes may rise to 10,000 protons/cm^3 with associated energies of 40 kev. In severe storms, particles with average energies of 100 Mev have been observed.

Geomagnetically trapped radiation has been found in two zones (Van Allen belts) girdling the earth. The inner zone (proton zone) ranges from about 500 miles to 6,000 miles in altitude, the outer zone (electron zone) ranges from about 1,500 miles to 50,000 miles. The inner zone is quite stable, varying slightly with solar activity and having reported maximum fluxes of about 40,000 protons/(cm^2)(sec). The outer zone, however, varies considerably with solar activity. Fluxes on quiet days of 3 \times 10^8 electrons/(cm^2)(sec) have been reported. With solar-induced activity, maximum flux is increased to 10^9 electrons/(cm^2)(sec).

Radiation of the kind discussed here might be expected to result in some degree of radiation damage to certain materials. The severity would be expected to be a function of the dosage and the time of exposure.

**Solid Particles.** In addition to gas particles, a vehicle orbiting the earth or traversing interplanetary space would encounter solid particles either as meteoroids or
meteoric dust. Depending upon the size and density of particles, their impinging velocity, and the duration of exposure, damage in the form of surface roughening or penetration may occur.

Solid particles of three kinds are known to exist: (1) iron-nickel particles, 7.8-8.0 g/cm³; (2) stoney particles, 3.4-4.0 g/cm³; and (3) so-called "dust balls", 0.01-2.0 g/cm³. They move generally in the ecliptic plane at velocities in the range 10 to 70 km/sec (also assumed impact velocity range). The particles are not distributed uniformly, rather they occur sporadically and as "showers". Estimate of the spatial mass density at earth orbit is of the order of $10^{-14}$ to $10^{-15}$ g/cm³. Depending upon an assumed particle size and density, this corresponds roughly to a particle density of $10^{-14}$ to $10^{-15}$ particles/cm³.

Vacuum. The decrease in gas pressure with altitude is indicated by the simple tabulation below (3).

<table>
<thead>
<tr>
<th>Altitude, miles</th>
<th>Pressure, mm Hg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea level</td>
<td>760</td>
</tr>
<tr>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>125</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>500</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>4,000</td>
<td>$10^{-13}$</td>
</tr>
<tr>
<td>&gt;14,000</td>
<td>$&lt;10^{-12}$</td>
</tr>
</tbody>
</table>

It is noted that at an altitude of only 125 miles, the vacuum is about that obtained by usual techniques in the laboratory. It is quite obvious also that a really hard vacuum exists above about 4000 miles. Factors to be considered as a consequence of this hard vacuum include sublimation of surfaces, breakdown or degradation of organic compounds, and changes in mechanical properties.

Table 3 summarizes the environments and lists factors that must be assessed in design of rocket-motor cases and selection of materials when considering vehicles for long-time service outside of the earth or planetary atmospheres.

**TABLE 3. ENVIRONMENTAL FACTORS FOR INTERPLANETARY MISSIONS**

<table>
<thead>
<tr>
<th>Environment</th>
<th>Important Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic radiation</td>
<td>Intermittent heating</td>
</tr>
<tr>
<td></td>
<td>(cyclic thermal stress)</td>
</tr>
<tr>
<td></td>
<td>Radiation damage</td>
</tr>
<tr>
<td>Particle radiation</td>
<td>Radiation damage</td>
</tr>
<tr>
<td></td>
<td>Sputtering</td>
</tr>
<tr>
<td>Solid-particle impingement</td>
<td>Erosion of surfaces</td>
</tr>
<tr>
<td></td>
<td>Penetration, with attendant flaws</td>
</tr>
<tr>
<td>Vacuum environment</td>
<td>Sublimation of surfaces</td>
</tr>
<tr>
<td></td>
<td>Degradation of organic compounds</td>
</tr>
<tr>
<td></td>
<td>Changes in mechanical strength</td>
</tr>
<tr>
<td></td>
<td>Absorptivity and emissivity</td>
</tr>
</tbody>
</table>
Material selection for rocket cases depends on the behavior of potential case materials when they are exposed to various environmental factors discussed in later sections on Environmental Effects; however, in addition, it is highly dependent upon the mechanical properties required. The mechanical-property requirements are governed by the critical stress conditions set up in the case by the various loadings, both mechanical and thermal. The critical stress conditions are those which govern the design. In order to determine the critical stress conditions reliably, it is usually necessary to make a complete stress analysis. This requires detailed information on the distribution of all of the loads applied to the structure, as a function of time, throughout the entire operational life.

It is important to keep in mind that, even for a given stage within a vehicle having a specified mission profile, the loads and stresses are not only functions of location and time, but also of design, including structural configuration, material thicknesses, and material elastic moduli and thermal properties. This aspect is discussed in detail later in connection with thermal stress and in connection with glass-filament-wound composites.

Also, there are many interactions between loading conditions; some of these may even reduce the critical stresses under combined conditions. For example, in a three-stage vehicle, during firing of the first stage, the thrust and inertia forces induce compressive loads in the first-stage motor case which are offset to a certain extent by the axial tension produced by the internal pressure in the case. This relief usually does not occur in the motor cases of the second and third stages, since they are at ambient pressure during first-stage burning.

The critical stresses in rocket cases are those due to internal pressure and those due to external loadings. The paragraphs which follow describe how these stresses arise, their nature, and their general effects on design. Since the stresses which arise in monolithic cases are more simple than those for composite cases, it is convenient to discuss monolithic cases first.

Monolithic Cases

Burning of the solid-propellant grain produces a combustion pressure within the port area (usually an internal-star-shaped opening) of the grain (see Figure 3). This pressure, not necessarily uniform along the length of the grain, acts on the grain, which behaves as a low-modulus viscoelastic* material somewhat similar to a polymer. When a monolithic metallic case is used, the case stiffness is so high in comparison to the long-time elastic shear modulus of the propellant that nearly the full combustion pressure is transmitted through the propellant grain to the case.

In addition to the axial variation in pressure, there is another aspect in which the pressure acting on a solid-rocket case differs from that in an ordinary uniform-pressure vessel. This is the circumferential nonuniformity in pressure distribution which is due to local variations in combustion pressure and nonuniformity in the propellant-grain wall

* A viscoelastic material is one with time-dependent elastic properties.
Figure 3: Typical Solid-Propellant Rocket Motor Including Case, Propellant, and Nozzle. The case is shown in black.
thickness (both initially due to the internal star design and during burning). However, for simplicity, in design the pressure distribution is often assumed to be uniform.

Within the limitations described above, a rocket case can be considered to be a thin-walled, cylindrical-shell-type, internal-pressure vessel*. In any thin-walled shell, stresses due to internal pressure are of two different types: membrane stresses and discontinuity stresses.

Just as the name implies, membrane stresses are those due to membrane action such as exhibited in an inflated balloon. For shells with thin walls, these stresses are essentially uniform through the case thickness and act only in directions perpendicular to the thickness direction. In a thin-walled cylindrical vessel, such as a rocket case, the largest membrane stress is the hoop stress, a tension stress acting in the circumferential direction. The smallest membrane stress, which is a tension stress only one-half of the hoop stress in magnitude, acts in the longitudinal direction. The hoop stress is easily computed by the formula \( pD/2t \), where \( p \) is the pressure acting on the case, \( D \) is the mean diameter of the case, and \( t \) is the case wall thickness. Although this formula is not exact, it is sufficiently accurate for most design purposes**.

Discontinuity stresses** are those which are due to differences in membrane displacements of various elements of a pressurized shell when these elements are pressurized separately. In general, a difference in membrane displacements of two individual elements produces, at the junction, both a shear force normal to the shell surface and biaxial bending moments. These, in turn, produce a uniaxial shear stress (which varies parabolically from zero at the surface to a maximum in the middle) and biaxial bending stresses (which vary from compression on one surface to tension on the other). Fortunately, discontinuity stresses are of a rather localized nature, since they tend to diminish rapidly as the distance from the discontinuity is increased.

In rocket cases, the major discontinuities are the junctions of the cylinder with the end closures and the skirts and of the aft closure with the nozzle ports. For ease of manufacture, it is the practice, for rocket cases, to use the same wall thickness throughout the entire case, except possibly in the closures. Various geometrical configurations have been used for closures. The most popular configurations are approximately ellipsoids of revolution, with the minor axis coinciding with the axis of the cylindrical portion of the case. In general, for a multistage vehicle, the interstage structure, which connects two adjacent stages, is a highly loaded structure and thus is relatively heavy. Therefore, in order to achieve a high mass fraction*** for the stage, it is generally necessary to use a closure which is as shallow as possible. The limiting factors here are maximum discontinuity stresses and circumferential buckling (structural instability) due to compressive hoop stresses in the highly curved "knuckle" region of the dome if the closure is too flat.

There is no choice in nozzle location for a single-nozzle design; however, for a multinozzle configuration, from a stress standpoint, it is desirable to locate the nozzle ports where the maximum effective stress in the original unported dome is smallest. Of course, this may not always be possible due to internal-gas-flow considerations or to nozzle-port rotation when the case is pressurized.

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*The discussion is limited to cylindrical cases, which are by far the most prevalent, although spherical cases are currently in use, primarily on small sizes for auxiliary purposes.

**A more comprehensive, yet basic, discussion of membrane and discontinuity stresses in thin shells is given in Chapter 8 of Reference (1).

***The term mass fraction is defined at the beginning of the section on Structural Weight.
Since the nozzles introduce discontinuity stresses in the aft closure, it is necessary to provide some kind of local reinforcement of each nozzle port. This can be done by adding a thick ring reinforcement or by gradually increasing the wall thickness in the vicinity of the ports. The latter method is more efficient structurally, but sometimes increases the fabrication costs.

By way of review, the external loads acting on a rocket case may include axial compression, longitudinal shear at the case-propellant interface*, longitudinal bending, a torque varying linearly along the axis**, external pressure (as in a blast), localized loads along a circumferential band (such as occur in ground handling), and thermal gradients. Each of these loads may affect the strength of the case in two ways:

1. By combining with the stresses due to internal pressure so as to either increase or decrease the maximum effective stress

2. By inducing buckling (structural instability).

Monolithic metallic cases generally have sufficient "wall stiffness" (the product of elastic modulus and wall thickness) to prevent buckling from being critical, except in a few designs having extremely high compression or bending loads. There is some additional buckling resistance over that of the case itself by virtue of the internal burning pressure and the propellant grain (even before burning starts). For example, for a typical motor case, internal pressure gives approximately a 76 per cent increase in buckling resistance over the unpressurized case alone(6) and a case containing a case-bonded propellant has approximately 52 per cent more buckling resistance than the case alone(7). The former increase of course is available to the first-stage case at all times, while the latter increase is available to all of the higher stages during the early boost period when the external loads are most critical.

The total buckling load applied as a uniform longitudinal shear stress (due to axial acceleration of the propellant) depends on the case length, diameter, thickness, and material. However, for cases of current interest, it is sufficiently accurate from the standpoint of buckling to consider the total inertia force to act as a simple axial compressive load.(8)

Thermal stresses are produced by over-all changes of temperature in structures composed of materials with different thermal-expansion coefficients, by thermal gradients in restrained structures, and by nonlinear thermal gradients. Even in a monolithic metallic case, the thermal expansions of three different materials are involved: the propellant, the insulation, and the case itself. Thermal stresses are in general proportional to the product of the elastic modulus and the thermal-expansion coefficient. To date, the thermal degradation of strength and of elastic modulus (important in connection with buckling) with increasing temperature has been more critical in case design than thermal stresses; the former topic is discussed in the section on Environmental Effects. However, thermal stresses have been an important factor in determining the lower limit on storage temperature. Solid propellants usually have a much higher thermal-expansion coefficient than the case materials, and curing temperatures for case-bonded propellants are well above room temperature; thus, the tensile thermal strains set up in the propellant grain must be limited to a permissible value to prevent fracture of the grain. Recent suggestions to circumvent this limitation are to continuously wind glass filaments(4)

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*The load is due to the axial acceleration acting on the propellant mass.  
**The load is due to rotational acceleration acting on the propellant mass. This would occur in spin-stabilized stages.
or high-strength steel bars of a special cross section directly on the propellant at the launch site.

