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ERRATA SHEET No. 1
Frankford Arsenal Report R-159, June 1961
"Analog Computer Study of Reactionless Launchers" by S. Goldstein and S. Horst.

Please make the following corrections in subject report.

1. Page 7, 3d paragraph, end of second line -
   Insert: backwards
   To read: ....... integrating the curve backwards from the point ......

2. Page 10, 4th and 5th lines below Equation (5) -
   Change: R is the molar gas constant and it is equal to 2782
   ft-lb-mole-°K;
   To read: R is the universal gas constant and it is equal to
   \( \frac{2782 \text{ ft-lb (energy)}}{\text{lb (weight)-mole-°K}} \);

3. Page 40, 3d paragraph, 2nd line -
   Change: This was probably due to failure of ......
   To read: This was probably due to malfunction of ......

4. Page 40, 3d paragraph, 5th line -
   Change: ....... of measurement) was less than the charge weight.
   To read: ....... of measurement) was more than the charge weight.

5. Page 45, Table IV, 3d line -
   Change: F Propellant impetus 3.78 ft-lb/lb
   To read: F Propellant impetus 3.78 x 10^5 ft-lb/lb
ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

BY

S. GOLDSTEIN
S. NARISI

ARPA Order No. 39-59, Task No. 8
ABMA Control No. 1D-295-00460
SPB 2-2249-60

JUNE 1961

FRANKFORD ARSENAL
RESEARCH AND DEVELOPMENT GROUP
PHILADELPHIA 37, PA
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REPORT R-1589

ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

FA Subproject No. C409
ARPA Order No. 39-59

Task No. 8  ABMA Control No. ID 295-00-60
           SPB 2-2249-60

PREPARED BY:

S. GOLDSTEIN
Physicist

REVIEWED BY:

J. J. DONELLY
Chief
Ballistics Branch

H. P. MANNING
Acting Chief,
Ballistic Systems Division

APPROVED BY:

THOMAS J. RABER
Colonel, Ordnance Corps
Chief
Research and Development Group

FOR:

A. R. CYR
Colonel
Ordnance Corps Commanding

FRANKFORD ARSENAL
Research and Development Group,
Philadelphia 37, Pa.

June 1961
OBJECT

To verify the assumptions and determine the empirical approximations needed to simulate the Davis Gun principle on an analog computer.

SUMMARY

Standard closed-borech interior-ballistic equations were modified to make them applicable for predicting the performance at reactionless launchers. Following this, a series of test firings were made using two reactionless launchers of the Davis Gun type for obtaining the test firing data required. Direct and high-low launchers were considered. Concurrent with the actual test firings, an analog computer was programmed to predict the performance of the guns using the same charge weight, propelled weight and shot-start. The test firing data was compared with the curves obtained from the computer. Using the differences noted, certain factors of the original equations programmed into the computer were varied in order to bring the computer curves into close agreement with actual test results. These factors included heat loss coefficient, initial charge burnt, and in the case of the high-low gun the effective mean impetus of the propellant during the ballistic cycle. It was concluded that the knowledge gained can be used to predict and optimize the performance of reactionless systems and recommended that research be conducted, using the computer, to establish the feasibility of high-muzzle-velocity launchers, the ejection of unequal and/or large weights, the simultaneous firing of multiple tube systems, and the dynamic breaking behavior of the shot-start rod and cartridge case. It was further recommended that a factorial type program be used when it is desired to obtain data for analytical studies of ballistic devices.

AUTHORIZATION

ARPA Order No. 39-59, Task No. 8

ABMA Control No. ID 295-00-60

SPB 2-2249-60
# Analog Computer Study of Reactionless Launchers

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ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

INTRODUCTION

As part of the Advanced Research Projects Agency (ARPA) Program which envisions ejecting diversionary objects from ballistic missiles, Frankford Arsenal was authorized to investigate methods for packaging and launching such objects, under the administration of the Army Ballistic Missile Agency (ABMA). The work was subsequently assigned to Pitman-Dunn Laboratories. An investigation of the patents on the Davis Gun and various texts on ballistics revealed that very little published work had been done on this type of reactionless gun. One phase of the project was an analog computer study to investigate the interior ballistics of reactionless launchers. This report covers the computer study. It gives a brief description of designs tested, followed by explanations of the test firing setups, ballistic theories, computer setups and computer results associated with each design.

REACTIONLESS LAUNCHER DESIGNS TESTED

Launcher Design: No. 1

Launcher No. 1 shown on Figures 1, 2, and 3 is a direct* system of the full caliber** type(1). This launcher is 24 inches long with a smooth bore diameter of 4.134 inches. It is used to eject two projectiles with equal force in opposite directions without transmitting reaction to its mounting platform. For a shot-start device

*A direct system is one in which the projectile acquires its velocity as a result of direct pressure from the gases of the burning propellant.

**In a full caliber system the projectile diameter is the same size as the bore of the gun, i.e. sabots or over caliber projectiles with spigots are not used.

1. Frankford Arsenal Technical Memorandum #60-26-1 "Decoy Packaging Launching Studies" (4) -- Secret
Figure 1. Assembly Drawing of Launcher No. 1
Figure 3. View Showing Propellant Igniter for Launcher No. 1.
ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

it uses a projectile connecting rod calculated to break when subjected to a predetermined pull force. The operation of launcher No. 1 is initiated by two electrically fired detonators attached to opposite ends of a 1-foot length of pyrocore detonating cord which is wound around the necked-down portion of the rod. When the pyrocore cord is detonated it ignites the propellant which surrounds it, causing a pressure to be built up between the two projectiles. When the pressure reaches the level required to break the connecting rod at the necked-down section, the projectiles start to move and are eventually ejected from the launcher tube.

Launcher Design No. 2

Direct System

Launcher No. 2 shown on Figure 4 (with the exception of the orifice) was originally designed as a direct system of both the open and closed spigot types.* The launcher is 31 inches long, with a bore diameter of 2.25 inches. Launcher No. 2 operation is initiated by gas pressure from an external source such as an initiator. The gas enters the firing head and propels the firing pin into the launcher cartridge. When the cartridge fires the propellant gas generated flows directly into the expansion chamber where it acts when on the projectiles, propelling them from the chamber.

High-Low System

The launcher No. 2 design using only a closed spigot was later modified as seen in Figure 4 by adding an orifice plate between the cartridge housing and the launcher tube section. The purpose of the orifice plate was to produce the high-low ballistic type situation where the gas is generated in a confined volume under high pressure and vented through an orifice into a volume of lower pressure.

