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TRANSIENT PRESSURE MEASURING
METHODS

Effects of Tubing Connection on Transducer Response

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I. GENERAL

For a number of years, Princeton University has been extremely active in liquid propellant combustion instability research through the Jet Propulsion Group of the Department of Aeronautical Engineering. Experimental rocket chamber operation has required from the start complete and exhaustive instrumentation for transient pressure fluctuations, and the research program has been heavily dependent on the results of this testing in terms of actual transient pressure records. As the sophistication of the test program increased, the requirements for a more advanced means of obtaining and analyzing the necessary data increased as well. To the end that the technique and knowledge gained in this research effort could be further advanced and disseminated to others in the field, a contract with NASA was entered into in March 1961 to further define and improve the ability to obtain transient pressure data in rocket chamber testing. This report is the first of several which are the outcome of this program. There will be reports published shortly on basic transient transducer design and limitations placed by this specialized application as well as the design and development of evaluation techniques and equipment to discover and demonstrate the ability and shortcomings inherent in transducer systems. Evaluation of several existing systems and the suggested improvements including the results of practical testing of these systems will be covered. Some preliminary designs of new transducers oriented toward the specifications which resulted from the cooperation of a number of users of transient pressure instrumentation will be discussed also.
TECHNICAL REPORT ON TUBING CONNECTED TRANSDUCERS

II. INTRODUCTION

Under this contract, the prime effort is toward the more advanced phases of the measurement of transient pressure in rocket chambers. It is necessary from a practical standpoint, however, to investigate the measurement of the slower transients, and oscillatory phenomena by means which, although inadequate, represent the common practice in the rocket industry and in most research and development establishments. The validity of such practices as using several inches or more of tubing to connect a transducer into the chamber depends upon the frequency range required, which, of course, is related to the size of the chamber and the mode of instability being examined. Theoretical and experimental treatment of tubing connected transducers has been included in the overall reporting related to this contract with NASA, NASr 36, because the technique of transient pressure measurement is imperfect enough and is beset with sufficient obstacles that many testing facilities simply do not attempt to record this important data, or they resort to accelerometer data, which is misleading due to its extremely poor signal-to-noise ratio. It will be shown that under certain limited conditions, transducers can be connected by short lengths of tubing and render useful data on a restricted frequency basis.

III. NECESSITY OF USING A TUBING CONNECTED TRANSDUCER

Transient pressures are most faithfully recorded by means of a flush-mounted transducer. Presently available transducers, however, are large in diaphragm size, they have generally poorer
accuracy, more hystorisals are subject to more zero drift than
a cavity type, and they have been notoriously poor in dependability when
subjected to high heat flux densities. To install a transient pressure
transducer in a regeneratively cooled chamber is an extremely difficult
undertaking simply because of these problems and the mechanical diffic-
culties involved in a suitable pressure boss. A pressure boss with
a 1/4" hole or less being otherwise commonly supplied, a useful approach to
this measurement of transient pressure would then be to determine the
effect of the necessary length of tubing to connect a transducer to
the chamber and restrict the recorded response to a value well below
this resonant frequency. A number of researchers have published papers
in regard to the effect of tubing on pressure transients, primarily in
the slower ranges (1, 2, 3). The treatment in this paper is designed
to supplement the theoretical treatments therein supplied.

IV. THEORETICAL TREATMENT

The tubing with which a transducer is connected into a rocket
chamber will result in serious distortion of the frequency response.
This distortion is caused by resonance of the connecting tubing and the
distortion is therefore far more serious than simple attenuation. Further-
more, unless a transducer of minimal volume is used, resonance of the
tubing-transducer system will occur at a lower frequency than with
tubing alone.

The tubing may be considered as an organ pipe, open at one
dend, closed at the other:

$$f_{oc} = \frac{C}{4l}$$

where

$c = \text{velocity of sound}$

$l = \text{tubing length}$

* Numbers in parentheses refer to references list at the end of this report.
The frequency found in (1) is correct in short tubes only for the case where zero volume transducers are used at the termination of the tube.

When a transducer having a finite volume is used with a length of tubing short enough to constitute merely an orifice, the result is a Helmholtz resonator (Ref. 4), similar in construction to the whistle, ocarina or the body of a violin. Its resonant frequency is as follows:

$$\frac{1}{2\pi} \sqrt{\frac{1}{MCa}}$$

(2)

where

$M = \text{inertance (circular orifice only)}$

$Ca = \text{acoustical capacitance}$

The analogy of (2) to the formula of a lumped parameter series resonant circuit will be noted:

![Diagram of a series resonant circuit]

