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CALCULATION OF FACTORS, AFFECTING THE ACCURACY IN
DETERMINING THE DRIFT OF FLOATING INTEGRATING GYROSCOPES

By

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Calculation of Factors, Affecting the Accuracy in Determining the Drift of Floating Integrating Gyroscopes

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We will assume, that when testing for drift the investigated gyroscope and platform of the stand are situated, as is shown in fig.1. For the sake of commonness we will assume that the platform of the stand is driven (brought into rotation) through a gap-less reduction gear. The motor of the platform is coupled to an amplifier to the input of which is fed the output voltage of the sensing element of the angle of the investigated gyroscope, fastened to the platform of the stand (fig.1).

In fig.1 and further on were adopted the following three systems of axis, originating in point 0 where the axes of rotation of frame and rotor of examined gyroscope do intersect:

1. The system of $xyz_0$ axes is connected with the body of the investigated gyroscope. The $x$ and $y$ axes are the output and input axes of the gyroscope; axis $z_0$ is perpendicular to axes $x$, $y$ and forms together with them a right trihedral.

2. The system of $x\eta z$ axes, connected with the earth and is oriented geographically. Axes $\chi$ and $\eta$ are horizontal; axis $\chi$ faces eastward, axis $\eta$ northward, and axis $z$ into the zenith.

3. The system of $XYZ$ axes is connected with the platform of the testing stand. Axis $Y$ appears to be the axis of rotation of the platform; axes $X$ and $Z$ are perpendicular to axis $Y$ and form with it a right trihedral.

In fig.1 and further on designates the angle of inclination of the figurational

Fig.1, Variant of platform and gyroscope arrangement when checking drift of gyroscope.

SEE PAGE 1a FOR FIGURE 1.
axis of the gyroscope $z$ from axis $z_0$, $\alpha$ and $\Omega$ - the angle and angular velocity of platform rotation about axis $Y$. Omega designates the angular velocity of diurnal rotation of the Earth, and $\varphi$ - latitude of test stand disposition.

In an ideal case when testing according to fig.1, the axes $y$ and $Y$ should coincide with axes eta; axes $X$ and $Z$ should lie in plane xi zeta. Axis $x$ should coincide with axis $X$, and $z_0$ with axis $Z$.

The moment of interferences $M$, affecting the gyroscope around axis $x$, will be assumed as consisting of constant and variable components. The constant component will be designated by $M^0$. The variable component, which is to a large extent an accidental value, to simplify the analysis will be considered as varying in accordance with the harmonics law with amplitude $M^\infty$, frequency $p$ and initial phase $\theta$. The coefficient $\xi$ is equal to the ratio of amplitude of variable component of moment $M$ to it constant component $M^0$.

In this way, considering moment $M$ as positive, when the direction of its vector coincides with positive direction of axis $x$, it is assumed that the moment of interferences $M$ is a function of time $t$ and is determined by formula

$$M = M^0 \left[ 1 + \xi \sin(pt + \theta) \right]$$

In this case the angular velocity of the drift $\omega_1$ will also be a function of time, whereby its instantaneous values will be determined by equation

$$\omega_1 = \omega_1^c \left[ 1 + \varepsilon \sin(pt + \theta) \right] \quad (\omega_1^c = \frac{M^0}{H}) \quad (I)$$

Here $\omega_1^c$ - constant component of angular velocity of drift ($H$ - eigen-moment of gyroscope).

Assuming that the tested gyroscope, amplifier and platform motor are inertialess we can write their equation in the following form:

Equation of gyroscope (when compiling same we have disregarded the gyroscopic moment, due to the projection of velocity $\omega$ on axis $z_0$, because this moment is small in comparison with other moments because the ordinary angle $\alpha \approx 0$)

$$\mathbf{U} - \mathbf{0} = K(\omega_x + \omega_z - \omega_1^c [1 + \varepsilon \sin(pt + \theta)]), \quad K = \frac{M^0}{H^2} \quad (2)$$
Equation of amplifier

\[ U_{(2)} = K_1[1 + \delta_1 \sin(\nu t + \theta_1)] U_{(1)} \]  

(3)

Equation of motor

\[ \dot{\alpha}_0 = -K_d U_{(1)} \]  

(4)

Here \( K_2 \) - specific damping moment of gyroscope; \( K_1 \) and \( U_{(1)} \) - curvature of characteristic (sensitivity) and output voltage of gyroscope sensing element; \( \omega_\gamma \) - projection of angular velocity of diurnal rotation of the Earth \( \omega \) on axis \( y \); \( K_3 \) and \( U_{(2)} \) - nominal (rated) coefficient of amplification and output voltage of amplifier; \( K_4 \) - curvature of velocity characteristic of motor; \( \alpha_0 \) - angle of rotation of motor shaft.