DIRECT SYSTEMS

Firing Test Setup

Launcher No. 1

To initiate the program, calculations of charge weight, pressure, time, projectile velocity, acceleration and travel were made

*A spigot is a device for firing a projectile from a gun tube which is of a smaller caliber than the projectile. It is usually a hollow metal cylinder with the ends closed. If the end of the spigot facing the combustion chamber is closed, the design using this spigot is referred to as a closed spigot design. Conversely, if the end is open the design is referred to as an open spigot design.
based on projectile diameter, weight and travel. Test launcher No. 1 was then fired to verify the calculations. Thirteen firings were made, including four exploratory firings to check out the system. In all of the test firings, two 25-point weights were ejected simultaneously from the launcher. The charge values used for these firings were 50, 60, and 70 grams and the static shot-start breaking pressures were 500 and 1000 psi.

Two kinds of shot-start rods were tested in this system. One kind was designed to break under a static tensile stress of 500 psi, the other at 1000 psi. Since the tensile strength increases at high rates of loading, the actual shot-start pressures in the test firings were substantially greater than the static breaking strengths of the rods.

The pressure time curves for the firings were measured and plotted. Correct travel was obtained by double integrating the curves from the point of muzzle exit time (which appears as a sharp dip in the pressure curve at the end of the stroke). The zero points for travel thus gave the points on the pressure-time curves at which the projectiles began to move, and hence the approximate shot-start breaking pressures. The ballistic cycle times and velocities were confirmed by evaluating high-speed movies taken during the tests.

Rounds 7, 11, 12 were considered representative, and the curves for these rounds are shown in the "Computer Results" section of this report.

Launcher No. 2

The firing program for Launcher No. 2 (direct system) was divided into two parts, factorial programs B and C. In factorial program B, the charge weight, propellant web, and shot-start rod were varied. The spigot type and projectile weight were held constant. In factorial program C, the shot-start rod, spigot type, and projectile weight varied, and the charge weight and web were held constant. The values of these variables and constants are given in Table I. Since there were three variables for each program and two values per variable, there were eight possible combinations. Each combination was fired three times; thus, the total of rounds for each part of the program was 24.

Pressure, velocity, and travel data were obtained in the same manner as for Launcher No. 1. The curves for representative rounds are also shown in "Computer Results." The complete firing results are given in Reference No. 3.

1. Frankford Arsenal Technical Memorandum 460-26-1 "Decoy Packaging Launching Studies" (U) - Secret
Table I. Factorial Programs (Direct System)

Factorial Program B
(Projected weight 60 lb closed spigot)

<table>
<thead>
<tr>
<th>Charge Weight (gm)</th>
<th>Propellant Web (in.)</th>
<th>Shot-Start Static Breaking Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.016</td>
<td>800</td>
</tr>
<tr>
<td>7</td>
<td>0.016</td>
<td>1500</td>
</tr>
<tr>
<td>7</td>
<td>0.030</td>
<td>800</td>
</tr>
<tr>
<td>7</td>
<td>0.030</td>
<td>1500</td>
</tr>
<tr>
<td>20</td>
<td>0.016</td>
<td>800</td>
</tr>
<tr>
<td>20</td>
<td>0.016</td>
<td>1500</td>
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<tr>
<td>20</td>
<td>0.030</td>
<td>800</td>
</tr>
<tr>
<td>20</td>
<td>0.030</td>
<td>1500</td>
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</tbody>
</table>

Factorial Program C
(Charge weight 20 gm, propellant web 0.016 in.)

<table>
<thead>
<tr>
<th>Projectile Weight (lb)</th>
<th>Shot-start Static Breaking Pressure (psi)</th>
<th>Spigot Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>None</td>
<td>Open</td>
</tr>
<tr>
<td>20</td>
<td>None</td>
<td>Closed</td>
</tr>
<tr>
<td>20</td>
<td>800</td>
<td>Open</td>
</tr>
<tr>
<td>20</td>
<td>800</td>
<td>Closed</td>
</tr>
<tr>
<td>60</td>
<td>None</td>
<td>Open</td>
</tr>
<tr>
<td>60</td>
<td>None</td>
<td>Closed</td>
</tr>
<tr>
<td>60</td>
<td>800</td>
<td>Open</td>
</tr>
<tr>
<td>60</td>
<td>800</td>
<td>Closed</td>
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Ballistic Theory

Assumptions made for this study are as follows:

1. All of the propellant is ignited simultaneously, and burning proceeds identically on each grain.
2. The burning of the ends of the propellant grains is not important.
3. The heat loss is proportional to the kinetic energy of the projectile.
4. There is no pressure or temperature gradient behind the projectile.
5. Frictional effects are negligible.

6. The kinetic energy of the gases is negligible.

7. The effect of the initial body of air in the chamber on the temperature at any time is negligible.

8. The effects of the igniter charge on the pressure-time history were considered negligible.*

9. The molecular mass of the gaseous products of combustion is constant during the burning period.

10. The specific heat of the propellant is constant.

11. Work done on the chamber and shot-start device by the expansion of propellant gases is negligible and breaking of the rod occurs instantaneously when the specified pressure is reached.

The equations used in the computer study are as follows:**

**Acceleration** - The projectile acceleration at any time is:

\[ \ddot{x} = \frac{A \frac{dA}{dt}}{W} P \]  \hspace{1cm} (1)

where

- \( A \) is the bore area (in.\(^2\))
- \( g \) is the acceleration caused by gravity (ft/sec\(^2\))
- \( W \) is the projectile weight (lb)
- \( P \) is the pressure at any time (lb/in.\(^2\))

In this equation, \( \dot{x} > 0 \) when \( P > P_{ss} \), the shot-start pressure.

*Exception to this was made in the case of the small charge (2.5 gms) used in the high low study. (See p 40, para 3.)

**See Appendix I for List of Symbols.
Velocity - The projectile velocity at any time is:

\[ \ddot{x} = \int x \, dt \]  

(2)

Travel - The projectile travel at any time is:

\[ x = 12 \int \dot{x} \, dt \]  

(3)

where \( x \) is in inches, and \( \dot{x} \) is in feet per second

Volume - The volume filled by the propellant gas at any time is:

\[ V = V_0 + 2A \frac{C - N}{\rho} \rho A N \]  

(4)

where the quantity \( \frac{C - N}{\rho} \) represents the volume filled by the charge remaining unburned at any time, and \( \rho A N \) represents the volume filled by the molecules of the gas. The factor 2 in the quantity \( 2Ax \) appears because two projectiles move in opposite directions, each moving \( x \) distance.

Equation of State - The equation of state is:

\[ PV = N n R T \]  

(5)

The quantity \( N \) is the mass of the gas in the chamber at any time. This is equal to the mass of the charge burned at any time. The quantity \( n R T \) contains the units of specific energy: \( n \) represents the number of moles (formed by burning one gram of propellant); \( R \) is the molar gas constant and it is equal to 2782 ft-lb-mole-\(^\circ\)K; \( T \) is the temperature of the propellant gas at any time, \( t \), after the cartridge case bursts.

In ballistics, a quantity known as the "impetus," \( F \), of the propellant is a characteristic of its chemical composition. \( F \) is in units of ft-lb/\( n \). It is defined by the equation:

\[ F = n R T_0 \]

In this equation, \( T_0 \) is the isochoric adiabatic flame temperature.