To reduce (2) to practical terms, $M$ and $Ca$ may be evaluated:

$$M = \rho \frac{(l + 1.7r)}{\pi r^2}$$

(3)

where

$l = \text{length of entrance orifice}$

$\rho = \text{density of gas}$

$r = \text{radius of entrance orifice}$
\[ Ca = \frac{V}{\rho c^2} \]  
(4)

where \( V \) = volume of chamber  
\( c \) = velocity of sound in gaseous media

Substituting (3) and (4) in (2):

\[
\begin{align*}
F_h &= \frac{1}{2\pi} \int \frac{\rho c^2 (\pi r^2)}{\nu \rho (l+1.7r)} \\
&= \frac{C}{2\pi} \int \frac{\pi r^2}{\nu (l+1.7r)}
\end{align*}
\]

(2a)

Obviously as \( l \) is increased, at some value of \( l \), \( 2a \) becomes invalid, because when \( l \) is large enough \( 2a \) reduces to:

\[
F_h = \frac{C}{2\pi l}
\]

(2b)

This is in disagreement with (1) and the transition range is sufficiently resistant to calculation that it is beyond the scope of this section to attempt it. This difficulty stems from the fact that the derivation of (2) was from lumped parameters as previously shown, while (1) is analogous to a transmission line, having distributed parameters throughout its length.

Figure 1 shows the calculations of (1) and (2), \( F \) plotted against resonant frequency for 2 values of sound velocity.

The velocity of sound in the combustion gases at a pressure \( P_o \) in a rocket chamber is described by:

\[
C = \sqrt{\frac{\gamma P_o T_i}{\rho}}
\]

(5)
LENGTH VS FREQUENCY (HELIUM) FOR HELMHOLTZ RESONATOR AND ORGAN PIPE THEORETICAL CURVES + EXPERIMENTAL POINTS
where \( \gamma = \) ratio of specific heats
\( P_0 = \) pressure in gas
\( \rho = \) density of gas
\( T_1 = \) temperature of gas

A simple reduction of units results in:

\[
C = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{T_1}{T_0}}
\]

(5a)

where \( E = \) bulk modulus of elasticity:
\( \frac{T_1}{T_0} = \) ratio of absolute temperatures

Fortunately, \( E \) and \( \rho \) are mutually affected by pressure and the effect upon \( C \) of pressure is therefore negligible. Temperature, however, does affect \( C \) and must be estimated to predict response.

V. USE OF FIGURE 1 TO PREDICT RESPONSE

Calculation of resonant frequency of a line terminated in zero volume is simple as the only variables are \( \gamma \) and \( C \). This curve then is not affected by tubing diameter. In the curve computed by (2a), however, both transducer volume and tubing diameter must be considered. The curve resulting therefrom is a very special case, as the parameters were chosen to fit the case of 1/4" tubing, having an I.D. of .173 and a transducer volume similar to that of a Tabor transducer, Model 601624, .03 cubic inches. When computing for other transducers and tubing diameters, appropriate corrections must be made.

An obvious difficulty lies in obtaining really adequate...
knowledge of $C$, when neither temperature nor composition of the combustion gases is really well established in the connecting tubing. It is certain that temperature is lower than the chamber temperature, but is probably somewhat above ambient. Depending on what cooling methods for the connecting tubing are used and upon the composition of the combustion products finding their way into the tubing, an estimate must be made. Since $C$ only varies as the square root of the absolute temperatures, however, not too large an error will be introduced owing to an error in estimation of the temperature of $100^0$, about 20%. If the tubing is cooled by a water spray on its outer surface, this error will be lowered somewhat although, of course, $C$ will be lower. The composition of the combustion products, dependent upon what propellants are in use, should not be too difficult to determine. Tubing connections should be located as much as possible in a vertical direction downwards, so collection of liquids will not be likely to occur. The presence of liquefied propellant, water, or extremely high density material, even if gaseous will, of course, cause serious degradation of response. It should be possible to predict $F$ with an expected error of no more than 25%.

VI. WATER-FILLED TUBING

Occasionally, in order to isolate the transducer from the hot gases, water filled tubing has been tried. The results from this expedient usually appear to exhibit much higher response than in a gas filled tube. Upon further investigation, however, the apparently high response is spurious and is accounted for by the fluid mass, the elasticity of the transducer chamber and connecting tubing, and the low damping of the relatively large tubing usually employed. The damping
factor of a typical liquid filled system is far lower than the same system gas-filled, because of low values of flow occurring due to the incompressible fluid used. The output of any transducer so installed is subject to extreme ringing at high amplitude and any increase in frequency response will be masked by it. The resonant frequency of a line, filled with water is about 2 to 3 times higher in any case, the value of \( C \) for water being 4760 ft/sec, compared to 2000-3000 for combustion gases.

