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TO: Recipients of DOFL Report No. TR-1031
FROM: W. H. Pepper/AMXIDO-RBA

The following corrections and additions should be made in the above-referenced report:

Page 7, 6th line from bottom: Change "90°" to "180°" to read "...shifting the null 180°."

Pages 9, 10 and 12; tables 1, 2, and figure 4: Interchange "A" and "B".

Page 9, table 1: Add minus sign before first parenthesis in arm "D" to read "D -(-α sin..."

Pages 10 and 16, figures 3 and 7: Interchange "E" and "F".

Page 14, figure 6 (upper-right corner): Change "Eφ = 270°" to read "Fφ = 270°."
A MINIATURE AUTOMATIC DIRECTION FINDER

J. Little
W. H. Peiper
J. A. Kaiser
H. B. Smith, Jr.

14 May 1962
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(2) To perform the necessary research, development, and engineering on components, subsystems, and systems of the type referred to in Item (1) to achieve maximum immunity of such systems to the conditions prevalent in a battlefield environment.

(3) To conduct a research program in fluid amplification for military applications.

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The findings in this report are not to be construed as an official Department of the Army position.
A MINIATURE AUTOMATIC DIRECTION FINDER

H. B. Smith, Jr.
J. A. Kaiser
W. H. Pepper
J. Little

FOR THE COMMANDER:
APPROVED BY

H. Sommer
Chief, Laboratory 200

Qualified requesters may obtain copies of this report from ASTIA.
Frontispiece. Spiral Array.
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Figure 20. Azimuthal plots of A and B for θ = 60°.
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ABSTRACT

A miniaturized unambiguous direction finder with no moving parts is described. At a frequency of 1000 Mc, a complete system, consisting of a two-wire spiral antenna, with cavity backing, operating in the first two radiation modes can be built into a cylinder 10 in. in diameter and 3 in. deep. Direct scaling laws apply, so that at 2000 Mc, for example, the cylinder would be 5 in. in diameter and 1 1/2 in. deep. Since there are no moving parts, and printed-circuit techniques can be used, the total weight (excluding power supply and readout or display) will be less than 1/2 lb.

The pattern of this antenna array gives hemispheric coverage. The output, fed into an analog device, gives a determination of the elevation and azimuth of a received signal, with no ambiguities, in the hemisphere bounded by the plane of the spiral. The processing network consists of two 3-way power dividers and a pair of hybrids, which can be mounted on the back of the cavity by using strip-line techniques. An experimental version of the system has been built and preliminary data are presented.

1. INTRODUCTION

A miniaturized direction finder with no moving parts has been designed and constructed for guided missile application to meet the need for a compact, lightweight device with no directional ambiguities, to determine both elevation and azimuth. It uses a two-wire Archimedean spiral antenna operating in the first two modes simultaneously. Although the antenna may also be used as a scanning device, discussion herein is limited to the direction finder.

The directional finder has not only the required deep null broadside for accurate null finding near the axis, but also the capability of uniquely determining the signal direction in terms of the conventional antenna spherical coordinate angles $\theta$ and $\phi$ in a wide cone about the antenna axis by using relative amplitudes of signals out of six terminals of a network. This network consists of two 3-way power dividers and two hybrid ring networks. By the theorem of reciprocity, if coherent signals are fed into the six output arms of the network, the direction of the main lobe of the antenna may be uniquely positioned by varying the relative amplitudes of the six signals. Further research will be required to increase the precision of the direction finder.

---

2. THEORETICAL DEVELOPMENT

To describe the manner in which the device functions, a spiral antenna as a radiator is considered first. A two-wire, self-complementary, Archimedean spiral antenna, connected to terminals 5 and 6 of the ring network is fed from either terminal 1 or 2 as illustrated in figure 1. The diameter of the antenna must be at least 2\(\lambda/\pi\) but less than 3\(\lambda/\pi\), where \(\lambda\) is the operational wavelength, so that the antenna will operate in the first two radiation modes only.

![Diagram of a ring network](image)

Figure 1. Ring network.