By substitution, the equation of state can be written as:

\[ PV = NFT/T_0 \]  

(5a)

If the volume, \( V \), is in cubic inches, and if it is desired to determine the pressure in lb/in.\(^2\), the equation of state can be written as:

\[ P = \frac{12NFT}{VT_0} \]  

(5b)
Energy - The powder potential is defined by the equation:

\[ e = C_v T_0 \]

where \( C_v \) is the specific heat of the gas at constant volume and is averaged between the adiabatic flame temperature and a temperature 1000K lower. Its significance lies in the fact that it is approximately proportional to the useful energy obtained from a unit mass of propellant used in a given ballistic system.

\( C_v \) is defined as:

\[ C_v = \frac{nR}{\gamma - 1} \]

Thus, the powder potential,

\[ e = C_v T_0 \]

\[ = \frac{nRT_0}{\gamma - 1} \]

\[ = \frac{F}{\gamma - 1} \]

The internal energy per unit mass, \( e' \), of the gas at any temperature, \( T \), may be defined as:

\[ e' = C_v T \]

\[ = \frac{nRT}{\gamma - 1} = \frac{FT}{(\gamma - 1) T_0} \]

The total energy available for doing work is:

\[ E = E' = N C_v T_0 - N C_v T \]

\[ = N \frac{F}{\gamma - 1} - \frac{FT}{(\gamma - 1) T_0} \]

\[ = \frac{NF}{\gamma - 1} \left( 1 - \frac{T}{T_0} \right) \]

It has been shown that it is a valid assumption that the heat loss, \( E_h \), at any time in the ballistic cycle, is proportional to the kinetic energy of the two projectiles at that instant:\(^2\)

\(^2\) Corner, "The Theory of Internal Ballistics of Guns," Willey, 1950
The kinetic energy of the two projectiles is:

\[ K.E. = \frac{W^2 x}{g} \]

The complete energy equation may be derived by equating the total energy released by the propellant minus the total internal energy of the gas at any time, to the sum of the heat lost and the kinetic energy:

\[ E - E' = E_n + K.E. \]  

\[
\frac{N f}{\gamma - 1} (1 - \frac{T}{T_0}) = \frac{\beta W^2 x}{g} + \frac{W^2 x}{g} 
\]  

Equation of energy and state - A combined equation of energy and state may be obtained by eliminating \( T/T_0 \) between equations (5b) and (6a). This gives:

\[
P = \frac{12NF - 12 (\gamma - 1) (\beta + 1) \frac{W^2 x}{g}}{V} 
\]

Gas production - The equation of gas production is:

\[
\hat{N} = \rho Sr
\]

Where \( \hat{N} \) = the rate of production of gas

\( S \) = the burning surface area of the propellant

\( \rho \) = the density of the propellant

\( r \) = the linear burning rate of the propellant

In this discussion, only single-perforation propellant grains will be considered. For these, the burning surface, \( S \), remains constant since burning of the ends of the grains can be ignored if the ratio of the grain web, \( w \), to the grain length, \( L_g \), is sufficiently small.
To determine the total burning surface area, $S$, of a propellant charge, assuming that the individual grains ignite isochronally and burn uniformly, consider an individual grain with a surface (excluding its ends) of $S_i$. This surface is equivalent to:

$$S_i = 2\pi L_i (R_1 + R_2)$$

The density, $\rho$, of the propellant may be found in the following manner. The volume, $V_i$, of the individual grain of propellant is:

$$V_i = \pi L_i (R_1^2 - R_2^2)$$

The density of a propellant grain, $\rho_i$, is obtained by dividing the weight of the individual grain, $C_i$, by its volume:

$$\rho_i = \frac{C_i}{V_i} = \frac{C_i}{\pi L_i (R_1^2 - R_2^2)}$$

The mass of the gas produced per unit time by "n" grains of solid propellant of density, $\rho$, each with a burning surface, $S_i$, and density, $\rho_i$, can be written:

$$\dot{N} = \rho Sr = r \sum_{i=1}^{n} \rho_i S_i$$

The sum, $\sum_{i=1}^{n} \rho_i S_i = \sum_{i=1}^{n} \frac{2C_i}{R_1 - R_2} = \frac{2C}{R_1 - R_2}$

However, the web is:

$$w = R_1 - R_2$$

Therefore, \[ \dot{N} = \frac{2C r}{w} \]

In addition, the linear burning rate is: \[ r = B p^a \]

Substituting for $r$:

$$\dot{N} = \frac{2CBp^a}{w}$$

$$\int_{N_0}^{N} dN = \frac{2CB}{w} \int_{0}^{t} p^a dt$$
where \( N_0 \) is the mass of the propellant burned when \( t = 0 \) (at the time the cartridge case bursts).

The mass of the gas produced at any time is:

\[
N = N_0 + \frac{2CB}{w} \int_0^t P^2 dt
\]  \hspace{1cm} (8a)

Computer Setup

The operation of Reactionless Launchers Numbers 1 and 2 (direct system) was simulated with a Goodyear Electronic Differential Analyzer (GEDA), Model GN 215-L3. An electrical analog (Figure 5) was devised which would exhibit the same dynamic behavior as this type of launcher. The computer was then programmed to solve the ballistic equations for these launchers. Comparison between computer and experimental curves was made by making the points at the end of the two travel curves coincident. Once this point (and thus the length of stroke) was established the difference in shape of the pressure curves and any phase shift between them could be determined.

The analog employed 10 amplifiers, 10 potentiometers, 4 diodes, 19 fixed resistors, 2 variable resistors, 5 capacitors, 2 multipliers, 1 function generator, and 1 relay. Appendix I shows the symbols which denote these components on the analog schematic. The resistor values are in megohms, and the capacitor values are in microfarads. The potentiometer values represent the inputs to the computer. Table II shows the constants for Launchers Numbers 1 and 2 (direct system). Table III shows the variables for each round selected for simulation.