**Composite Cases**

Although there are many different kinds of composite structures, filament-wound structures are the most popular type for rocket-motor cases. Filament winding is the process of continuously wrapping individual wires or strands of glass filaments (usually wetted with resin) on a mandrel; then curing the resin and removing the mandrel to leave a filament-wound composite structure.

Perhaps the greatest advantage of filament-wound composite structures in general (regardless of filament material) is the inherent potential advantage due to the flexibility of design and manufacture. If a greater strength (on a stress basis) is required in a given direction, either more windings can be oriented in that direction or the windings can be oriented at such a helix angle that the required directional strength characteristics can be achieved. Thus, for a motor case in which stresses due to internal pressure are critical, one can either use twice as many circumferential windings as axial or one can wrap the windings at an angle of 54.7 degrees (arc tangent of √2) measured from the axis of the case. Also, one can provide more windings at highly loaded locations. Thus, it is possible, in theory, to achieve relatively easily equal-stress configurations, which are highly efficient from a weight and material-utilization standpoint. Analyses of this type are usually called netting analyses, since they consider only the windings and even consider them to be perfectly flexible (i.e., to have no bending or shearing stiffness). The resulting netting configurations, some of which are applicable to cylindrical motor cases having end closures with polar openings only, are geodesics, often called geometric isotensoids. However, in practice, some of this efficiency is lost due to: (1) an excess of bonding material filling the voids but not contributing to the strength, (2) the netting geometrical requirements for more windings near the axis, and (3) equal openings at each end of the case, and complexity of the winding pattern.

Recently, in order to circumvent some of the practical difficulties in the use of geodesic netting patterns, various other end-closure configurations and helix angles have been proposed.

It has long been known that drawing of steel into very small-diameter wire (music wire) increases its strength considerably, resulting in wire strength/density of slightly over 2 million psi/pci. Similarly, small filaments of glass (often termed fiber glass) exhibiting strength/density of nearly 3 million psi/pci have long been known. These strength/density values of filaments have presented a real challenge to materials engineers to utilize some of their potential in composites so as to achieve lower weights than possible with monolithic cases.

Although wire-wrapped metallic pressure vessels have been used at least since before World War II, their introduction into motor-case use has been quite recent. Typical values which have been obtained in wire-wrapped resin-bonded motor cases are as follows.
Composite strength* / density, psi/pci 692,300 to 1,102,000
Longitudinal elastic modulus, psi 5 x 10^6 to 9 x 10^6
Circumferential elastic modulus, psi 12 x 10^6 to 18 x 10^6

Due to the rapid development of glass-filament-wound motor cases, there has been a decrease in interest in wire-wrapped cases. However, the use of beryllium wire shows sufficient potential that there may be some interest in its use as a wire wrap for motor cases of the future.

Although glass-filament-wound rocket cases have been under intensive development since 1947(17) from several aspects (design and materials, including glass, resin, and coupling agent), reliably attainable composite strength/density values are still only approximately 1.67 million psi/pci.**

Since the procedures and terminology used in glass-filament winding is somewhat peculiar to this field, it is well to define some terminology as follows:

A glass filament is an individual, long (essentially continuous) piece of glass approximately 4 to 12 microns in diameter. These very fine filaments are gathered into a bundle called an end or strand. There are usually either 204 or 408 filaments per end.

There are three major methods of packaging these strands; the most widely used one for filament winding results in a loose, very slightly twisted group of strands known as a roving.

There are two basic methods of filament winding:

1. **Wet winding**, in which the resin is applied wet to the roving just prior to winding.

2. **Dry winding**, in which a preimpregnated roving ("prepreg"), already containing some resin and partially cured, is used. Then final curing is accomplished by application of pressure and temperature for an adequate time.

The resins usually used in filament-wound structures are of either the epoxy or polyester types. Epoxy resins are the more popular at present due to their lower shrinkage and stronger resin/glass bond.

There are additional bonuses and also many problems associated with the use of glass-filament-wound cases. One bonus is the lack of catastrophic (brittle) failures often associated with cases of certain low-ductility metals. This aspect is discussed in the section on Static Mechanical Behavior. Another bonus is a pressure drop across the propellant grain on the order of 30 to 100 psi. (18) This is due to a decrease in the ratio of the case radial stiffness to the propellant stiffness compared with the same ratio for a metallic case, thus, the propellant grain carries some of the pressure load. Optimum advantage can be taken of this factor only when (1) the combustion pressure is relatively low or when the propellant has high ductility (otherwise grain cracking may occur during firing), and (2) the highest burning pressure occurs early in the firing (otherwise, when the grain has burned away and thus cannot carry much pressure, the case would be excessively loaded).

*Composite strength is defined in the section on Structural-Weight Material Indexes.
**This value is for Type E glass with an HTS finish, wound by numerical control. In the near future, with the introduction of high-strength X554 glass fiber, even higher values, probably over 2 million psi/pci, can be expected.
Problems associated with the use of glass-filament-wound cases include:

(1) The need for metallic inserts, filament-wound in place, for the forward-boss and skirt attachments, and for nonpolar openings, when required, in the end closures. In addition to the weight penalty of the attachment, difficulties have been encountered with shear failures at the bond, but these have been eliminated in at least one instance by use of an elastomeric adhesive.

(2) The severe degradation of the elastic modulus and strength at moderately elevated temperatures (discussed in more detail in the section on Environmental Effects). This problem may be minimized somewhat by the use of more insulation.

(3) Stress crazing of the resin and resin failures due to the excessive strain concentration in filament-wound resin structures loaded in tension perpendicular to the filament direction (i.e., the axial tension in a hydrotest of a circumferentially wrapped motor case).

These phenomena do not necessarily cause premature failure, and several approaches toward eliminating them, or at least reducing their severity, are under way.

(4) Detrimental effects of breakage of one or more individual filaments within the composite. These are closely related to the distance required to transfer shear through the resin across the gap between the broken parts of the filaments.

(5) Abrasion damage to glass filaments during winding processes. The present state of the art is such that this problem has not been completely eliminated. This abrasion damage can be minimized by the application of suitable coupling agents and sizes.

(6) Scatter (large standard deviation in strength values. In a motor case this could be due to items (3) and (4) above, plus voids in the resin, inadequate coupling agents and resin coatings, and the inherent scatter in the tensile strength of the glass filaments themselves. However, standard deviations as low as 3.5 per cent of the mean have recently been achieved. This is comparable to values obtained with monolithic metallic structures.

(7) Resin shrinkage, largely thermal shrinkage due to cooling from the curing temperature to room temperature.

(8) The present difficulty in making a filament-wound motor case with a removable end closure. With a monolithic metallic case, however, this is being done rather easily.
The determination of the critical buckling loads for any filament-wound structure is complicated by the fact that its structural behavior is anisotropic. Although the relationship between axial and hoop strengths against internal pressure can readily be determined by the number of windings in each direction, the picture is much more complicated for buckling. Also glass-filament-wound cases have had a poor reputation in regard to buckling resistance. However, on the basis of weight to resist buckling due to a given load (as represented by the structural index for buckling, \(\sqrt{E/d}\)), glass-filament cases are currently more efficient than steel ones. (See the section on Structural-Weight Material Indexes).

New developments in glass-filament winding include investigation of preimpregnated ("pre-preg") rovings to replace the wet-winding method currently used. Results do not appear to be favorable enough yet to warrant a change at this time. (27) Another new development of promise is high-strength glass fibers with a fiber strength/density of 7.8 million psi/pci. (28)

A recent milestone in the development of glass-filament winding is the fabrication of a 30,000 gallon, 13-foot-diameter booster case. (29) Serious consideration is being given to the advantages of filament winding Nova-class SPR space boosters at the launch site.

Some interesting design concepts to try to improve the buckling strength of glass-filament-wound motor cases include hollow filaments (to achieve greater bending stiffness per unit weight) and ordinary filaments wrapped over a series of longitudinal metallic tubes welded together in a cylindrical configuration.

A design concept that would eliminate the filament-wound structural problems which are due to resin weakness is an all-metal filament-wound construction. The basic idea here would be to eliminate the resin and substitute either a welded or brazed joint instead. There are obviously many problems of a fabricational and metallurgical nature which would have to be worked out before a motor case could be successfully made in this fashion.

In view of the limitations of glass-filament-wound cases as described above, several new composite-structural concepts have recently been under development for rocket-case applications. One of these is the glass-filament-wound aluminum-alloy case which has achieved strength/density and buckling indexes slightly better than those for current glass-filament cases. Other advantages claimed for glass-aluminum composite cases are cost, weight, and reliability advantages in short cases with multiple openings and in motor cases which are so large that a segmented design is mandatory. (30) In such instances, the glass-aluminum composite should give a much more simple design at the attachments and fittings.

Another new composite-structural concept which is currently under development is a sandwich case consisting of glass-fiber-resin facings and an aluminum honeycomb core. Although this type of structure would not be advantageous for designs in which internal pressure is the critical loading, it appears to be quite competitive, on a weight basis.
basis, with glass-aluminum composites for designs governed by buckling. It is claimed\(^{(5)}\) that the weight required for such a case is independent of axial load; this is unlike monolithic, filament-wound, and other nonsandwich-type cases in which the weight increases with axial load at sufficiently high load values.

Conventional all-metal sandwich structures (even when the metal is beryllium) are not as light in weight, for the same load-carrying capacity, as the glass-filament/aluminum sandwich structure described above. However, in certain instances such as in very large first-stage boosters, where minimum cost is more important than minimum weight, this type of composite construction should be considered. Various core geometrical configurations have been investigated, including honeycomb core, truss or corrugated core, and dimpled core.

**STRUCTURAL SAFETY AND RELIABILITY**

In current missile design usage, there are two general philosophies or methods of approach to determining the relationship between mechanical properties of structural materials and calculated structural parameters. The older of the two philosophies is the uniform factor-of-safety approach and the other is the statistical reliability approach, which also utilizes factors of safety.

**Uniform Factor of Safety Approach**

In designing aerospace structures, following the long-established practice in civil engineering structures, it has become customary to incorporate so-called factors of safety. A factor of safety is really a contingency and ignorance factor to provide for unexpected contingencies and uncertainties in determining loading spectra, computing stress or buckling phenomena, and determining material properties. Numerous factors of safety are in current use; these depend on the kind of service, type of loading, and mode of failure involved.

A factor of safety can be defined broadly as the ratio of the level of a quantity which will produce failure (by a particular mode and for a particular type of service and kind of loading) to the design value (or expected value) of the same quantity. The "quantity" referred to may be a load (such as the axial compressive load or bending moment which will cause buckling) or a stress value (such as the yield strength in tension, \(F_{ty}\), or ultimate tensile strength, \(F_{tu}\)).

For the case of combined loadings which induce buckling, various interaction equations are in use\(^*\). For loadings which produce failure due to excessive stress, principles of combined stresses** and an appropriate combined-stress failure envelope (such as the one defined for yielding in the section on Biaxial Stress-Strain Behavior) is used.

\(^*\)See Eq. (3) in Appendix A of this report for an example of an interaction equation. Numerous interaction equations are presented on pages 106 of Reference (1).

\(^**\)See any text on "Failure-stress analysis (strength of materials)".
The values used for factors of safety in missile design are carryovers from the values used successively for many years in the aircraft industry. In aircraft design, a factor of safety of 1.5 is used for ultimate failure and a factor of safety of 1.0 is used for yielding. The design of a given structural component is then governed by either $F_{tu}/1.5$ or $F_{ty}/1.0$, whichever is the smaller of the two. For materials with ratios of $F_{tu}$ to $F_{ty}$ of approximately 1.5, which was typical of many common aircraft materials used in the 1930's and 1940's, there was little difference between the values of $F_{tu}/1.5$ and $F_{ty}/1.0$. However, as materials of higher strength were developed, the $F_{tu}/F_{ty}$ ratio continually decreased. Thus, modern aircraft design came to be governed by $F_{tu}$ rather than $F_{ty}$.