---


Assume that a radio frequency signal is applied to terminal 1 of the ring network. The energy of this signal divides equally as it enters the ring and emerges from the network at terminals 3 and 4 with equal amplitudes, but 180° out of phase. Consider that the lines from 3 to 5 and 4 to 6 are of equal length so that there is no relative phase shift of the two signals entering the terminals of the antenna. In the vicinity of the center of the antenna the currents are out of phase; for this reason little radiation occurs. As the currents proceed away from the center, the relative phase is shifted because of the difference in path lengths along the two spiral filaments. When the difference in path lengths between adjacent conductors is λ/2, the currents in these conductors are in phase and the radiation vectors add. Even for fairly coarse Archimedean spiral antennas, the difference in path length of \( \lambda/2 \) can be approximated as occurring in a thin annulus of radius \( \lambda/2\pi \).

At some point in this circular region, the currents in adjacent filaments will be in phase with the currents entering into terminal 1. Using this as a reference point, one can establish a relative phase diagram of the currents about this \( \lambda/2\pi \) radius circle. [See inner ring in (A) of fig.2] This mode of operation is called the first radiation mode.

Similarly, if a signal is applied to terminal 2 of the network shown in figure 1, the currents entering the antenna are in phase and of equal amplitude, and the radiation vectors will add on the \( \lambda/2\pi \) radius circle. Again one can establish the relative phase diagram [outer ring in (A) of fig. 2] with this mode of operation, called the second radiation mode. Note, however, that the "0" phase of the second mode may or may not lie on the same radius as the "0" phase point of the first radiation mode.

The phase of the currents establish the phase of the radiation fields. At some angle \( \phi \), the radiation fields of the first mode and the second mode are in phase. This direction is referred to as \( \phi = 0 \), and the phase diagram is redrawn as shown in (B) of figure 2, indicating the direction of in-phase fields as "0" - "0".

If an rf signal is fed into both terminals 1 and 2, both modes will be excited simultaneously. If these signals are in phase, the radiation field will be such that the radiation vectors of the first and second mode will add in the \( \phi = 0 \) direction and cancel in the \( \phi = 180 \) direction, thus forming a null in the \( \phi = 180 \) direction. If the phase into terminal 1 is shifted by 180°, the first mode is rotated through 180°, shifting the null to the direction of \( \phi = 0 \). If the signal into terminal 2 is shifted by 180°, the physical rotation of the second mode is 90°, shifting the null 90°. By the theorem of reciprocity, if the antenna is used as a receiver, the phase of the signals out of terminals 1 and 2 can be used to determine the direction of the source.

The exact field equations for the spiral antenna operating in the first or second mode have not been derived. As an approximation, based on pattern measurements, the first mode antenna pattern can be...
(A) RELATIVE PHASE DIAGRAM

R = \frac{2\pi}{M} (1ST MODE)

R = \frac{\lambda}{M} (2ND MODE)

(B) DIRECTION OF IN-PHASE FIELDS

Figure 2. Spiral array phasing diagrams.
represented by $\beta \cos \theta e^{(j\phi + jwt)}$, and the second mode by $\alpha \sin \theta e^{(j2\phi + jwt)}$, where $\phi$ is defined in the range 0 to $2\pi$, and $\theta$ is defined between 0 and $\pi/2$. These are in phase at $\phi = \phi_0$, as required by the method of defining the radiation fields of the first and second modes. Also, if the crossover point of the relative gain patterns of the two modes occurs at $\theta = \theta_0$, then

$$\alpha \sin \theta_0 = \beta \cos \theta_0$$

Assume a signal received by the antenna from a direction $(\theta, \phi)$, fed into a network, shown in figure 3, consisting of two 3-way power dividers and two hybrid ring networks. The line lengths of the network are such that $a = b$, $c = d$, $e = f + \lambda/4$, where $\lambda$ is the system design wavelength. The signals in the arms, before detection, are of the form shown in table 1.