The circuit containing amplifiers 23 and 24 (Figure 5) simulates the effect of the necked-down rod. Initially, potentiometer 17 keeps the plate of the associated diode negative, so that the diode does not conduct. When the pressure reaches a certain value (proportional to the predetermined shot-start pressure), the diode conducts. The resulting output of amplifier 23, energizes the relay coil, switching in the circuit containing amplifiers 20, 17, and 19, which control the acceleration, velocity, and travel.
Figure 5. Computer Analog for Simulating Direct System
Table II. Constants for Launchers Numbers 1 and 2

<table>
<thead>
<tr>
<th>Launcher No. 1</th>
<th>Launcher No. 2 (Direct System)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>Burning rate exponent</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>Burning rate coefficient</td>
</tr>
<tr>
<td><strong>d</strong></td>
<td>Bore diameter</td>
</tr>
<tr>
<td><strong>F</strong></td>
<td>Impetus of the propellant</td>
</tr>
<tr>
<td><strong>g</strong></td>
<td>Acceleration caused by gravity</td>
</tr>
<tr>
<td><strong>T_o</strong></td>
<td>Adiabatic flame temperature</td>
</tr>
<tr>
<td><strong>V_o</strong></td>
<td>Initial volume</td>
</tr>
<tr>
<td><strong>W</strong></td>
<td>Projectile weight</td>
</tr>
<tr>
<td><strong>W</strong></td>
<td>Web of the propellant</td>
</tr>
<tr>
<td><strong>X_t</strong></td>
<td>Total travel (stroke)</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>Specific volume of the gas</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>Heat loss coefficient</td>
</tr>
<tr>
<td><strong>γ</strong></td>
<td>Ratio of specific heats</td>
</tr>
<tr>
<td><strong>ρ</strong></td>
<td>Density of the propellant</td>
</tr>
</tbody>
</table>
Table III. Variables for Launchers Numbers 1 and 2

<table>
<thead>
<tr>
<th>Launcher No. 1 (Direct System)</th>
<th>Rd 7</th>
<th>Rd 11</th>
<th>Rd 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>C Charge weight</td>
<td>60 gm</td>
<td>60 gm</td>
<td>70 gm</td>
</tr>
<tr>
<td>$P_{ss}$ Shot-start breaking*</td>
<td>2500 psi</td>
<td>3325 psi</td>
<td>4000 psi</td>
</tr>
<tr>
<td>$P_{ss}^1$ Static shot-start breaking pressure</td>
<td>500 psi</td>
<td>1000 psi</td>
<td>1000 psi</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Launcher No. 2 (Direct System)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factorial B</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Launch No.</th>
<th>Rd 4</th>
<th>Rd 11</th>
<th>Rd 12</th>
<th>Rd 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>C Charge weight</td>
<td>7 gm</td>
<td>20 gm</td>
<td>20 gm</td>
<td>7 gm</td>
</tr>
<tr>
<td>$P_{ss}$ Shot-start breaking*</td>
<td>1250 psi</td>
<td>1900 psi</td>
<td>2000 psi</td>
<td>860 psi</td>
</tr>
<tr>
<td>$P_{ss}^1$ Static shot-start breaking pressure</td>
<td>800 psi</td>
<td>1500 psi</td>
<td>0.016 in.</td>
<td>0.030 in.</td>
</tr>
<tr>
<td>w Propellant web</td>
<td>0.016 in.</td>
<td>0.030 in.</td>
<td>0.016 in.</td>
<td>0.030 in.</td>
</tr>
</tbody>
</table>

| **Factorial C** |

<table>
<thead>
<tr>
<th>Launch No.</th>
<th>Rd 4,10,24</th>
<th>Rd 4,8,23</th>
<th>Rd 5,20,22</th>
<th>Rd 6,15,19</th>
</tr>
</thead>
<tbody>
<tr>
<td>V Initial volume</td>
<td>82.9 in.$^3$</td>
<td>82.9 in.$^3$</td>
<td>82.9 in.$^3$</td>
<td>82.9 in.$^3$</td>
</tr>
<tr>
<td>w Projectile weight</td>
<td>60 lb</td>
<td>20 lb</td>
<td>20 lb</td>
<td>60 lb</td>
</tr>
</tbody>
</table>

Potentiometer 18 limits the maximum output of amplifier 23. At a predetermined output of amplifier 23, the diode temporarily conducts, shorting out the amplifier. The circuit containing amplifier 24 prevents the output of amplifier 23 from dropping below the level necessary to keep the relay coil energized.

*These values were determined from the experimental P-T (see p. 7, para 3).
Computer Results

Launcher No. 1

Curves were computed for three representative rounds fired with Launcher No. 1. The heat loss coefficient of 2.0 was finally selected. This was found to produce the best agreement with the experimental curves.

Figures 6, 7, and 8 show both the computed and experimental curves for rounds 8, 12, and 13, respectively. The rates of pressure increase in the computed and the experimental curves at the shot-start breaking point were virtually identical. The times from shot-start breaking point-to-peak pressure in the computed and experimental curves were virtually identical. In addition, there was good agreement between the computed and the experimental curves for velocity.

There is, however, a significant difference between the early parts of the computed and experimental pressure-time curves in all of the figures. Adjustments in initial charge burnt can be made to compensate for these differences but it was not done at this point.

The principal factors which affected ignition in Launcher No. 1 are as follows:

1. The propellant in Launcher No. 1 firings was ignited by a coil of Pyrocore.

2. The cartridge case which held the propellant and Pyrocore was made out of cardboard and could not retain the gas. Consequently, the ignition delay times tended to be long. The unusually long delay time for round 12 shows that the performance of this type of ignition system is marginal.

Launcher No. 2

When the computer was first programmed to solve the ballistic equations for Launcher No. 2, the amount of charge burned initially \((N_0)\) was at first assumed to be zero. A standard aluminum cartridge case was used, and it was thought that this cartridge case would have a negligible effect on propellant burning.

In the B factorial program, the initial slopes of the computer pressure-time curves were less than those for the experimental curves. (See sheet 1 of Figures 9, 10, 11, and 12). The peak pressures were nearly the same. The time from the initial pressure rise-to-peak pressure was considerably longer for the computer curves. The velocity and travel curves were in close agreement.
Figure 6. Launcher No. 1, Round No. 7

- PROJECTILE WT - 25LB
- SPIGOT WT - CLOSED
- CHARGE WT - 60 GM
- SHOT - START - 500 PSI (STATIC)
- INITIAL CHARGE BURNT - NONE
- PRESSURE

TIME (MILLISECOND INTERVALS)

TRAVEL (IN.)

VELOCITY (FT/SEC)

PRESSURE (PSI)
Figure 10. Launcher No. 2, Factorial B, Round No. 11 (Sheet 1)
Figure 10. Launcher No. 2, Factorial B, Round No. 11 (Sheet 2)
**ANALOG COMPUTER STUDY OF REACTIONLESS Launchers**

**Figure 11.** Launcher No. 2, Factorial B, Round No. 12 (Sheet 2)

- Projectile WT = 60 lb
- Spigot = Closed
- Charge WT = 20 gm
- Web = 0.016 in.
- Shot-Start = 800 PSI (Static)
- Initial Charge Burnt = 4 gm

Graph details:
- Pressure (PSI)
- Time (ms)
- Travel (in.)
- Velocity (ft/sec.)

Legend:
- Experimental
- Computer
Figure 12. Launcher No. 2, Factorial B, Round No. 20 (Sheet 1)
In factorial program C, the initial slopes of the computer pressure-time curves were much less than those of the experimental curves. (See sheet 1 of Figures 13, 14, 15, and 16.) The difference between peak pressures was very large, as much as 1700 psi in one case. The ballistic cycle times differed by as much as 17 milliseconds, and the velocity and travel curves also were far from agreeing with their experimental counterparts.