The general philosophy behind the establishment of factors of safety for missile structural design was that, if failure of a structural component would result in loss of life to either crew members or ground personnel, the factor of safety should be the same as for aircraft design. However, for unmanned-vehicle structural components in which failure would not result in loss of life, it was believed that the ultimate factor of safety could be decreased; this resulted in some weight savings. Later, due to some early experience with pressurized components which would cause loss of life if they failed, it was decided to use an ultimate safety factor of 2.0 for pressurized components.

Typical minimum structural factors of safety in current missile-design use are summarized in Table 4.

### Table 4. Typical Minimum Factors of Safety for Missile Structures

<table>
<thead>
<tr>
<th>Condition</th>
<th>Based on Yield Strength</th>
<th>Based on Ultimate Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unmanned vehicles and no hazard to personnel involved</td>
<td>1.00</td>
<td>1.10 to</td>
</tr>
<tr>
<td>Unpressurized components in manned vehicles or involving personnel hazard</td>
<td>1.00</td>
<td>1.50</td>
</tr>
<tr>
<td>Pressurized components in manned vehicles or involving personnel hazard</td>
<td>1.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

It is the usual practice to use a factor of safety of 1.0 for buckling failures, regardless of whether the phenomenon occurs in the elastic, elastoplastic, or plastic ranges of the material. In the past it has been customary to use buckling loads determined experimentally, since theoretically determined buckling loads or shell-type structures have been found to be "unconservative" (i.e., higher than the actual values determined experimentally). These discrepancies are probably due to geometrical imperfections in actual structures and to the large (nonlinear) deflections that actually occur, while the theory assumes small (linear) deflections.

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*In aircraft parlance, the maximum expected load divided by 1.5 was called the limit load, and it was expected that no permanent deformation would result from this load.

† In a more technical sense, this is not to be confused with nonlinearity of the stress-strain relation, such as that associated with strain hardening.*
Due to minimum-weight considerations, there is considerable impetus toward reducing the factors of safety, where consistent with maintaining an acceptable reliability level. Lowering the factor of safety can result in a decrease in structural reliability. With the growing application of statistical reliability concepts to electronic components and systems, particularly in missile applications, it was natural to apply this approach to missile structures. In this approach to structural integrity, structural reliability is quantitatively defined as the probability of success of the structure in performing its function under the operational environment assumed for the design.

Since structural reliability is defined as a probability, in order to assess structural reliability it is necessary to consider the statistical variation of both applied stresses and the strength of the structure. As a hypothetical example, Figure 4 illustrates results of a number of experimental measurements of stress due to applied load and of failure strength of a structure. For convenience, both types of data are plotted in the same units, viz., stress, expressed in ksi. Individual stress values are plotted on the left. The resulting frequency distribution is plotted on the right.

In the following considerations, it is assumed that the frequency distributions are normal (Gaussian) distributions* and changes with respect to time are neglected. A normal distribution is completely defined by the mean value and the standard deviation, which is a measure of variability or scatter about the mean.

For the normal distributions shown in Figure 4, there is an overlap of the applied-stress and strength distributions. The definition of structural reliability given previously can now be restated as follows: structural reliability is the probability that the strength exceeds the applied stress. Then, still assuming normal distributions, structural reliability depends upon the standard deviations and the difference between the means. The crux of designing a structural component to a desired reliability level is the proper selection of this difference between the means. This difference between the means is called the margin of safety and can be expressed in the following ways:

1. As a fixed stress difference in psi (or ksi).
2. As a fixed ratio** of the above difference to the applied stress.
3. As a fixed number of standard deviations, \( \sigma \), of the differences between the individual values of the strength and applied load. Statistically, this quantity is given by

\[ \tau = \sqrt{\sigma_1^2 - \frac{1}{\sigma_2^2}} \]

where \( \sigma_1 \) and \( \sigma_2 \) are the respective standard deviations of the strength and applied stress.

---

*Any other mathematical function which fits the actual data better can be used, if desired.
**This ratio is directly related to the factor of safety. In fact, it is the factor of safety minus one.
FIGURE 4. EXAMPLE SHOWING VARIATIONS IN STRENGTH AND APPLIED STRESS

Factor of safety is 1.33; structural reliability is 0.9871, assuming normal distributions.
To ensure a uniform structural reliability, it is necessary to use the third method of specifying the margin of safety. As a guide in selecting the margin of safety appropriate to any desired structural reliability level, the following tabulation is useful:

<table>
<thead>
<tr>
<th>Structural Reliability Level</th>
<th>Margin of Safety, numbers of standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000</td>
<td>0</td>
</tr>
<tr>
<td>0.9000</td>
<td>1.28</td>
</tr>
<tr>
<td>0.9500</td>
<td>1.64</td>
</tr>
<tr>
<td>0.9900</td>
<td>2.33</td>
</tr>
<tr>
<td>0.9990</td>
<td>3.09</td>
</tr>
<tr>
<td>0.9999</td>
<td>3.71</td>
</tr>
</tbody>
</table>

In this tabulation it is noted that any increase in margin of safety beyond approximately three standard deviations provides very little increase in reliability.

As an example, it is conceivable that a structure could have a higher mean strength (and thus a higher factor of safety, assuming the same load distribution) than the structure in Figure 4 and yet have a lower reliability, provided that the higher-mean-strength structure has a wider scatter in its strength values. In other words, factors of safety are still used in the statistical reliability approach, but they vary depending upon the frequency distributions of the strength and applied-stress values.

Although the approach described here is a sound one, it is only one of many variations of the same statistical reliability approach. Some of the other approaches in use are more sophisticated; others are quite simple (and more uncertain) — for instance, use of material-property minimum values is a simple attempt to achieve a more uniform reliability level throughout a structure. The growing use of the statistical reliability approach to structural integrity has given considerable impetus to statistical analysis of applied load spectra, buckling loads, burst strengths, and material properties. As the use of materials with lower ductility increases, it is expected that the statistical reliability approach will become increasingly important.

The above discussion was limited to a simple structural element and did not consider complicating factors such as series components, parallel components, and multiple modes of failure. These factors are briefly discussed in the next few paragraphs.

When structural components function in series fashion, such as the links of a chain, the over-all structural reliability, \( R \), is equal to the product of the structural reliabilities, \( r_i \), of the individual components. Thus,

\[
R = r_1 \times r_2 \ldots \times r_n
\]

where \( n \) is the total number of components in series. In this instance, if there is a large number of components, the reliabilities of the individual components must be quite high in order to achieve a reasonable over-all reliability.

*Note: The terms "series" and "parallel" refer to functional considerations, not to physical arrangement. Thus, the technical definition of a series system is one in which each component functions independently of any other component, and the system fails when any one component fails. Similarly, a parallel (or redundant) system is one in which each component function independently of any other component, but the system does not fail unless all of the components fail (since the components have diverse functions).
Another way of arranging multiple components is to put them in parallel. Such a system is called a redundant system and its over-all reliability is given by

\[ R = 1 - (1 - r_1)(1 - r_2) \ldots (1 - r_n). \]

In a redundant structural system, the over-all reliability is greater than the reliabilities of any of the individual-component reliabilities.

In actual structures, it takes considerable judgment on the part of the designer, backed by sufficient experimental failure data, to determine the type of interactions, if any, which exist between various modes of failure. For example, if all of the modes of failure of a structure are independent of each other and there are no parallel load paths, the structure can be considered to be a series system. However, if there are interactions between the various failure modes, the strength of the component may be determined by the worst combination of failure modes interacting with each other.

**STATIC MECHANICAL BEHAVIOR**

**Biaxial Stress-Strain Behavior**

A biaxial stress field is a stress system in which two of the three principal stresses are not negligible. In a motor case, the negligible principal stress is the one in the thickness direction. This is in contrast to a uniaxial stress field, such as that acting in an ordinary tensile specimen, which has only one principal stress of significance.

The biaxial ratio is defined as the ratio of one of the biaxial principal stresses to the other, the latter being taken as a reference. It is customary to take the circumferential direction as the reference direction. Thus, in the cylindrical portion of a motor case, the biaxial ratio is simply the ratio of the axial stress to the circumferential stress. Using elementary statics, it can be shown that the biaxial ratio in the cylindrical portion of a case under internal pressure only is 0.5, except near the end closures. The biaxial ratio in the vicinity of the closures and in the closures themselves depends on the geometrical configurations of the end closures. When a case is subjected to axial compression in addition to internal pressure, the biaxial ratio decreases from the 0.5 value, the exact amount depending on the relative magnitude of the externally applied compressive stress.

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*Here the terms "series" and "parallel" refer to functional considerations, not to physical arrangement. Thus, the technical definition of a series system is one in which each component functions independently of any other component, and the system fails when any one component fails. Similarly, a parallel (or redundant) system is one in which each component functions independently of any other component, but the system does not fail unless all of the components fail, since the components have duplicate functions.

**If one properly selects the orientation of a small element of a structure, there is always some orientation in which there are no shear forces acting, only normal (direct) stresses. The values of the normal stresses acting on an element oriented in such a fashion is called principal stress. The numerically largest of these is termed the maximum principal stress.*
The effective modulus of elasticity is defined as the slope of the elastic (straight-line) portion of the maximum principal stress – principal strain curve. For tension-tension loading (biaxial loading in which the biaxial principal stresses are both tension stresses), the effective modulus is always larger than the uniaxial modulus, while for tension-compression loading, the modulus is always smaller than the uniaxial one. In the plastic range, a tension-tension biaxiality raises the stress ordinate of the stress-strain curve and decreases the strain abscissa. Typical stress-strain curves for AISI 4340 steel, heat-treated to an ultimate tensile strength of 260,000 psi and subjected to biaxial tension loading, are shown in Figure 5. (33)

In uniaxial tension, the usually accepted criterion for determining the yield strength is the stress value corresponding to a plastic strain (offset value) of 0.2 per cent. However, in biaxial stress fields, the effect of this same offset is not equivalent, in terms of yielding, to its effect under uniaxial conditions. Therefore, a different criterion, which gives a biaxial offset strain equivalent to a given uniaxial offset strain, must be used. (34) This offset under biaxial conditions is always less than the uniaxial value. For a biaxial ratio of 0.5 (and 2.0), the offset strain is 86.6 per cent of the uniaxial offset; for a biaxial ratio of unity, the offset strain reaches its minimum value of one-half of the uniaxial offset.

Theoretically, for a perfectly ductile material, there is an increase in yield strength of 15 per cent (compared to the uniaxial tension yield strength) at a biaxial ratio of 0.5. However, for many actual rocket-case materials, the increase in yield strength under such conditions is slightly less. For example, AISI 4340 steel over a range of heat treatments from 180,000 to 260,000 psi $F_{tu}$ has an increase of approximately 12 per cent, as shown in Figure 6. Since the yield strength depends only upon the material, condition, size, biaxial ratio, and test temperature, it can be considered to be strictly a property of the material.

In the motor-case field, the use of a proof test has become customary. In connection with this test, there is a pressure and a corresponding hoop stress at which no appreciable yielding takes place in hydrotest. In actual practice, it is difficult to determine the exact stress corresponding to proof conditions; thus, the use of a slightly modified definition is necessary. The proof stress is then defined as the stress corresponding to an arbitrary small plastic strain, usually 0.01 per cent. The proof stress can be determined in various ways, such as from full-scale motor-chamber hydrotest or from small-scale cylindrical specimen tests (such as used to obtain the curves in Figure 6).

Unlike the ultimate tensile strength as determined in a simple tensile test, the ultimate strength of a motor case is not a fundamental material property. In motor-case design, the burst strength, as determined in hydrotests, is the ultimate strength property used for design purposes. This subject is treated in the next section.