**Table 1. Antenna Signal Forms**

<table>
<thead>
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<tr>
<td>A</td>
<td>$(2 \beta \cos \theta e^{j\phi}) e^{jwt}$</td>
</tr>
<tr>
<td>B</td>
<td>$(2 \alpha \sin \theta e^{j2\phi}) e^{jwt}$</td>
</tr>
<tr>
<td>C</td>
<td>$(\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{jwt}$</td>
</tr>
<tr>
<td>D</td>
<td>$(- \alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{jwt}$</td>
</tr>
<tr>
<td>E</td>
<td>$(\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{j(\omega t - \pi/2)}$</td>
</tr>
<tr>
<td></td>
<td>$+ (- \alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{jwt}$</td>
</tr>
<tr>
<td>F</td>
<td>$(\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{j(\omega t - \pi/2)}$</td>
</tr>
<tr>
<td></td>
<td>$- (- \alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{jwt}$</td>
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After square law detection, these signals are of the form shown in table 2.
Table 2. Signal Forms After Square Law Detection

<p>| | | | | | |</p>
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<tr>
<td>A</td>
<td>$\alpha^2 \cos^2 \theta$</td>
<td>B</td>
<td>$\alpha^2 \sin^2 \theta$</td>
<td>C</td>
<td>$\alpha^2 \sin^2 \theta + \beta^2 \cos^2 \theta + 2 \alpha \beta \sin \theta \cos \theta \cos \phi$</td>
</tr>
<tr>
<td>D</td>
<td>$\alpha^2 \sin^2 \theta + \beta^2 \cos^2 \theta - 2 \alpha \beta \sin \theta \cos \theta \cos \phi$</td>
<td>E</td>
<td>$\alpha^2 \sin^2 \theta + \beta^2 \cos^2 \theta + 2 \alpha \beta \sin \theta \cos \theta \sin \phi$</td>
<td>F</td>
<td>$\alpha^2 \sin^2 \theta + \beta^2 \cos^2 \theta - 2 \alpha \beta \sin \theta \cos \theta \sin \phi$</td>
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</tbody>
</table>

It can be seen that the sum of the indications (power) in arms A and B is equal to the sum of the indications in arms C and D, and is also equal to the sum of the indications in arms E and F, which must be the case considering the method in which the power is split up. These six arms have signals that are apparently a function of four variables: $\alpha$, $\beta$, $\theta$, and $\phi$. However, since the products $\alpha \sin \theta$ and $\beta \cos \theta$ are the only way in which these parameters appear, the signals in the six arms can actually be expressed in terms of three variables: $\alpha \sin \theta$, $\beta \cos \theta$, and $\phi$. Also, the detected signals in arms A and B are seen to be independent of $\phi$.

To find the direction from which an incident signal is coming, an algebraic function of the signals in arms, A, B, C, D, E, and F (for simplicity, the amplitude of the detected signal in an arm is represented by the name of the arm), combined with the knowledge of $\theta$ (which can be set by calibration) is all that is necessary. By simple manipulation

$$\phi = \sin^{-1} \left[ \frac{E-F}{\sqrt{(E-F)^2 + (C-D)^2}} \right]$$

(1)

and

$$\theta = \tan^{-1} \left[ \frac{(A)}{B} \right]^{1/2} \tan \theta_o$$

Theoretical plots of the amplitudes of the detected signals as a function of variations in $\theta$ and $\phi$ are given in figures 4, 5, and 6. There are, of course, many other relationships $^2$ that can be used to calculate

$^2$ A simpler relation for $\phi$ would be $\phi = \tan^{-1} \left( \frac{E-F}{(C-D)} \right)$ but this would be difficult to handle in the region in which $C-D \rightarrow 0$, in which case one could consider $\phi = \cot^{-1} \left( \frac{(C-D)}{(E-F)} \right)$. 

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Figure 4. Theoretical detected signal in arms A and B as a function of elevation angle.
Figure 5. Theoretical detected signal in cases C, D, E, F as a function of elevation angle, for φ equal to π/4.
Figure 6. Theoretical detected signal in arms C, D, E, F as a function of elevation angle, for $\phi$, $\pi/2$, $\pi$, $(3\pi)/2$. 
\( \phi \) and \( \Theta \), and a study of the deviation of the actual first and second mode antenna patterns from the assumed relations might be useful in determining the probable deviation of the actual source direction from the calculated source direction.

From the redundancy in observations, it might seem that fewer detectors could be used to determine \( \Theta \) and \( \phi \). In this section it will be shown that, in general, this is not possible, but that under certain assumptions it might be possible to eliminate arms A and B (and thus replace the three-way power dividers with two-way power dividers) from the system.

