NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
TECHNICAL NOTE
D-708

THEORY AND DESIGN CURVES FOR A YO-YO DE-SPIN MECHANISM FOR SATELLITES

J. V. Fedor
Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON
August 1961
THEORY AND DESIGN CURVES FOR
A YO-YO DE-SPIN MECHANISM
FOR SATELLITES

by
J. V. Fedor
Goddard Space Flight Center

SUMMARY

Equations are developed for the yo-yo de-spin mechanism. Under conditions usually met in a de-spin application, the equations can be greatly simplified. A computation sheet is presented based on the simplified equations. The design of a de-spin mechanism is thus reduced to reference to a chart and a few simple calculations.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>ANALYSIS</td>
<td>3</td>
</tr>
<tr>
<td>Phase 1 Analysis</td>
<td>5</td>
</tr>
<tr>
<td>Phase 2 Analysis</td>
<td>6</td>
</tr>
<tr>
<td>Simplification of Equations</td>
<td>8</td>
</tr>
<tr>
<td>Distributed Mass of Wire</td>
<td>9</td>
</tr>
<tr>
<td>APPLICATION OF DESIGN CURVES AND EQUATIONS</td>
<td>9</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>10</td>
</tr>
<tr>
<td>Reference</td>
<td>11</td>
</tr>
<tr>
<td>Appendix A - Yo-Yo De-Spin Calculation Sheet (Radial Release)</td>
<td>11</td>
</tr>
</tbody>
</table>
INTRODUCTION

The yo-yo de-spin mechanism is essentially two pieces of wire with weights on the ends (Figure 1). These wires are symmetrically wrapped around the equator of the satellite and the weights are secured by a release mechanism (Figure 2). At a pre-selected time after satellite spin-up and separation from the launching vehicle, the weights are released, thus discarding enough momentum to reduce the spin of the satellite to the desired value. It is the object of this note to present the yo-yo de-spin theory in equation and curve form so that the designer can readily apply the theory to the design of the de-spin mechanism.

For given values of the satellite moment of inertia and radius, and for a given amount of spin reduction, the designer wants to determine the weights that must be used, the length of wire, and the maximum tension in the wire. In what follows, equations will be developed for these quantities. It will also be seen that under certain conditions usually met in de-spin applications, a great simplification of the equations is possible. With this simplification, all of the design curves can be put onto one dimensionless graph.

ANALYSIS

After being placed in orbit, the satellite travels at essentially constant velocity and the centrifugal action is cancelled out by the gravity force. Since any equation of motion written for the satellite would be independent of this constant velocity, the satellite can be considered stationary and spinning about a fixed axis for the purposes of the present analysis.
Figure 1 - De-spin weights and wire

Figure 2 - Release mechanism
There are two phases to the spin reduction process considered. In Phase 1 the wire is changing in length and is tangent to the satellite. In Phase 2 the length of the wire is constant but its position is changing from tangent to perpendicular to the satellite. With proper design, the wire can then be released when it is perpendicular to the satellite. It should be noted that releasing the wires after Phase 2 is more efficient, weight-wise, than releasing them after Phase 1.

**Phase 1 Analysis**

A sketch of Phase 1 with its coordinate system is shown in Figure 3. Only one wire is shown, as the system is considered symmetrical. Also, the system is considered torque-free; small moments due to the earth’s magnetic or gravitational fields are neglected.

The total kinetic energy of the system, and also the Lagrangian, is

\[ T = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) , \]

where \( \phi \) is the total mass of both weights and \( T \) is the moment of inertia of the satellite about the spin axis. Initially the wires are considered weightless; later, a method will be developed to take the weight of the wire into account without disturbing the form of the equations. Using the transformation equations

\[ x = a \cos \theta \cdot \sin \phi , \]
\[ y = a \sin \theta - \ell \cos \theta , \]

the Lagrangian can be put into the form

\[ T = \frac{1}{2} I \dot{\phi}^2 + \frac{1}{2} m (\ell^2 \dot{\theta}^2 + \ell^2 \dot{\phi}^2) . \]

The length of wire unwound at any time during this phase is

\[ \ell = a (\theta - \phi) . \]

The equations of motion in Lagrangian notation are

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\phi}} \right) - \frac{\partial T}{\partial \phi} = Q_\phi = 0 , \]
\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} = Q_\theta = 0 . \]
where $\phi$ and $\theta$ are considered to be generalized coordinates. Equations 5, 6, and 7 can be integrated once to yield an equation which expresses the conservation of momentum. Equation 4, which is equal to a constant and represents the conservation of energy, can be combined with the conservation of momentum equation to yield $\dot{\phi}$ and $\dot{\theta}$ as a function of the length of the wire during Phase 1:

$$\dot{\phi} = \frac{\dot{\phi}_0 (1 - \ell^2/\lambda^2)}{1 + \ell^2/\lambda^2}$$