VII. EXPERIMENTAL VERIFICATION

An experiment was initiated whereby several lengths of tubing were excited sinusoidally by means of a sine wave pressure modulator,* developed under a contract with NASA, both with and without a terminating volume. The data derived from this portion of the experimental program are plotted on the same axes as the calculated curves in Figure 1. Both nitrogen and helium were used in the experiment, and both give reasonable agreement with calculated values. The response curve for a typical tubing length is shown in Figure 2 with and without the added transducer volume. The chamber of the modulating device which was designed for response measurement of flush transducers actually constitutes a portion of the resonant system and so amplitude results at the higher oscillatory modes, should be considered doubtful. A small transducer having very high frequency response, an SLM Quartz transducer, is used to monitor the modulator chamber and the response of the lines, without transducer volume added, was obtained by means of a second SLM transducer mounted as shown in Figure 3. Response is plotted in per cent response of the terminating

* This device will be described completely in a technical report to be issued shortly.
TUBING ALONE
TUBING TERMINATED IN \(0.08\text{m}^3\) VOLUME

\begin{align*}
\text{PERCENT RESPONSE} & \quad \text{DRIVING FREQUENCY} \\
0 & \quad 0 \quad 200 \quad 600 \quad 800 \quad 1000 \quad 2000 \quad 4000 \quad 6000 \\
\text{200 CPS} & \quad \text{600 CPS} & \quad \text{800 CPS} & \quad \text{1000 CPS}
\end{align*}

\begin{align*}
\text{TUBING ALONE} \\
\theta = 0^\circ & \quad \theta = 36^\circ & \quad \theta = 180^\circ & \quad \theta = 180^\circ \\
\text{TUBING VOLUME} \\
\theta = 50^\circ & \quad \theta = 180^\circ & \quad \theta = 180^\circ & \quad \theta = 360^\circ
\end{align*}

UPPER TRACE - TEST SECTION
LOWER TRACE - REFERENCE

RESPONSE OF 5\(\frac{3}{8}\)" TUBING
CLOSED END AND VOLUME TERMINATION

FIGURE 2
FIGURE 3

SECTIONAL VIEW OF EXPERIMENTAL APPARATUS
transducer referred to the monitor. The summary of results shows the close agreement between calculated and theoretical response.

To verify further the experimental results, each combination was tested on the shock tube. The response thus obtained is shown in Figure 4, the shock tube excited step function response of the tubing-transducer assembly for a given tubing length. The shock tube was operated on nitrogen, at 50 to 1 pressure ratio, 30 psi shock front at tubing entrance. It was assumed that no shock would reach the transducer, because of boundary effects in the relatively small size tubing, so a simple isentropic compression was assumed and temperature correction of $C$ was made:

$$T_2 = T_0 \left( \frac{P_2}{P_1} \right)^{\gamma/C_p}$$

$$C_2 = C_0 \sqrt{\frac{T_2}{T_1}} = 1.62 C_0$$  \hspace{1cm} (6)

where $C_1 = \text{velocity of sound} = 520^\circ \text{K}$, $C_2 = \text{velocity at new temperature}$, $P_2/P_1 = \text{shock tube pressure ratio} = 31$.

The results are in agreement with the calculated and sine wave response.

Figure 5 is a photograph of the Sine Wave Modulator and Figure 6 is a block diagram of the system.

VIII. CONCLUSION

Although there are better methods available to measure transient
FIGURE 4a.
Pressure Step Coupled to Zero Volume Transducer by 5-3/8" Tube
938 cps Indicated Before Temperature Correction*
Upper Trace Unfiltered, 140KC Transducer Resonance
Lower Trace Low Pass Filter 20KC

FIGURE 4b.
Same as 4a, but Transducer with .02 in³ Volume was Used
Upper Trace Unfiltered, Transducer Resonance 6.5KC
Lower Trace 9/2 cps Indicated Before Temperature Correction*

*See text.
BLOCK DIAGRAM OF SINE WAVE MODULATOR SYSTEM
pressures in a rocket motor than by using tubing connected transducers, when discretion in data analysis is used and a reasonably low response is sufficient, this method can successfully deliver usable data, and circumvent the obvious difficulty of a large chamber irregularity and difficulties with burnout due to marginal cooling under unstable burning conditions. Where higher response is necessary, however, than herein predicted, flush mounted transducers are essential.
REFERENCES


2. Li, Y. T., "Dynamic Measurement," Chapter II, Summer Session course, 16,369, Department of Aeronautical Engineering, Massachusetts Institute of Technology.


### SUMMARY OF RESULTS

<table>
<thead>
<tr>
<th>TUBING LENGTH (IN)</th>
<th>TERMINATION VOLUME (IN³)</th>
<th>HELIUM CALCULATED</th>
<th>HELIUM SINE WAVE MODULATOR²</th>
<th>NITROGEN CALCULATED</th>
<th>SHOCK TUBE</th>
<th>NITROGEN SINE WAVE MODULATOR²</th>
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<td></td>
<td>255</td>
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<td>137</td>
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</table>

1. Nitrogen driving nitrogen - Includes isentropic temperature correction. Estimated accuracy ± 15%
2. Estimated accuracy ± 5%
3. Probable reason for larger discrepancies in longer tubes is the temperature gradient along its length.