Burst Behavior of Motor Cases

The ultimate tensile strength as determined in a simple uniaxial tensile test depends only upon the material, condition, specimen size, and test temperature. The
FIGURE 5. TYPICAL BIAXIAL STRESS-STRAIN CURVES AT ROOM TEMPERATURE FOR AISI 4340 ALLOY-STEEL CYLINDERS, $F_{tu} = 260$ KSI

A biaxial ratio, $B$, of zero corresponds to the circumferential direction. Data from Reference 33.
Figure 6. Biaxial Yield-Strength Envelope at Room Temperature for AISI 4340 Alloy-Steel Cylinders, $F_{tu} = 180$ to 260 KSI

$F_{ty}$ and $F_{tu}$ measured in the circumferential direction. Based on data from Reference 33.
burst strength* of a motor case is known to depend upon all of these factors and also upon the geometrical configuration (cylinder, sphere, or flat sheet). For a cylindrical case, it is further dependent on the ratio of the cylinder length to its diameter and on the shape, thickness, material, and condition of the end closures. Although a general theoretical analysis taking all of these factors into account has not yet been accomplished, experimental evidence suggests that the shorter the vessel in relation to its diameter and the stiffer its end domes in relation to the cylinder stiffness, the higher will be the burst strength. (35) This is in keeping with the concept that a short vessel owes its additional strength to the girdle-restraint effect of the end closures. (36) Also, from theoretical and experimental studies, for instance Reference (37), it is known that the lower the strain hardening**, the greater will be the burst strength/ultimate tensile strength ratio.

Effect of Small Flaws

For internal-pressure vessels made of materials with relatively high ductility***, there is usually no difficulty in reaching the burst strength of the vessel even when fairly severe local flaws are present. However, steels with high strength/density, achieved by tempering at low temperatures, have low strain-hardening and ductility as measured in a uniaxial tensile test. This means that stress raisers due to local flaws of a metallurgical or mechanical nature do not have sufficient opportunity to "smooth out" and decrease in value. Consequently, the high local stresses reached at such flaws, even at quite low values of the nominal stress, can be sufficient to produce premature failure. The appearance of failures due to such flaws gave little if any evidence of plastic deformation, so such flaw-initiated failures are termed brittle or cleavage fractures. Furthermore, such failures occur catastrophically, that is, with one or more cracks propagating at high speeds. This topic has been treated extensively in previous DMIC reports. (38-40) Some of its design implications are discussed briefly here.

The flaw-failure phenomenon very definitely limits the maximum heat-treatment level practicable for sheet materials, as shown qualitatively in Figure 7 for a typical high-strength, low-ductility material.**** Several different approaches have been used to predict this phenomenon quantitatively on the basis of theory. One of these approaches is based on ductility considerations in conjunction with the concentration of strain at a flaw. (41) Another approach is based on fracture toughness as measured in notched-sheet tensile tests. (42) Although brittle failure has been observed in uniaxial tensile tests on flat sheet specimens containing a flaw, as well as in rocket-motor cases, the effect of various amounts of biaxiality on the phenomenon is not known quantitatively at present.

Some of the early problems of brittle fracture in rocket cases have been minimized with increased experience in welding high-strength, low-ductility metals. Improved case manufacturing methods which eliminate longitudinal welds have also improved the situation. Additional relief has been provided by improved inspection techniques.

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* Burst strength refers to the nominal hoop-stress level corresponding to the maximum (burst) pressure in hydro-test of a vessel failing with sufficient ductility rather than in a local flaw. Strength in the presence of flaws is discussed in the next section.

** Strain hardening refers to the slope increase in stress as the strain is increased in the plastic range of the stress-strain curve.

*** The standard quantitative estimation of ductility, it can be expressed by total elongation in a tensile test per cent. In steel, it is more useful to rate-type fracture appearance or various measures of fracture toughness.

**** For this qualitative effect, has been found to occur in titanium alloys as well as in ferrous and stainless steels.
FIGURE 7. EFFECT OF ULTIMATE TENSILE STRENGTH AS OBTAINED BY HEAT TREATMENT ON THE HOOP STRESS AT BURST FOR MOTOR CASES OF HIGH-STRENGTH, LOW-DUCTILITY MATERIAL CONTAINING SMALL FLAWS.
The strength in the presence of a flaw of a given type decreases quite severely as the length of the initial flaw is increased. A number of studies are being conducted to predict such effects. It appears that definite progress is being made in determining the strength of a given sheet material in the presence of a flaw of given length. Unfortunately, this is only half of the problem. The other half is concerned with inspection of sheet material to determine the severity of flaws present. Factors affecting the severity include the flaw type, configuration, and dimensions, particularly either the sharpest radius or the sharpest radius oriented perpendicular to the largest tensile principal stress (usually the hoop stress in a rocket case).

The problems involving failure initiation in flaws in monolithic cases of high-strength, low-ductility metals have helped to focus attention on alternative materials—notably glass-filament-wound composites.

**FATIGUE BEHAVIOR**

In considering a missile or a space vehicle, the problems of premature failure of components by fatigue might seem a remote possibility. However, this has not been true in practice. The severe environment is frequently more significant than the rather short exposure times. Failure can be generated by very-high-frequency stresses from acoustic sources. On the other hand, in some cases, failure has occurred in a low number of cycles. These two aspects—low-cycle fatigue and acoustic fatigue—can be of great significance in design of the missile system.

**Low-Cycle Fatigue**

Low-cycle fatigue is failure that occurs from repeated cycles of high stresses or strains. The cycles to cause failure may range from 1 to about 10,000 cycles. In view of the short lifetimes observed and the high stresses that are required to cause such failure, it is agreed that low-cycle fatigue results from an accumulation of repeated plastic strain.

One major problem in rocket-motor cases has been concerned with low-cycle fatigue resulting from mechanical loading.

The present problem has grown out of attempts to achieve high performance in rocket-motor cases. In service these cases are subjected to internal pressure high enough to produce nominal hoop stresses near the yield strength of the case material. At various stages of manufacture, the cases are proof tested to design stress with a number of internal pressure cycles. Sometimes the magnitude of the internal pressure is increased with each cycle. Some failures of cases have occurred in very few pressure cycles and frequently at stress levels substantially below design pressure. Depending upon the material, these failures may be "brittle" in nature. This type of behavior represents more than a fatigue phenomenon in the usual sense. The mechanism of case fracture, while stimulated by the high repeated pressure, may actually result from the growth of microscopic flaws to a size where rapid crack propagation (associated with some critical flaw length) may occur (this was discussed more fully in the preceding
Failures have occurred in these hydrotests in the vicinity of weld discontinu-
ities, either in the case body or in closures. These types of failures lend credence to
the thought that hydrotesting may actually decrease reliability since existing flaws may
progressively grow with successive pressure cycles. At the end of hydrotesting, the flaw may be at a critical size to propagate rapidly during the next pressure cycle
(namely, firing of the booster).

In addition to improvements in welding techniques and elimination of longitudinal
welds, the use of an improved proof-testing procedure, such as the one suggested by
Corten may be helpful in alleviating this type of low-cycle fatigue.

To provide some insight into how fatigue considerations may affect material selec-
tion, Figure 8 has been prepared from data in the literature. Three materials are
represented on the figure: (1) D-6ac steel heat treated to $F_{tu} = 270$ ksi, (2) Ti-6Al-
4V heat treated to 170 ksi, and (3) glass filament-resin composite. These materi-
als are representative of those currently considered for use in rocket-motor cases.

The fatigue curves in the figure are presented with units of stress (as a per cent of
ultimate tensile strength) as the ordinate and $\log N$ as the abscissa, where $N$ is the num-
ber of cycles to failure. In the case of the steel and titanium alloys, the data were ob-
tained from axial-load fatigue tests of simple specimens (i.e., uniaxial loading) where
the stress for a given test ranged from zero to some maximum tensile value. The glass
filament-resin composites actually were filament-wound spherical pressure vessels,
repeatedly pressurised from ambient pressure to some maximum pressure. Under this
condition, the stresses also ranged from zero to a maximum value. The state of stress
was balanced biaxial tension (i.e., the biaxial ratio was 1, as defined in the section on
Biaxial Stress-Strain Behavior).

The data clearly show that, over all the lifetime range studied, there is an appreci-
able loss in relative fatigue strength of a filament-wound composite in comparison with
that of the titanium and steel alloys. It is possible that the biaxial stresses in the com-
posite may have some effect.

Acoustic Fatigue

Another source of fatigue damage that must be considered in the design of missiles
and space vehicles is that associated with fluctuating sound pressure. This problem is
of interest in rocket-motor-case design since pressure fluctuations occur during the fir-
ing of the engine and during early powered flight. The pressure fluctuations are random in nature and cover a wide range of frequencies. It is these random pres-
sure fluctuations that can excite resonant dynamic response in the structure. Since fre-
cuencies may range in excess of 10,000 cycles per second at sound levels greater than
170 decibels, accumulation of large numbers of stress cycles (associated with typical
response frequencies) can occur in relatively short periods of time.

There are a number of noise environments that may contribute to the over-all
pressure field. One of the significant sources is the rocket engine itself. During
launch and while the vehicle is slowly gaining speed and altitude, this is the major
source. Since the sound pressure levels associated with each frequency are different
along the length of the vehicle, it is important in considering structural response to know
the characteristics of the sound field. This includes not only the root mean square sound
FIGURE 8. COMPARISON OF FATIGUE BEHAVIOR OF ROCKET-CASE MATERIALS
pressure levels, but also their statistical distribution, since it may be those levels higher than the root mean square level that contribute to fatigue. It should be noted that vehicles launched from a silo experience significantly higher sound levels than do pad-launched vehicles. (49)

At subsonic and supersonic speeds, a turbulent boundary layer exists over much of the vehicle surface. Associated with this turbulent condition are pressure fluctuations. If there is structural response to certain frequencies, there can be damage from the standpoint of acoustic fatigue.

In a missile in supersonic flight, abrupt changes in static pressure (called supersonic shocks) may occur at certain locations, usually at points where marked changes in external geometry occur. These shocks are known to move or oscillate and hence contribute to the noise environment. These pressure fluctuations can be intense and thus constitute an important factor in acoustic fatigue.

There are a number of other noise sources either about which little is known or which are much less important than those described above. These sources include noise radiation from the turbulent flow in the wake of a vehicle, convection of atmospheric turbulence through shock waves, etc.

There are several problems that must be considered in connection with acoustic fatigue. Although it is possible to indicate the various sources of pressure fluctuations, it is not possible at present to describe with high confidence the nature of the pressure fluctuations. This kind of information is now being generated; without it, the problem of evaluating structural response is not precise.

The pressure fluctuations, covering a wide range of frequencies, no doubt excite resonant vibrations in the structure. This response could involve more than one mode of vibration and would be at discrete frequencies in the broad noise spectrum. It is these resonant vibrations that can lead to fatigue failure. One of the problems then is to determine these critical modes and frequencies, and the stresses associated with the likely modes of response.

In order to determine the stresses, the analysis must include an estimate of the vibration energy dissipation (damping). This is necessary because if the structure has high damping, the stresses experienced under resonant vibration will be significantly reduced. Although basic structural materials under oscillating load will dissipate small amounts of energy by internal hysteresis, material damping represents an insignificant effect as contrasted to other damping mechanisms.

Structural damping is one of the more effective mechanisms for dissipating vibration energy. (49, 50) It is frequency insensitive and depends upon amplitude of oscillations. It probably arises chiefly from slip at interfaces of joints. The mode of vibration is an important factor here, since the mode determines which joints are highly loaded and thus contribute significantly to damping.

Techniques which significantly increase damping of a structure are currently being evaluated. (49) These involve the addition of plastic-material, "damping compounds", which when added at proper points in the structure, provide significant reduction in panel amplitudes under resonant vibration. (Some experiments have shown 90 per cent reductions in amplitude and stresses compared with those experienced without damping.)
These materials tend to offer high damping capacity over limited frequency and temperature ranges. In view of the significant effect on acoustic fatigue life, current effort is being directed toward broadening of the frequency and temperature ranges for which such materials have high damping capacity. Also, effort is being made to determine the most efficient way of using these materials.