(8a)

and

$$\dot{\theta} = \frac{2\dot{\phi}_0}{1 + \ell^2/\lambda^2}$$

(8b)

where $\dot{\phi}_0$ is the initial spin rate and $\lambda^2 = 1/m + a^2$. The above-mentioned equations can also be integrated to give the length of wire as a function of time:

$$\ell(t) = a\dot{\phi}_0 t$$

(9)

The total tension in the wires is simply the product of the mass of the weights and the acceleration:

$$F_1 = mA_m = -m(n\ddot{\phi} + \ell\dot{\phi}^2)$$

(10a)

where $A_m$ is the acceleration of the mass $m$ along the wire. Substituting Equations 8a and 8b into Equation 10a yields

$$F_1 = \frac{4m\dot{\phi}_0^2 \ell (1 - a^2/\lambda^2)}{(1 + \ell^2/\lambda^2)^2}$$

(10b)

It can easily be shown that the maximum tension during Phase 1 is

$$F_{1_{\text{max}}} = \frac{3}{4}\sqrt{3m\lambda} \dot{\phi}_0^2 (1 - a^2/\lambda^2)$$

(11)

or in dimensionless form

$$\frac{F_{1_{\text{max}}}}{ma_0^2 \lambda} = 1.3(1 - a^2/\lambda^2)$$

(12)

where $a_{0} = \dot{\phi}_0$. 
Phase 2 Analysis

A sketch of the Phase 2 arrangement is shown in Figure 4. The total kinetic energy in this phase is

\[ T = \frac{1}{2} I \dot{\phi}^2 + \frac{1}{2} m [a^2 \dot{\phi}^2 + 2 a \ell \cos (\theta - \gamma) \dot{\phi} \dot{\gamma} + \ell^2 \dot{\gamma}^2] \tag{13} \]

where \( \dot{\theta} = \dot{\phi} \) (from Equation 5) since \( \ell \) is a constant. This phase is much more complicated analytically than the previous one. Fortunately, we are mainly interested in the spin at the end of Phase 2 (at the release of the wire, when \( \theta - \gamma = 0 \)) and the tension in the wire. They both can be obtained by applying the conservation of momentum and of energy, and an equation similar to Equation 10.

The conservation of energy (at \( \theta - \gamma = 0 \)) gives

\[ \frac{1}{2} I \dot{\phi}^2 + \frac{1}{2} m (a \dot{\phi}^2 + \ell \dot{\gamma}^2) = \text{const.} = \frac{1}{2} (I + ma^2) \dot{\phi}_0^2 \tag{14} \]

and the conservation of momentum gives

\[ I \dot{\phi}_2 + m (\ell + a \gamma) (a \dot{\phi}_2 + \ell \dot{\gamma}) = \text{const.} = (I + ma^2) \dot{\phi}_0 \tag{15} \]

Combining Equations 14 and 15 we obtain

\[ \frac{\ell}{a} + 1 = \frac{(1 - r)I}{ma^2} + 1 \sqrt{\frac{(1 - r^2)I}{ma^2}} \cdot 1 \tag{16} \]

or, rearranging a bit,

\[ r = \frac{(G + 1)^2}{(\ell/a + 1)^2 - (G + 1)} \frac{G}{G} \tag{17} \]

where \( r = \dot{\phi}_0 \dot{\phi}_0 \) and \( G = (1 - r)I/ma^2 \) for conciseness of notation and later development. The force in the wire just at release is

\[ F_2 = m (a \dot{\phi}^2 + \ell \dot{\gamma}^2) \]

\[ = m \dot{\phi}_0^2 \ell \left( \frac{ar^2}{\ell} + \frac{\dot{\gamma}^2}{\dot{\phi}_0^2} \right) \tag{18} \]
In terms of $G$, $r$ and $\ell/a$, the force is

$$F_2 = ma^2 \left\{ \frac{ar^2}{\ell} + \left[ \frac{G + 1 - r(\ell/a + 1)}{(\ell/a + 1)f/a} \right]^2 \right\}. \quad (19)$$

The force given by Equation 18 or 19 is a maximum for Phase 2 because the length of the wire is constant and the angular velocity of the wire $(\gamma)$ is a maximum just before release (the $ar^2/\ell$ term contributes very little to the force).