Equation (3) takes into consideration the possibility of fluctuations of the amplification factor relative to its nominal value with amplitude \( \delta K_3 \) \( (\delta = \text{const} \ - \text{small parameter}) \) and frequency \( \nu \); \( \theta_1 \) designates the initial phase.

Because of error in reduction gear the platform during uniform rotation of motor shaft will rotate irregularly. The dependence of the angle of rotation of stand platform upon the angle of rotation of motor shaft is determined by expression

\[ \alpha = \frac{\delta_2}{i} + \delta_2 \sin(\nu t + \theta_2) \]  

(5)

Here \( i \) - nominal value of reductor gear ratio; \( \delta_2 \) = const - small parameter, equal to the maximum absolute error in angle of rotation of the platform, due to errors of the reduction gear; \( \nu_2 \) - dimensionless frequency; \( \theta_2 \) - initial phase.

Having solved equation (3) relative to \( U_{(1)} \) and substituting in it subsequently equations (4) and (5), we further assume in it

\[ \frac{1}{1 + \delta_1 \sin(\nu t + \theta_1)} \approx 1 - \delta_1 \sin(\nu t + \theta_1) \]  

and disregard the member, containing products of small values \( \delta_1 \delta_2 \). We differentiate the expression for \( U_{(1)} \) obtained in this way once with respect of time and substitute in equation (2)

In consequence the equation of motion of the platform (stand platform) will be obtained in following form:

\[ T \ddot{\alpha} + \ddot{\alpha} = -\omega_\gamma \omega_\gamma + \omega_\gamma \omega_\gamma \sin(\mu t + \theta) + \delta_1 T \frac{d}{dt}[\ddot{\alpha} \sin(\nu t + \theta_1)] + \delta_2 T \frac{d}{dt}[\ddot{\alpha} \cos(\nu t + \theta_2)] \]  

(6)
where the time constant

\[ T = \frac{i}{K_i N_i} \quad (7) \]

Integrating equation (6) for the first time at initial conditions \( \alpha = \alpha = 0 \) and at \( t \neq 0 \), we will obtain

\[ T \dot{\alpha} + \alpha = -(\omega - \omega_i) t - \frac{\cos \theta}{p} \left[ \cos (pt + \theta) - \cos \theta \right] + \]

\[ + T \dot{\alpha} \left[ \delta_1 \sin (v_1 t + \theta_1) + \delta_2 \sin (v_2 t + \theta_2) \right] \quad (8) \]

The given nonlinear equation will be integrated approximately (but with a fully sufficient degree of accuracy), assuming that in its right side

\[ \alpha = -(\omega - \omega_i) t \quad \dot{\alpha} = -(\omega - \omega_i) \quad (9) \]

One can easily see that these values for \( \alpha \) and \( \dot{\alpha} \) are derived in stable state, provided \( \xi = \delta_1 = \delta_2 = 0 \); when \( T = \xi = 0 \) this would happen everywhere. Having completed the integration, we obtain:

\[ \alpha = -(\omega - \omega_i) (t - T) - \frac{\cos \theta}{p} \left[ \sin \theta \left[ \cos (pt + \theta) - \cos \theta \right] + \right. \]

\[ + T (\omega - \omega_i) \left[ \delta_1 \sin (v_1 t + \theta_1) + \delta_2 \sin (v_2 t + \theta_2) \right] - \]

\[ - \delta_2 \sin (v_2 t) - (\omega - \omega_i) \left[ \delta_1 \sin (v_1 t + \theta_1) + \delta_2 \sin (v_2 t + \theta_2) \right] \]

\[ \times (1 + \delta_1 \sin (v_1 t + \theta_1) + \delta_2 \sin (v_2 t + \theta_2)) \right] \quad \dot{\alpha} = \omega - \omega_i \quad (10) \]

Here

\[ \cot \theta = \frac{p}{T}, \quad \cot \theta_1 = \frac{T}{v_1}, \quad \cot \theta_2 = \frac{T}{v_2} (\omega - \omega_i) \]

From expression (9) is evident that the time constant \( T \) should be possibly smaller.