As a result of these discrepancies, it was assumed that the amount of charge burned at the beginning of the ballistic cycle was 20 to 25 percent of the total charge. (The beginning of the ballistic cycle was then assumed to be the time when the aluminum cartridge case bursts.) The initial amount of the charge burned was varied between these limits to produce the closest agreement. This was accomplished by varying the setting of potentiometer 9. (See Figure 5.) An increase in the setting of potentiometer 9 represented a greater amount of charge initially burned.

The resulting computer curves showed closer agreement with the experimental curves. (See sheet 2 of Figures 9, 10, 11, 12, 13, 14, 15, and 16.) The slope of the pressure-time curve increased and the time from initial pressure rise to peak pressure decreased.

An experimental program is now being conducted to determine at what pressures the standard CAD cartridge cases burst under different rates of loading. In the instance described above, it seems feasible that 20 to 25 percent of the charge was burned before the cartridge burst.

All-Burnt - It is possible to determine all-burnt points readily from gas-produced-time curves. The amount of gas produced rises to a maximum and then levels off at the all-burnt point. The all-burnt points were determined for a number of rounds in the factorial A and B programs of Launcher No. 2. The location of the all-burnt point may be affected by three of the variables in this series of experiments:

1. Increasing the strength of the shot-start rod causes the propellant to burn more rapidly. This tends to displace the all-burnt point nearer to peak pressure.

2. Increasing the amount of charge initially burnt tends to displace the point nearer to peak pressure. See round 11, factorial B, for example. (Figure 10, sheets 1 and 2)

3. Increasing the web tends to displace the all-burnt point farther from peak pressure, since more time is required to burn through a larger web.
Figure 13. Launcher No. 2, Factorial C, Round No. 1, 10, 24 (Sheet 1)
Figure 14. Launcher No. 2, Factorial C, Round No. 4, 8, 23 (Sheet 2)
Figure 15. Launcher No. 2, Factorial C, Round No. 5, 20, 22 (Sheet 1)
Figure 15. Launcher No. 2, Factorial C, Round No. 5, 20, 22 (Sheet 2)
Figure 16. Launcher No. 2, Factorial C, Round No. 6, 15, 29 (Sheet 1)
Figure 15. Launcher No. 2, Factorial C, Round No. 6, 15, 19 (Sheet 2)
All-burnt points were not noted for all computer curves, since the primary purpose of this study was to develop a method for simulating the performance of a reactionless launcher on a computer. In simulating the performance of a theoretical launcher, however, it would be of interest to know the all-burnt points for the following reasons:

1. In a closed system (i.e., a system in which the piston is stopped by a shoulder, so that it does not leave the tube), the launcher tube may burst if the propellant continues to burn after the piston has completed its stroke.

2. A better velocity uniformity may be obtained when the all-burnt point occurs at or close to peak pressure.

HIGH-LOW SYSTEM

Firing Test Setup

Launcher No. 2 was modified by adding an orifice plate. It was then fired in factorial program A, in which the charge weight, shot-start rod, and orifice were varied. The projectile weight was 20 pounds. The values for the variables were as follows:

<table>
<thead>
<tr>
<th>Charge Weight</th>
<th>Shot-Start Rod</th>
<th>Total Orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 grams</td>
<td>0 psi</td>
<td>0.0463 in.²</td>
</tr>
<tr>
<td>6.0 grams</td>
<td>0 psi</td>
<td>0.06486 in.²</td>
</tr>
</tbody>
</table>

The total orifice area was determined by the number of holes in the orifice plate. (See Figure 4.) One orifice plate contained 35 holes, giving it a total open area of 0.0463 in.²; the other plate contained 49 holes, giving it a total open area of 0.06486 in.².

A number of difficulties were encountered in the instrumentation for the high-low phase of the test program; therefore, the rounds chosen to be simulated were those for which the total amount of gas discharged through the orifice was closest to the initial charge weight. This condition was determined as follows:
The applicable equation for the charge is:

\[ C = K_c A_t \int Pdt \]

where \( C \) is the charge weight

\( K_c \) is the gas-discharged coefficient for sonic flow

\( A_t \) is the total orifice area

\( P \) is the pressure in the high-pressure chamber

\( t \) is time.

The ratio of the pressures in the two chambers determines the value of the discharge coefficient.* In this calculation, \( K_c \) was assumed to be the sonic discharge coefficient for M2 propellant, which has the value, \( 6 \times 10^{-3}/\text{sec} \).

It was assumed that the charge was completely burned, and that all of the gas produced eventually passed from the high-pressure chamber to the low-pressure chamber. The pressure-time curves for the high-pressure chamber were integrated by planimeter to obtain \( \int Pdt \). The product, \( K_c A_t \int Pdt \), represents the amount of gas discharged through the orifices. This product should be approximately equal to the charge weight.

In some cases, the amount of gas discharged was less than the charge weight. This was probably due to failure of the pressure-measuring instrumentation. In the case of the rounds in which 2.5-gram charges were used, the amount of gas discharged (calculated by the aforementioned method of measurement) was less than the charge weight. This was due to the effect of the 1-gram black-powder igniter. For these rounds, the charge was increased by approximately 10 percent to compensate for the additional gas contributed by the igniter.

The curves for the rounds selected in this manner (gas discharged is approximately equal to charge weight) are shown in "Computer Results." The percentages of error between gas discharged and charge weight for the rounds whose curves were simulated are:

*See Appendix II, "Gas Discharge in a High-Low System."
# Analog Computer Study of Reactionless Launchers

<table>
<thead>
<tr>
<th>Round</th>
<th>Percentage of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (2.7 gms**)</td>
<td>+1.0%</td>
</tr>
<tr>
<td>21 (2.7 gms**)</td>
<td>+7.5%</td>
</tr>
<tr>
<td>14 (6 gms)</td>
<td>-2.6%</td>
</tr>
<tr>
<td>18 (6 gms)</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

A plus sign means that the calculated quantity of gas discharged was greater than the charge weight.

## Ballistic Theory

The assumptions made for the direct-system study were also considered to apply to the high-low system (including the assumption of initial charge burnt). Additional assumptions are discussed below.

A dual isothermal model was considered to represent the high-low ballistic system. In such a model, the temperature of the gases within both chambers is constant throughout the entire ballistic cycle. The assumptions made from this model are as follows:

1. The temperatures of the burning gases in the high-pressure chamber can be replaced by a mean value corresponding to an effective mean impetus, \( \lambda_H \), of constant value. The value of \( \lambda_H = 0.92 F \) (where \( F = n R T_0 \) is the actual impetus of the propellant used) gave the best results.

2. The temperature of the gases in the low-pressure chamber, because of turbulence, gas expansion, and heat loss, can be replaced by a mean value corresponding to an effective mean impetus, \( \lambda_L \), of constant value, \( \lambda_L = 0.7 \lambda_H \).