Let
\[
\frac{1}{2} \left[ (C-D)^2 + (E-F)^2 \right]^{1/3} = K_1 = 2 \alpha \beta \sin \phi \cos \phi
\]

Then
\[
K_2 = \frac{1}{2} (C + D) + K_1 = \Gamma \sin \phi + \beta \cos \phi + 2 \alpha \beta \sin \phi \cos \phi
\]

\[
= \left[ \alpha \sin \phi + \beta \cos \phi \right]^2
\]

and, similarly
\[
K_3 = \frac{1}{2} (C + D) - K_1 = \left[ \alpha \sin \phi - \beta \cos \phi \right]^2
\]

Now, if it is known how \( \phi \) compares with \( \phi_0 \), that is, whether it is larger or smaller, then \( \Theta \) can be found. This might be the case in a direction finder or homing device. For example, if \( \phi > \phi_0 \), then
\[
\Theta = \tan^{-1} \left[ \frac{K_2^{1/2} + K_3^{1/2}}{K_2^{1/2} - K_3^{1/2}} \tan \phi_0 \right]
\]

if \( \Theta < \Theta_0 \), then \( \alpha \sin \phi - \beta \cos \phi < 0 \), and
\[
\Theta = \tan^{-1} \left[ \frac{K_2^{1/2} - K_3^{1/2}}{K_2^{1/2} + K_3^{1/2}} \tan \phi_0 \right]
\]

Thus, if it is known from which general direction (in elevation) the received signal is expected, four detectors can be used instead of six. Also, by adjusting the pattern gains in the first and second modes (i.e., \( \alpha \) and \( \beta \)), \( \Theta_0 \) can be controlled and thus give better control over the region from which signals are permissible. Making \( \Theta_0 \) near \( 0^\circ \) or \( 90^\circ \), however, will introduce a large uncertainty in the calculated direction for a small measurement error.
3. EXPERIMENTAL PROCEDURE

A two-wire Archimedean spiral antenna $2\lambda/\pi$ in diameter was mounted $\lambda/4$ in front of a 24-in. square ground plane. This assembly was then mounted on a two-axis rotator. The two outputs of the antenna were fed into the networks illustrated in figure 7.

The antenna was mounted so that it could be rotated about its axis of symmetry ($\theta = 0$). The antenna was illuminated with a circularly polarized continuous wave signal at 1020 Mc. The terminals at A through F were all terminated in matched dummy loads, except for one that was terminated in a square law detector. The detected signal amplitudes at C, D, E, F were recorded as a function of $\phi$ for $\phi = 5^\circ$, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45° (figures 8, 9, 10, 11, 12, 13, 14). The amplitudes at A and B were recorded as a function of $\phi$ for the $\theta$'s listed above (fig. 15, 16, 17, 18, 19, 20, and 21).

Figure 7. Experimental direction finder.
Figure 8. Azimuthal plots of C, D, E, F for θ = 5°.
Figure 9. Azimuthal plots of C, D, E, F for $\theta = 10^\circ$. 

$f_0 = 1022$ mas - LEFT CIRCULAR POLARIZATION
Figure 10. Azimuthal plots of C, D, E, F for $\theta = 15^\circ$. 

$\theta = 15^\circ$
Figure 11. Azimuthal plots of C, D, E, F for $\theta = 30^\circ$. 

$f_0 = 1022$ mzo - LEFT CIRCULAR POLARIZATION
Figure 12. Azimuthal plots of C, D, E, F for $\theta = 45^\circ$. 

$\phi = 1022$ mrad - LEFT CIRCULAR POLARIZATION
Figure 13. Azimuthal plots of C, D, E, F for $\theta = 60^\circ$. 

$f_0 = 1022$ mg - LEFT CIRCULAR POLARIZATION

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Figure 14. Azimuthal plots of C, D, E, F for $\theta = 75^\circ$. 

$\ell^\circ = 1022$ mas - LEFT CIRCULAR POLARIZATION
FIRST MODE PATTERN (A)

SECOND MODE PATTERN (B)

\( \theta = 50^\circ \) - LEFT CIRCULAR POLARIZATION

These patterns were recorded at a gain setting 10dB below the others

Figure 15. Azimuthal plots of A and B for \( \Theta = 50^\circ \).
Figure 16. Azimuthal plots of A and B for $\theta = 10^\circ$. 