When \( \xi = \delta_1 = \delta_2 = 0 \) in stable state

\[ \dot{\alpha} = -(\omega - \omega_i) \quad (9a) \]

In this case from equations (5), (4), (3), considering that \( U_x(1) = K_i \beta \) and using equation (7), we obtain

\[ T = \frac{\beta}{N \left( \omega - \omega_i \right)} \quad (N = \frac{0}{K_i}) \quad (11) \]

It is evident therefrom, that \( T \) will be smaller the smaller the angle \( \beta \) will be, the angle necessary to produce the rotating motion of the platform of the stand to revolve at an angular velocity - \( (\omega - \omega_i) \). In addition, \( T \) is inversely proportional to coefficient \( N \), which appears to be the parameter of the gyroscope.
We will assume, that in the moments of time \( t_1 \) and \( t_2 \) \( (t_2 \geq t_1, t_2 - t_1 = \gamma) \) the angle \( \gamma \) was equal to \( \alpha_1 \) and \( \alpha_2 \) respectively. Then using expression (9), assuming that \( t_1 \gg T \), and disregarding on the basis of this the member, containing \( \exp (-t_2/T) \) - \( \exp (-t_1/T) \), we obtain:

\[
-w_1^{\text{o}} + w_o^{\text{a}} + \frac{2\gamma - \alpha_1}{\tau} + \Delta_1 + \Delta_2 + \Delta_3 = 0
\]  

(12)

In equation (12) have been adopted the following designations

\[
\omega_i^{\text{o}} = \omega_i^{\text{o}}[1 - \frac{\gamma}{\nu^2} \left\{ \cos \left( \nu t_2 + 0 \right) - \cos \left( \nu t_1 + 0 \right) \right\}],
\]

(13)

\[
\Delta_1 = \frac{\cos \gamma}{\nu^2} \cos \theta \left\{ \sin \left( \nu t_2 + 0 \right) - \sin \left( \nu t_1 + 0 \right) \right\},
\]

(14)

\[
\Delta_2 = -\frac{T_0}{\nu^2} (\omega_0 - \omega_i^{\text{a}}) \sin \delta \left\{ \cos \left( \nu t_2 + 0 + 0 \right) - \cos \left( \nu t_1 + 0 + 0 \right) \right\},
\]

(15)

\[
\Delta_3 = \frac{T_0 \nu}{\tau} (\omega_0 - \omega_i^{\text{a}}) \sin \delta \left\{ \sin \left( \nu t_2 + 0 + 0 \right) t_2 - 0 + 0 \right\} - \sin \left( \nu t_2 + 0 + 0 \right) t_2 - 0 + 0 \right\},
\]

(16)

Through \( \omega_i^{\text{o}} \) was designated the actual mean integral value of the angular velocity of the drift during time \( \gamma \). From equation (13) is evident that this velocity depends upon \( \gamma \) and upon \( t_1 \) as well, which appears to be an random value. The maximum \( \omega_i^{\text{o}} \) value is obtained at \( t_1 = (2n+1)/p \) \((n = 0, 1, 2, ... \) and \( \gamma = (2k+1)p/(k=0,1,2,...) \); in this case it will be equal to

\[
\omega_i^{\text{o}} \max = \omega_i^{\text{o}} \left\{ 1 + \frac{2\gamma}{p^2} \right\} = \frac{\omega_i^{\text{o}}}{1 + \frac{2\gamma}{(2k+1)p}} \]

(17)

The minimum value \( \omega_i^{\text{o}} \min \), obtainable at the very same values \( \gamma \), but at \( t_1 = (2n+1)p - \theta \), equals

\[
\omega_i^{\text{o}} \min = \omega_i^{\text{o}} \left\{ 1 - \frac{2\gamma}{p^2} \right\} = \frac{\omega_i^{\text{o}}}{1 - \frac{2\gamma}{(2k+1)p}} \]