3. The heat loss coefficient, \( \beta \), is equal to 2.0.

4. The propellant consists only of single-perforation grains with no end burning.

5. The equations developed for programming on the GEDA analog computer are as follows:

**Charge adjusted to account for the igniter effect. In programming these rounds, the effect of the igniter material was accounted for by adjusting the value of the initial charge burnt.
Acceleration:

\[ \ddot{x} = \frac{A g \ P_L}{W} \]  

where \( \ddot{x} > 0 \) when \( P_L > P_{ss}^* \)

Velocity:

\[ \dot{x} = \int \ddot{x} \ dt \]  

Travel:

\[ x = 12 \int \dot{x} \ dt \]  

Equation of state (high-pressure chamber):

\[ P_H V_H = 12 (N - N') \ \lambda_H \]  

where \( V_H = V_{OH} - \frac{C-N}{6} - a \ (N-N') \)

Equation of state (low-pressure chamber):

\[ P_L V_L = 12 N' \ \lambda_L \]  

where \( V_L = (V_{OL} + 2AX - aN') \)

Gas production in the high-pressure chamber:

\[ N = N_0 + \frac{2CB}{W} \int P_H^a \ dt \]  

Gas discharge into the low-pressure chamber**:

\[ N' = A_t \int_0^t \xi K_c P_H \ dt \]  

*\( P_{ss} \) is the actual shot-start rod breaking pressure.

**The derivation of and special assumptions for this equation are given in Appendix II.
ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

\[
K_c = \left[ \frac{9\gamma}{F} \left( \frac{2}{\gamma + 1} \right) \right]^{\frac{1}{2}} \sqrt{\frac{p}{\lambda_H}}
\]

Equation of energy balance:

\[
\frac{N'}{\gamma - 1} \lambda_L = \frac{N'}{\gamma - 1} \left( \frac{\gamma}{\gamma - 1} \right) \lambda_H - (1 + \beta) \frac{W}{g} v^2
\]

where \( \frac{N'}{\gamma - 1} \lambda_L \) represents the total internal energy of the propellant gas in the low-pressure chamber at any time.

\( (1 + \beta) \frac{W}{g} v^2 \) represents the total heat loss and the kinetic energy of the projectiles.

\( \frac{N'\gamma}{\gamma - 1} \lambda_H \) represents the kinetic and thermal energy of the gas discharged into the low-pressure chamber.*

Computer Setup

The operation of Reactionless Launcher No. 2, high-low system, was simulated by an electrical analog (Figure 17). Table IV shows the constants and Table V the variables which determined the values of the potentiometers for each round.

*This term is equivalent to \( N'\gamma F p T_H \) (where \( T_H \) is the effective mean temperature in the high-pressure chamber during the ballistic cycle) as is apparent from:

\[
N' \gamma C v T_H = N' \gamma \left( \frac{nR}{\gamma - 1} \right) T_H = \frac{N'\gamma}{\gamma - 1} \lambda_H \quad \text{where} \quad \lambda_H = nR T_H.
\]
Figure 17. Computer Analog for Simulating High-Low System Fittings
Table IV. Constants for Launcher No. 2 (High-Low System)

<table>
<thead>
<tr>
<th>A</th>
<th>Bore area</th>
<th>3.97 in.²</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Bore diameter</td>
<td>2.25 in.</td>
</tr>
<tr>
<td>F</td>
<td>Propellant impetus</td>
<td>3.78 ft-lb/1b</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration caused by gravity</td>
<td>32.2 ft/sec²</td>
</tr>
<tr>
<td>Kc</td>
<td>Gas discharge coefficient for sonic flow</td>
<td>6 x 10⁻³/sec²</td>
</tr>
<tr>
<td>W</td>
<td>Projectile weight</td>
<td>20 lb</td>
</tr>
<tr>
<td>w</td>
<td>Web of the propellant grain</td>
<td>0.016 in.</td>
</tr>
<tr>
<td>VoH</td>
<td>Initial volume of the high pressure chamber</td>
<td>2.9 in.³</td>
</tr>
<tr>
<td>VoL</td>
<td>Initial volume of the low-pressure chamber</td>
<td>31.0 in.³</td>
</tr>
<tr>
<td>Xt</td>
<td>Total travel (stroke)</td>
<td>11.5 in.</td>
</tr>
<tr>
<td>α</td>
<td>Heat loss coefficient</td>
<td>2.0</td>
</tr>
<tr>
<td>γ</td>
<td>Ratio of specific heats</td>
<td>1.25</td>
</tr>
<tr>
<td>λ_H</td>
<td>Effective mean impetus in the high-pressure chamber</td>
<td>3.48 x 10⁵ ft-lb/1b</td>
</tr>
<tr>
<td>λ_L</td>
<td>Effective mean impetus in the low-pressure chamber</td>
<td>2.43 x 10⁵ ft-lb/1b</td>
</tr>
</tbody>
</table>

Table V. Variables for Launcher No. 2 (High-Low System)

<table>
<thead>
<tr>
<th></th>
<th>Rd 4</th>
<th>Rd 21</th>
<th>Rd 14</th>
<th>Rd 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Charge weight</td>
<td>2.5 gm</td>
<td>2.5 gm</td>
<td>6.0 gm</td>
</tr>
<tr>
<td>Pss</td>
<td>Shot-start breaking pressure</td>
<td>200 psi</td>
<td>200 psi</td>
<td>0 psi</td>
</tr>
<tr>
<td>A_t</td>
<td>Total orifice area</td>
<td>0.0463 in.²</td>
<td>0.06486 in.²</td>
<td>0.06486 in.²</td>
</tr>
</tbody>
</table>

This analog is similar to the analog for the direct system. Its operation is straightforward. The amount of charge initially burnt, the shot-start rod, and the back-pressure factor were compensated for as follows:

1. The amount of charge initially burnt was set by a potentiometer which was added between the reference voltage and amplifier 11 (potentiometer 3 in Figure 17).

2. The switch on the output of amplifier 18 is part of a comparator circuit (Figure 5) which compensated for the effect of the shot-start rod. This separate circuit (not shown in Figure 17) energizes the coil of a relay when the pressure reaches a predetermined value, thus switching in the velocity and travel circuits.
3. The circuit enclosed by dotted lines compensates for any change in the back-pressure factor. As the pressure ratio, $P_l/P_H$, changes, the back pressure factor, $\xi$, may change. (See Appendix II, "Gas Discharge in a High-Low System.") A change in $\xi$ affects the mass of gas discharged to the low pressure side and thus the energy available to propel the projectiles. However, a number of tests showed that $P_l/P_H$ was always less than 0.56 for this launcher. The back-pressure factor, $\xi$, then was always equal to 1, and the flow depended only on the value of $P_H$, the pressure in the high-pressure chamber. Consequently, this circuit could have been eliminated from the analog.