- First Mode Pattern (A)
- Second Mode Pattern (B)

$\phi = 1022$ mce - Left Circular Polarization

These patterns were recorded at a gain setting 10 dB below the others.
Figure 17. Azimuthal plots of A and B for $\theta = 15^\circ$
Figure 18. Azimuthal plots of A and B for $\theta = 30^\circ$. 

FIRST MODE PATTERN (A)  
SECOND MODE PATTERN (B)  
$f_0 = 1022$ MHz - LEFT CIRCULAR POLARIZATION

THESE PATTERNS WERE RECORDED AT A GAIN SETTING 10dB BELOW THE OTHERS.
Figure 19. Azimuthal plots of A and B for $\theta = 45^\circ$. 

--- FIRST MODE PATTERN (A) 
--- SECOND MODE PATTERN (B) 

$\theta = 45^\circ$ 

$f_0 = 1022$ mcs—LEFT CIRCULAR POLARIZATION
Figure 20. Azimuthal plots of A and B for $\theta = 60^\circ$. 

---

FIRST MODE PATTERN (A)  
SECOND MODE PATTERN (B)  
$f_0 = 1032$ mc - LEFT CIRCULAR POLARIZATION
Figure 21. Azimuthal plots of A and B for $\theta = 75^\circ$. 

--- FIRST MODE PATTERN (A) 
--- SECOND MODE PATTERN (B) 

$\phi = 1022$ mce - LEFT CIRCULAR POLARIZATION
4. CONCLUSIONS AND RECOMMENDATIONS

Since data given in figures 8 through 21 indicate that the system behaves approximately as assumed in the formulation tabulated in table 2, it is assumed that the antenna can be used to determine the spherical coordinate angles $\phi$ and $\theta$. It is estimated that the determination of $\theta$ will be most precise for $15^\circ < \theta < 75^\circ$.

No attempts have been made to optimize the circuit and antenna. However, it has been demonstrated that the spiral antenna is a precise phasing device, suggesting the possibility of engineering an accurate direction finder. Ultimate system accuracy will also be a function of the frequency dependence and losses of the processing networks. One source of azimuth error is discussed in appendix A.

Investigation of the three arrangements listed below may provide a method of increasing the precision of the direction finder.

(1) Increasing the $\lambda/4$ spacing between the antenna and the ground plane, causing a partial null in the first mode radiation pattern.

(2) Adding a similar antenna element in front of the antenna, increasing the gain of both first and second modes.

(3) Adding an antenna for the first mode only in front of the spiral antenna, thus increasing the first mode gain only.

No attempt has been made up to this time to evaluate the relative efficacy of the various arrangements.

---

APPENDIX A

FREQUENCY ERROR ANALYSIS

One possible source of error in the spiral antenna direction finder is the effect of a change in frequency from the design value. If the antenna is used as a direction finder, a shift in the source frequency would cause lines e and f of the network to have lengths that differ by more or less than a quarter wavelength. Also, if the system is to be used to search for and locate unknown signal sources, it would be necessary to know over what frequency range the system can be designed to search without exceeding an acceptable error in the calculated direction.

Equations (1) are rewritten for ease of analysis, as

\[ \phi = \tan^{-1} \left( \frac{E-F}{C-D} \right) \]

and

\[ \theta = \tan^{-1} \left( \left( \frac{A}{B} \right)^{1/2} \tan \theta_0 \right) \]

The only arms affected by a shift in frequency are E and F, so that the observed elevation angle is not affected by a change in frequency. If the difference in line length is equivalent to an angular rotation of \( \gamma \) degrees, then the input signals to arms E and F can be written (neglecting \( e^{j\omega t} \)) as

\[
E: \ (\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{-j\gamma} \\
\quad + \alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi} \\
F: \ (\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi}) e^{-j\gamma} \\
\quad -(-\alpha \sin \theta e^{j2\phi} + \beta \cos \theta e^{j\phi})
\]