(18)

In this way we have \( \omega_i^{\text{o}} \min < \omega_i^{\text{o}} \gamma \max \), whereby

\[
\omega_i^{\text{o}} \min = \omega_i^{\text{o}} \left\{ 1 - \frac{2\gamma}{p^2} \right\} = \omega_i^{\text{o}} \left\{ 1 - \frac{2\gamma}{(2k+1)p} \right\}
\]

(17)

With rise in \( \gamma \) \( m \) decreases, and \( \omega_i^{\text{o}} \) tends toward \( \omega_i^{\text{o}} \)

If the stand would be ideal (for this, under the assumptions made by us, it is necessary to have \( T = \delta_1 = \delta_2 = 0 \), then \( \Delta_1 = \Delta_2 = \Delta_3 = 0 \), and \( \omega_i^{\text{o}}, \alpha_i^{\text{a}}, \alpha_i^{\text{a}} \), and \( \gamma \) known to be absolutely accurate, then the velocity values \( \omega_i^{\text{o}} \), calculated by formula
obtainable from expression (12), would be equal to its actual values. The angle \( \alpha_1 \) and \( \alpha_2 \) should be substituted in formula (18) with consideration of their sign.

The sign of the angle \( \alpha \) should be identical with the velocity sign \( \alpha \), which is considered to be positive, when the direction of its vector coincides with the positive direction of axis \( Y \) (fig.1). During tests \( \omega_y \geq 0 \), if \( \omega_y > 0 \), then angle \( \omega \) is positive (see term (9)).

It is clear from the preceding, that -- even during the fulfillment of the mentioned ideal conditions of value \( \omega_1 \), obtainable during repeated measurements, realized within one and the same time \( \gamma \), they will be different as result of the dependence of \( \omega_1 \) upon \( \gamma \). As is evident from equation (17) the magnitude of total disagreement of value's \( \omega_1 \) will not exceed

\[
m_{\text{max}} = \frac{4\delta_0^2}{\pi} \frac{t}{18a}.
\]

In actuality \( T, \delta_1 \) and \( \delta_2 \) are not equal to zero and the adopted value \( \omega_y \) and the obtainable by measurement values \( \alpha_1 \) and \( \alpha_2 \) and \( \gamma \) differ from the actual values by certain small values \( \Delta \omega_y, \Delta \alpha_1, \Delta \alpha_2 \) and \( \Delta \gamma \). We will explain, how this reflects itself on the accuracy of determining \( \omega_1 \) by formula (18). We will analyze the right side of equation (18) into a Taylor series in powers \( \Delta \omega_y, \Delta \alpha_1, \Delta \alpha_2 \) and \( \Delta \gamma \), confining ourselves to members, containing the given small values in zero and first degree.

Utilizing the obtained expansion and formula (12), we obtain, that the absolute error in the determination of \( \omega_1 \) according to formula (18) is equal

\[
\Delta \omega_1 = \Delta_1 + \ldots + \Delta_3,
\]

where

\[
\Delta_1 = \Delta \omega_y, \quad \Delta_2 = -\frac{\Delta \alpha_1}{t}, \quad \Delta_3 = \frac{\Delta \gamma}{t}, \quad \Delta_4 = -\frac{\Delta \gamma}{t} \Delta t \approx \frac{\omega_y - \omega_1}{t} \Delta t.
\]

and \( \Delta_1, \Delta_2, \Delta_3 \) are determined by formulae (14), (15), (16). Their maximum values are equal to:

\[
\Delta_1 = \pm 2 \omega_1 \frac{T}{t}, \quad \Delta_2 = \pm 2 \Delta \omega_y \frac{T}{t}, \quad \Delta_3 = \pm 2 \Delta \alpha_1 \frac{T}{t}, \quad \Delta_4 = \pm 2 \Delta \gamma \frac{T}{t}.
\]

The maximum absolute error in determining \( \omega_1 \) by formula (18) can be assumed to
\[ \Delta \omega_i = \pm \left( \Delta \omega_1 + \ldots + \Delta \omega_2 \right)^{\frac{1}{2}} \]