The channels and variables recorded were as follows:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Variable Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pressure in the low-pressure chamber</td>
</tr>
<tr>
<td>2</td>
<td>Velocity</td>
</tr>
<tr>
<td>3</td>
<td>Travel</td>
</tr>
<tr>
<td>4</td>
<td>Gas discharged into the low-pressure chamber</td>
</tr>
</tbody>
</table>

Computer Results

In order to make a proper comparison between the experimentally obtained curves and those obtained from the computer, the experimental pressure-time curves were analyzed to determine the amount of gas discharged through the orifices.* Rounds 4, 14, 18, and 21 (see Figures 18, 19, 20, and 21) were chosen for simulation because the analysis showed that the weight of the gas discharged through the orifices in these rounds was closest to the original charge weight.

As in the B and C factorial programs, it was necessary to assume that a significant amount of the charge was initially burned. The percentage of the charge burnt before travel begins was inversely proportional to the total charge. Since the same type of cartridge case (M29A2) was used for both charges (2.5 and 6.0 grams), a larger percentage of the 2.5-gram charge was required to burst the cartridge case. The values assumed varied from 3.8 to 13.6 percent, for the rounds which were simulated, the smaller percentage being for the larger charge.

In general, the curves determined by analog simulation agree closely with the curves determined by actual firing tests.

*See "High-Low System, Firing Test Setup"
ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

Figure 18. Launcher No. 2,Factorial A, Round 4.
Figure 19. Launcher No. 2, Factorial A, Round No. 21
Figure 20. Launcher No. 2, Factorial A, Round No. 14
CONCLUSIONS

Direct System

The reasonably close agreement obtained for the direct system launchers shows that it is now possible to simulate direct system designs on the computer. The facts listed below are applicable to this type of system.

1. The amount of charge initially burnt (20-25 percent) is an important factor in the interior ballistics of a direct-system reactionless launcher where an aluminum cartridge case is used.

2. A heat loss coefficient of about 2 is representative of this system and propellant (M2).

High-Low System

The very close agreement obtained for the high-low reactionless launcher shows that the dual isothermal model is a useful representation of this type of system. It is now possible to simulate other high-low system designs on the computer.

For this particular design of high-low launcher, the use of the back-pressure factor to account for subsonic flow is unimportant and can be neglected.

The amount of charge initially burnt at the time the cartridge bursts has a significant effect on pressure in the high-pressure chamber.

RECOMMENDATIONS

It is recommended that:

1. The computer be used in the future to design and establish and charge for reactionless launchers. This should greatly reduce the time and money spent on experimental design and evaluation.
2. Further research be conducted using the computer to establish the feasibility of high-muzzle-velocity launchers, the ejection of unequal weights, and the simultaneous firing of multiple tube systems.

3. Further investigation be made on the ejection of large weights.

4. A program be conducted with proper instrumentation to determine the exact time and breaking pressure of the shot-start rod.

5. The dynamic breaking pressure and the exact amount of propellant burned before rupture of the cartridge case be determined by experiment.

6. That a factorial type experimental program be conducted whenever it is desired to obtain data for an analytical study of any ballistic device.

Jacobs, D., and C. Litz, Decoy Packaging and Launching Studies (U), Technical Memorandum M60-26-2, Frankford Arsenal, October 1960 (SECRET).


APPENDIX I

LIST OF SYMBOLS

A -- Bore area (in.²)

A_t -- Total area of the discharge orifices in the high-low ballistic system (in.²)

a -- Burning rate exponent

B -- Burning rate coefficient (in./sec-psi²)

C -- Charge weight (lb)

C_p -- Specific heat at constant pressure (ft-lb/lb-°K)

C_v -- Specific heat at constant volume (ft-lb/lb-°K)

d -- Bore diameter (in.)

e -- Powder potential (ft-lb/lb)

e' -- Internal energy per unit mass at any temperature, T (ft-lb/lb)

F -- Impetus of a propellant (ft-lb/lb)

g -- Acceleration caused by gravity (ft/sec²)

K_c -- Sonic gas discharge coefficient, a constant (sec⁻¹)

K_o -- Subsonic gas discharge coefficient, a variable (sec⁻¹)

L_i -- Length of a propellant grain (in.)

N -- Mass of propellant burned, or gas produced (lb)

N' -- Mass of the gas ejected from the high-pressure chamber into the low-pressure chamber in a high-low system (lb)

N_o -- Initial charge burnt, or gas produced, when t is zero (lb)

n -- Number of gram-moles of gas per gram of propellant, the inverse of molecular weight (gm-moles/gm)

P -- Pressure (lb/in.²)

P_H -- Instantaneous pressure in the high-pressure chamber in a high-low system (lb/in.²)
ANALOG COMPUTER STUDY OF REACTIONLESS LAUNCHERS

\( P_L \) -- Instantaneous pressure in the low-pressure chamber in a high-low system \((\text{lb/in.}^2)\)

\( P_{SS} \) -- Shot-start pressure \((\text{lb/in.}^2)\)

\( R \) -- Molar gas constant \((2732 \text{ ft-lb/lb-mole-°K})\)

\( r \) -- Linear burning rate of a propellant \((\text{in./sec})\)

\( R_1 \) -- Outside radius of a single-perforation propellant grain \((\text{in.})\)

\( R_2 \) -- Inside radius of a single-perforation propellant grain \((\text{in.})\)

\( S \) -- Burning surface area of the total propellant charge \((\text{in.}^2)\)

\( S_1 \) -- Surface area of a propellant grain, except for its ends \((\text{in.}^2)\)

\( T \) -- Space mean temperature at any time, \( t \) \((°K)\)

\( T_H \) -- Effective mean temperature in the high-pressure chamber during the ballistic cycle \((°K)\)

\( T_0 \) -- Isochoric Adiabatic flame temperature \((°K)\)

\( t \) -- Time; \( t \) is zero when the cartridge case bursts \((\text{sec})\)

\( V \) -- Volume at time, \( t \) \((\text{in.}^3)\)

\( V_H \) -- Instantaneous free volume in the high-pressure chamber \((\text{in.}^3)\)

\( V_L \) -- Instantaneous free volume in the low-pressure chamber \((\text{in.}^3)\)

\( V_0 \) -- Initial volume \((\text{in.}^3)\)

\( V_{0H} \) -- Initial volume of the high-pressure chamber, without propellant \((\text{in.}^3)\)

\( V_{0L} \) -- Initial volume of the low-pressure chamber \((\text{in.}^3)\)

\( V_i \) -- Volume of a propellant grain \((\text{in.}^3)\)

\( W \) -- Projectile weight \((\text{lb})\)

\( w \) -- Web of a propellant grain \((\text{in.})\)

\( x \) -- Travel \((\text{in.})\)

\( X_t \) -- Total travel; stroke length \((\text{in.})\)