For square law detection, the output of E is proportional to

\[ \alpha^2 \sin^2 \theta (1 - \cos \gamma) + \beta^2 \cos^2 \theta (1 + \cos \gamma) + 2 \alpha \beta \sin \theta \cos \theta \sin \phi \sin \gamma \]

and the output of F is proportional to

\[ \alpha^2 \sin^2 \theta (1 + \cos \gamma) + \beta^2 \cos^2 \theta (1 - \cos \gamma) - 2 \alpha \beta \sin \theta \cos \theta \sin \phi \sin \gamma \]
Thus the observed value of $\phi$ (call it $\phi'$) is found to be

$$\phi' = \tan^{-1} \left[ \frac{E - F}{C - D} \right]$$

$$= \tan^{-1} \left[ \frac{(\delta^2 \cos^2 \theta - \alpha^2 \sin^2 \theta) \cos \gamma + 2 \alpha \beta \sin \theta \cos \theta \sin \phi \sin \gamma}{2 \alpha \beta \sin \theta \cos \theta \cos \phi} \right]$$

$$= \tan^{-1} \left[ \frac{(\delta^2 \cos^2 \theta - \alpha^2 \sin^2 \theta) \cos \gamma}{2 \alpha \beta \sin \theta \cos \theta \cos \phi} + \tan \phi \sin \gamma \right]$$

The error $\phi' - \phi$ is thus

$$\phi' - \phi = \tan^{-1} \left[ \frac{(\delta^2 \cos^2 \theta - \alpha^2 \sin^2 \theta) \cos \gamma}{2 \alpha \beta \sin \theta \cos \theta \cos \phi} + \tan \phi \sin \gamma \right] - \phi$$

The error in azimuth angle is seen to be strongly dependent upon the elevation angle $\theta$.

Substituting the crossover point relation

$$\alpha \sin \theta_c = \beta \cos \theta_c$$

gives, after some rearrangement

$$\phi' - \phi = \tan^{-1} \left[ \frac{\tan^2 \theta_c - \tan^2 \theta}{2 \tan \theta_c \tan \theta} \frac{\cos \gamma}{\cos \phi} + \tan \phi \sin \gamma \right] - \phi$$

If the gains of the first and second modes are equal ($\tan \theta_c = 1$), this reduces to

$$\phi' - \phi = \tan^{-1} \left[ \frac{\cot 2\theta \cos \gamma}{\cos \phi} + \tan \phi \sin \gamma \right] - \phi$$

Thus, for elevation angles ($\theta$) near 0° (on axis) or near 90° (in the plane of the spiral array), a large error in the calculated value of $\phi$ may occur for values of $\gamma \pm \pi/2$.

Plotting the error in azimuthal angle as a function of $\gamma$, as shown in figures A1 and A2, reveals that the maximum error occurs at values of $\phi$ near 0°, and the possible error increases greatly with increasing deviation of the line length difference from 90°. If this device is to be used to locate unknown signal sources, it is desirable to have a single device operate over a wide range of frequencies. By using newly
Figure A1. Theoretical azimuthal error for $\gamma = 92^\circ$. 
Figure A2. Theoretical azimuth error for $\gamma = 95^\circ$. 

$\theta = 80^\circ, 70^\circ, 60^\circ, 50^\circ, 45^\circ$. 

(Degrees) $\phi - \phi$
developed phase shifters, in which the phase difference between two channels is very nearly 90° over a broad frequency range, it is theoretically possible to hold γ to the range of 88° to 92° over a 2:1 range in frequency. Assuming that this value can be approximately held in practice and that other sources of error do not exist, the maximum error in calculated azimuthal angle for elevation angles between 10° and 80° is held to approximately 5°.

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(Two pages of abstract cards follow)
A miniaturized unambiguous direction finder with no moving parts is described. At a frequency of 1000 kc, a complete system, consisting of a two-wire spiral antenna, with cavity backing, operating in the first two radiation modes can be built into a cylinder 10 in. in diameter and 3 in. deep. Direct scaling laws apply, so that at 500 kc, for example, the cylinder would be 5 in. in diameter and 1 1/2 in. deep. Since there are no moving parts and printed-circuit techniques can be used, the total weight (excluding power supply and readout or display) will be less than 1/2 lb. The pattern of this antenna array gives hemispheric coverage. If operated in an analog device, a determination of the elevation and azimuth of a received signal, with no ambiguities, in the hemisphere bounded by the plane of the spiral. The processing network consists of two 3-way power dividers and a pair of hybrids which can be mounted on the back of the cavity by using strip-line techniques. An experimental version of the system has been built and preliminary data are presented.