Having substituted here the values \( \Delta \omega_1, \ldots, \Delta \omega_2 \), we obtain:
\[ \Delta \omega_i = \pm \frac{1}{\sigma} \left( \Delta \omega_i \sigma^2 + 2 \Delta \omega_i (a_y - o_1) \times \right. \]
\[ \left. \times \left[ \Delta \sigma_i^2 + 4T \left( \delta_i^2 + \delta_i^2 \gamma_i^2 + \delta_i^2 \gamma_i^2 \right) \right] \right)^{\frac{1}{2}} \]

(19)

where \( \Delta \omega_1 \) and \( \Delta \gamma_1 \) - maximum values of absolute errors \( \Delta \omega \) and \( \Delta \gamma \); maximum values of absolute errors \( \Delta \alpha_1 \) and \( \Delta \alpha_2 \) assumed to be identical, equaling \( \Delta \alpha \).

If it is necessary, that the maximum absolute error in determining \( \omega_1 \) by formula (18) should not exceed a certain given value, for which we will preserve designation \( \Delta \omega_1 \), then time \( \tau \) should satisfy the following condition, obtainable from formula (19)
\[ \tau > \left( \frac{2 \Delta \sigma_i^2 + (a_y - o_1)^2 (\Delta \sigma_i^2 + 4T \left( \delta_i^2 + \delta_i^2 \gamma_i^2 + \delta_i^2 \gamma_i^2 \right) \right)}{\Delta \omega_i^2 - \Delta \omega_i^2} \]

(20)

Hence it is evident, that the inequality should be fulfilled
\[ \Delta \sigma_i^2 < \Delta \omega_i^2 \]

(21)

When this inequality is not satisfied no greater time \( \gamma \) will be able to assure determination of drift with required accuracy, characterized by value \( \Delta \omega_1 \). In this case the actual value \( \Delta \omega_1 \) will always be greater than \( \Delta \omega_1 \), and at greater values \( \gamma \) it will be practically equal to \( \Delta \omega_1 \), which plainly evident from formula (19).

When inequality (21) is satisfied there will everywhere exist such a time value \( \gamma \), at which the maximum absolute error in determining \( \omega_1 \) by formula (18) will not exceed the given value \( \Delta \omega_1 \). In this case the required time value \( \gamma \) will be the lower the higher the satisfaction of inequality (21). Consequently the value \( \Delta \omega_1 \) should be reduced by all possible means.

We will determine the value \( \Delta \omega_1 \) for the case of testing drift, according to fig.1. The position of the platform, and consequently, the axes \( X, Y, Z \) connected with it relative to axes \( xi, eta, zeta \), will be determined with the aid of angles \( \psi, \zeta, \alpha \) (fig.2). The angle \( \psi \) is the angle formed by the projection of axis \( Y \) on the plane of horizon with plane of the meridian; \( \zeta \) - angle of inclination of axis \( Y \) from plane of the horizon; \( \alpha \) - angle of rotation of the platform around axis \( Y \).
When \( \psi = \varphi = \alpha = 0 \) the axes \( X,Y,Z \) coincide with axes \( x_1, \eta, \zeta \) respectively.

We will assume that the tested instrument is fastened on the platform in corresponding fixing elements and that in general case the position of its axes \( x,y,z \) relative to axes \( X,Y,Z \) is determined by three angles \( \gamma, \lambda, \kappa \) (fig. 2). Gamma - angle formed by the projection of axis \( y \) with plane \( XY \), \( \lambda \) - angle of rotation of instrument body around axis \( y \).

In the process of testing the angles \( \gamma, \lambda, \kappa \) remain unchanged.

It is evident when examining fig 2, that the projection of angular velocity of the instrument

\[
\omega_y = \omega \cos \psi \cos \eta \gamma + \omega \sin \psi \cos (\eta, y)
\]

where

\[
\cos (\eta, y) = - \left( \sin \psi \cos \eta - \cos \psi \sin \varphi \sin \alpha \right) \sin \gamma - \cos \psi \cos \varphi \cos \gamma \cos \lambda +
\]

\[
+ \left( \sin \psi \sin \eta - \cos \psi \cos \varphi \sin \alpha \right) \sin \lambda
\]

\[
\cos (\zeta, y) = \left( \cos \varphi \sin \zeta \sin \gamma + \sin \varphi \cos \gamma \cos \lambda \right) +
\]

\[
+ \cos \varphi \cos \alpha \sin \lambda
\]

(23)

(24)

From expressions (22) - (24) is evident that \( \omega_y \) does not depend upon angle \( \lambda \).