\( \dot{x} \) -- Velocity \((\text{ft/sec})\)
\* - Acceleration (ft/sec^2)
\(a\) - Specific volume (in.\(^3\)/lb)
\(\beta\) - Heat loss coefficient (dimensionless)
\(\gamma\) - Ratio of specific heats (dimensionless)
\(\zeta\) - Back-pressure factor (dimensionless)
\(\lambda_H\) - Effective mean impetus of the gases in the high-pressure chamber (ft-lb/lb)
\(\lambda_L\) - Effective mean impetus of the gases in the low-pressure chamber (ft-lb/lb)
\(\rho\) - Density of propellant (lb/in.\(^3\))
\(\Delta\) - Amplifier No. 1
\(\bigcirc\) - Potentiometer No. 1
\(\square\) - Recorder Channel No. 1
\(\bigwedge\) - Resistor (in ohms)
\(-\) - Capacitor (in microfarads)
\(\bigtimes\) - Multiplier
\(\bigcirc\) - Function Generator
\(-\) - Diode
APPENDIX II
GAS DISCHARGE IN A HIGH-LOW SYSTEM

The equation for gas discharge into the low-pressure chamber of a high-low system is:

\[ N' = \int_0^t \beta K_c A_t P_H dt \]

where \( N' \) is the mass of the gas which reaches the low-pressure chamber.

\( \beta \) is the back-pressure factor
\( K_c \) is the sonic gas discharge coefficient, a constant 6.0 x 10^{-3}/sec for M2 Propellant
\( A_t \) is the total area of the discharge orifices
\( P_H \) is the pressure in the high-pressure chamber
\( t \) is time

The value of the back-pressure factor, \( \beta \), is defined as:

\[ \beta = \frac{K_o}{K_c} \]

when \( \frac{P_L}{P_H} \geq \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \) or for \( \gamma = 1.25 \), \( \frac{P_L}{P_H} \geq 0.56 \)

where \( K_o \) is the discharge coefficient for subsonic flow, a variable.

When the ratio of the pressures, \( \frac{P_L}{P_H} < 0.56 \), the gas flow is said to be "sonic." Then \( \beta \) is defined as 1, and:

\[ \beta K_c = \left[ \frac{g \gamma}{\lambda_H} \left( \frac{\gamma + 1}{\gamma - 1} \right) \right]^{\frac{1}{2}} \]

When \( \frac{P_L}{P_H} \geq 0.56 \), the gas flow is said to be "subsonic."

*For additional information on gas discharge see Corner, "Theory of Interior Ballistics of Guns," Wiley and Sons, 1951.
Then, $K_o$ varies with the pressure ratio as follows:

$$K_o = \left\{ \left[ \frac{2}{\gamma - 1} \frac{\frac{9}{\lambda H}}{\left( \frac{P_L}{P_H} \right)^2} \frac{2}{\gamma} \right] \left[ 1 - \left( \frac{P_L}{P_H} \right)^{\frac{\gamma - 1}{\gamma}} \right] \right\}^{1/2}$$

where $\gamma$ is the ratio of specific heats, assumed to be 1.25
$\lambda H$ is the effective mean impetus of the gases in the high-pressure chamber

The resulting values of $f$ for $\gamma = 1.25$ are given in Table VI and plotted in Figure 22.

Table VI. Variation of Back-Pressure Factor with Pressure Ratio

<table>
<thead>
<tr>
<th>$P_L/P_H$</th>
<th>$f$</th>
<th>$P_L/P_H$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56</td>
<td>1.000</td>
<td>0.94</td>
<td>0.507</td>
</tr>
<tr>
<td>0.60</td>
<td>0.995</td>
<td>0.96</td>
<td>0.419</td>
</tr>
<tr>
<td>0.70</td>
<td>0.948</td>
<td>0.98</td>
<td>0.300</td>
</tr>
<tr>
<td>0.80</td>
<td>0.840</td>
<td>0.99</td>
<td>0.214</td>
</tr>
<tr>
<td>0.85</td>
<td>0.755</td>
<td>0.997</td>
<td>0.117</td>
</tr>
<tr>
<td>0.90</td>
<td>0.638</td>
<td>1.00</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Figure 22. Back Pressure Factor vs Pressure Ratio
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Authors: S. Goldstein and S. Harisi

Research and Development Group, Frankford Arsenal, Phila.
57, Pa. ANALOG COMPUTER STUDY OF EXPERIMENTAL LAUNCHER

Modified standard closed-breath interior ballistic equations were applied to predict the performance of reactionless launchers. Following this, a series of test firings were made using two Davis-Gun-type reactionless launchers. Both low- and high-velocity launchers were considered. The results obtained with the curves calculated from the computer. Using the differences noted, certain factors were varied in the computer curves into close agreement with actual test results. These factors included test loss coefficient, initial charge burnt, and the effective mass and length of the propellant during the ballistic cycle.

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The knowledge gained can be used to predict and optimize the performance of reactionless systems and research should be conducted using the computer to establish the feasibility of high-muzzle-velocity launchers, the ejection of unequal and/or large weights, the simultaneous firing of multiple tube systems and the dynamic breaking behavior of the shot-star rod and cartridge case. A factorial type program should be used to obtain the best experimental data for an analytical study.
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The knowledge gained can be used to predict and optimize the performance of reentry vehicle systems and research vessels by conducting using the computer to establish the feasibility of high-velocity, launchers, the ejection of special re-entry vehicles, the simultaneous firing of multiple re-entry vehicles and the dynamic modeling behavior of the ship-start rocket and propulsion cases. A factorial type program should be used to obtain the best experimental data for an analytical study.
FOR ERRATA

AD 263 057

THE FOLLOWING PAGES ARE CHANGES

TO BASIC DOCUMENT
ERRATA SHEET No. 1
Frankford Arsenal Report R-1589, June 1961
"Analog Computer Study of Reactionless Launchers,"
by S. Goldstein and S. Narisi

Please make the following corrections in subject report.

1. Page 7, 3d paragraph, end of second line -
   Insert: backwards
   To read: ....... integrating the curves backwards from the point ......

2. Page 10, 4th and 5th lines below Equation (5) -
   Change: R is the molar gas constant and it is equal to 2782 ft-lb-mole-°K;
   To read: R is the universal gas constant and it is equal to
   \[ \frac{2782 \text{ ft-lb (energy)}}{\text{lb (weight) mole-°K}} \]

3. Page 40, 3d paragraph, 2nd line -
   Change: This was probably due to failure of ..... 
   To read: This was probably due to malfunction of ..... 

4. Page 40, 3d paragraph, 5th line -
   Change: ..... of measurement) was less than the charge weight.
   To read: ..... of measurement) was more than the charge weight.

5. Page 45, Table IV, 3d line -
   Change: F Propellant impetus 3.78 ft-lb/lb
   To read: F Propellant impetus \[ 3.78 \times 10^5 \text{ ft-lb/lb} \]

30 November 1961