ENVIRONMENTAL EFFECTS

Several factors are of concern in considering possible effects of environment as it influences design of rocket-motor cases. First, certain thermal inputs may result in relatively steady thermal stress; for example, aerodynamic heating, propellant burning, and radiant heat received during long-range interplanetary travel. Cyclic heat input may set up periodic thermal stresses in vehicle structure when the vehicle is orbiting a planetary body passing into and from the shadow side to the sun side of the body repeatedly. In the former instance, where the motor case is heated aerodynamically or by propellant burning during liftoff and climb, thermal stresses will be superimposed upon high stresses due to external and internal loads. The motor-case-material selection, design stresses, and certain design details under such conditions probably will be governed by the mechanical strength of the case material at the anticipated working temperature of the structure. In the latter case, if orbiting is for long periods of time, an analysis of the periodic thermal stresses would be required to determine whether thermal fatigue would be a problem in the booster of such a vehicle.

Second are the considerations of the space environment—electromagnetic and particle radiation, solid-particle impingement, and vacuum effects. These considerations are discussed in the sections that follow. In these discussions it is assumed that the vehicle under consideration is one that will be orbiting the earth for long periods of time or is on an interplanetary mission and that a rocket booster attached to the vehicle can be fired on command. Further, the discussion is directed primarily to the motor case and end closures but not to hardware such as nozzles, gimballing devices, etc. In this context, primary interest is in the effect of space environment on mechanical behavior of case materials and certain design features associated with loads generated by firing the booster. Environmental effects on electrical and optical properties, carrier effects for semiconductor devices, and other characteristics are not of interest here.

Mechanical Strength as Affected by Temperature

Rocket-motor-case temperatures from aerodynamic heating and propellant burning are limited to the range 300 F to 600 F. However, for high-performance vehicles, it is probable that higher temperatures may be required. Therefore, the effect of temperature on material behavior is important. Figures 9 through 13 show the effect of temperature on \( F_{tu} \), \( F_{cy} \), \( F_{bru} \), \( F_{su} \), and \( E \) and \( E' \), respectively, of a number of materials that are considered and used in motor cases.\(^{(51)}\) The materials include a high-strength steel, a titanium alloy, and glass filament-epoxy resin composite.
FIGURE 9. EFFECT OF TEMPERATURE ON THE TENSILE STRENGTHS ($F_{tu}$ AND $F_{ty}$) OF ALLOY STEELS, TITANIUM ALLOYS, AND GLASS FILAMENT-EPOXY RESIN COMPOSITES
FIGURE 10. EFFECT OF TEMPERATURE ON THE COMpressive STRENGTH ($F_{cy}$) OF ALLOY STEELS, TITANIUM ALLOYS, AND GLASS FILAMENT-EPOXY RESIN COMPOSITES
FIGURE 11. EFFECT OF TEMPERATURE ON THE BEARING STRENGTHS ($F_{bru}$ AND $F_{bry}$) OF ALLOY STEELS, TITANIUM ALLOYS, AND GLASS FILAMENT-EPOXY RESIN COMPOSITES
Figure 12. Effect of Temperature on the Shear Strength (FSu) of Alloy Steels, Titanium Alloys, and Glass Filament-Epoxy Resin Composites.
Several features are immediately evident from these figures. All mechanical characteristics of all materials decrease as the temperature increases. However, this decrease is most marked for the glass filament-epoxy resin composite. For example, the tensile strength of the glass-filament material at a temperature of 500°F is about 60 per cent that at room temperature. At the same temperature, the steel has a tensile strength 90 per cent of that at room temperature; titanium, 80 per cent. As illustrated in Table 5, shear and bearing strength (which can be important at openings and skirts) and modulus of elasticity (which seriously influences stiffness and buckling resistance) of glass-filament composites are more drastically reduced than tensile strength at a temperature of only 400°F.

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Shear Strength, ( F_{su} )</th>
<th>Ultimate Bearing Strength, ( F_{bru} )</th>
<th>Modulus of Elasticity, ( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>96</td>
<td>92</td>
<td>96</td>
</tr>
<tr>
<td>Titanium</td>
<td>82</td>
<td>86</td>
<td>88</td>
</tr>
<tr>
<td>Glass Filament/Epoxy</td>
<td>45</td>
<td>20</td>
<td>42</td>
</tr>
</tbody>
</table>

From such a tabulation, it is obvious that certain structural indexes applied in preliminary screening (based frequently on room-temperature behavior) should be carefully examined at realistic operating temperatures.

The designer may exercise certain options in design to favor use of a lightweight efficient material. If heating is primarily aerodynamic, investigation of external insulation may show that case temperatures can be reduced to tolerable levels. A similar situation may be helpful if major heating is from propellant burning. Other trade-offs may be possible to alleviate the temperature problem.

**Thermal Stresses and Thermal-Stress Fatigue**

Thermal stresses arise when the temperature of a part of a structure is heated above or cooled below the temperature of the supporting structure. This structure may be designed so that the heated or cooled portion is completely or partially restrained from expanding (if heated) or contracting (if cooled). The thermal stresses may be transients or more or less steady state.

One damaging effect of thermal stresses could arise if thermal strains are high enough to cause plastic deformation. If these strains are repeated, for example during long-time earth orbiting, the possibility of thermal fatigue should be considered. There are a number of influencing factors here that are important to the problem. They relate quite specifically to material selection and certain design details.

With a given temperature rise, materials with large values of thermal conductivity and subject to mechanical constraints develop small thermal strains and stresses (for the
case of skin heating). Low conductivity infers large thermal strains. Thus, conductivity affects the maximum thermal-stress value.

Thermal diffusivity (conductivity divided by specific heat and density) on the other hand determines the rate of development of thermal gradients. Therefore, in transient thermal response, a material with a low diffusivity is desirable.

Thermal-expansion coefficient also determines the magnitude of thermal strains. For repeated plastic thermal strains, a low coefficient of expansion is desirable.

Directly related to the problem of thermal stresses is the maximum temperature. If the temperature is cycled also, the cyclic temperature range influences fatigue behavior. In both cases, the lower the maximum temperature or the smaller the temperature range, the lower is the possibility of fatigue failure.

It is quite likely that materials may be selected on the basis of other characteristics than thermal stress and fatigue. In that event, the physical constants conductivity, diffusivity, and expansion coefficient cannot be arbitrated. As demonstrated by experience with a number of satellite vehicles, the designer has at his option direct means to limit maximum surface temperature and fluctuating temperature for an orbiting vehicle. This can be achieved by insulation and can be further enhanced by special coatings and finishes on the surface. (3)

As stated in an earlier section, the electromagnetic radiation received by a vehicle is composed primarily of that from the sun and secondarily by solar radiation reflected from the earth or another planet and also that radiated by the earth or planet. Internally, heat is generated in the vehicle by communication and other electronic gear, life-support equipment, and the personnel in the vehicle. The stable temperature of the vehicle then is related to the heat balance established between the external heat load, the internal heat load, and the heat loss by radiation from the vehicle.

Since the major external heat load is from the sun (radiating essentially as a black body at 5820 K), retention of this radiant heat in the vehicle is dependent upon the surface area exposed to the sun and the absorptivity of the surface layer. At the same time, the radiant heat loss to interstellar space is a function of the vehicle surface area that is radiating and the emissivity of the surface layer (a low-temperature emission).

It is seen then that the principal factor to establish vehicle temperature in space environment is the ratio of absorptivity, \( a_s \), to low-temperature emissivity, \( e \).

Thus, in order to achieve a low equilibrium temperature, the designer will be looking for surface finishes or treatments that have low solar absorptivity and high values of low-temperature emissivity. It is for this reason that certain oxides used as pigments and techniques such as sandblasting of surfaces have been employed, and are still being examined. (3)

Electromagnetic and Particle Radiation

There are two factors that may be of interest: (1) sputtering and (2) radiation damage. Since thermal effects from electromagnetic radiation have been discussed in the preceding section, this will not be considered a factor in this section.
As a space vehicle orbits the earth or travels in space, it will collide with atoms or ions having a broad range of energy. On impact, these particles tend to knock atoms from the surface. This is the process that is called sputtering. There has been some concern that, over extended periods of exposure, sputtering may result in appreciable loss in metal from the surface and reduction in thickness. Jaffe and Rittenhouse(3) have made some estimates of surface loss based on plausible estimates of the flux of atomic particles impacted and sputtering efficiencies. The following tabulation summarizes their results.

<table>
<thead>
<tr>
<th>Particle Source</th>
<th>Loss in Thickness per Year</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low earth satellite</td>
<td>10^{-10}</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>Peak of inner Van Allen belt</td>
<td>10^{-9}</td>
<td>10^{-1}</td>
</tr>
<tr>
<td>Solar flare protons</td>
<td>10^{-6}</td>
<td>10^{2}</td>
</tr>
<tr>
<td>Steady solar wind protons</td>
<td>3 \times 10^{-8}</td>
<td>3.0</td>
</tr>
<tr>
<td>Cosmic rays</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

It should be apparent on the basis of their estimates that loss in metal thickness as a consequence of sputtering is not an important consideration for present rocket-case designs.

Radiation damage can occur when high-energy atomic particles penetrate into structural materials. The depth of penetration is proportional to the energy level; however, greater damage may occur with lower energy particles of high flux, since damage would occur in a thin layer of material.

Damage can occur by two mechanisms. One of these, ionization, involves the removal of electrons from atoms. For rocket-case materials, this is characteristic of plastics. The second mechanism is displacement of atoms from their positions in the crystal lattice. This type of damage is characteristic for inorganic insulators and metals.

Jaffe and Rittenhouse(3), McCoy(4), and others have concluded generally that mechanical properties of metals are not affected by radiation encountered in space, with the exception that a thin surface layer of metal may be damaged when exposed in the inner Van Allen Belt or when exposed by radiation from solar flares.

Organic materials, on the other hand, are radiation sensitive, and mechanical properties may be significantly altered by long exposure. King, et al.(52), have tabulated considerable information on life expectancy of a large number of polymeric materials based upon an assumed radiation field and an assumed rate of energy absorption. The data are only indicative; however, furane resins, phenolics, polyesters, and styrene polymers showed appreciably long lives to threshold damage and to 25 per cent change in mechanical strength.
In selecting case materials, where damage may be undesirable, it is probably prudent to use the more resistant material in a given class. In some cases, nominal shielding may be adequate to prevent radiation damage.

**Solid-Particle Impingement**

The problem of solid-particle impingement does not appear to apply to rocket-motor cases except for those vehicles that may have a booster attached for subsequent firing in space. The factors to consider are erosion of surface and penetration.

On the basis of available information, surface erosion from meteoric dust does not appear to be a structural problem. Erosion rates in the range 1 to 1000 microns per year have been deduced on the basis of estimates of the assumed mass and distribution of particles. Only if the case surfaces were treated to provide certain absorptivity and emissivity for temperature control might erosion become significant.

Penetration, partial penetration, internal spalling, or delamination may be of structural significance. However, danger from meteoroid puncture is not as great as might be expected, since particles large enough to penetrate also are few in number.

Impact of large particles on high-strength metal cases, even if the case is not completely penetrated, may produce critical flaws that might lead to rupture of the case during firing. Glass-fiber cases also may be weakened with such impacts by minute cracking at the impacted region and by delamination of the fiber layers.

These considerations and others concerned with explosive decompression of pressurized and manned regions of vehicles have led to consideration of shields or bumpers that would be located some distance above the vehicle surface. These bumpers would interrupt impinging meteoroids and serve to fragment them. Thus, the fragments would be dispersed over a wide area of the underlying shell, and presumably decrease the possibility of penetration.

**Vacuum Environment**

A number of factors have been considered important for various components of space vehicles exposed to the hard vacuum of space: sublimation, degradation of organic compounds, changes in mechanical strength, and changes in absorptivity and emissivity. Some of these may be important in considering rocket-motor cases, others, not important.

For example, sublimation of metal and inorganic compounds has been of some interest for some vehicle components. For the practical metals of interest (steels, titanium alloys, aluminum alloys, even beryllium), the temperatures at which sublimation rates are high enough to be of engineering significance are generally much higher than temperatures at which these materials normally are used. Hence, sublimation is not of concern. Similar statements apply to the inorganic compounds. Also, for metals and inorganic compounds, the vacuum environment at temperatures to which the rocket-motor cases will be subjected will not affect the mechanical strength of the materials. Since
sublimation does not occur, absorptivity and reflectivity of the case materials cannot be impaired.