But this still does not mean that no attention should be paid to the magnitude of \( \lambda \). It must be remembered that upon a change of this angle there is also a change in position of the instrument relative to the field of gravitational force, and consequently, there will also be a change in its drift. It is evident from these expressions, that \( \omega_y = \omega \cos \gamma \) at \( \gamma = \varphi = \lambda = 0 \) and any given value \( \alpha \). This value \( \omega_y \) equal to the horizontal component of the diurnal rotation of the Earth, appears to be for the case under question the nominal value \( \omega_y \), substituted in formula (18) during

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the calculation of angular velocity of drift $\omega$. Upon changing angle alpha as well as angle $\lambda$, there is a change in position of the instrument relative to the field of gravitation, and consequently, there is also a change in its drift.

Ordinarily angles $\psi, \phi, \gamma$ and $\lambda$ appear to be so small, that if they are expressed in radians, they can at least be considered as small values of first order relative to unity. As to the angle alpha, it can generally not be considered as small.

Decomposing the right side of equation (22) into a Taylor series by degrees of small values $\psi, \phi, \gamma, \lambda$ and $\Delta \psi$ ($\Delta \psi$ - absolute error of determining latitude of point $\varphi$ in the zone $\psi = \phi = \gamma = \lambda = 0, \varphi = \varphi$) and confining ourselves to members, containing $\psi, \phi, \gamma, \lambda, \Delta \psi$ only in zero and first degrees, we obtain

$$\omega = \omega \cos \psi + \omega \sin \psi (0 + \gamma \sin x + \lambda \cos x - \Delta \psi)$$

It is evident herefrom that in case under consideration the absolute error in determining $\omega$ equals

$$\Delta \omega = \omega (0 + \gamma \sin x + \lambda \cos x - \Delta \psi)$$

The maximum value of this error can be accepted as equal

$$\Delta \omega_{max} = \pm \omega \cos \psi \sqrt{\gamma^2 + \lambda^2 \sin^2 x + \lambda \cos x + \Delta \psi^2}$$

where $\psi, \phi, \gamma, \lambda$ and $\Delta \psi$ - maximum values of $\psi, \phi, \gamma, \lambda$, and $\Delta \psi$.

The values $\Delta \omega_{x}$ for the most widely known variants of testing floating integrating gyroscopes for drift, calculated by an analogous method, are listed in table given below. At the fifth and sixth testing variants the angle alpha remains small.

At the beginning of the report we have assumed, that the reduction gear of the test stand appears to be gapless. It is apparent that this is absolutely necessary for the fifth and sixth testing variants, because in these cases the instantaneous angular velocity of the stand platform is equal to the instantaneous angular velocity of the drift of the examined gyroscope, which can generally be changed not only in value, but also in sign. At remaining five testing variants the platform of the testing stand rotates in one direction with instantaneous angular velocity

$$\dot{\alpha} = - \omega_{x} + \omega_{y}$$
which can change only somewhat in value on account of change in magnitude, and in general case also in sign of instantaneous angular velocity of drift \( \omega_d \).

It is evident from the table, that at the first four test variants the maximum absolute error \( \Delta \omega_d \), calculated with an accuracy of up to small values of first magnitude, is determined only by \( \phi \) values, \( \Delta \phi \), \( \gamma \), \( \lambda \) values and appears to be a function of \( \phi \) and \( \alpha \) angles. In case of the fifth and sixth variants the value \( \Delta \omega_d \) is affected also by \( \psi \), but from angle \( \alpha \) and \( \Delta \phi \) the maximum absolute error \( \Delta \omega_d \) is independent in these cases. At the seventh variant \( \Delta \omega_d = 0 \) with an accuracy to small values of first magnitude.