### Simplification of Equations

The equations developed thus far are exact in that no analytical approximations have been made in their derivations. A great simplification can be made by noting the following: $G$ in Equation 17 is large (in the order of 200 or more) for most practical spin weights and spin reductions. Thus if $G$ is replaced by $G + 1$ in the denominator of Equation 17, we have

$$r = \frac{(G + 1)^2}{(\ell/a + 1)^2} - \frac{(G + 1)}{G - 1} = \frac{G + 1}{(\ell/a + 1)^2} - 1. \quad (20)$$

Rearranging Equation 20 into a dimensionless form gives

$$\frac{I}{ma(a + \ell)^2} = \frac{1 + r - (\ell/a + 1)^{-2}}{1 - r}. \quad (21)$$

Now, if $\ell/a$ is greater than $2\pi$ ($\ell$ is one circumference length) the $(\ell/a + 1)^{-2}$ term in Equation 21 is negligible. Hence, we have the very simple expression

$$\frac{I}{ma(a + \ell)^2} \approx \frac{1 + r}{1 - r}. \quad (22)$$

Equation 22 has been plotted in Figure 5.

Calculations indicate that the maximum tension in the wire occurs in Phase 1 in most cases; hence the simple expression of Equation 12 can be used to calculate this maximum force. Since $a^2/\lambda^2$ is small in most de-spin designs, the maximum force is simply

$$F_{max} = 1.3 ma_0^2 \lambda. \quad (23)$$

Consistent with Equation 22, Equation 19 can be simplified to

$$F_2 = ma^2 \left\{ \frac{ar^2}{\ell} + \left[ \frac{(r + 1)(\ell/a + 1) - r\gamma^2}{\ell/a} \right]^2 \right\}. \quad (24)$$
Figure 5 - Yo-yo design curves; plot of $I/m(t + a)^2$ as a function of $r = \omega/\omega_0$
Distributed Mass of Wire

We shall now develop an approximate method to include the distributed weight of the wires without disturbing the form of the derived equations. From Figure 6 the kinetic energy of $i^{th}$ particle is $\frac{1}{2} \rho A \xi (\dot{x}_i^2 + \dot{y}_i^2)$, where $\rho$ is the mass density per unit length of the wire. Summing over all the particles, we have

$$\frac{1}{2} \sum \rho A \xi (\dot{x}_i^2 + \dot{y}_i^2).$$

By using the transformation equations

$$x_i = a \cos \theta + \xi \cos \gamma, \quad (26)$$
$$y_i = a \sin \theta + \xi \sin \gamma, \quad (27)$$

and replacing the summation by integration, the total kinetic energy of the wire is found to be

$$KE_{wire} = \frac{1}{2} \int_0^\xi \rho \left( a^2 \dot{\phi}^2 + 2a \xi \cos (\phi - \gamma) \ddot{\phi} \gamma + \xi \dot{\gamma}^2 \right) \, d\xi$$

$$= \frac{1}{2} \rho \left( \ell^2 \dot{\phi}^2 + 2at \cos (\theta - \gamma) \ddot{\phi} \dot{\gamma} + \ell^2 \dot{\gamma}^2 \right). \quad (28)$$

Since $t$ is large just before release, $\ell^2 \dot{\gamma}^2$ is the largest term in Equation 28. In any approximate formula we should include all of this term. Separating Equation 28 into two parts results in the following:

$$KE_{wire} = \frac{1}{2} \rho \left[ \frac{\ell^2}{3} \dot{\phi}^2 + 2at \cos (\theta - \gamma) \ddot{\phi} \dot{\gamma} + \ell^2 \dot{\gamma}^2 \right]$$

$$+ \frac{\rho \ell}{2} \left[ \frac{2a^2}{3} \dot{\phi}^2 + \frac{at}{3} \cos (\theta - \gamma) \ddot{\phi} \dot{\gamma} \right]. \quad (29)$$

Just before wire release, $\dot{\phi}$ is also small, so the second part of Equation 29 is small compared to the first part. Hence,

$$KE_{wire} \approx \frac{1}{2} \rho \frac{\ell}{3} \left[ \dot{\phi}^2 + 2at \cos (\theta - \gamma) \ddot{\phi} \dot{\gamma} + \ell^2 \dot{\gamma}^2 \right]. \quad (30)$$

where $\rho \ell/3$ is one-third the mass of the wire. Note that Equation 30 is similar to the second term of Equation 13. Note also that when $\theta - \gamma = \pi/2$ and $\gamma = \dot{\theta}$ (that is, Phase 1), Equation 30 is similar to Equation 4. With this approximation in Phase 1, the kinetic energy due to the change of wire length is implicitly neglected. The change of length of the wire does not contribute much to the total kinetic energy of the wire. Thus we can
include the effect of the weight (mass) of the wire by including one-third the weight of
the wire in the weight of the mass term of the derived equations. We therefore define
\( m \) in the derived equations in the following way:

\[
m = m_0 + \frac{\rho \ell}{3} ,
\]

(31)

where \( m_0 \) is the mass of the spin weights and \( \rho \) is twice the normal mass density of the
wire (to take into account both wires). This equation should be valid for large spin
reductions (\( \tau \) small).