The situation for glass-fiber reinforced plastics is not as optimistic as it is for metals. In the vacuum environment, degradation of long-chain polymeric compounds occurs and results in formation of more volatile fragments. Therefore, with time, significant loss in weight (10 per cent) can occur which results in changes in mechanical properties of engineering significance. Degradation is a function of temperature, higher loss rates occurring at higher temperatures. Consequently, if these materials are used in motor cases that may be subjected to long-time exposure in vacuum, consideration must be given to the operating temperature and to the strength loss. Jaffe and Rittenhouse present available information summarizing the decomposition of polymers on the basis of temperature to produce a weight loss of 10 per cent per year in vacuum. They caution, however, that much of the data are not too reliable and suggest that if detailed information is required, it is necessary to test experimentally.

STRUCTURAL WEIGHT

The objective of a rocket system, whether it has a defense or research mission, is usually to place a desired payload at a given location with a certain velocity. Thus, a dimensionless parameter which is a measure of the efficiency of a vehicle to achieve this objective has come into wide use in the rocket field. This is the mass fraction (or mass ratio), which is defined as the ratio of the propellant mass (before burning) to the total mass (propellant plus inert components) for the particular stage concerned. Obviously, the mass fraction can be thought of as a measure of the weight efficiency. For example, a mass fraction of 0.9 is within current technology; this means that the weight of inert parts accounts for 10 per cent of the total. The higher the mass fraction, the more efficient is the design from a weight standpoint.

It is imperative in current technology that a high mass fraction be achieved in the upper stages of a multistage vehicle if an adequate payload is to be carried. However, in the first stage, and to a certain extent in intermediate stages, the mass fraction is not as critical, so that some other factor, such as thrust-to-weight ratio or cost, is usually the governing criterion.

In order to achieve a high mass fraction, it is obvious that the structural materials must be used efficiently. This means selecting materials and structural configurations so as to minimize the structural weight for a given propellant mass.

Structural-Weight Material Indexes

The total inert-part weight of a solid-propellant rocket includes, in addition to the motor case itself, such items as the skirts, adapters, liners, insulation, grain restrictor, and igniter. Thus, the mass fraction is difficult to evaluate for motor cases made of different materials without going into considerable detail in regard to the various materials used. It is noted that the purpose of using multiple stages is to reduce the energy expended upon masses after they have served their useful purposes. In other words, the purpose of multistaging is to achieve a higher mass fraction.
accessory items. Furthermore, it is difficult to estimate the weight of end closures, particularly the aft one which is a function of nozzle size and arrangement. Therefore, it is desirable to use another means of evaluating structural-weight efficiency. Since a given mass of propellant can be assumed to occupy a given volume, it is often convenient to evaluate motor-case materials on the basis of weight, \( W \), of the cylindrical portion of the case per unit volume, \( V \), enclosed by this portion.*

If it is assumed that the critical loading is internal pressure, it is shown in Appendix A that the \( W/V \) ratio for a monolithic case constructed of an isotropic material** is directly proportional to the pressure and inversely proportional to the ratio of the design stress level to the density, \( d \), of the case material. As discussed in the sections on Structural Safety and Reliability and Static Mechanical Properties, the design stress level is usually approximately proportional to the ultimate tensile strength.*** Thus, strength/density \( (F_{tu}/d) \) is the structural-material index of minimum-weight design for internal pressure.

For conditions where buckling governs the design, it is shown in Appendix A that the structural-material index of minimum-weight design is merely \( \sqrt{E/d} \), where \( E \) is the modulus of elasticity.

For various materials, basic mechanical properties and the two structural-weight indexes are listed in Table 6. These are based upon minimum strength values as given in Reference (53), or as estimated from available data.

Aluminum alloy 2014-T6 is listed primarily for historical interest, since it is typical of the case materials used in small ordnance-type rocket cases in the early 1950's, and for comparative purposes, since it is currently in use in liquid-fuel rocket tankage. Aluminum alloy 7178-T6 is listed as a representative of the higher-strength-level aluminum alloys currently available.

The magnitudes of axial compression and bending loads in monolithic rocket-case applications in the past have been sufficiently low that case weight has been governed by internal-pressure considerations rather than buckling. Thus, there has been an intensive search for materials with high strength/density. This quest first motivated a change from aluminum alloys to familiar aircraft steels like AISI 4130 and 4340, then to special alloy steels such as D-6ac, MBMC-1, 5Cr-Mo-V, and 300 M. Appreciable weight reductions have recently been achieved by redesigning rocket cases to permit a material change from high-strength steel to titanium alloy. The two major competing alloys are Ti-6Al-4V, which is considered to be the "4340" of the titanium field, and Ti-13V-11Cr-3Al, which has slightly higher strength but is less ductile and more difficult to weld.

Recently, there has been considerable interest in low-carbon, martensitic, 18-per cent-nickel steels, known as Mar-aging steels, for rocket cases. (54) The Mar-aging steels can be heat treated, by a simple aging treatment, to higher \( F_{tu} \) levels than the medium-carbon alloy steels. These steels offer no unusual problems in fabrication and welding, and they have good fracture toughness at high \( F_{tu} \) levels.

---

*Another way to evaluate case-material structural weight efficiency is on the basis of weight per unit surface area for the cylindrical portion. However, this has the disadvantage that it is dependent upon case diameter.

**An isotropic material is one which has the same elastic properties in all directions.

***It is preferable to use basis strengths, rather than uniaxial values, since many ductile materials exhibit an increase in strength of 5 to 10 per cent under such a condition. However, such data are highly dependent upon configuration and are available for only a limited number of materials. Thus, for comparative purposes \( F_{tu} \) values are used here.
## TABLE 6. BASIC MATERIALS PROPERTIES AND STRUCTURAL INDEXES OF SELECTED MATERIALS

(Approximate Values at Room Temperature for Comparative Purposes)

<table>
<thead>
<tr>
<th>Material</th>
<th>Minimum Yield (0.2% Offset), $F_{ty}$, ksi</th>
<th>Minimum Ultimate, $F_{tu}$, ksi</th>
<th>Density, $d$, pci</th>
<th>Modulus, $E$, millions of psi</th>
<th>$F_{tu}/d$, millions of psi/pci</th>
<th>$\sqrt{E}/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Alloys (Monolithic)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2614-T6</td>
<td>60</td>
<td>68</td>
<td>0.10</td>
<td>10.5</td>
<td>0.68</td>
<td>32.4</td>
</tr>
<tr>
<td>7178-T6</td>
<td>74</td>
<td>84</td>
<td>0.10</td>
<td>10.3</td>
<td>0.84</td>
<td>32.1</td>
</tr>
<tr>
<td>AISI 4340</td>
<td>217</td>
<td>260</td>
<td>0.28</td>
<td>29</td>
<td>0.93</td>
<td>19.2</td>
</tr>
<tr>
<td>5Cr-Mo-V</td>
<td>220</td>
<td>260</td>
<td>0.28</td>
<td>29</td>
<td>0.96</td>
<td>19.2</td>
</tr>
<tr>
<td>300 M</td>
<td>230</td>
<td>270</td>
<td>0.28</td>
<td>29</td>
<td>0.97</td>
<td>19.2</td>
</tr>
<tr>
<td>18Ni-Co-\text{\textomega} (300)</td>
<td>295 typ.</td>
<td>300 typ.</td>
<td>0.29</td>
<td>26.5</td>
<td>1.02</td>
<td>17.8</td>
</tr>
<tr>
<td>Titanium Alloys (Monolithic)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>150</td>
<td>170</td>
<td>0.161</td>
<td>16</td>
<td>1.06</td>
<td>24.8</td>
</tr>
<tr>
<td>Ti-13V-11Cr-3Al</td>
<td>170</td>
<td>190</td>
<td>0.175</td>
<td>16</td>
<td>1.09</td>
<td>22.8</td>
</tr>
<tr>
<td>Composites</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Music wire and resin, helically wound</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass filament (Type \text{\textomega}) and resin, helically wound(^{(b)})</td>
<td>--</td>
<td>94-120</td>
<td>0.072</td>
<td>2.5 typ. (a)</td>
<td>1.31-1.67</td>
<td>22.0</td>
</tr>
<tr>
<td>Glass filament, resin and aluminum alloy 7178-T6</td>
<td>--</td>
<td>128 typ.</td>
<td>0.097</td>
<td>6.7 typ. (a)</td>
<td>1.32</td>
<td>26.7</td>
</tr>
</tbody>
</table>

\(^{(a)}\) Effective values applicable to axial-compression buckling only.

\(^{(b)}\) The strength values depend upon the type of glass finish, the resin system, the netting design, and winding method.
Before structural-material indexes for composite materials can be discussed intelligently, it is necessary to discuss the basis for comparison with monolithic materials. The design basis used for composite materials varies widely. In wire-wound-resin cases, it has been the practice to refer strength, modulus, and density values to the wire; however, in glass-filament-wound resin-bonded aluminum cases, the design is based on the aluminum. In order to make legitimate comparisons with monolithic materials, it is necessary to use strength, modulus, and density values of the composite. This has been done for the composites listed in Table 6. Then all of the equations presented in Appendix A for monolithic cases can be used directly.*

The primary loading present in all rocket-motor cases is internal pressure; furthermore, in motors currently either available or under consideration, the pressure varies only from 200 to 1000 psi. The other major loading is an external loading consisting of a combination of axial compression and bending which tends to produce failure by buckling. It is readily obvious that larger vehicles will have larger applied external loads and also larger buckling resistance in terms of total external load (in pounds). Therefore, in order to compare external loads on an equitable basis relative to the structural weight required, the parameter \( P_{eq}/D^2 \) is used in the derivation in Appendix A. Here \( P_{eq} \) is the equivalent axial load (an axial compression load, equivalent so far as buckling is concerned, to the actual combination of axial compression and pure bending) and \( D \) is the motor-case diameter. **

It is desirable to present a graphical picture of the effect of external loading on the structural weight. First of all, Equation (4) in Appendix A shows that, for a motor case which is critical for internal pressure, the cylindrical motor-case weight per enclosed volume \( (W/V) \) is unaffected by external loading. Thus, in a plot depicting \( W/V \) versus \( P_{eq}/D^2 \), this relationship is represented by a horizontal straight line (Line A) as shown in Figure 14.

Using \( P_{eq} \) for \( P \) in Equation (7) in Appendix A, it is seen that when buckling is critical, the relationship between \( W/V \) and \( P_{eq}/D^2 \) is parabolic, i.e., \( W/V \) is proportional to the square root of \( P_{eq}/D^2 \), as shown by Curve B in Figure 14. The actual weight per enclosed volume is given by the higher value of the two relationships. Thus, for example, at a \( P_{eq}/D^2 \) value represented by vertical Line 1 in Figure 14, the higher \( W/V \) is the one determined by internal pressure (Line A), so this portion of Line A is shown as a solid line, rather than dotted. Conversely, at higher external loads exemplified by vertical Line 2, the higher \( W/V \) is the one determined by buckling, i.e., parabolic Curve B. Thus, this portion of Curve B is shown solid. The weight/volume relationship over the entire range of external loadings covered by Figure 14 is represented by the solid portions of Line A and Curve B.

Figure 15 summarizes the effect of material selection on the weight of a rocket-motor case of monolithic construction; Figure 16 covers composite motor cases. These figures show the effects of case material and equivalent load parameter (equivalent axial load \( P_{eq} \) divided by the square of the case diameter) on the weight per volume enclosed for the cylindrical portion of a case for a rocket with a 400-psi burning pressure. This

---

* If the design is based entirely on the diameters, the constant \( c \) in Equation (4) of Appendix A must be replaced by 3, then the modified equation is applicable to optimum filament-wound shells of any geometrical configuration.