We will assume, that \( \Delta \phi = 90^\circ = \lambda = \gamma = \varphi \). then, for the first two variants

\[
\Delta \omega_d = (\Delta \omega_d)_{1,2} = \pm q \sqrt{3} \omega \sin \varphi
\]

for third and fourth variants

\[
\Delta \omega_d = (\Delta \omega_d)_{3,4} = \pm q \sqrt{2} \omega \cos \varphi
\]

for fifth and sixth variants

\[
\Delta \omega_d = (\Delta \omega_d)_{5,6} = \pm q \sqrt{2} \omega
\]

Superimposing equation (25) over (27), we obtain

\[
\frac{(\Delta \omega_d)_{1,2}}{(\Delta \omega_d)_{5,6}} = \sqrt{\frac{3}{2}} \sin \varphi = 1.23 \sin \varphi
\]

At a latitude \( \varphi = 54^\circ 20' \) we have

\[
(\Delta \omega_d)_{1,2} = (\Delta \omega_d)_{5,6}
\]

On all \( \varphi > \varphi \) \( (\Delta \omega_d)_{5,6} \leq (\Delta \omega_d)_{1,2} \) and, consequently, the fifth and sixth test variants are more preferred than the first and second variants.

A characteristic feature of the seventh variant appears to be this that in this case the practically maximum absolute error \( \Delta \omega_d = 0 \).

The analysis made gives basis of arriving at a conclusion in selecting the time interval \( \gamma \) between two subsequent measurements of angular position of the platform of the test stand when determining the drift of floating integrating gyroscopes. This time should be selected by formula (20). The value \( \Delta \omega_d \) should be determined by

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the corresponding formula shown in the table or by one of the corresponding formulas (25)-(27).

Table. Nominal values of $\omega_1$ and values of maximum absolute error $\Delta \omega_1$ for various variants of platform and gyroscope arrangements during testing of floating integrating gyroscopes for drift.

<table>
<thead>
<tr>
<th>Nominal schema of platform and gyroscope arrangement</th>
<th>Nominal value $\omega_1$</th>
<th>Value of absolute maximum error $\Delta \omega_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\pm \omega \sin \varphi (\Delta \varphi_1^2 + \varphi^2 +$ $+ \gamma_1 \cos a + \lambda_1 \sin a)^{1/2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pm \omega \sin \varphi (\Delta \varphi_1^2 + \varphi^2 +$ $+ \gamma_1 \sin a + \lambda_1 \cos a)^{1/2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pm \omega \cos \varphi (\Delta \varphi_1^2 + \varphi^2 +$ $+ \gamma_1 \sin a + \lambda_1 \cos a)^{1/2}$</td>
</tr>
</tbody>
</table>

$\gamma_1$ - maximum value of angle of inclination of axis Y to vertical line

$\gamma$ and $\gamma_1$ - maximum values of angles of inclination of axis Y to the plane of the horizon and to plane x zeta.

At other conditions being equal for the seventh time variant $\gamma$ will be the lowest, because in this case $\Delta \omega_1 = 0$. At the fifth and sixth variants $\omega_1 = 0$ and consequently for their time $\gamma$ at other conditions being equal, will be somewhat smaller.
than for the first four variants.

Fundamentally the time magnitude $\gamma$ is determined by values $\Delta \alpha_x, \Delta \omega_y$, and $\Delta \omega_y$. Consequently, $\Delta \alpha_y$ and $\Delta \omega_y$ should be reduced by all means possible. It is very important instead of separate measuring two angles $\alpha_1$ and $\alpha_2$ to measure only one angle, equaling to the difference $\alpha_2 - \alpha_1$, because this allows to reduce the time $\gamma$ by approximately $\sqrt{2}$ times.

If the time $\gamma$ corresponds to formula (20), then the experimental value of the mean integral angular velocity of the drift during the time $\gamma$ equals

$$\dot{\omega}_i = \omega_{i0} \pm \Delta \omega_i \quad (30)$$

where $\omega_{i0}$ is the value of the velocity determined by formula (18) and $\Delta \omega_i$ is the value of maximum absolute error in determining $\omega_i$ by formula (18), given by equation (19).

In conclusion we wish to mention, that the author knows of no investigations, devoted directly to problems discussed in this report. Reports which do have some kind of close relationship to the subject in discussion is the work, e.g. of [1].

Submitted Dec. 26, 1960

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