**APPLICATION OF DESIGN CURVES AND EQUATIONS**

We shall now apply the simplified equations to a practical case. In Reference 1, the
following information is given for the S-30 Ionosphere Probe Satellite:

\[
\begin{align*}
I &= 2.5 \text{ slug ft}^2 \\
\alpha &= 1.25 \text{ ft} \\
\omega_0 &= 15\pi \text{ rad/sec} \\
\omega_f &= 10\pi/3 \text{ rad/sec}
\end{align*}
\]

A calculation for \( \tau \) gives

\[
\tau = \frac{10}{45} = 0.222
\]

From Figure 5, with \( \tau = 0.222 \), we read the value 157 for \( 1/m(\ell + a)^2 \). If we let \( \tau = \frac{306}{157} \), a simple calculation for the weight of the spin weights and wire gives

\[
w = mg = \frac{mg}{1.57(\ell + a)^2} = \frac{2.5(32.2)}{157 \left( \frac{221}{12} \right)} = 0.151 \text{ lb.}
\]

A check of the assumption that \( G \) is large gives a value of 266 which is more than ade-
quate. Reference 1 reports a value of 0.150 lb. calculated from the theoretically correct
equations.

Taking into account the weight of the wire, we have for the spin weights alone

\[
m_{0g} = mg - \frac{1}{3} \rho \ell \]

\[
= 0.151 - \frac{1}{3} (0.005) \frac{206}{12} = 0.121 \text{ lb.}
\]

where a double weight density of 0.005 lb per foot has been assumed.

The calculated maximum tension in one wire is

\[
F_{\max} = 1.3 \frac{m}{2} \omega_0^2 \lambda = \frac{1.3(0.151)(47)^2 (23.12)}{7(32.2)} = 156.6 \text{ lbs}
\]
The value read from a graph in Reference 1 is 155 lbs. A calculation of the maximum force in Phase 2 (Equation 24) gives 150 lbs.

To aid in systematizing de-spin calculations, a computation sheet has been developed (Appendix A). It has been found convenient to use in de-spin calculations. It is believed, also, that this sheet helps to reduce human errors.

CONCLUSION

The simplified equations derived here greatly facilitate the application of de-spin mechanism theory to a particular design. With the calculation sheet, the procedure has been reduced to reading a graph and making a few routine calculations.

REFERENCE

DEFINITION OF SYMBOLS AND UNITS:

1 - moment of inertia about spin axis (slug ft²)

a - radius of satellite (ft)

l - length of one yo-yo wire (ft)

m - total mass of both spin weights + 1/3 mass of both wires (slugs)

Fmax - maximum tension in wire (lb)

ω₀ - initial spin rate (rad/sec)

ωf - final spin rate (rad/sec)

r - final spin rate divided by initial spin rate

g - acceleration of gravity (ft/sec²)

TO CALCULATE THE TOTAL MASS (WEIGHT) OF SPIN WEIGHTS AND WIRE (m):

Record

I = __________ slug-ft²

a = __________ ft

l = __________ ft.

ω₀ = __________ rad/sec

ωf = __________ rad/sec

Calculate

r = ωf/ω₀ = __________.

With this value of r, read the value of Le(θ + a)² from the design curve. Then calculate the following:

w = mg = \frac{Ig}{R(\theta + a)²} = \left( \frac{32.2}{(\theta)²} \right) = __________ = __________ lbs.

TO CALCULATE MAXIMUM TENSION IN ONE WIRe. Calculate λ by

λ² = \frac{I}{m} + a² = __________ = __________,

or

λ = __________ ft.

Also

ω₀² = __________/sec²;

Fmax = 1.3 \frac{m}{2} ω₀² λ = 1.3 \left( \frac{2}{1} \right) ( )² ( ) = __________ lbs.

CHECK OF UNDERLYING ASSUMPTION OF THE EQUATIONS: Calculate G as follows:

G = \frac{(1 - r)I}{ma²} = __________ = __________.

If G ≥ 100 and l/a > 2π, the answers are accurate to about 1-1/2 percent of the theoretically correct value.

Equations are developed for the yo-yo despin mechanism. Under conditions usually met in a despin application, the equations can be greatly simplified. A computation sheet is presented based on the simplified equations. The design of a despin mechanism is thus reduced to reference to a chart and a few simple calculations.
Equations are developed for the yo-yo despoin mechanism. Under conditions usually met in a despoin application, the equations can be greatly simplified. A computation sheet is presented based on the simplified equations. The design of a despoin mechanism is thus reduced to reference to a chart and a few simple calculations.

I. Fedor, J. V.
II. NASA TN D-708

(Initial NASA distribution: 22, Guidance and homing systems; 24, Launching dynamics; 47, Satellites; 50, Stability and control; 52, Structures.)

Copies obtainable from NASA, Washington

NATO