** The equation used for calculating \( P_{eq} \) is Equation (12), Appendix A.
FIGURE 14. SCHEMATIC DIAGRAM EXPLAINING THE EFFECT OF EQUIVALENT EXTERNAL LOADING ON THE DESIGN WEIGHT OF A MOTOR CASE DESIGNED FOR A GIVEN INTERNAL PRESSURE
FIGURE 15. WEIGHT OF CYLINDRICAL PORTION OF ROCKET-MOTOR CASE PER UNIT OF ENCLOSED VOLUME VERSUS AN EQUIVALENT EXTERNAL LOAD PARAMETER FOR MONOLITHIC CONSTRUCTION OF VARIOUS MATERIALS.
FIGURE 16. WEIGHT OF CYLINDRICAL PORTION OF ROCKET-MOTOR CASE PER UNIT OF ENCLOSED VOLUME VERSUS AN EQUIVALENT EXTERNAL LOAD PARAMETER FOR VARIOUS COMPOSITE CONSTRUCTIONS
burning pressure has been selected because it is intermediate between the usual limits of 200 to 1000 psi; also, it is typical burning pressure for second-stage rockets, in which buckling is apt to be critical. The assumptions on which Figures 15 and 16 are based include:

1. Room-temperature minimum properties as given in Table 6 are used.
2. No allowance is made for increase in $E_tu$ due to biaxiality.
3. Ultimate safety factor is 1.25.
4. Insulation weight is not included.
5. Effect of case length/diameter ratio on buckling is neglected.
6. Credit is given for a 52 per cent increase in buckling resistance due to a case-bonded propellant.
7. For the ordinary glass filament/resin case, a 40-psi decrease in pressure acting on the case is taken into consideration.
8. Values shown for the sandwich motor case are approximate ones based on Reference (31). Since the core-material weight is constant per square foot of case surface area, its weight per volume enclosed decreases with increasing case diameter. A weight value is shown for only one motor-case diameter: 120 inch.
9. The limiting values (0.185 and 0.300) of the buckling factor $C$ (see Appendix A) were used for each material.

Typical ranges of the equivalent external load parameter (the abscissas of Figures 15 and 16) are also indicated in the figures by horizontal brackets (in the lower part of the figures) for three classes of vehicles, the characteristics of which are given in Appendix B. In the range of values for each type of vehicle, the higher values are applicable to only second-stage motor cases and the lower values are primarily applicable to the first and third stages. Equivalent load parameter values corresponding to the fourth stage of Vehicle C have been omitted in the figures, since it is not a "large" rocket.

In Figures 15 and 16 some of the data are represented as bands or areas rather than lines. The reason for the range of values for high-strength steels and for titanium alloys in the left-hand (horizontal-line) portion of Figure 15 and for Composite X in Figure 16 is to cover the range of strength and correspondingly $F_{tu}$/d for each of the classes of materials in Table 6. The ranges of values in the right-hand portion of Figure 15 (i.e., nonhorizontal lines) for these materials, as well as for Composites W, X, and Y in Figure 16, are related to the use of a range of values for the buckling coefficient $C$ (defined in Equation (5) in Appendix A). The upper boundaries correspond roughly to mean values of $C$, while the lower boundaries represent the 90 per cent probability value (at 95 per cent confidence level).

Since only one material is presented in each class for aluminum alloy and composites W, X, and Y, each of these is represented in the left-hand portion as a single line. However, due to the combination of a low $F_{tu}$/d index and a high $\sqrt{E}$/d index for the aluminum alloy (7178-T6), buckling does not become critical within the load range covered.
by Figure 15. In other words, the weight per unit volume for 7178-T6 is determined by internal pressure only and is unaffected by load over the range of values shown.

The structural-weight material index for pressure-critical design, $F_{tu}/d$, is significant in the left-hand portion of the figures, i.e., when the external loads are relatively small. Therefore, for monolithic structures, titanium alloys have the highest $F_{tu}/d$ and thus the lowest $W/V$ (see Figure 15). Similarly, glass-filament/resin composite (Composite X) having an even higher $F_{tu}/d$ is still lighter (see Figure 16).

As higher and higher final-stage velocities are required, the equivalent external load parameter $P_{eq}/D^2$ will also increase, as shown by the increases in the ranges of $P_{eq}/D^2$ values in going from Vehicle A through Vehicle B to Vehicle C (see Appendix B). As a result of their combination of high $F_{tu}/d$ and high $\sqrt{E/d}$, titanium alloys will result in lighter: (lower $W/V$) motor cases at high loads than will any of the other materials shown for monolithic construction. It is important to note that it is the combination of $F_{tu}/d$ and $\sqrt{E/d}$ which makes titanium alloys best in the high-load range. Thus, for monolithic construction and regardless of the combination of loadings, use of titanium alloys will result in the lightest motor case per enclosed volume.

The choice of the type of composite construction which is best from a $W/V$ standpoint is somewhat more complicated. Reference to the lowest values for each of the composites represented in Figure 16 shows that the order of merit for various ranges of the load parameter are as shown in Table 7.***

### TABLE 7. ORDER OF MERIT(a) FOR VARIOUS COMPOSITE MATERIALS AT VARIOUS RANGES OF THE EQUIVALENT LOAD PARAMETER, ASSUMING A 400-PSI BURNING PRESSURE

<table>
<thead>
<tr>
<th>Order of Merit for Indicated Ranges of Equivalent Load Parameter, psi</th>
<th>0 - 50</th>
<th>50 - 74</th>
<th>74 - 89</th>
<th>89 - 110</th>
<th>110 and Up</th>
</tr>
</thead>
</table>

(a) Lightest composites (i.e., smallest $W/V$) are listed first.
(b) A 120-inch-diameter motor case is assumed.

Several significant conclusions can be drawn from Table 7 in regard to composite-material development programs oriented toward rocket-case application:

(1) At present Composite X (glass filament/resin) is best (from a motor-case weight per enclosed volume standpoint) only for low external loads (i.e., low values of $P_{eq}/D^2$). Any reductions in weight at higher

---

*Beryllium is not included in the discussion because of the considerable difficulties in fabrication which must be overcome before beryllium motor cases are practical. However, typical property values for a rolled beryllium sheet are: $F_{tu} = 500$ ksi, $f_{tu} = 30,000$ psi, $E = 360,000$ psi, $v = 0.25$. On this basis, beryllium would be located in Figure 15 by a horizontal line at $F_{tu}/d = 50$, and a vertical line at $W/V = 0.06$, and $I/d = 15$ in $\text{in}^3$. At low and intermediate external loadings, this would be quite comparable to titanium alloys. But, at high external loadings, it would represent a weight saving over titanium alloys.

**Aluminum alloys have a much higher $F_{tu}/d$ index than do titanium alloys, but they are at a disadvantage due to lower than $F_{tu}/d$ index, as shown in Figure 15 (except for very large final-stage velocity requirements - more than that of Vehicle C in Appendix B).

*** Considerable further work is indicated on a weight basis for the load ranges covered by Figures 15 and 16. See also, second column of Table 7 and the ensuing discussion.
loads must be achieved by increasing \( \sqrt{E/d} \); increasing \( F_{tu}/d \) will not help at higher loads. This indicates that the successful development of high-modulus glass filaments and hollow glass filaments should result in lighter cases at higher loads.

(2) Composite Y (glass filament/aluminum/resin) is best for intermediate loads; thus it should come into more prominence in the next few years than it has so far. Its simplicity in comparison with sandwich construction should make it desirable from a cost standpoint even at quite high loads.

(3) Composite Z (glass-filament/aluminum honeycomb sandwich) looks the best for very high loads, so its further development should be pursued if solid-propellant rockets are to be used for boosters of the Nova class and beyond.

(4) Regardless of the combinations of loads considered, Composite W (music wire/resin) is inferior, from a weight standpoint, to other composites. Thus, unless it has an overriding economic advantage, its further development is not justified. However, this does not preclude investigation of wire materials other than steel, i.e., beryllium.

By comparing the envelopes of lowest values in Figures 15 and 16, it can be concluded that the best material (titanium alloy) considered for monolithic construction is not as good as the best composites in their respective ranges of application (see Table 7). However, it should be remembered that this conclusion and others presented in this section are based upon the cylindrical case weight/enclosed volume concept presented in detail in the Appendix.

There are some environmental modifying effects that are not included in such an analysis as presented here; these effects could change the conclusions in some instances. These effects include fatigue, temperature-sensitive properties, and space environmental effects (radiation, vacuum).

At the same time the analysis does not consider effect of closure weights and other fitting weights which may result in a considerable weight penalty when certain materials (composites in particular) are employed.

All of the factors involved in determining the performance objective (placing a given payload at a specified location with a desired velocity) are subject to various trade-offs. These factors include motor-case weight versus insulation weight, three stages versus four, etc.

REFERENCES


APPENDIX A

MATHEMATICAL DERIVATIONS OF STRUCTURAL-WEIGHT MATERIAL INDEXES
APPENDIX A

MATHEMATICAL DERIVATIONS OF STRUCTURAL-WEIGHT MATERIAL INDEXES

Internal Pressure Critical

The largest stress in the cylindrical portion of an internally pressurized motor case is the hoop stress, \( S \), given by

\[
S = k p D / 2 t,
\]

where \( k \) is a factor which depends upon the design of the end closures, \( p \) is the pressure, and \( D \) and \( t \) are the mean diameter and wall thickness of the motor case.

The weight, \( W \), of a unit length in the cylindrical portion of a motor case is approximately

\[
W = \pi D d t,
\]

where \( d \) is the density of the case material.

The volume, \( V \), enclosed by a unit length of a cylinder is approximately

\[
V = \pi D^2 / 4.
\]

If the maximum stress, \( S \), is now assumed to be the design stress, \( F_d \), for the material, Equations (1), (2), and (3) can be combined to give the following expression for the \( W/V \) ratio:

\[
W/V = \frac{2 k p}{F_d / d}.
\]

Thus, when internal pressure governs the design, \( W/V \) is inversely proportional to \( F_d / d \).

Buckling Critical

The two most critical buckling loads for a motor case are usually either axial compression or bending. For simplicity, only axial compression will be treated here in detail. The critical buckling stress \( (F_c)_{CR} \) for a monolithic, isotropic cylindrical shell in axial compression is computed by

\[
(F_c)_{CR} = 2 G E t / D,
\]

where \( E \) is the elastic modulus and \( G \) is a coefficient which depends somewhat on the \( D/t \) ratio.\(^9\)

\(^9\) In a pressurized case, \( D/t \) ratios less than 100 (includes all modern motor cases), recommended values of \( C \) range from \( 0.1 \) to \( 0.3 \) (95 percent probability value at a 95 percent confidence level) to 0.3 (mean value) (see Reference (1) on page 1).
A-2

Now for design, \((F_c)_{cr}\) is also simply the critical axial load, \(P\), multiplied by a factor of safety, \(N\), and divided by the cross-sectional area of the case \((πDt)\):

\[
(F_c)_{cr} = \frac{PN}{πDt}.
\]  

(6)

Solving Equations (5) and (6) for the wall thickness and using this to compute the weight, we obtain the following expression for the weight per enclosed volume:

\[
W/V = \frac{1}{\sqrt{\frac{E}{d}}} \sqrt{\frac{8PN}{D^2}}.
\]  

(7)

The requirement that buckling take place in the elastic range means that \((F_c)_{cr}\) as given by Equations (5) and (6), with \(N = 1\), cannot exceed the compressive yield strength \(F_{cy}\). This imposes the following upper limit on the axial load \(P\):

\[
P_{max}/D = (\pi/2C) \left(\frac{F_{cy}^2}{E}\right).
\]  

(8)

Fortunately, for values of \(F_{cy}^2/E\) for all monolithic materials and composites currently under consideration and for external loads on rocket cases for vehicles in current use or contemplated for the foreseeable future, it is believed that Equation (8) will not be a real limitation. Thus, the structural-material index of minimum-weight design for external loads is merely \(\sqrt{E/d}\).

When both compressive and bending loads are acting, the interaction equation is\(^{(1)}\):

\[
\frac{F_c}{(F_c)_{cr}} + \frac{F_b}{(F_b)_{cr}} = 1,
\]  

(9)

where \(F_c\) are \(F_b\) are the direct compressive and bending stresses for buckling under combined-load conditions, \((F_c)_{cr}\) is as defined previously, and \((F_b)_{cr}\) is the bending stress for buckling under bending alone.

Now the relationship between \((F_b)_{cr}\) and \((F_c)_{cr}\) is given approximately by\(^{(1)}\)

\[
(F_b)_{cr} = 1.35 (F_c)_{cr}.
\]  

(10)

Also, we have, from elementary bending theory applied to a thin-walled cylinder,

\[
F_b = Mc/I = 4M/πD^2t,
\]  

(11)

where \(M\) is the bending moment and \(I/c\) is the section modulus.

Now, combining Equations (9), (10), and (11), we arrive at the following expression for the equivalent combined loading \(P_{eq}\), as used in Figures 15 and 16 and in Appendix B:

\[
P_{eq} = P + \frac{4M}{1.35 D}.
\]  

(12)
APPENDIX B

EXAMPLE SHOWING EFFECTS OF MISSION AND STAGE LOCATION ON MOTOR-CASE SIZE, LOADS, AND UNIT WEIGHT
APPENDIX B

EXAMPLE SHOWING EFFECTS OF MISSION AND STAGE LOCATION ON MOTOR-CASE SIZE, LOADS, AND UNIT WEIGHT

Three hypothetical vehicles, each with the same payload (300 pounds) but performing a different mission, are considered here. They are as follows.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Mission</th>
<th>Last-Stage Burnout Velocity, fps</th>
<th>Number of Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>ICBM (5000-nautical-mile range)</td>
<td>2,150</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>Earth orbit (300-nautical-mile altitude, 96-minute circular orbit)</td>
<td>23,000</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>Earth escape</td>
<td>36,500</td>
<td>4</td>
</tr>
</tbody>
</table>

The following basic data are taken from Reference (31).

<table>
<thead>
<tr>
<th>Stage</th>
<th>Case Diameter, D, in.</th>
<th>Loading at Maximum External Loading, P, kilopounds</th>
<th>Condition for Occurrence of Maximum External Loading, M, in-kilopounds</th>
<th>Maximum External Loading Station, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Launch)</td>
<td>A</td>
<td>38.8</td>
<td>52.0</td>
<td>102</td>
</tr>
<tr>
<td>2 (Intermediate)</td>
<td>B</td>
<td>38.0</td>
<td>48.0</td>
<td>70</td>
</tr>
<tr>
<td>3 (Final for A &amp; B)</td>
<td>C</td>
<td>26.0</td>
<td>38.0</td>
<td>48</td>
</tr>
<tr>
<td>4 (Final for C)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>38</td>
</tr>
</tbody>
</table>

(a) Maximum external loading refers to the external loading conditions at which $P_{eq}$ (defined below) is maximum.
(b) The symbol 1B refers to the end of first-stage boost; 2B refers to the end of second stage boost; max q refers to the maximum-dynamic-pressure condition.
(c) Location measured from aft (lower) end of each stage.

The maximum equivalent axial load $P_{eq}$ is defined as follows:

$$P_{eq} = P + \frac{4}{1.35} \frac{M}{D}$$

where

$P$ = axial load, and $M$ = bending moment.
The maximum values of $P_{eq}$ and of its parametric version $P_{eq}/D^2$ are:

<table>
<thead>
<tr>
<th>Stage</th>
<th>$P_{eq}$ (kilob.)</th>
<th>$P_{eq}/D^2$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>47.0</td>
<td>143</td>
</tr>
<tr>
<td>2</td>
<td>51.4</td>
<td>145</td>
</tr>
<tr>
<td>3</td>
<td>20.5</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

The following values for internal pressure were assumed for each stage.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Pressure, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>300(a)</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
</tr>
</tbody>
</table>

(a) Stated in Reference (31).

As an example for the weight per enclosed volume ($W/V$) of the cylindrical portion of the motor case, a single material is chosen throughout. It is assumed to have a biaxial yield strength of 250 ksi, a biaxial ultimate strength of 280 ksi, an elastic modulus of 29,000 ksi, and a density of 0.28 pci. The factors of safety are 1.00 on yield and 1.25 on ultimate. Then the design hoop stress is 224 ksi.

It is assumed that the stiffness of the propellant grain increases the equivalent axial-load-carrying capacity by 52 per cent. For the conditions in which internal pressure is acting during occurrence of maximum external load, a 7b per cent increase in equivalent axial-load-carrying capacity is assumed. Thus, the "design" values of equivalent axial load for this example are:

$$P_d = P_{eq}/1.76 \text{ for 1st-stage motor cases}$$

$$P_d = P_{eq}/1.52 \text{ for motor cases for all other stages.}$$

For numerical values, see Table B-1.
TABLE B-1. EXTERNAL LOAD PARAMETERS, DESIGN CRITERIA, AND CYLINDRICAL-PORTION WEIGHT PER ENCLOSED VOLUME FOR MOTOR CASES OF VARIOUS STAGES OF THREE HYPOTHETICAL VEHICLES

<table>
<thead>
<tr>
<th>Stage</th>
<th>Actual $P_d/D^2$</th>
<th>Transitional $P_d/D^2$</th>
<th>Design Governing Criterion[a]</th>
<th>$W/V$, lb/ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>17.6</td>
<td>30.1</td>
<td>28.6</td>
<td>67.8</td>
</tr>
<tr>
<td>2</td>
<td>23.4</td>
<td>41.4</td>
<td>63.1</td>
<td>24.4</td>
</tr>
<tr>
<td>3</td>
<td>20.0</td>
<td>32.0</td>
<td>53.3</td>
<td>24.4</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>10.2</td>
<td>--</td>
</tr>
</tbody>
</table>

(a) IP denotes internal pressure; B denotes buckling.

Then, assuming a buckling factor $C$ of 0.3, the transitional value of $P_d/D^2$ at which the $W/V$ for internal pressure and for buckling would be identical is given by the following equation:

$$(P_d/D^2)_{trans.} = 0.471 \left(\frac{\rho \sqrt{E/d}}{S_{d}/d}\right)^2.$$  

As shown in the table, the transitional value of $P_d/D^2$ exceeds the actual value in only two situations (the third stages of Vehicles B and C). Thus, buckling governs only in these two situations.

The values of $W/V$ are also shown in Table B-1. It is to be noted that, for a given vehicle, $W/V$ decreases in going from the first stage to the last stage. Also, as the escape velocity is increased (i.e., going from Vehicle A to Vehicle C), $W/V$ for a given stage increases.
<table>
<thead>
<tr>
<th>DAMC Report Number</th>
<th>Title</th>
</tr>
</thead>
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<tr>
<td>158A</td>
<td>The Effects of Alloying Elements in Titanium, Volume A: Constitution, September 15, 1960 (PB 151094 $1.50)</td>
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<td>137</td>
<td>Design Information on 17-7 PH Stainless Steels for Aircraft and Missiles, September 22, 1960 (PB 151096 $1.00)</td>
</tr>
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<td>Availability and Mechanical Properties of High-Strength Steel Extrusions, October 26, 1960 (PB 151097 $1.75)</td>
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<td>139</td>
<td>Melting and Casting of the Refractory Metals Molybdenum, Columbium, Tantalum, and Tungsten, November 18, 1960 (PB 151095 $1.00)</td>
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<td>Physical and Mechanical Properties of Commercial Molybdenum-Base Alloys, November 30, 1960 (PB 151099 $1.00)</td>
</tr>
<tr>
<td>141</td>
<td>Titanium-Alloy Forgings, December 18, 1960 (PB 151100 $2.25)</td>
</tr>
<tr>
<td>142</td>
<td>Environmental Factors Influencing Metals Applications in Space Vehicles, December 27, 1960 (PB 151101 $1.29)</td>
</tr>
<tr>
<td>143</td>
<td>High-Strength Steel Forgings, January 6, 1961 (PB 151102 $1.75)</td>
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<td>144</td>
<td>Stress-Corrosion Cracking – A Nontechnical Introduction to the Problem, January 6, 1961 (PB 151103 $0.75)</td>
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<td>145</td>
<td>Design Information on Titanium Alloys for Aircraft and Missiles, January 10, 1961 (PB 151104 $2.25)</td>
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<td>146</td>
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<td>The Factors Influencing the Fracture Characteristics of High-Strength Steel, February 6, 1961 (PB 151106 $1.29)</td>
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<td>149</td>
<td>Testing for High Temperature Service, February 21, 1961 (PB 151108 $1.00)</td>
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<td>150</td>
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<td>151</td>
<td>Environmental and Metallurgical Factors of Stress-Corrosion Cracking in High-Strength Steels, April 14, 1961 (PB 151110 $0.75)</td>
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<td>152</td>
<td>Binary and Ternary Phase Diagrams of Columbium, Molybdenum, Tantalum, and Tungsten, April 28, 1961 (AD 297739 $3.00)</td>
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<td>153</td>
<td>Physical Metallurgy of Nickel-Base Superalloys, May 5, 1961 (AD 280401 $1.25)</td>
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<td>155</td>
<td>Oxidation of Tungsten, July 17, 1961 (AD 263388 $3.00)</td>
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<td>156</td>
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<td>A Summary of the Theory of Fracture in Metals, August 7, 1961 (PB 181068 $1.75)</td>
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<td>158</td>
<td>Stress-Corrosion Cracking of High-Strength Stainless Steels in Atmospheric Environments, September 10, 1961 (AD 266255 $1.25)</td>
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<td>159</td>
<td>Gas-Pressure Bonding, September 25, 1961 (AD 263133 $1.25)</td>
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<td>160</td>
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<td>161</td>
<td>Status Report No. 1 on Department of Defense Refractory Metals Sheet-Rolling Program, November 2, 1961 (AD 260777 $1.00)</td>
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<td>162</td>
<td>Coatings for the Protection of Refractory Metal from Oxidation, November 24, 1961 (AD 271334 $2.50)</td>
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<td>Control of Dimensions in High-Strength Heat-Treated Steel Parts, November 29, 1961 (AD 270045 $1.00)</td>
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<td>164</td>
<td>Semiinterstitial Precipitation-Hardenable Stainless Steels, December 6, 1961 (AD 270045 $1.00)</td>
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<td>165</td>
<td>Methods of Evaluating Welded Joints, December 28, 1961 (AD 272008 $2.25)</td>
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<td>166</td>
<td>The Effect of Nuclear Radiation on Structural Metals, September 15, 1961 (AD 265829 $2.50)</td>
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<td>167</td>
<td>Summary of the Fifth Meeting of the Refractory Companies Working Group, March 12, 1962</td>
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<td>169</td>
<td>The Effect of Motion Alkali Metals on Containment Metals and Alloys at High Temperatures, May 28, 1961</td>
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<td>170</td>
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<td>171</td>
<td>The Physical Metallurgy of Cobalt-Base Superalloys, July 6, 1962</td>
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<td>172</td>
<td>Background for the Development of Materials To Be Used in High-Strength-Steel Structural Weldments, July 31, 1962</td>
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<td>173</td>
<td>New Developments in Welded Fabrication of Large Solid-Fuel Rocket-Motor Cases, August 6, 1962, June 2, 1963</td>
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<td>174</td>
<td>Live-Materials Forgings, October 10, 1963</td>
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<tr>
<td>175</td>
<td>Summary of the Sixth Meeting of the Refractory Companies Working Group, September 24, 1962</td>
</tr>
<tr>
<td>176</td>
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<tr>
<td>177</td>
<td>Thermal Radiative Properties of Selected Materials, November 16, 1962</td>
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<td>178</td>
<td>Steels for Large Solid-Propellant Rocket-Motor Cases, April 12, 1962</td>
</tr>
<tr>
<td>179</td>
<td>A Guide to the Literature on High-Velocity Metalworking, December 1961</td>
</tr>
</tbody>
</table>
This report covers the selection of materials from the standpoint of design for the inert parts of large solid-propellant rocket-motor cases. The main intent of the report is to clarify considerations of design for the benefit of materials specialists in the aerospace industry and for the benefit of the materials industries which supply the aerospace industry. There is a brief description of types of missions and mission profiles and a discussion of the loads and environment to which the missile is subjected. Since mechanical and thermal loads are reacted by the motor case acting as a structure, a discussion of stresses and other design factors is included. Structural reliability, static and fatigue behavior, environmental effects, and structural weight are discussed from a design point of view.